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TPI-MINN-95/14-T
UMN-TH-1344-95

Two - Dimensional Electron Liquid in a Weak Magnetic Field

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(May 7, 1995)

Abstract

We present an effective theory describing the low-energy properties of an interacting 2D electron gas at large non-integer filling factors $\nu \gg 1$. Assuming that the interaction is sufficiently weak, $r_s < 1$, we integrate out all the fast degrees of freedom, and derive the effective Hamiltonian acting in the Fock space of the partially filled Landau level only. This theory enables us to find two energy scales controlling the electron dynamics at energies less than $\hbar\omega_c$. The first energy scale, $(\hbar\omega_c/\nu)\ln(\nu r_s)$, appears in the one electron spectral density as the width of a pseudogap. The second scale, $r_s\hbar\omega_c$, is parametrically larger; it characterizes the exchange-enhanced spin splitting and the thermodynamic density of states.

PACS numbers: 73.20.Dx, 73.40.Hm, 73.40.Gk

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I. INTRODUCTION

Since the discovery of the fractional quantum Hall effect[1], the properties of two-dimensional (2D) interacting electron systems in a strong magnetic field have attracted persistent attention. Historically, almost all the efforts were directed toward the study of the strong magnetic field case when only the lowest Landau level is occupied. Some attempts were undertaken[2] to extend the analysis to a larger filling factor $\nu > 1$. However, they were all limited to the case when the energy of the electron-electron interaction is much smaller than the inter-Landau level spacing, $\hbar\omega_c \gg e^2/(\kappa\ell)$, with ℓ and κ being the magnetic length and the background dielectric constant respectively. Under this condition, one can describe the system in terms of one Landau level only; the effect of mixing of the other Landau levels was either neglected or taken into account by perturbation theory[3].

In a weak magnetic field, the typical Coulomb energy exceeds the cyclotron energy $\hbar\omega_c$. In this case, one can start from the Fermi liquid theory in zero magnetic field. The concept of Fermi liquid usually enables one to neglect the interactions between quasiparticles when determining their energy spectrum in the vicinity of the Fermi level. This concept is based, first, on the screening of Coulomb interaction, and, second, on the constraint on the phase space allowed for a scattering event[4]. Landau quantization of the non-interacting quasiparticles in two dimensions creates a system of discrete levels. The existence of energy gaps between the levels affects adversely the screening, which involves the low-energy excitations of the electron system. In addition, if the Fermi level coincides with a Landau level, the system of non-interacting quasiparticles becomes infinitely degenerate, and even a weak interaction lifts this degeneracy. This may result in a non-trivial fine structure of the partially filled Landau level (PFLL).

In this paper, we study the low-energy properties of the system with a partially filled high Landau level[5] (the level index $N \gg 1$ equals to the integer part of $[\nu/2]$). By integrating out all the other degrees of freedom, we derive the effective interaction between the electrons occupying this level. This procedure is valid for a weakly interacting electron gas $r_s \lesssim 1$, and at sufficiently large filling factors, $Nr_s \gg 1$. (Here $r_s \equiv \sqrt{2}e^2/\kappa\hbar v_F$ is the conventional parameter characterizing the interaction strength.)

The effective Hamiltonian enables us to develop a microscopic theory of the thermodynamic and tunnelling densities of states, and a description of spin excitations. In the tunnelling density of states, we find a pseudo-gap with a characteristic width $(\hbar\omega_c/2N)\ln(Nr_s)$, which confirms the result of the hydrodynamic approach[6]. The thermodynamic density of states and spin excitations are characterized by the energy scale $r_s\hbar\omega_c$. Both energies are smaller than $\hbar\omega_c$. It makes our theory complementary to the conventional Fermi liquid picture[7] which is valid in the energy range $E \gtrsim \hbar\omega_c$.

The paper is arranged as follows. Sec. II is a qualitative discussion of the energy scales relevant for the problem, and the summary of our main results. Sec. III is devoted to the rigorous derivation of the effective Hamiltonian describing the low-energy physics. In Secs. IV and V we apply the effective Hamiltonian to study the thermodynamic and the tunnelling density of states. The spin excitations are discussed in Sec. VI.

II. QUALITATIVE DISCUSSION AND RESULTS

Let us first consider an incompressible electron liquid with an integer filling factor $\nu = 2N$. Because of the gap $\hbar\omega_c$ in the spectrum, an external in-plane electric field $\mathcal{E}$ can not be screened by the 2D electron system. Instead, it causes only a finite polarization per unit area, $\mathcal{P} = \chi\mathcal{E}$. The polarizability of the incompressible 2D electron liquid reduces the interaction $U(r)$ between two point charges embedded into it,

$$U(r) = \int \frac{d^2q}{(2\pi)^2} \frac{2\pi e^2}{\kappa q} \frac{1}{\varepsilon(q)} e^{i\mathbf{q}\mathbf{r}}, \tag{2.1}$$

where the two-dimensional dielectric function $\varepsilon(q)$ is related to the polarizability χ by[8]

$$\varepsilon(q) = 1 + \frac{2\pi q}{\kappa}\chi(q). \tag{2.2}$$

At small wave vectors, the matrix elements of the dipolar moment between adjacent Landau levels, $d_{N,N-1}$, give the main contribution to the polarizability, $\chi \sim n_L|d_{N,N-1}|^2/\hbar\omega_c$; here $n_L = 1/2\pi\ell^2$ is the electron density on a Landau level. The characteristic spreading of the electron wave function on a high Landau level is equal to the cyclotron radius $R_c \equiv v_F/\omega_c$, and the estimate of the dipolar moment is $d_{N,N-1} \sim eR_c$. Substituting χ into Eq. (2.2), one finds[9]:

$$\varepsilon(q) = 1 + \frac{R_c^2}{a_B} q, \tag{2.3}$$

where $a_B = \hbar^2\kappa/me^2$ is the Bohr radius. Eq. (2.3) is valid only at small wave vectors, $qR_c \ll 1$. In the opposite limit, $qR_c \gg 1$, a large number of Landau levels participate in polarization of the 2D electron liquid. Therefore, the standard Thomas-Fermi screening holds:

$$\varepsilon(q) = 1 + \frac{2}{a_B q}. \tag{2.4}$$

Formulas (2.3) and (2.4) match at $qR_c \sim 1$; the corresponding value of the dielectric constant, $\varepsilon(q \sim 1/R_c) \sim R_c/a_B$, is large in the weak magnetic field limit, where $R_c \gg a_B$. The dielectric function for arbitrary q is given by Eq. (3.26), see Sec. III.

As it follows from Eq. (2.3), polarization is irrelevant only for interactions on a very large length scale, $r \gg R_c^2/a_B$, where $U(r)$ is given by the unscreened Coulomb interaction, see Fig. 1. At a smaller scale, $R_c \ll r \ll R_c^2/a_B$, polarization is important, and Eqs. (2.1) and (2.3) yield:

$$U(r) \simeq \frac{\hbar\omega_c}{2N} \ln\left(\frac{R_c^2}{a_B r}\right). \tag{2.5}$$

A t $a_B \ll r \ll R_c$, according to Eq. (2.4), Thomas-Fermi screening takes placer, and the potential has the form

$$U(r) \simeq \frac{e^2 a_B^2}{r^3}. \tag{2.6}$$

The renormalized potential Eq. (2.1) is significantly smaller than $\hbar\omega_c$. Therefore, unlike the bare Coulomb potential, the renormalized interaction does not mix Landau levels. This observation enables us to construct an effective theory of the low energy properties of the electron system. Similar to the strong magnetic field case, $\nu < 1$, the corresponding Hamiltonian is just the energy of interaction between the electrons restricted to a single, partially filled Landau level. The main difference is that in our case the interaction potential (2.1) is much smaller than bare Coulomb potential, and the PFLL has a high index $N \gg 1$. Although still unsolvable, this low-energy theory is much simpler than the original one, where all the Landau levels were relevant.

Armed with the effective theory, we are able to estimate the characteristic energy scales which control the low-energy dynamics. In order to make these estimates, we employ the Hartree-Fock trial function, based on the one-electron wave functions of the N-th Landau level. (See Secs. IV and V for more details.) At small electron concentration on the PFLL, $n_e^N \lesssim 1/R_c^2$, the separation between electrons on this level exceeds the size of the wave function, R_c. In this case the system under consideration is equivalent to a classical crystal of point charges interacting via potential (2.1). In the extremely dilute limit, $n_e^N \ll a_B^2/R_c^4$, the polarization is not important, and the usual Coulomb repulsion between the electrons of the PFLL is not modified. In this limit the electron system of the PFLL does not differ from a Wigner crystal of electrons of the lowest Landau level in the strong magnetic field regime. However, due to the polarization effect, there is a wide region of electron densities in the weak field regime, $a_B^2/R_c^4 \lesssim n_e^N \lesssim 1/R_c^2$, where the Coulomb repulsion in the crystal is replaced by the logarithmic potential (2.5). In this case, the static properties of the electron crystal are equivalent to those of the vortex lattice in a thin superconducting film[10].

We apply the developed picture to study of the energy dependence of the tunnelling density of states. We find a suppression of the tunnelling density of states (one-electron spectral density) at energies close to the Fermi level E_F, which is similar to the known results[11-13] for a small filling factor, $\nu \lesssim 1$. In the latter case, despite rather different approaches, all the authors[11-13] find a "gap" region of width $\Delta_g \simeq e^2 n_e^{1/2}/\kappa$ (here $n_e \equiv n_e^{N=0}$ is the total electron density at $\nu < 1$). Such a gap can be easily understood: Δ_g equals the Coulomb interaction energy of an extra electron with a "frozen" 2D electron system. If there were no relaxation of the system at all, the density of states would vanish at energies $|E - E_F| \lesssim \Delta_g$. The relaxation processes smear out the threshold in the density of states; however, the suppression is still strong at these energies, because the characteristic relaxation time exceeds $\hbar/\Delta_g$ in a strong magnetic field. For a weak magnetic field, there are two stages in the evolution of the system after an electron tunnelled in. On the short time scale, $t \sim \omega_c^{-1}$, the polarization of the medium is formed, and the potential induced by the additional electron acquires the form (2.1). At larger time scales, only the electrons of the PFLL can be re-distributed. The latter processes are slow, which causes the suppression of the tunnelling density of states. The corresponding gap is determined by the interaction of the tunneled electron with its nearest neighbors, $\Delta_g \simeq U(r \simeq 1/\sqrt{n_e^N})$. For a broad range of the filling factors of the PFLL, $a_B^2/R_c^4 \ll n_e^N \ll 1/R_c^2$, the width of the gap is:

$$\Delta_g \simeq \frac{\hbar\omega_c}{4N} \ln\left(\frac{R_c^4 n_e^N}{a_B^2}\right). \tag{2.7}$$

It is worth noticing that Δ_g depends on the interaction strength and on the electron density

on the PFLL only logarithmically.

