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Zero Mode Corrections in Perturbative Light Cone Scalar Field Theory

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Abstract:
We reinvestigate perturbative light cone ϕ^4-theory, concentrating on the zero mode sector with longitudinal momentum $p^+ = 0$. We show that an appropriate quantisation program, taking the constraints of the theory carefully into account, necessarily yields a *nonvanishing* zero mode of the scalar field. This zero mode operator enters the hamiltonian and induces additional nonlocal interactions. We explicity calculate the second order mass correction stemming from a nontrivial zero mode sector. We show that these corrections manifest themselves as finite size effects and tend to zero in a continuum version of the theory. As long as there is no evidence that the zero mode corrections may be neglected a priori, light cone perturbation theory has to be regarded more involved than previously believed.

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1. Introduction

Light cone quantisation has been regarded as a powerful tool for the perturbative treatment of field theories[1]. The requirement that the spectrum of the Poincaré generators P^μ should be contained in the forward light cone implies that for massive particles the longitudinal momentum p^+ must be positive. This ensures that the exact ground state of the system is the bare Fock space vacuum of the canonical quanta. The triviality of the light cone ground state is much preferred for the calculation of deep inelastic structure functions, form factors and many other subjects of particle physics in the perturbative domain. A trivial vacuum state, however, seems to exclude nonperturbative phenomena, due to the common knowledge that nonperturbative physics is intrinsically related to a nontrivial ground state.

In the last few years attempts have been made to extend light cone quantisation to the nonperturbative domain of field theories. This has been done within the discretisation method of Brodsky, Pauli et. al.[2], later followed by light cone Tamm-Dancoff techniques[3]. In a recent series of publications the light cone vacuum puzzle has been resolved for the Schwinger model[4,5] and for self-interacting scalar field theories[6,7]. This success is closely related to the observation that the study of nonperturbative light cone physics requires a careful reinvestigation of the quantisation procedure: Treating light cone field theories as constrained systems and carefully taking the zero mode structure at $p^+ = 0$ into account, permits to discuss quantum induced vacuum expectation values, effective potentials, phase transitions etc.[7]. In contrast to the conventional approach, nonperturbative phenomena on the light cone are no longer related to a complicated structure of the vacuum state vector, but are determined by a highly nontrivial operator structure of the theory's zero mode sector[7].

The appearence of zero mode operators, however, is not restricted to the nonperturbative domain. These operators are also present in perturbation theory, which we are going to show in this paper. In section 2, we properly regulate the characteristic light cone infrared singularities[8] by enclosing the system in a spatial box of length $2L$. Furthermore, we derive the complete set of constraints and review the Dirac brackets. In section 3, we present the perturbative structure of the zero mode sector and show, that quantising via a correspondence principle is plagued by an operator ordering problem. In the last section, we discuss the consequences of a nontrivial zero mode sector for the hamiltonian and calculate the second order mass corrections stemming from a nontrivial zero mode of ϕ. We illustrate, that these corrections are finite size effects, which tend to zero after performing the infinite volume limit, $L \to \infty$. We finally conclude, that light cone perturbation theory — within a consistent quantisation procedure — is technically more difficult than usually believed[1,9].

2. Dirac quantisation

In this section, we briefly review the Dirac quantisation program of light cone ϕ^4-theory in 1+1 dimensions. For a more detailed discussion the reader is referred to ref.[6,7].

The lagrangian density reads

$$\mathcal{L} = 2\,\partial_+\phi\,\partial_-\phi - \frac{1}{2}m^2\,\phi^2 - \frac{\lambda}{4!}\phi^4\,, \tag{2.1}$$

where $\partial_+ = \frac{1}{2}(\partial_0 + \partial_1)$ is the time derivative and $\partial_- = \frac{1}{2}(\partial_0 - \partial_1)$ is the spatial derivative in light cone coordinates. Note that (2.1) is linear in the velocity, $\partial_+\phi$, i.e. maximally singular[10]. Therefore, other than in the conventional approach, light cone ϕ^4-theory is a constrained system. Quantising such a system is known[11] to be a nontrivial task: To obtain the ***physical*** subspace of the (classical) phase space, the complete set of constraints has to be deduced. Since these constraints reduce the number of independent degrees of freedom, the Poisson brackets no longer represent the canonical brackets of the physical phase space, but have to be replaced by the so-called Dirac brackets[10]. The corresponding quantum theory is finally obtained via a correspondence principle[10].