At larger concentrations, $n_e^N \gtrsim R_c^{-2}$, the single electron wave functions are still orthogonal to each other, but the densities corresponding to these wave functions have a significant spatial overlap. Because of the strong repulsion between electrons at short distances, one could naively expect that the interaction on these distances gives the main contribution to the energy of the many-electron state. However, for the fully spin polarized electron system of the PFLL, the orbital part of the many-electron wave function is antisymmetric, and vanishes whenever two electrons have the same coordinates. This suppression of the wave function amplitude compensates the large value of the interaction potential at $r \lesssim a_B$. At larger distances, $a_B \lesssim r \lesssim R_c$, the interaction (2.6) decays rapidly, and the corresponding contribution to the Hartree-Fock energy turns out to be smaller than the contribution of the long-range interaction (2.5), see Secs. IV and V. This results in the saturation of the logarithmic growth of the gap Δ_g with the electron density: at $n_e^N \gtrsim R_c^{-2}$ the gap in the tunnelling density of states is

$$\Delta_g \simeq \frac{\hbar\omega_c}{2N} \ln(r_s N). \tag{2.8}$$

This estimate was obtained previously in the framework of the hydrodynamic approach[6].

We turn now to the discussion of the spin-flip excitations. The orbital part of the many-electron wave function of such an excited state is no longer antisymmetric. Therefore, the energy of such an excitation, Δ_s, is determined not only by the bare Zeeman energy but also by the extra interaction energy associated with the change in the structure of the orbital wave function. Because the orbital part of the wave function is not antisymmetric, the interaction on a short range ($r \lesssim a_B$) now does contribute to the energy of the state. This corresponds to the well known exchange enhancement of the g-factor[14]. Following Ref. 15 we analyze the exactly solvable case of a PFLL which has a completely filled spin-polarized sublevel. Flipping the spin of a single electron affects the exchange interaction of this electron with all the other electrons of the *same* Landau level. All the lower Landau levels carry equal number of electrons of both spin polarizations, and do not contribute to the energy associated with the spin flip. Therefore, only $1/2N$-th fraction of the total electron density participates in the exchange enhancement, and the corresponding contribution to Δ_s is approximately N times smaller than the exchange energy per electron at zero magnetic field. For a 2D electron gas with Fermi wave vector k_F, the latter energy is of the order $e^2 k_F/\kappa$, and the contribution to Δ_s is of the order $e^2 k_F/\kappa N$. The rigorous result obtained in Sec. VI differs from this estimate only by a logarithmic factor. For the effective g-factor defined by the relation

$$g_{\rm eff} \equiv \frac{\Delta_s}{\hbar\omega_c}, \tag{2.9}$$

we obtain

$$g_{\rm eff} = g_0 + \frac{r_s}{\sqrt{2\pi}} \ln\left(\frac{2^{3/2}}{r_s}\right), \tag{2.10}$$

if the filling factor ν is odd. Here g_0 is determined by the Zeeman splitting in the absence of the interaction, and it is usually small (in $GaAs$ the value of g_0 is only 0.029). For the even filling factors, the value of Δ_s is determined solely by g_0[14].

We see that in the weak magnetic field, the energy scale of charge excitations $\hbar\omega_c/N$ is parametrically smaller than the energy scale for spin excitations $r_s\hbar\omega_c$. This is qualitatively different from the situation at low filling factor, $\nu \lesssim 1$, where both these excitations are characterized by the same energy scale $e^2/\kappa\ell$.

III. EFFECTIVE HAMILTONIAN

We start from the full Hamiltonian of the system

$$\hat{H} = \hat{H}_0 + \hat{H}_{int}, \tag{3.1}$$

where the Hamiltonian for non-interacting electrons, $\hat{H}_0$, is given by

$$\hat{H}_0 = \sum_{n,k} [n\hbar\omega_c - \mu]\, \psi^\dagger_{n,k}\psi_{n,k}. \tag{3.2}$$

Here n and k are the Landau level index and the guiding center coordinate respectively (we adopt the Landau gauge). The fermionic field operators in Eq. (3.2) satisfy the standard anticommutation relations $\{\psi_{n,k}, \psi_{m,q}\} = 0$, $\left\{\psi^\dagger_{n,k}, \psi_{m,q}\right\} = \delta_{mn}\delta_{kq}$, and μ is the chemical potential (we include the energy $\hbar\omega_c/2$ into μ). To simplify the notation, we omit the spin indices. We will also neglect the Zeeman term $g_0\sigma\hbar\omega_c/2$ in all the intermediate calculations. This is legitimate because $g_0 \ll 1$ and therefore all the effects associated with the small Zeeman energy can be included in the effective Hamiltonian after the fast degrees of freedom are integrated out.

Electron-electron interaction is described by

$$\hat{H}_{int} = \frac{e^2}{2\kappa} \iint \frac{d^2\boldsymbol{r}_1 d^2\boldsymbol{r}_2}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|} : (\hat{\rho}(\boldsymbol{r}_1) - n_e)(\hat{\rho}(\boldsymbol{r}_2) - n_e) : \quad , \tag{3.3}$$

where $: \ldots :$ stands for the normal ordering, $\hat{\rho}$ is the electron density operator, $\hat{\rho}(\boldsymbol{r}) = \Psi^\dagger(\boldsymbol{r})\Psi(\boldsymbol{r})$, and n_e is the average electron density. The electron annihilation operator Ψ is related to the operators $\psi_{n,k}$ by $\Psi(\boldsymbol{r}) = \sum_{n,k} \psi_{n,k}\varphi_{n,k}(\boldsymbol{r})$, where $\varphi_{n,k}(\boldsymbol{r})$ are the single electron wave functions in a magnetic field.

In order to develop the effective low-energy theory, we notice that the expression for the partition function, $\mathcal{Z} = Tr\exp(-\beta\hat{H})$, can be written as

$$\mathcal{Z} = Tr_N\left(Tr'_N \exp(-\beta\hat{H})\right), \tag{3.4}$$

where Tr_N is the trace over the Fock space of the PFLL (N is the index of the PFLL, β is the inverse temperature) and Tr'_N means trace over all the other Landau levels. At low temperatures, $\beta\hbar\omega_c \gg 1$, Eq. (3.4) acquires the form

$$\mathcal{Z} = Tr_N \exp(-\beta\hat{H}_{eff}), \tag{3.5}$$

where the effective Hamiltonian $\hat{H}_{eff}$ is defined as

$$\hat{H}_{eff} = -\lim_{\beta_0\to\infty} \frac{1}{\beta_0} \ln\hat{\Lambda}(\beta_0), \quad \hat{\Lambda}(\beta_0) = Tr'_N\left\{e^{-\beta_0\hat{H}}\right\}. \tag{3.6}$$

Let us emphasize that, in general, the low frequency properties can be described only by means of the effective action which includes a retarded interaction. The description by an effective Hamiltonian with *instantaneous* interaction is possible only if the characteristic time of the retardation is much smaller than $\hbar/E_c$, where E_c is the maximal energy scale which is considered within the effective theory. This condition is met for the problem under consideration. Indeed, the summation in Eq. (3.6) involves inter-Landau level transitions only. These transitions are associated with the energy gaps not smaller than $\hbar\omega_c$. Correspondingly, the characteristic time of the retardation related to such a transition is $t \lesssim \omega_c^{-1}$. On the other hand, we saw at the Sec. II that the largest characteristic energy $E_c = \Delta_s$ is still parametrically smaller than $\hbar\omega_c$, which enables us to neglect the retardation.

We now turn to the calculation of the effective Hamiltonian (3.6). The simplest way to proceed is to use the Hubbard-Stratonovich transformation: first, we introduce auxiliary scalar field $\phi(\tau, \boldsymbol{r})$ to decouple quartic in Ψ interaction part of the Hamiltonian; second, we perform the summation over all the fermionic degrees of freedom not belonging to PFLL. After that, we integrate out auxiliary fields by means of the saddle point approximation and thus obtain the effective Hamiltonian H_{eff}.

Below we implement this procedure. Performing the first step of the Hubbard-Stratonovich transformation, we find for Λ from Eq. (3.6)

$$\hat{\Lambda}(\beta_0) = \int \mathcal{D}\phi \exp\left(\frac{1}{2}\int d^3\boldsymbol{\xi}\int d^3\boldsymbol{\xi}'\phi(\boldsymbol{\xi})\phi(\boldsymbol{\xi}')K(\boldsymbol{\xi}-\boldsymbol{\xi}')\right) \times$$
$$Tr'_N\left\{e^{-\beta_0\hat{H}_0}T_\tau \exp\left(-\int d^3\boldsymbol{\xi}\phi(\boldsymbol{\xi})\left(\hat{\rho}(\boldsymbol{\xi})-n_e\right)\right)\right\}, \tag{3.7}$$

where T_τ is the imaginary time ordering, and 3D vector $\boldsymbol{\xi}$ represents time and space coordinates, $\boldsymbol{\xi} = (\tau, \boldsymbol{r})$. The domain of integration over $\boldsymbol{\xi}$ is: $\tau \in [0, \beta_0]$, $x \in [0, L_x]$, $y \in [0, L_y]$, where $L_x \times L_y$ is the geometrical size of the 2D system. We will omit the Planck constant in all the intermediate calculations.

The measure of the functional integral in Eq. (3.7) is determined by the condition

$$\int \mathcal{D}\phi \ \exp\left(\frac{1}{2}\int d^3\boldsymbol{\xi}_1\int d^3\boldsymbol{\xi}_2\phi(\boldsymbol{\xi}_1)\phi(\boldsymbol{\xi}_2)K(\boldsymbol{\xi}_1-\boldsymbol{\xi}_2)\right) = 1, \tag{3.8}$$

and the function K is defined by the equation

$$\int d^3\boldsymbol{\xi}_3 K(\boldsymbol{\xi}_1-\boldsymbol{\xi}_3)\frac{e^2\delta(\tau_3-\tau_2)}{\kappa|\boldsymbol{r}_3-\boldsymbol{r}_2|} = \delta(\boldsymbol{\xi}_1-\boldsymbol{\xi}_2). \tag{3.9}$$

The density operator is

$$\hat{\rho}(\boldsymbol{r},\tau) = \bar{\Psi}(\boldsymbol{r},\tau+\epsilon)\Psi(\boldsymbol{r},\tau), \quad \epsilon \to +0, \tag{3.10}$$

where the fermionic operators in Matsubara representation are $\Psi(\tau) = e^{\tau\hat{H}_0}\Psi e^{-\tau\hat{H}_0}$, and $\bar{\Psi}(\tau) = \Psi^\dagger(-\tau)$. By introducing the infinitesimal positive time shift ϵ into definition (3.10) we preserve the normal ordering of the fermionic operators in the interaction Hamiltonian, see Eq. (3.3).

After the decoupling is performed, the fermionic part of the Hamiltonian in Eq. (3.7) becomes quadratic, which enables us to make the second step of the Hubbard-Stratonovich

transformation, i.e. to carry out the trace Tr'_N over the "fast" fermionic degrees of freedom. The result depends solely on the variables belonging to the PFLL:

$$\hat{\Lambda}(\beta_0) = e^{-\beta_0\hat{H}_0^N}\int \mathcal{D}\phi \ e^{-F_0\{\phi\}}T_\tau\left\{e^{-\hat{F}_1\{\phi,\hat{\rho}_N\}}e^{-\hat{F}_2\{\phi,\Psi_N,\bar{\Psi}_N\}}\right\}, \tag{3.11a}$$

$$F_0\{\phi\} = \int d^3\boldsymbol{\xi} f_\phi(\boldsymbol{\xi},\boldsymbol{\xi}) - \frac{N}{\pi\ell^2}\int d^3\boldsymbol{\xi}\phi(\boldsymbol{\xi})$$
$$-\frac{1}{2}\int d^3\boldsymbol{\xi}_1\int d^3\boldsymbol{\xi}_2\phi(\boldsymbol{\xi}_1)\phi(\boldsymbol{\xi}_2)\left(K(\boldsymbol{\xi}_1-\boldsymbol{\xi}_2) - 4G_N(\boldsymbol{\xi}_2,\boldsymbol{\xi}_1)\tilde{G}_\phi(\boldsymbol{\xi}_1,\boldsymbol{\xi}_2)\right), \tag{3.11b}$$

$$\hat{F}_1\{\phi,\hat{\rho}_N\} = \int d^3\boldsymbol{\xi}\phi(\boldsymbol{\xi})\left(\hat{\rho}_N(\boldsymbol{\xi}) - n_e^N\right), \tag{3.11c}$$

$$\hat{F}_2\left\{\phi,\Psi_N,\bar{\Psi}_N\right\} = \int d^3\boldsymbol{\xi}_1\int d^3\boldsymbol{\xi}_2\left[\bar{\Psi}_N(\boldsymbol{\xi}_1^+)\Psi_N(\boldsymbol{\xi}_2) - 2G_N(\boldsymbol{\xi}_2,\boldsymbol{\xi}_1)\right]\phi(\boldsymbol{\xi}_1)\tilde{G}_\phi(\boldsymbol{\xi}_1,\boldsymbol{\xi}_2)\phi(\boldsymbol{\xi}_2), \tag{3.11d}$$

$$G_N(\boldsymbol{\xi}_1,\boldsymbol{\xi}_2) = -P_N(\boldsymbol{r}_1,\boldsymbol{r}_2)e^{(\tau_2-\tau_1)(N\omega_c-\mu)}\theta(\tau_1-\tau_2-\epsilon), \tag{3.11e}$$

where the 3D vector $\boldsymbol{\xi}^+$ is $(\tau+\epsilon, \boldsymbol{r})$. In Eqs. (3.11) Ψ_N is the projection of the fermionic operator (in Matsubara representation) on the PFLL, $\Psi_N(\boldsymbol{\xi}) = e^{\tau(\mu-N\omega_c)}\sum_k \psi_{N,k}\varphi_{N,k}(\boldsymbol{r})$, operator $\bar{\Psi}_N(\tau)$ is defined as $\bar{\Psi}_N(\tau) = \Psi_N^\dagger(-\tau)$. Projections of the density operator $\hat{\rho}$ and of the Hamiltonian $\hat{H}_0$ on the PFLL are given by $\hat{\rho}_N(\boldsymbol{\xi}) = \bar{\Psi}_N(\boldsymbol{\xi}^+)\Psi_N(\boldsymbol{\xi})$ and by $\hat{H}_0^N = (N\omega_c - \mu)\int d^2\boldsymbol{r}\hat{\rho}_N(\boldsymbol{r})$ respectively. In Eq. (3.11e), $P_N(\boldsymbol{r}_2,\boldsymbol{r}_1)$ is the projection operator onto the PFLL[16]; $n_e^N = n_e - N/\pi\ell^2$ is the average electron concentration on the PFLL. Operator $\hat{F}_2$ in Eq. (3.11d) is defined in such a way $T_\tau\hat{F}_2 = \hat{F}_2$. This follows from the anticommutation relation for the fermionic operators $\left\{\Psi_N(\boldsymbol{r}_1), \Psi_N^\dagger(\boldsymbol{r}_2)\right\} = P_N(\boldsymbol{r}_1,\boldsymbol{r}_2)$.

The Green's function $\tilde{G}_\phi(\boldsymbol{\xi}_1,\boldsymbol{\xi}_2)$ is the kernel of the integral operator:

$$\hat{G}_\phi = \hat{P}_N^\perp\left[-\frac{\partial}{\partial\tau} + \mu + \frac{\omega_c}{2} - \hat{P}_N^\perp\hat{\mathcal{H}}_\phi\hat{P}_N^\perp\right]^{-1}\hat{P}_N^\perp, \tag{3.12}$$

Here $\hat{P}_N^\perp$ is the projection operator onto the space of functions *orthogonal* to the states of the PFLL[16], and $\hat{\mathcal{H}}_\phi$ is the Hamiltonian of an electron in the magnetic field and in the external potential ϕ:

$$\hat{\mathcal{H}}_\phi = \frac{\left(-i\nabla + \frac{e}{c}\boldsymbol{A}\right)^2}{2m} + \phi(\boldsymbol{r},\tau). \tag{3.13}$$

The function $\tilde{G}_\phi$ describes the evolution in the external field ϕ of electron states constrained to the Landau levels $n \neq N$. This constraint is implemented by the introduction of the projection operators $\hat{P}_N^\perp$ into Eq. (3.12). At $\tau_1 = \tau_2$ the function $\tilde{G}_\phi$ is defined as

$$\tilde{G}_\phi(\tau_1,\tau_1) \to \tilde{G}_\phi(\tau_1,\tau_1+\epsilon) \tag{3.14}$$

in accordance with Eq. (3.10).

The first term in Eq. (3.11b) results from the summation over the fermionic states with $n \neq N$, and represents the "thermodynamic potential" of electrons in these states in the field ϕ. The function $f_\phi(\boldsymbol{\xi}_1,\boldsymbol{\xi}_2)$ is defined as the kernel of the operator

$$\hat{f}_\phi = 2\ln\hat{G}_\phi, \tag{3.15}$$

with $\tilde{G}_\phi$ from Eq. (3.12). Here, the factor 2 comes from the spin degeneracy.

In order to perform the integration over the auxiliary field in Eq. (3.11a), we employ the saddle point approximation. It means that one has to expand the functional $F_0\{\phi\}$, see Eq. (3.11b), and the operator $\hat{F}_2\left\{\phi, \Psi_N, \bar{\Psi}_N\right\}$, see Eq. (3.11d), up to the second order in ϕ.

We start from the expansion of the operator $\hat{F}_2\left\{\phi, \Psi_N, \bar{\Psi}_N\right\}$. It contains explicitly a factor, which is bilinear in ϕ, and therefore we can replace $\tilde{G}_\phi$ in Eq. (3.11d) by the Green's function $\tilde{G}_0$ of an electron in the absence of an external field, $\phi = 0$. Then, Eq. (3.12) enables us to find the explicit form of $\tilde{G}_0$:

$$\tilde{G}_0(\boldsymbol{r}_1, \tau_1; \boldsymbol{r}_2, \tau_2) = \sum_{k, n \neq N} \varphi^*_{n,k}(\boldsymbol{r}_2)\varphi_{n,k}(\boldsymbol{r}_1) e^{(\tau_2 - \tau_1)(n\omega_c - \mu)}$$
$$\times \left[\theta(\tau_2 - \tau_1)\theta(N - n) - \theta(\tau_1 - \tau_2)\theta(n - N)\right]. \tag{3.16}$$

When expanding the first term in $F_0\{\phi\}$, we use definition (3.15) and the solution of Eqs. (3.12), (3.13) up to the second order in ϕ, and we obtain:

$$\int d^3\boldsymbol{\xi} f_\phi(\boldsymbol{\xi}, \boldsymbol{\xi}) \approx \beta_0\Omega_0 + \frac{N}{\pi\ell^2}\int d^3\boldsymbol{\xi}\phi(\boldsymbol{\xi}) +$$
$$+ \int d^3\boldsymbol{\xi}_1 \int d^3\boldsymbol{\xi}_2 \phi(\boldsymbol{\xi}_1)\phi(\boldsymbol{\xi}_2)\tilde{G}_0(\boldsymbol{\xi}_2, \boldsymbol{\xi}_1)\tilde{G}_0(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2), \tag{3.17}$$

where $\Omega_0 = [(N-1)\hbar\omega_c/2 - \mu]\, N L_x L_y/\pi\ell^2$ is the thermodynamic potential of the system of noninteracting electrons on the completely filled Landau levels. The third term in the functional $F_0\{\phi\}$, see Eq. (3.11b), is already bilinear in ϕ.