The light cone infrared singularity at longitudinal momentum $p^+ = 0$ requires a careful treatment of the system's zero mode sector[4-7,12]. To this end, we enclose the system in a spatial box extending from $x^- = -L$ to $x^- = L$ and impose periodic boundary conditions for all the phase space variables in $x^- \equiv x$. In a finite spatial box the longitudinal momenta become discrete and a clear cut separation of the zero mode ($p^+ = 0$) and the nonzero mode sector ($p^+ > 0$) is possible. This prescription permits to properly regulate the infrared singularity and simultaneously takes the zero mode contributations into account. At the phase space level, this is technically achieved by decomposing the scalar field variable ϕ into two orthogonal components, a zero mode part

$$\omega = \frac{1}{2L}\int_{-L}^{L} dx\,\phi(x)\,, \tag{2.2}$$

and a nonzero mode part

$$\varphi(x) = \phi(x) - \omega\,. \tag{2.3}$$

The conjugate canonical momenta π_ω and π_φ do not depend on the velocity and yield two primary constraints:

$$\begin{aligned} \theta_1(x) &= \pi_\varphi(x) - \partial_-\varphi(x) \approx 0\,, \\ \theta_2 &= \pi_\omega \approx 0\,. \end{aligned} \tag{2.4}$$

By applying the Dirac-Bergmann algorithm[10], we obtain a secondary constraint:

$$\theta_3 = m^2\,\omega + \frac{\lambda}{3\,!}\,\frac{1}{2L}\int\limits_{-L}^{L} dx\,\left[\,\varphi(x)+\omega\,\right]^3 \approx 0\,. \tag{2.5}$$

The existence of θ_3 has first been recognised by Maskawa and Yamawaki[13] and later by Wittman[14]. This constraint is the zero mode of the equation of motion,

$$\left[\,4\,\partial_+\,\partial_- + m^2\,\right]\phi(x) + \frac{\lambda}{3!}\,\phi^3(x) = 0\,, \tag{2.6}$$

and permits to express the zero mode ω of the scalar field variable in terms of the fluctuating field φ. Due to θ_3 the light cone ϕ_2^4-zero mode sector is no longer stuck at the classical level once and forever, but will be influenced by loop corrections[7]. Note, that integrating the equation of motion provides an alternative approach to θ_3 and to calculate ω. For light cone Yukawa theory, this has been done by McCartor and Robertson[12] to determine the system's bosonic zero mode. In the framework of the Dirac-Bergmann algorithm θ_3 completes the set of constraints[6,7] and the Dirac brackets may now be evaluated. The field algebra of the nonzero mode sector is

$$\begin{aligned}
\{\varphi(x),\,\varphi(y)\}^* &= -\frac{1}{2}\left[\frac{1}{2}\mathrm{sgn}(x-y) - \frac{x-y}{2L}\right]\,,\\
\{\varphi(x),\,\pi_\varphi(y)\}^* &= \frac{1}{2}\left[\delta(x,y) - \frac{1}{2L}\right]\,,\\
\{\pi_\varphi(x),\,\pi_\varphi(y)\}^* &= \frac{1}{2}\,\partial_-\left[\delta(x,y) - \frac{1}{2L}\right]\,.
\end{aligned} \tag{2.7}$$

In the zero-mode sector, we obtain an abelian field algebra:

$$\{\omega,\,\omega\}^* = \{\omega,\,\pi_\omega\}^* = \{\pi_\omega,\,\pi_\omega\}^* = 0\,. \tag{2.8}$$

In addition, we get a nontrivial Dirac bracket

$$\{\omega,\,\varphi(x)\}^* = \frac{1}{2L}\,\frac{1}{M^2(\omega,\varphi)}\int\limits_{-L}^{L} dy\,\{\varphi(x),\,\varphi(y)\}^*\,\frac{\lambda}{2!}\,\left[\,2\,\omega\,\varphi(y) + \varphi^2(y)\right]\,, \tag{2.9}$$

with the abbreviation

$$M^2(\omega,\varphi) = m^2 + \frac{\lambda}{2!}\left[\omega^2 + \frac{1}{2L}\int\limits_{-L}^{L} dy\,\varphi^2(y)\right]\,. \tag{2.10}$$