Substitution of Eq. (3.17) into Eq. (3.11b) yields

$$F_0\{\phi\} = \beta_0\Omega_0 - \frac{1}{2}\int\int d^3\boldsymbol{\xi}_1 d^3\boldsymbol{\xi}_2 \phi(\boldsymbol{\xi}_1)\mathcal{S}(\boldsymbol{\xi}_1 - \boldsymbol{\xi}_2)\phi(\boldsymbol{\xi}_2), \tag{3.18}$$

where the second term describes fluctuations of the field ϕ which are renormalized due to the integrated out degrees of freedom. The kernel $\mathcal{S}$ equals to

$$\mathcal{S}(\boldsymbol{r}, \tau) = \int \frac{d\omega d^2 q}{(2\pi)^3}\frac{\kappa q}{2\pi e^2}\varepsilon(q, \omega) e^{i\boldsymbol{q}\boldsymbol{r} - i\omega\tau}, \tag{3.19}$$

where $\varepsilon(q, \omega)$ coincides with the dielectric function calculated in the random phase approximation[17]:

$$\varepsilon(q, \omega) = 1 - \frac{2\pi e^2}{\kappa q}\Pi(q, \omega). \tag{3.20}$$

Here, the polarization operator $\Pi(q, \omega)$ in Eq. (3.20) is given by

$$\Pi(q, \omega) = 2\int d^2\boldsymbol{r} d\tau e^{-i\boldsymbol{q}\boldsymbol{r} + i\omega\tau}\left[\tilde{G}_0(\boldsymbol{r}, \tau; 0, 0) + \right.$$
$$\left. G_N(\boldsymbol{r}, \tau; 0, 0)\right]\left[\tilde{G}_0(0, 0; \boldsymbol{r}, \tau) + G_N(0, 0; \boldsymbol{r}, \tau\right], \tag{3.21}$$

where the functions $\tilde{G}_0$ and G_N are defined by Eqs. (3.16) and (3.11c) respectively. The explicit form and the asymptotic behavior of $\Pi(q, \omega)$ are presented in Appendix A.

Within the approximations made, we can transform Eq. (3.11a) to the form of a Gaussian integral over the auxiliary field ϕ. The result is

$$\hat{\Lambda}(\beta_0) = e^{-\beta_0(\Omega_0 + \hat{H}_0^N)}\int \mathcal{D}\phi\, T_\tau\left[e^{-\hat{F}_1\{\phi, \hat{\rho}_N\} - \hat{F}_2^0\{\phi, \Psi_N, \bar{\Psi}_N\}}\right]$$
$$\times \exp\left(\frac{1}{2}\int d^3\boldsymbol{\xi}_1 \int d^3\boldsymbol{\xi}_2 \phi(\boldsymbol{\xi}_1)\phi(\boldsymbol{\xi}_2)\mathcal{S}(\boldsymbol{\xi}_1 - \boldsymbol{\xi}_2)\right), \tag{3.22}$$

where $\hat{F}_2^0$ is obtained from operator $\hat{F}_2$, see Eq. (3.11d), by replacing $\tilde{G}_\phi$ with $\tilde{G}_0$.

Before we proceed further, let us emphasize that the saddle point approximation can be justified only if the characteristic value of the fluctuations of the auxiliary field ϕ is small enough. The estimate which will be presented later in this section shows that the expansion quadratic in ϕ is parametrically valid for the weakly interacting electron system at large filling factors.

Now, we are in the position to perform the actual integration in Eq. (3.22). Operator $\hat{F}_1$, see Eq. (3.11c), is a linear functional of ϕ, whereas operator $\hat{F}_2^0$, see Eq. (3.11d), is quadratic in ϕ; because the fluctuations of field ϕ are small, the typical value of $\hat{F}_2$ is much smaller than that of $\hat{F}_1$. If we neglect the term $\hat{F}_2^0$ at all, the functional integration in Eq. (3.22) can be easily performed. The calculation is further simplified if we approximate $\mathcal{S}(\tau) \simeq \delta(\tau)\int \mathcal{S}(\tau)d\tau$. It is valid approximation for the description of the low-energy dynamics of electrons belonging to PFLL: $\mathcal{S}$ is rapidly decaying function of time: $\mathcal{S}(\tau) \propto \exp(-\omega_c|\tau|)$ at $\tau \gtrsim \omega_c^{-1}$, as it follows from Eqs. (3.19) - (3.21). The integration over field ϕ then results in an exponential $\exp\left(-\beta_0\hat{H}_{int}^{eff}\right)$, with the Hamiltonian of density-density interaction,

$$\hat{H}_{int}^{eff} = \frac{1}{2}\int\int d^2\boldsymbol{r}_1 d^2\boldsymbol{r}_2 U(\boldsymbol{r}_1 - \boldsymbol{r}_2) \tag{3.23}$$
$$\times : \left(\hat{\rho}_N(\boldsymbol{r}_1) - n_e^N\right)\left(\hat{\rho}_N(\boldsymbol{r}_2) - n_e^N\right) : .$$

The operator (3.23) acts within the Fock space of PFLL, and the renormalized pair interaction $U(r)$ is related to $\mathcal{S}(\boldsymbol{\xi})$ by

$$\int d^3\boldsymbol{\xi}_3 \mathcal{S}(\boldsymbol{\xi}_1 - \boldsymbol{\xi}_3)U(|\boldsymbol{r}_3 - \boldsymbol{r}_2|) = \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2). \tag{3.24}$$

Substitution of Eq. (3.19) into Eq. (3.24) yields:

$$U(r) = \int \frac{d^2 q}{(2\pi)^2}\frac{2\pi e^2}{\kappa q}\frac{1}{\varepsilon(q)}e^{i\boldsymbol{q}\boldsymbol{r}}. \tag{3.25}$$

Here the static dielectric function $\varepsilon(q) \equiv \varepsilon(q, \omega = 0)$ describes the renormalization of the bare Coulomb potential due to the integrated out degrees of freedom. This function can be easily found with the help of Eqs. (3.20) and (A7a), and it has the form

$$\varepsilon(q) = 1 + \frac{2}{q a_B}\left[1 - \mathcal{J}_0^2(qR_c)\right] \tag{3.26}$$

for the large filling factor $N \gg 1$ and for values of wave vector q much smaller than Fermi wave vector, k_F. In Eq. (3.26), $a_B = \hbar^2\kappa/me^2$ is the Bohr radius and $\mathcal{J}_0(x)$ is the zeroth

order Bessel function. Asymptotic behavior of the renormalized potential $U(r)$ was discussed in Sec. II, see also Fig. 1.

The term $\hat{F}_2^0$ from Eq. (3.22) can be taken into consideration by means of the perturbation theory using the small parameters r_s and $1/N$. We show in Appendix B that this operator generates term of the effective Hamiltonian which is linear in density. This linear term can be trivially included into the free electron Hamiltonian $\hat{H}_N^0$ by a shift of the chemical potential.

Finally, substituting the resulting expression for Λ in Eq. (3.6) and restoring the Zeeman energy term, we obtain the expression for the effective Hamiltonian:

$$\hat{H}_{eff} = \Omega - \mu^* \int d^2\boldsymbol{r}\hat{\rho}_N(\boldsymbol{r}) + \hat{H}_{int}^{eff} + \frac{1}{2}g_0\hbar\omega_c \int d^2\boldsymbol{r}\left(\Psi_{N,\downarrow}^\dagger(\boldsymbol{r}), \Psi_{N,\downarrow}(\boldsymbol{r}) - \Psi_{N,\uparrow}^\dagger(\boldsymbol{r}), \Psi_{N,\uparrow}(\boldsymbol{r})\right), \tag{3.27}$$

where $\hat{H}_{int}^{eff}$ is defined by Eq. (3.23), the explicit expression for the shifted chemical potential μ^* is presented in Appendix B, see Eq. (B30), and the thermodynamic potential Ω of the electrons on the filled Landau levels is given by Eq. (B26). We will not need concrete values of μ^* and Ω in the further calculations.

Hamiltonian (3.27) is the main result of this section. The physical meaning of this Hamiltonian is that the low-frequency dynamics of the system is described by the electrons belonging to the upper Landau level. Interaction between these electrons is renormalized due to the large polarizability of all the other Landau levels.

Let us now discuss the condition of the validity of the approximations we made. As we mentioned earlier, the saddle point approximation can be justified only if the characteristic value of the fluctuations of the auxiliary field ϕ is small enough. Below we show that the fluctuations at all the spatial and time scales are parametrically small for a weakly interacting electron system at large filling factors.

The magnitude of the fluctuations localized within the spatial range r and within the time interval τ can be estimated from Eqs. (3.22) and (3.19) as

$$\left\langle\phi^2\right\rangle_{\tau,r} \simeq \frac{\hbar}{\tau}\left(\frac{e^2}{\kappa r\varepsilon(r^{-1},\tau^{-1})}\right). \tag{3.28}$$

Further estimates depend on the relation between the scale r and the cyclotron radius R_c.

If this spatial scale is large, $r \gg R_c$, only the dipole transitions between the nearest Landau levels are induced. The corresponding off-diagonal matrix elements of the Hamiltonian are of the order of $(R_c/r)|\phi|$. The mixing of the Landau levels by the field ϕ is small if

$$\frac{\langle\phi^2\rangle_{\tau,r}}{(\hbar\omega_c + \hbar/\tau)^2}\left(\frac{R_c}{r}\right)^2 \ll 1. \tag{3.29}$$

Combining Eq. (3.28) with Eq. (3.29) and using the results (3.20) and (A7c) for the dielectric function, we find the condition of small fluctuations

$$r_s N \gg 1. \tag{3.30a}$$

Therefore, the long-range fluctuations are not "dangerous" in the weak magnetic field regime.

Let us now analyze the short range fluctuations, $r \ll R_c$. In this case transitions between distant Landau levels are possible, and the requirement of the smallness of fluctuations coincides with the standard one for the validity of the random phase approximation at zero magnetic field,

$$r_s \ll 1. \tag{3.30b}$$

Thus, the saddle point approximation is valid for the weakly interacting electrons in a weak magnetic field.

In the following sections, we will apply the effective Hamiltonian (3.27) to describe various physical effects associated with the PFLL.

IV. GROUND STATE ENERGY AND THERMODYNAMIC DENSITY OF STATES

In this section, we evaluate the ground state energy and thermodynamic density of states $\partial n_e/\partial\mu$ as a function of the filling factor of the partially filled Landau level $\Delta\nu = \nu - 2N$. We consider explicitly the case $\Delta\nu \leq 1$. System at the filling factors $1 < \Delta\nu \leq 2$ can be analyzed with the help of the electron-hole symmetry, and the the thermodynamic density of states for this case can be obtained from the results for $\Delta\nu \leq 1$ by the replacement $\Delta\nu \to 2 - \Delta\nu$.

We assume that the ground state is spin polarized at $\Delta\nu \leq 1$, and thus omit the irrelevant Zeeman term[18]. The chemical potential μ^* for the given electron concentration on the PFLL $n_e^N = M/L_xL_y$ is found by differentiating the ground state energy of Hamiltonian (3.27) with respect to the number of electrons M:

$$\mu^* = \frac{\partial E_0(M)}{\partial M}, \quad E_0(M) = \frac{\left\langle 0|\hat{H}_{int}^{eff}|0\right\rangle_M}{\langle 0|0\rangle_M}, \tag{4.1}$$

where $|0\rangle_M$ is the wave function of the ground state of the system with M electrons on the PFLL.

Evidently, it is sufficient to consider only the filling factors $\Delta\nu \leq 1/2$. At larger filling factors, $1/2 < \Delta\nu \leq 1$, one can use the electron-hole transformation within the spin sublevel, $\Psi_N^h = \Psi_N^\dagger$, and study the system of holes with the filling factor $1 - \Delta\nu$ described by the Hamiltonian

$$\hat{H}_{eff} = \Omega - \mu^* \int d^2\boldsymbol{r}\left(\frac{1}{2\pi\ell^2} - \hat{\rho}_N^h(\boldsymbol{r})\right) + \hat{H}_{int}^{eff}, \tag{4.2}$$

$$\hat{H}_{int}^{eff} = -\Delta_{ex}\int d^2\boldsymbol{r}\left(\frac{1}{4\pi\ell^2} - \hat{\rho}_N^h(\boldsymbol{r})\right) + \frac{1}{2}\iint d^2\boldsymbol{r}_1 d^2\boldsymbol{r}_2 U(\boldsymbol{r}_1 - \boldsymbol{r}_2) : \left[\hat{\rho}_N^h(\boldsymbol{r}_1) - n_h^N\right]\left[\hat{\rho}_N^h(\boldsymbol{r}_2) - n_h^N\right] : .$$

Here $\hat{\rho}_N^h = \left(\Psi_N^h\right)^\dagger \Psi_N^h$ is the hole density operator $\hat{\rho}_N^h$ and $n_h^N = 1/(2\pi\ell^2) - n_e^N$ is the average density of holes at the PFLL. The first term in $\hat{H}_{int}^{eff}$ corresponds to the shift of the chemical potential of the completely filled Landau level due to the exchange interaction[14,15]:

$$\Delta_{ex} = 2\pi\ell^2 \int d^2r U(r) P_N(r,0) P_N(0,r). \tag{4.3}$$

Here $P_N(r_1, r_2)$ is the projector operator on the PFLL[16]. The integration in Eq. (4.3) can be performed with the help of the explicit form (B21) for $P_N(r_1, r_2)$. Under the conditions (3.30), the calculation yields

$$\Delta_{ex} = \hbar\omega_c \frac{r_s}{\sqrt{2\pi}} \ln\left(\frac{2^{3/2}}{r_s}\right). \tag{4.4}$$

It follows from Eq. (4.2) that filling of one spin sublevel results in the shift $-\Delta_{ex}$ of the chemical potential. This enables us to find the average thermodynamic density of states which is defined by the relation

$$\overline{\left(\frac{\partial n_e}{\partial \mu}\right)} = \frac{1/2\pi\ell^2}{\mu(\Delta\nu = 1-\delta) - \mu(\Delta\nu = \delta)}, \quad \delta \to +0. \tag{4.5}$$

With the help of Eq. (4.4), we obtain

$$\overline{\left(\frac{\partial n_e}{\partial \mu}\right)} = -\frac{m}{\hbar^2}\left[\sqrt{2} r_s \ln\left(\frac{2^{3/2}}{r_s}\right)\right]^{-1}. \tag{4.6}$$

It is worth noticing, that the thermodynamic density of states (4.6) does not depend on magnetic field at all. We will see below that for a broad range of filling factors of the PFLL $\Delta\nu$, the actual value of the thermodynamic density of states is close to the average one given by Eq. (4.6).

The true ground state of the Hamiltonian (3.27) for non-integer filling factors is not known. In order to estimate the ground state energy, we use the Hartree-Fock trial function analogous to that used by Maki and Zotos[19] for the lowest Landau level

$$|0\rangle_M = \prod_{j=1}^{M} \hat{\psi}^\dagger_{\mathbf{R}_j} |0\rangle_0, \tag{4.7}$$

$$\hat{\psi}^\dagger_{\mathbf{R}} = \int d^2r \Phi_{\mathbf{R}}(r) \Psi^\dagger_N(r), \tag{4.8}$$

where $\Phi_{\mathbf{R}}(r)$ is the normalized one-electron wave function of a coherent state[20] on the PFLL with the guiding center localized about point $\mathbf{R}$. In the Landau gauge, $A_y = -Bx$, $A_x = 0$, this function has the form

$$\Phi_{\mathbf{R}}(r) = \frac{1}{\sqrt{2\pi\ell^2}} e^{-i[r\times\mathbf{R}]_z/2\ell^2 + ixy/2\ell^2} g(r - \mathbf{R}),$$
$$g(r) = \frac{1}{\sqrt{N!}}\left(\frac{x-iy}{\sqrt{2}\ell}\right)^N e^{-r^2/4\ell^2}. \tag{4.9}$$

The overlap between two functions (4.9) rapidly decreases with the distance between their guiding centers:

$$\int d^2r \Phi^*_{\mathbf{R}_1}(r)\Phi_{\mathbf{R}_2}(r) = e^{-i[\mathbf{R}_1\times\mathbf{R}_2]_z/2\ell^2} e^{-(\mathbf{R}_1-\mathbf{R}_2)^2/4\ell^2}. \tag{4.10}$$

Equation (4.10) shows that the overlap is exponentially small even when the distance between the guiding centers is smaller than R_c and the electrons are not separated in space.

For small filling factors $\Delta\nu < 1/2$, we can choose the guiding centers $\boldsymbol{R}_i$ separated by the distance much larger than the magnetic length ℓ which enables us to neglect the non-orthogonality of the coherent states. Then, the expression for the energy $E_0(M)$ given by Eq. (4.1) acquires a simple form

$$E_0(M) = \frac{1}{2}\sum_{i\neq j}^{M} V_{HF}(|\boldsymbol{R}_i - \boldsymbol{R}_j|) - \frac{M n_e^N}{2}\int d^2r U(r),$$
$$V_{HF}(R) = V_H(R) - V_F(R), \tag{4.11}$$

where Hartree V_H and Fock V_F potentials are given by

$$V_H(R) = \iint d^2r\, d^2r'\, U(r-r')\, |\Phi_0(r)|^2\, |\Phi_{\mathbf{R}}(r')|^2$$
$$= \int \frac{d^2q}{(2\pi)^2}\frac{2\pi e^2}{\kappa q}\frac{1}{\varepsilon(q)}\left[L_N\left(\frac{q^2\ell^2}{2}\right)\right]^2 e^{-q^2\ell^2 + i\mathbf{q}\mathbf{R}} \tag{4.12}$$

and

$$V_F(R) = \iint d^2r\, d^2r'\, U(r-r')\Phi^*_0(r)\Phi^*_{\mathbf{R}}(r')\Phi_0(r')\Phi_{\mathbf{R}}(r)$$
$$= \int \frac{d^2q}{(2\pi)^2}\frac{2\pi e^2}{\kappa q}\frac{1}{\varepsilon(q)}\left[L_N\left(\frac{q^2\ell^2}{2}\right)\right]^2 e^{-q^2\ell^2 - \mathbf{q}\mathbf{R} - \mathbf{R}^2/2\ell^2}, \tag{4.13}$$

respectively. Here $L_N(x)$ is the Laguerre polynomial[21]. We used Eqs. (3.25) and (4.9), when deriving Eqs. (4.12) and (4.13). Small corrections appearing due to the non-orthogonality of the functions (4.9) were discussed in Ref. 19.

Now, the energy given by Eq. (4.11) should be minimized with respect to the positions of the guiding centers $\boldsymbol{R}_j$. The best configuration corresponds to the guiding centers arranged in a triangular lattice[19]. Below we present analytical results for the energy of the variational ground state (4.7) in different domains of filling factor $\Delta\nu$.

Dilute system, $\Delta\nu \ll N^{-1}$ – In this case, the lattice constant $a = \left(\sqrt{3}n_e^N/2\right)^{-1/2}$ is large, $a \gg R_c$, so that the Fock potential (4.13) is exponentially small and the Hartree term $V_H(R)$ coincides with the interaction potential $U(R)$. In this limit, two situations may be distinguished.