The complicated Dirac bracket in (2.9) is due to a coupling of the zero mode and the nonzero mode sector, which is caused by θ_3. Since the quantisation of the system is performed by applying the correspondence principle,

$$[\hat{A}\,,\,\hat{B}] = i\,\{\widehat{A\,,\,B}\}^*\,, \tag{2.11}$$

for any two operators $\hat{A}, \hat{B}$, this highly nontrivial coupling is preserved at the quantum level. In the nonperturbative approach of ref.[7] the infinite volume limit, $L \to \infty$, has to be performed at the very end of any calculation, in order to determine the quantum induced vacuum contributions, correctly. Since in this paper we study the corresponding perturbative domain, the box description has also to be retained for the *quantum* ϕ^4-theory. The prize one has to pay for this consistent regularisation is an operator ordering problem for $[\hat{\omega}\,,\,\hat{\varphi}(x)]$: Applying the correspondence principle to the Dirac bracket in (2.9), the R.H.S of this formula becomes operator-valued and an appropriate operator ordering is by no means clear.

In the following section, we solve $\hat{\theta}_3$ for $\hat{\omega}$ within perturbation theory. Provided with an explicit solution for $\hat{\omega}$, we calculate the *correct* quantum commutator $[\hat{\omega}\,,\,\hat{\varphi}(x)]$ to $\mathcal{O}(\lambda^2)$. The result is compared to the quantum analogue of (2.9), illustrating that the *correct* operator ordering can not be obtained from the correspondence principle.

3. Perturbative structure of the zero mode sector

In a recent paper[7] we have shown that the nonperturbative regime of light cone ϕ^4-theory is dominated by the zero mode sector of the system. The key role is played by the quantum version of θ_3,

$$\hat{\theta}_3 = m^2\hat{\omega} + \frac{\lambda}{3!}\hat{\omega}^3 + \frac{\lambda}{3!}\,\frac{1}{2L}\int\limits_{-L}^{L} dx\,\left[\hat{\varphi}^3(x) + \hat{\varphi}^2(x)\,\hat{\omega} + \hat{\varphi}(x)\,\hat{\omega}\,\hat{\varphi}(x) + \hat{\omega}\,\hat{\varphi}^2(x)\right] = 0\,. \tag{3.1}$$

Formula (3.1) is again the zero mode of the equation of motion and therefore also present in perturbation theory. To solve it for $\hat{\omega}$ explicity, we define this operator as a series in λ:

$$\hat{\omega} := \sum_{n=1}^{\infty} \lambda^n\,\hat{\omega}_n\,. \tag{3.2}$$

The unknown operators, $\hat{\omega}_n$, are determined recursively, by inserting (3.2) into the identity (3.1). A straightforward calculation leads to

$$\hat{\omega}_1 = -\frac{1}{6m^2}\frac{1}{2L}\int_{-L}^{L} dx\,\hat{\varphi}^3(x)\,,$$

$$\hat{\omega}_2 = -\frac{1}{6m^2}\frac{1}{2L}\int_{-L}^{L} dx\left[\hat{\varphi}^2(x)\hat{\omega}_1 + \hat{\varphi}(x)\hat{\omega}_1\hat{\varphi}(x) + \hat{\omega}_1\hat{\varphi}^2(x)\right], \tag{3.3}$$

$$= \left[-\frac{1}{6m^2}\right]^2 \frac{1}{(2L)^2}\int_{-L}^{L} dx\,dy\left[\hat{\varphi}^2(x)\hat{\varphi}^3(y) + \hat{\varphi}(x)\hat{\varphi}^3(y)\hat{\varphi}(x) + \hat{\varphi}^3(y)\hat{\varphi}^2(x)\right].$$

All higher order contributions to $\hat{\omega}$ may be evaluated in the same way. It is easy to see that the operator structure of $\hat{\omega}$ excludes a nonvanishing vacuum expectation value, i.e.

$$\langle 0|\omega|0\rangle = \lambda\,\langle 0|\omega_1|0\rangle + \lambda^2\,\langle 0|\omega_2|0\rangle + \mathcal{O}(\lambda^3) = 0 \quad . \tag{3.4}$$

This is physically expected within a perturbative treatment of the zero mode sector and coincides with ref.[7]. Nevertheless, the operator structure of $\hat{\omega}$ is highly nontrivial and modifies light cone ϕ^4-theory. This will be explicitly demonstrated in the next section.