For an extremely dilute system, $\Delta\nu \ll N^{-3} r_s^{-2}$, the lattice constant is larger than R_c^2/a_B, and therefore the Coulomb potential is not renormalized. In this case, the ground state energy coincides with that of the Wigner crystal on a neutralizing background,

$$E_0(M) = -\alpha\frac{e^2}{\kappa\ell}\frac{M^{3/2}}{M_\Phi^{1/2}} \tag{4.14}$$

where $\alpha = 0.782\ldots$ is a numerical constant[22], and $M_\Phi = L_x L_y/2\pi\ell^2$ is the number of states on a Landau level. The use of Eq. (4.1) immediately yields for the thermodynamic density of states in this regime

$$\frac{\partial n_e}{\partial \mu} = -\frac{0.542}{r_s}\frac{m}{\hbar^2}\left(\frac{\Delta\nu}{N}\right)^{1/2}. \tag{4.15}$$

The negative thermodynamic density of states for an analogous system was considered earlier in Ref. 23.

For a moderately dilute system $N^{-3}r_s^{-2} \ll \Delta\nu \ll N^{-1}$, the typical distance between electrons on the PFLL is much smaller than R_c^2/a_B. At such distances the potential of interaction is strongly renormalized by the screening and it is given by Eq. (2.5). With the logarithmic accuracy the energy of the system is given by

$$E_0(M) \simeq -M\frac{\hbar\omega_c}{8N}\ln\left[N^3 r_s^2 \frac{M}{M_\Phi}\right] \tag{4.16}$$

and, correspondingly, the thermodynamic density of states for moderately dilute system takes the form:

$$\frac{\partial n_e}{\partial \mu} = -\frac{4}{\pi}\frac{m}{\hbar^2}(\Delta\nu N). \tag{4.17}$$

"Dense" limit, $N^{-1} \ll \Delta\nu \ll 1/2$ – In this case the distance between the nearest guiding centers is smaller than R_c. The asymptotic behavior of the Hartree and Fock potentials depends on the relation between the magnetic length ℓ and the Bohr radius a_B. We restrict ourselves to the case $\ell \gg a_B$, or

$$N r_s^2 \gg 1, \tag{4.18}$$

which enables us to simplify Eqs. (4.12) and (4.13),

$$V_H(R) = \frac{e^2 a_B}{2\pi R_c R} + \frac{3e^2 a_B}{4\pi^2 R_c^2}\ln\left(\frac{R_c}{R}\right) + \frac{e^2 a_B}{R_c^2}\ln\left(\frac{R_c}{a_B}\right),$$
$$V_F(R) = \frac{e^2 a_B}{2\pi R_c R\,(1 + R a_B/2\ell^2)} \tag{4.19}$$

for the range of distances $\ell \ll R \ll R_c$.

The resulting potential $V_{HF}(R) = V_H(R) - V_F(R)$ is a smooth function of R at $R \ll \ell^2/a_B$. This enables us to find the analytical expression for the ground state energy (4.11) in the region $N^{-1}r_s^{-2} \ll \Delta\nu \ll 1/2$. First, we can approximate $\sum_{j\neq 0} V_{HF}(\mathbf{R}_j) \approx n_e^N \int d^2R V_{HF}(R) - V_{HF}(a)$, as the potential varies only logarithmically over the lattice cell. Second, we notice that the spatial average of the Hartree potential is exactly equal to the average of the bare potential $U(r)$, and these two terms cancel each other in Eq. (4.11) for $E_0(M)$. Finally, the integral of the Fock potential is proportional to the exchange shift, Δ_{ex}, see Eq. (4.4). The resulting energy of the system is

$$E_0(M) = -\frac{M^2}{2M_\Phi}\Delta_{ex} - \tag{4.20}$$
$$M\frac{\hbar\omega_c}{4N}\left[\frac{3}{8\pi^2}\ln\left(\frac{M}{M_\Phi}N\right) + \ln(N r_s) + \frac{1}{\sqrt{2}\pi r_s}\right],$$

which yields the thermodynamic density of states of the form

$$\frac{\partial n_e}{\partial \mu} = -\frac{m}{\hbar^2}\left[\sqrt{2}r_s \ln\left(\frac{2^{3/2}}{r_s}\right)\right]^{-1} \times$$
$$\left[1 - \frac{3\sqrt{2}}{32\pi r_s \ln(r_s^{-1})}(\Delta\nu N)^{-1}\right]. \tag{4.21}$$

The leading term in Eq. (4.21) coincides with Eq. (4.6).

The filling factors $\Delta\nu > 1/2$ can be considered similarly by using the electron-hole symmetry, see Eq. (4.2), and the trial function of type (4.7). The overall dependence of the thermodynamic density of states on the filling factor is shown in Fig. 2.

V. SPECTRAL DENSITY

The value of the one electron spectral density $A(\epsilon)$ can be measured in the tunnelling experiments[24,25]. For the electron states on the PFLL, $A(\epsilon)$ is defined by relations:

$$A(\epsilon) = A^+(\epsilon) + A^-(\epsilon), \tag{5.1}$$
$$A^+(\epsilon) = \frac{1}{M_\Phi}\sum_{k,\tilde{m}}\left|\langle \tilde{m}\,|\psi^\dagger_{k,N}|\,0\rangle\right|^2 \delta(\tilde{E}_{\tilde{m}} - \tilde{E}_0 - \epsilon),$$
$$A^-(\epsilon) = \frac{1}{M_\Phi}\sum_{k,\tilde{m}}\left|\langle \tilde{m}\,|\psi_{k,N}|\,0\rangle\right|^2 \delta(\tilde{E}_0 - \tilde{E}_{\tilde{m}} - \epsilon).$$

Here, $|\tilde{m}\rangle$ is an eigenstate of the *full* Hamiltonian (3.1), $\tilde{E}_{\tilde{m}}$ is the corresponding eigenvalue and $\tilde{m} = 0$ stands for the ground state of the system. Functions $A^+(\epsilon)$ and $A^-(\epsilon)$ describe the introduction of an extra electron or an extra hole onto the PFLL respectively.

We are interested in the spectral density at energies $|\epsilon| \ll \hbar\omega_c$. All the behavior of the system at such low energies can be described by the effective Hamiltonian (3.27). Following the method of Sec. III, we can express the spectral density (5.1) in terms of the eigenstates of the *effective* Hamiltonian as

$$A^+(\epsilon) = \frac{Z}{M_\Phi}\sum_{k,m}\left|\langle m\,|\psi^\dagger_{k,N}|\,0\rangle\right|^2 \delta(E_m - E_0 - \epsilon),$$
$$A^-(\epsilon) = \frac{Z}{M_\Phi}\sum_{k,m}\left|\langle m\,|\psi_{k,N}|\,0\rangle\right|^2 \delta(E_0 - E_m - \epsilon), \tag{5.2}$$

where m denotes the eigenstates of the effective Hamiltonian (3.27) and E_m is the corresponding eigenvalue, $m = 0$ is the ground state. The Z-factor in Eq. (5.2) describes the overlap between the low-energy eigenstates of the full Hamiltonian and the eigenstates of the effective Hamiltonian. The value of Z-factor coincides with the quasiparticle weight for a 2D degenerate plasma in the absence of the magnetic field,

$$Z = 1 - \frac{3r_s}{\pi\sqrt{2}}.$$

Now we use Eq. (5.2) to evaluate the spectral density. Our analysis is based on the trial function (4.7). Even though we are not able to find the details of the energy dependence of the spectral density, this approach provides a reliable estimate for the energy scales involved.

Because the trial function (4.7) is constructed from the coherent states (4.9), it is convenient to rewrite Eq. (5.2) in terms of the operators $\psi_{\mathbf{R}}, \psi^{\dagger}_{\mathbf{R}}$, see Eq. (4.8), creating or annihilating an electron in the coherent state:

$$A^{\pm}(\epsilon) = \frac{Z}{2\pi}\int dt e^{-i\epsilon t}\int \frac{d^2\boldsymbol{R}}{L_x L_y}\mathcal{G}^{\pm}(\mathbf{R},t), \tag{5.3a}$$

$$\mathcal{G}^{+}(\mathbf{R},t) = \left\langle 0\left|e^{-it\hat{H}_{eff}}\psi_{\mathbf{R}}e^{it\hat{H}_{eff}}\psi^{\dagger}_{\mathbf{R}}\right|0\right\rangle, \tag{5.3b}$$

$$\mathcal{G}^{-}(\mathbf{R},t) = \left\langle 0\left|\psi^{\dagger}_{\mathbf{R}}e^{-it\hat{H}_{eff}}\psi_{\mathbf{R}}e^{it\hat{H}_{eff}}\right|0\right\rangle. \tag{5.3c}$$

Here we used the representation of the projection operator in terms of the coherent states (4.9):

$$P_N(\boldsymbol{r}_1,\boldsymbol{r}_2) = \int \frac{d^2\boldsymbol{R}}{2\pi\ell^2}\Phi^{*}_{\mathbf{R}}(\boldsymbol{r}_2)\Phi_{\mathbf{R}}(\boldsymbol{r}_1).$$

Let us concentrate below on the calculation of the spectral density for the hole excitations $A^{-}(\epsilon)$. (The calculation procedure for $A^{+}(\epsilon)$ is similar and will be briefly outlined later.)

At small time t, expression (5.3c) can be approximated by the formula

$$\mathcal{G}^{-}(\mathbf{R},t) \approx \mathcal{G}^{-}(\mathbf{R},0)\exp\left(itE^{-}(\mathbf{R}) - \frac{1}{2}[tD(\mathbf{R})]^2\right) \tag{5.4}$$

with parameters

$$E^{-}(\mathbf{R}) = \frac{\left\langle 0\left|\psi^{\dagger}_{\mathbf{R}},\left[\psi_{\mathbf{R}},\hat{H}_{eff}\right]\right|0\right\rangle}{\left\langle 0\left|\psi^{\dagger}_{\mathbf{R}}\psi_{\mathbf{R}}\right|0\right\rangle}; \tag{5.5}$$

$$[D(\mathbf{R})]^2 = \frac{\left\langle 0\left|\psi^{\dagger}_{\mathbf{R}}\left[\hat{H}_{eff},\left[\hat{H}_{eff},\psi_{\mathbf{R}}\right]\right]\right|0\right\rangle}{\left\langle 0\left|\psi^{\dagger}_{\mathbf{R}}\psi_{\mathbf{R}}\right|0\right\rangle} - \left[E^{-}(\mathbf{R})\right]^2.$$

Substitution of Eq. (5.4) into Eq. (5.3a) gives the result

$$A^{-}(\epsilon) = \frac{Z}{\sqrt{2\pi}}\int\frac{d^2\boldsymbol{R}}{L_x L_y}\frac{\mathcal{G}^{-}(\mathbf{R},0)}{D(\mathbf{R})}\exp\left\{-\frac{(E^{-}(\mathbf{R})-\epsilon)^2}{2[D(\mathbf{R})]^2}\right\} \tag{5.6}$$

for the spectral density.