In the remaining part of this section, we study the derivation of the commutator that couples the zero mode and the nonzero mode sector. We calculate the *correct* commutator to $\mathcal{O}(\lambda^2)$ using the knowledge of $\hat{\omega}$ from (3.3) and then compare this result with $[\hat{\omega}\,,\hat{\varphi}(x)]$, obtained from the corresponding Dirac bracket. (3.3) yields

$$\begin{aligned}
[\hat{\omega}\,,\hat{\varphi}(x)] &= \lambda\,[\hat{\omega}_1\,,\hat{\varphi}(x)] + \lambda^2\,[\hat{\omega}_2\,,\hat{\varphi}(x)] + \mathcal{O}(\lambda^3)\,,\\
&= \frac{\lambda}{2m^2}\frac{1}{2L}\int_{-L}^{L} dy\,[\hat{\varphi}(x)\,,\hat{\varphi}(y)]\,\hat{\varphi}^2(y) -\\
&\quad - \frac{\lambda^2}{12m^4}\frac{1}{(2L)^2}\int_{-L}^{L} dy\,dz\,[\hat{\varphi}(x)\,,\hat{\varphi}(y)]\,\left[\hat{\varphi}^3(z)\hat{\varphi}(y) + \hat{\varphi}(y)\hat{\varphi}^3(z)\right] -\\
&\quad - \frac{\lambda^2}{12m^4}\frac{1}{(2L)^2}\int_{-L}^{L} dy\,dz\,[\hat{\varphi}(x)\,,\hat{\varphi}(y)]\,\left[\hat{\varphi}^2(z)\hat{\varphi}^2(y) + \hat{\varphi}(z)\hat{\varphi}^2(y)\hat{\varphi}(z) + \right.\\
&\qquad\qquad \left. + \hat{\varphi}^2(y)\hat{\varphi}^2(z)\right].
\end{aligned} \tag{3.5}$$

Alternatively, the commutator, $[\hat{\omega}\,,\hat{\varphi}(x)]$, is obtained from the Dirac bracket in (2.9). Since the R.H.S of (2.9) becomes operator-valued, an operator ordering has to be defined. To ensure

hermicity a Weyl prescription seems to be the only appropriate operator ordering. We finally get

$$[\hat{\omega}\,,\,\hat{\varphi}(x)] = \frac{1}{4L}\left[\frac{1}{M^2(\hat{\omega},\hat{\varphi})}B(x) + \frac{1}{M^2(\hat{\omega},\hat{\varphi})}B(x)\right]\,, \tag{3.6}$$

with the abbreviation

$$B(x) := \frac{\lambda}{2}\int_{-L}^{L} dy\,[\hat{\varphi}(x)\,,\,\hat{\varphi}(y)]\,\left(\hat{\varphi}^2(y) + \hat{\omega}\,\hat{\varphi}(y) + \hat{\varphi}(y)\hat{\omega}\right)\,. \tag{3.7}$$

Expanding in the coupling constant, λ, we have

$$\begin{aligned}[\hat{\omega}\,,\,\hat{\varphi}(x)] =& \frac{\lambda}{2m^2}\frac{1}{2L}\int_{-L}^{L} dy\,[\hat{\varphi}(x)\,,\,\hat{\varphi}(y)]\,\hat{\varphi}^2(y) - \\ &- \frac{\lambda^2}{12m^4}\frac{1}{(2L)^2}\int_{-L}^{L} dydz\,[\hat{\varphi}(x)\,,\,\hat{\varphi}(y)]\,\left[\hat{\varphi}^3(z)\,\hat{\varphi}(y) + \hat{\varphi}(y)\,\hat{\varphi}^3(z)\right] - \\ &- \frac{\lambda^2}{8m^4}\frac{1}{(2L)^2}\int_{-L}^{L} dydz\,[\hat{\varphi}(x)\,,\,\hat{\varphi}(y)]\,\left[\hat{\varphi}^2(z)\,\hat{\varphi}^2(y) + \hat{\varphi}^2(y)\,\hat{\varphi}^2(z)\right]\,.\end{aligned} \tag{3.8}$$