Now we use Maki-Zotos trial state (4.7) to find the functions in the integrand in Eq. (5.6). At a small filling of the PFLL, $\Delta\nu \ll 1$, we obtain with the help of Eq. (4.10)

$$\mathcal{G}^{-}(\mathbf{R},0) = \sum_{\mathbf{R}_i} e^{-(\mathbf{R}-\mathbf{R}_i)^2/2\ell^2}, \tag{5.7}$$

which describes the "density of the centers of orbits" in the Wigner crystal. Energy $E^{-}(\mathbf{R})$ defined in Eq. (5.5) turns out to be independent on the position $\mathbf{R}$ because all the electrons in the Wigner crystal have the same energy:

$$E^{-} = \sum_{\mathbf{R}_i\neq 0} V_{HF}(|\mathbf{R}_i|) - n_e^N\int d^2\boldsymbol{r}U(r) - \mu^{*}. \tag{5.8}$$

Here the Hartree - Fock energy V_{HF} is defined by Eq. (4.11), and the second term describes the neutralizing background. The parameter $D(\mathbf{R})$ is also independent of R and it is given by

$$D^2 = \ell^2\sum_{\mathbf{R}_i\neq 0}\left[\nabla V_{HF}(|\mathbf{R}_i|)\right]^2. \tag{5.9}$$

Because none of the parameters in the exponent in Eq. (5.6) depend on $\mathbf{R}$, the integration can be easily performed, and it yields:

$$A^{-}(\epsilon) = \Delta\nu\frac{Z}{\sqrt{2\pi}D}\exp\left\{-\frac{(E^{-}-\epsilon)^2}{2D^2}\right\}. \tag{5.10}$$

Let us now discuss the physical meaning of the energies E^{-}, D. After an electron is removed from the PFLL, the system acquires extra energy equal to $-E^{-}(\mathbf{R})$ due to the interaction between the hole and all the electrons of the PFLL. If all the other electrons of the PFLL were "frozen", this state would give a contribution $\propto \delta(\epsilon - E^{-}(\mathbf{R}))$ to the spectral density. (Classical model of the frozen electrons was used in Ref. 12 for the electrons on the lowest Landau level.) However, the state formed right after the tunnelling is not an eigenfunction. The decay of this initial state leads to the finite quantum width D of the spectral density, see Fig. 3 a.

To find the region of applicability of expression (5.10) we notice that the expansion in t we used in Eq. (5.4) is valid for $|t| \ll |E^{-}/D^2|$. The characteristic time determining the value of the integral in Eq. (5.3a) can be estimated as $t \simeq |\epsilon - E^{-}|/D^2$. Combining these two conditions, we find that Eq. (5.10) is applicable for $|\epsilon - E^{-}| \ll |E^{-}|$. At the boundary of applicability, the spectral density (5.10) is proportional to $\exp[-(E^{-}/D)^2]$, and therefore Eq. (5.10) describes the main contribution to the spectral density only if $|E^{-}| \gg D$. Substituting the explicit expression for Hartree-Fock potential (4.11) into Eqs. (5.8) and (5.9), we see that the ratio $D/|E^{-}|$ is maximal at $\Delta\nu \simeq 1/2$, where its value is $D/|E^{-}| \simeq 0.1(\ln N)^{-1/2} \ll 1$. Furthermore, we believe that the use of the Hartree-Fock function sets the upper limit for D. This is because trial function (4.7) does not take into account the correlations in the motion of centers of orbits, and thus overestimates zero-point fluctuations of the electron density. The use of a more sophisticated[26] trial function, or of a phenomenological model that has the correct spectrum of excitations of the Wigner crystal[13] would restrict the summation in Eq. (5.9) to the nearest neighbors only, and further reduce the value of D.

The spectral density of holes has a sharp peak, because a hole corresponds to a vacancy in the Wigner crystal. In contrast, the tunnelling electron may create an interstitial at an arbitrary position in the elementary cell. This gives rise to a finite width of the spectral density $A^{+}(\epsilon)$, even for the "frozen", $D = 0$, crystal. The energy band for an interstitial is given by

$$E^{+}(\mathbf{R}) = \sum_{\mathbf{R}_i}\left[\frac{V_{HF}(|\mathbf{R}_i - \mathbf{R}|)}{1-e^{-(\mathbf{R}-\mathbf{R}_i)^2/2\ell^2}}\right] - n_e^N\int d^2\boldsymbol{r}U(r) - \mu^{*}. \tag{5.11}$$

The leading term in the quantum width D for electrons does not depend on $\mathbf{R}$, and it is still given by Eq. (5.9).

To calculate the spectral density $A^{+}(\epsilon)$, one can use Eq. (5.6) with $E^{-}(\mathbf{R})$ and $\mathcal{G}^{-}(\mathbf{R},0)$ replaced by $E^{+}(\mathbf{R})$ and $1-\mathcal{G}^{-}(\mathbf{R},0)$ respectively. The width of the electron spectral density

is determined by the energy band (5.11). The abrupt edge of the spectral density at $\epsilon = \min_{\mathbf{R}}\{E^+(\mathbf{R})\}$ is smeared only by a small quantum width D, see Fig. 3 a.

The minimal energy necessary to create an interstitial, $\min\{E^+(\mathbf{R})\}$, is of the order of energy $|E^-|$ needed to create a vacancy. Therefore the difference $\Delta_g = \min\{E^+(\mathbf{R})\} - E^-$ exceeds significantly D and thus determines the pseudogap in the spectral density (5.1). Using Eqs. (5.8), (5.11), and the asymptotic behavior of the Hartree-Fock potential $V_{HF}(R)$, see Sec. IV, we obtain the width of the pseudogap for different filling factors:

$$\Delta_g = \begin{cases} \simeq \dfrac{e^2}{\kappa\ell}\sqrt{\Delta\nu}, & \Delta\nu \ll N^{-3}r_s^{-2} \\ \dfrac{\hbar\omega_c}{4N}\ln\left[N^3 r_s^2 \Delta\nu\right], & N^{-3}r_s^{-2} \ll \Delta\nu \ll N^{-1} \\ \dfrac{\hbar\omega_c}{2N}\ln(Nr_s), & N^{-1} \ll \Delta\nu \ll 1/2, \end{cases} \tag{5.12}$$

The leading term in the width of the gap agrees with the result of Ref. 6 for the broad range of filling factors $\Delta\nu \gtrsim 1/N$. The overall dependence of the characteristic energy scales on the filling factor $\Delta\nu$ is shown in Fig. 3 b.

It is important to notice that in the dominant region of filling factors, $\Delta\nu > 1/N$, the width of the gap Δ_g is determined by the renormalized interaction (2.5) at distances $r \simeq R_c$, which are large compared to the characteristic distance between the electrons of PFLL. The gap is insensitive to the short range correlations in the wave function of the ground state. Therefore, we anticipate that the value of Δ_g we found is robust: it is not an artifact of the Maki-Zotos trial function, but it should be the same (with logarithmic accuracy) for any state homogeneous on the macroscopic scale $r \gtrsim R_c$.

VI. SPIN EXCITATIONS

In this section we analyze the simplest excitations of the PFLL at $\Delta\nu = 1$. This problem is similar to the one studied in Ref. 15. The only difference is that the interaction potential is renormalized due to the polarizability of the other Landau levels. Our goal is to study how this renormalization affects the spin excitations.

The energy of the fully spin polarized ground state[18] $|M = M_\Phi, S_z = M_\Phi/2\rangle$ at $\Delta\nu = 1$ is

$$E_p = \Omega - M_\Phi\left(\mu^* + \frac{\Delta_{ex} + g_0\hbar\omega_c}{2}\right), \tag{6.1}$$

where Δ_{ex} is defined by Eq. (4.4). The eigenstates of the Hamiltonian with an extra electron or hole

$$|e\rangle = \psi^\dagger_{N,k,\downarrow}\left|M_\Phi, \frac{M_\Phi}{2}\right\rangle, \quad |h\rangle = \psi_{N,k,\uparrow}\left|M_\Phi, \frac{M_\Phi}{2}\right\rangle$$

have the energies

$$\begin{aligned} E_\downarrow &= E_p - \mu^* + \tfrac{1}{2}g_0\hbar\omega_c, \\ E_\uparrow &= E_p + \mu^* + \Delta_{ex} + \tfrac{1}{2}g_0\hbar\omega_c \end{aligned} \tag{6.2}$$

respectively. Equations (6.2) enable us to relate the width of the spin gap $\Delta_s = E_\downarrow + E_\uparrow - 2E_p$ to the value of Δ_{ex}, i.e. $\Delta_s = \Delta_{ex} + g_0\hbar\omega_c$. Substituting Δ_s in Eq. (2.9) and using Eq. (4.4), we obtain the result (2.10) for the effective g-factor.

It is worth noticing that the effective g-factor is independent on N in the limit of a weak magnetic field due to the Thomas-Fermi screening[27]. (Without screening, this factor would logarithmically diverge with $N \to \infty$.)

The exchange enhancement is maximal if the filling factor is odd. When the both spin sublevels are either empty or completely filled (ν is even), the spin splitting is determined by the bare g - factor[14].