Comparing (3.5) with (3.8), one immediately sees that both commutators do not coincide. Formula (3.8) differs from the exact result in (3.5) by a term proportional to

$$\frac{\lambda^2}{(2L)^2}\int_{-L}^{L} dydz\,[\hat{\varphi}(x)\,,\,\hat{\varphi}(y)]\,\hat{\varphi}(z)\,\hat{\varphi}^2(y)\,\hat{\varphi}(z)\,. \tag{3.9}$$

This discussion explicitly demonstrates the shortcoming of the Dirac-Bergmann algorithm for theories with nonlinear interactions: The zero mode of the (nonlinear) equation of motion appears as a constraint, and yields a field dependent Dirac bracket, which couples the zero mode and the nonzero mode sector (c.f. (2.9)). Transforming this bracket into a commutator via the correspondence principle results in an operator ordering, however, which is far from being the correct one. Since we believe that the quantum description is more fundamental than the classical one, this ordering problem can only be resolved by first quantising the theory and then imposing and solving the (operator) constraints of the system. This has been done in a few cases for theories with a finite number of degrees of freedom[15]. For field theories, however, this is a very ambitious program, which has hardly been considered in the literature[16].

Fortunately, this operator ordering problem does not really impede an extensive discussion of ϕ^4-theory in the perturbative as well as in the nonperturbative domain, as we can obtain $[\hat{\omega}\,,\,\hat{\varphi}(x)]$, by solving $\hat{\theta}_3 = 0$ for $\hat{\omega}$.

4. Second order mass correction

A nonvanishing operator $\hat{\omega}$, as obtained in the previous section, definitely modifies the hamiltonian:

$$\hat{P}^- = \int_{-L}^{L} dx \left[\frac{1}{2} m^2 \hat{\varphi}^2 + \frac{\lambda}{4!} \hat{\varphi}^4 \right] + \hat{P}^-_{corr} \ , \tag{4.1}$$

with the abbreviation

$$\hat{P}^-_{corr} := \int_{-L}^{L} dx \left[\frac{1}{4} m^2 \hat{\omega}^2 + \frac{\lambda}{4!} \left(\hat{\varphi}\,\hat{\omega}\,\hat{\varphi}^2 + \hat{\omega}\,\hat{\varphi}^3 + \hat{\omega}^2\,\hat{\varphi}^2 + \hat{\varphi}^2\,\hat{\omega}\,\hat{\varphi} + \hat{\varphi}\,\hat{\omega}^2\,\hat{\varphi} + \hat{\omega}\,\hat{\varphi}\,\hat{\omega}\,\hat{\varphi} \right) \right] . \tag{4.2}$$

The additional term $\hat{P}^-_{corr}$ is derived making use of the operator identity $\hat{\theta}_3 = 0$. Note that in the lowest order of the coupling constant, λ, a pure $\hat{\varphi}^4$-hamiltonian is reproduced, since $\hat{\omega}$ is at least of order λ. The second order, however, is altered by a zero mode correction:

$$\hat{P}^{-(2)}_{corr} = -\frac{\lambda^2}{144 m^2} \frac{1}{2L} \int_{-L}^{L} dx\, dy \left[\hat{\varphi}(x)\,\hat{\varphi}^3(y)\,\hat{\varphi}^2(x) + \hat{\varphi}^2(x)\,\hat{\varphi}^3(y)\,\hat{\varphi}(x) \right] . \tag{4.3}$$

Due to a nontrivial $\hat{\omega}$, all orders of the coupling constant are involved in the hamiltonian and induce additional nonlocal interactions as in (4.3). From this observation, we conclude that a discretised version of light cone perturbation theory is no longer equivalent to the conventional ϕ^4-theory as previously believed[1,9].

To illustrate the consequences stemming from a nontrivial zero mode, we calculate the corresponding second order mass correction. To this end, we consider the full one-particle propagator in the framework of 'old-fashioned' perturbation theory. Since the zero mode of the scalar field operator has no vacuum expectation value, the full propagator reads

$$i\,\tilde{\Delta}_F(x,y) = \langle 0 | T^+ \left[\dot{\hat{\phi}}(x)\,\dot{\hat{\phi}}(y) \right] | 0 \rangle = \langle 0 | T^+ \left[\hat{\varphi}(x)\,\hat{\varphi}(y) \right] | 0 \rangle \ . \tag{4.4}$$

In terms of Fock operators the scalar field $\hat{\varphi}(x)$ is

$$\hat{\varphi}(x) = \sum_{n=1}^{\infty} \frac{1}{\sqrt{4\pi n}} \left[\hat{a}_n\, e^{-ik_n^+ x^-/2} + \hat{a}_n^\dagger\, e^{ik_n^+ x^-/2} \right] . \tag{4.5}$$

Here, $\hat{a}_n^\dagger$ ($\hat{a}_n$) creates (annihilates) a particle with discrete longitudinal momentum $k_n^+ = 2\pi n/L$, and

$$[\, \hat{a}_n \, , \, \hat{a}_m^\dagger \,] = \delta_{nm} \ . \tag{4.6}$$

With (4.5) the Fourier representation of the propagator is

$$i\,\tilde{\Delta}_F(x,y) = \sum_{n=1}^{\infty} \frac{1}{4\pi n}\,\frac{i}{2\pi}\int dk^-\left[e^{-i\,k_n\cdot x} + e^{i\,k_n\cdot x}\right]\langle n|\frac{1}{k^- - \hat{P}_s^- + i\,\epsilon}|n\rangle \tag{4.7}$$

where we have used the abbreviations $k_n := (k^-, k_n^+)$, and $|n\rangle := a^\dagger(k_n^+)|0\rangle$. We also introduced a subtracted hamiltonian, $\hat{P}_s^- := \hat{P}^- - E$, with E being the vacuum expectation value of the hamiltonian:

$$\hat{P}^-|0\rangle = E|0\rangle\ . \tag{4.8}$$

To proceed, we define the free one-particle propagator

$$\hat{G}_0 := \frac{1}{k^- - \hat{P}_{0s}^- + i\,\epsilon}\ , \tag{4.9}$$

with

$$\hat{P}_{0\,s}^- := \frac{1}{2}\,m^2\int_{-L}^{L} dx\ \left[\hat{\varphi}^2 - \langle 0|\hat{\varphi}^2|0\rangle\right] = \frac{1}{2}\,m^2\int_{-L}^{L} dx\ :\hat{\varphi}^2:\ . \tag{4.10}$$

The one-particle Fock space amplitude in (4.7) finally reads

$$\langle n|\frac{1}{k^- - \hat{P}_s^- + i\,\epsilon}|n\rangle\ =\ \langle n|\hat{G}_0|n\rangle + \langle n|\hat{G}_0\,\hat{V}_s\,\hat{G}_0|n\rangle + \langle n|\hat{G}_0\,\hat{V}_s\,\hat{G}_0\,\hat{V}_s\,\hat{G}_0|n\rangle + \ldots\ . \tag{4.11}$$

In formula (4.11) we have introduced a subtracted interaction hamiltonian, $\hat{V}_s := \hat{P}_s^- - \hat{P}_{0\,s}^-$. Note that in a properly regularised version of light cone quantisation, a single point, $p^+ = 0$, is excluded from the longitudinal momentum spectrum of the free one-particle amplitude, $G_0(k_n)$,

$$G_0(k_n) := \langle n|\hat{G}_0|n\rangle = \frac{1}{k^- - k_n^- + i\epsilon}\qquad,\qquad k_n^- = \frac{m^2}{k_n^+}\ . \tag{4.12}$$

This fact is preserved, even after performing the infinite volume limit, $L \to \infty$. The reader, who is interested in more technical details, is referred to ref.[4]. The free one-particle light cone propagator therefore differs from the corresponding conventional propagator in contrast to common knowledge[1,9].

The zero mode contribution to the second order mass shift is

$$\delta m^2_{corr} = k_n^+\left[\langle n|\hat{P}^{-(2)}_{corr}|n\rangle - \langle 0|\,\hat{P}^{-(2)}_{corr}|0\rangle\right]\ . \tag{4.13}$$

A straightforward calculation leads to

$$\begin{aligned}\delta m^2_{corr} &= -\frac{\lambda^2}{6m^2}\frac{L}{(4\pi)^3}\frac{1}{n}\left[\sum_{m=1}^{n-1}\frac{1}{(n-m)m}+4\sum_{m=1}^{\infty}\frac{1}{(n+m)m}\right]k_n^+ \\ &= -\frac{\lambda^2}{6m^2}\frac{L}{(4\pi)^3}\frac{2}{n^2}\left[3\gamma+\Psi(n)+2\Psi(1+n)\right]k_n^+ ,\end{aligned} \tag{4.14}$$

where γ is Euler's constant and Ψ the Digamma-function[17]. This example demonstrates that the zero mode corrections are indeed nonvanishing and definitely alter the results of discretised perturbation theory. This results contradicts the common knowledge that light cone scalar field theory is equivalent to the conventional approach[1,9]. There is, however, a loophole: It may be possible that the perturbative zero mode corrections only modify the box description of perturbation theory, i.e. these corrections are finite size effects. To check this argument, we study the continuum version of δm^2_{corr}. To study the L-dependence of (4.14) explicitly, we rewrite δm^2_{corr} in terms of the momenta k_n^+:

$$\delta m^2_{corr} = -\frac{\lambda^2}{3m^2}\frac{1}{16\pi k_n^+}\frac{1}{L}\left[3\gamma+\Psi(k_n^+\frac{L}{2\pi})+2\,\Psi(1+k_n^+\frac{L}{2\pi})\right] . \tag{4.15}$$

Since in the infinite volume limit, $L\to\infty$, the longitudinal momenta, k_n^+, remain finite, the Digamma function, Ψ, is dominated by its asymptotic behaviour[17]

$$\Psi(z)\simeq \ln(z)-\frac{1}{2z}-\frac{1}{12z^2}+\frac{1}{120z^4}+\ldots \qquad (z\to\infty \text{ in } |\arg(z)|<\pi) . \tag{4.16}$$

To determine the infinite volume limit of δm^2_{corr}, it is therefore sufficient to insert (4.16) into (4.15). A straightforward calculation yields

$$\lim_{L\to\infty}\delta m^2_{corr} = -\frac{\lambda^2}{3m^2}\frac{1}{16\pi k_n^+}\lim_{L\to\infty}\frac{1}{L}\left[3\gamma+\ln(k_n^+\frac{L}{2\pi})+2\,\ln(1+k_n^+\frac{L}{2\pi})+\ldots\right]=0 . \tag{4.17}$$

Note that the same results can be obtained by replacing the sums in (4.14) by their corresponding momentum integrals.

Due to its L-dependence the second order mass correction δm^2_{corr} is identified as a finite size effect and therefore, in a continuum version, the second order mass shift is equivalent to the conventional result. However, light cone perturbation theory becomes technically more difficult than previously believed[1,9]: Since there is no evidence that the perturbative zero mode structure of light cone ϕ^4-theory may be neglected a priori, all zero mode corrections have to be taken into account. Finally, at the very end of any calculation the infinite volume limit, $L\to\infty$, has to be performed, to decide which of the zero mode corrections drop out in a continuum version.

5. Conclusions

To handle the infrared structure of light cone field theories carefully, it is necessary to regulate the theory. The discretisation method, discussed in this paper, is a convenient procedure to regularise the infrared singularity of light cone field theory and additionally does not omit the zero mode structure. This is an important feature, since the zero mode sector is known to dominate nonperturbative light cone physics[7] and cannot be excluded in a perturbative approach a priori. The previous investigation has shown, that in a discretised version of light cone ϕ^4-theory, the zero mode sector is nontrivial. The corresponding contributions to the perturbative approach have been discussed for the second order mass shift. We have been able to identify the 2nd order zero mode mass corrections as finite size effects, but we have not been able to give a general proof, which permits to neglect zero mode corrections from the very beginning. As long as such a proof is missing, light cone perturbation theory is technically more difficult than previously believed[1,9].

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