Let us turn to the consideration of neutral excitations - spin waves[15,28] which at $\Delta\nu = 1$ are described by the wave functions:

$$|SW, \mathbf{k}\rangle = \sum_{k_1} e^{ik_x k_1 \ell^2}\psi^\dagger_{N,k_1,\downarrow}\psi_{N,k_1-k_y,\uparrow}\left|M_\Phi, \frac{M_\Phi}{2}\right\rangle. \tag{6.3}$$

Wave functions (6.3) are eigenstates of the Hamiltonian (3.27) with the energies $E_p + E_{SW}(k)$ where the energy of the spin wave $E_{SW}(k)$ is given by

$$\begin{aligned} E_{SW}(k) = g_{eff}\hbar\omega_c - \\ \int \frac{d^2q}{(2\pi)^2}\frac{2\pi e^2}{\kappa q}\frac{1}{\epsilon(q)}\left[L_N\left(\frac{q^2\ell^2}{2}\right)\right]^2 e^{-q^2\ell^2/2 + ikq\ell^2}. \end{aligned} \tag{6.4}$$

For small wave vectors, $kR_c \ll 1$, the spin wave has a quadratic dispersion relation which is typical for ferromagnets:

$$E_{SW}(k) = g_0\hbar\omega_c + \frac{e^2 R_c}{\pi\kappa}k^2. \tag{6.5}$$

At $k \to 0$ the energy of the spin wave is determined by the bare g-factor in agreement with Larmor's theorem. It is worth mentioning that the long-wavelength domain of the spin wave spectrum is controlled by the bare Coulomb interaction. This is because the spatial scale $k\ell^2$ important for the spectrum, see Eq. (6.4), is much smaller than the screening radius a_B.

In the region $R_c^{-1} \ll k \ll k_F$ the energy of the spin wave reveals oscillatory behavior

$$E_{SW}(k) = g_{eff}\hbar\omega_c + \frac{e^2}{\pi\kappa R_c}\left[C\left(\frac{k\ell^2}{a_B}\right) - \frac{\sin(2kR_c)}{2kR_c}\right]. \tag{6.6}$$

Here $C(x)$ is a smooth function with asymptotes

$$C(x) = \begin{cases} \ln x, & x \ll 1 \\ -\frac{1}{2x}, & x \gg 1 \end{cases}.$$

Spin waves with extremely large wave vectors, $k \gg k_F$, correspond to almost independent electron and hole, see Eq. (6.2). In this limit, the energy E_{SW} approaches Δ_s:

$$\Delta_s - E_{SW}(k) = U(k\ell^2) \lesssim \frac{\hbar\omega_c}{2N}\ln Nr_s. \tag{6.7}$$

VII. CONCLUSION

In this paper we constructed a theory describing low-energy excitations in a 2D electron liquid in a weak magnetic field. We have shown that all the excitations with energy smaller than cyclotron energy $\hbar\omega_c$ can be described by the effective Hamiltonian acting in the Fock space of the partially filled Landau level only. Starting from first principles, we obtained the explicit form of this Hamiltonian by integrating out all the other degrees of freedom.

Armed with this effective theory, we have been able to make important predictions:
1) We found that the *thermodynamic density of states* is negative for non-integer filling factors. For a broad range of filling factors, the value of the thermodynamic density of states was found to be independent on the value of magnetic field, see Eq. (4.6). This effect may be revealed in magnetocapacitance measurements[29] in the weak magnetic field regime.
2) The *tunnelling density of states* was shown to have a gap at the Fermi level. For a broad range of filling factors, the width of the gap, see Eq. (2.8), was shown to be consistent with that predicted by hydrodynamic approach of Ref. 6. The gap can be observed in tunnelling experiments[24] in a weak magnetic field. Evidence of suppression of the tunnelling density of states for the filling factors $\nu \lesssim 9$ was reported recently by Turner *et. al.*[25]. This suppression may be associated with the gap predicted by our theory. However, the observed width of the gap is twice larger than that given by Eq. (2.8).
3) The *exchange enhancement* of the effective g-factor remains strong in the weak field regime, and at all odd filling factors it takes a universal value, see Eq. (2.10). We found that the energy scale of charge excitations is parametrically smaller than the energy scale for spin excitations. This is qualitatively different from the situation at low filling factors, $\nu \lesssim 1$, where both these excitations are characterized by the same energy scale $e^2/\kappa\ell$.

None of the aforementioned effects could be obtained in the framework of the Landau quantization of the spectra of quasiparticles in the conventional Fermi liquid theory. This theory describes adequately the excitations with energies larger than $\hbar\omega_c$. Thus, our description of the low-energy properties of interacting 2D electron gas is complementary to the Fermi liquid picture.

ACKNOWLEDGMENTS

Helpful discussions with B. L. Altshuler, B. I. Halperin, A. I. Larkin, L. S. Levitov, Yu. B. Lyanda-Geller, A. H. MacDonald, B. I. Shklovskii, and X. Zhu are acknowledged with gratitude. We are thankful to F. W. J. Hekking for critical reading the manuscript and valuable comments. This work was supported by NSF Grant DMR-9423244.

APPENDIX A: EVALUATION OF $\Pi(\omega, Q)$.

Integration in Eq. (3.21) immediately yields

$$\Pi(q,\omega) = -\frac{2m}{\pi}\sum_{n_1=0}^{N-1}\sum_{n_2=N}^{\infty}\left[\frac{(-1)^{(n_2-n_1)}(n_2-n_1)}{(\omega/\omega_c)^2+(n_2-n_1)^2}\times\right.$$

$$\left. L_{n_1}^{n_2-n_1}\left(\frac{q^2\ell^2}{2}\right) L_{n_2}^{n_1-n_2}\left(\frac{q^2\ell^2}{2}\right) e^{-q^2\ell^2/2}\right], \tag{A1}$$

where $L_m^n(x)$ is the Laguerre polynomial[21].

Further simplification is possible for $q \ll k_F$. Under this condition, we can use the asymptotic expression for the Laguerre polynomial[21]

$$L_n^m(x) \simeq \frac{(n+m)!e^{x/2}\mathcal{J}_m\left(\sqrt{2x(2n+m+1)}\right)}{n!\left[x(n+m/2+1/2)\right]^{m/2}}, \tag{A2}$$

where $\mathcal{J}_m(x)$ is the Bessel function. This expression is applicable if $x \ll n$. Using Eq. (A2) and introducing a new index $j = n_2 - n_1$ we approximate expression (A1) as:

$$\Pi(q,\omega) = -\frac{2m}{\pi}\sum_{j=1}^{\infty}\left[\frac{j}{(\omega/\omega_c)^2+j^2}\right.$$
$$\left.\times\sum_{n_1=\max(N-j,0)}^{N-1}\left[\mathcal{J}_j\left(q\ell\sqrt{2n_1+j+1}\right)\right]^2\right]. \tag{A3}$$

The terms giving the main contribution to the sum in Eq. (A3) are characterized by value $j \sim q\ell\sqrt{N} \ll N$. This enables us to approximate $2n+j+1 \approx 2N$ and perform the summation over n_1. This yields[30]

$$\Pi(q,\omega) = -\frac{2m}{\pi}\sum_{j=1}^{\infty}\frac{j^2}{(\omega/\omega_c)^2+j^2}\left[\mathcal{J}_j(qR_c)\right]^2. \tag{A4}$$

Equation (A3) can be transformed into an integral form: substituting the identity

$$\left[\mathcal{J}_n(x)\right]^2 = \int_0^\pi \frac{dy}{\pi}\cos ny\ \mathcal{J}_0\left(2x\sin\frac{y}{2}\right) \tag{A5}$$

into Eq. (A3) and performing the summation over j, one easily obtains:

$$\Pi(q,\omega) = -\frac{m}{\pi}\left[1-\int_0^\pi dy\mathcal{J}_0\left(2qR_c\cos\frac{y}{2}\right)\frac{\frac{\omega}{\omega_c}\cosh\frac{\omega}{\omega_c}y}{\sinh\pi\frac{\omega}{\omega_c}}\right]. \tag{A6}$$

Equation (A6) enables one to obtain the asymptotic form of the polarization operator in different regimes. In the static limit, $\omega \ll \omega_c$, one finds from Eqs. (A6) and (A5):

$$\Pi(q,\omega) = -\frac{m}{\pi}\left[1-\mathcal{J}_0^2(qR_c)+\mathcal{O}\left(\frac{\omega^2}{\omega_c^2}\right)\right]. \tag{A7a}$$

For the high-frequency domain, $\omega \gg \omega_c$, only a small vicinity of the point $y = \pi$ contributes to the integral in Eq. (A6), and $\Pi(q,\omega)$ coincides with the result for 2D electron gas in the absence of the magnetic field:[31]

$$\Pi(q,\omega) = -\frac{m}{\pi}\left[1-\frac{|\omega|}{\sqrt{\omega^2+q^2v_F^2}}+\mathcal{O}\left(\frac{\omega_c^2}{\omega^2}\right)\right]. \tag{A7b}$$

In the hydrodynamic limit, $qR_c \ll 1$, one can expand the Bessel function in Eq. (A6) in a Taylor series, which yields

$$\Pi(q,\omega) = -\frac{m}{2\pi}\left[\frac{q^2 v_F^2}{\omega^2+\omega_c^2} + \mathcal{O}(q^4 R_c^4)\right]. \tag{A7c}$$

In the opposite limit, $R_c^{-1} \ll q \ll k_F$, the main contribution to the integral in Eq. (A6) comes from the vicinities of points $y = 0$ and $y = \pi$. The calculation gives

$$\Pi(q,\omega) \approx -\frac{m}{\pi}\left[1 - B(q,\omega)\frac{\omega\coth\left(\pi\frac{\omega}{\omega_c}\right)}{\sqrt{\omega^2+q^2v_F^2}} - \left(\frac{\omega}{qv_F}\right)\frac{\sin 2qR_c}{\sinh \pi\frac{\omega}{\omega_c}}\right],$$

$$B(q,\omega) = 1 + \frac{q^2 v_F^2 \omega_c^2\left(4\omega^2 - q^2 v_F^2\right)}{8\left(\omega^2+q^2v_F^2\right)^3}. \tag{A7d}$$

APPENDIX B: PARAMETERS OF THE EFFECTIVE HAMILTONIAN

In this Appendix we present the details of the derivation of the effective Hamiltonian (3.27).

We start from Eq. (3.22). We notice that operators $\hat{F}_1$ and $\hat{F}_2^0$, see Eqs. (3.11c) and (3.11d), are integrals of the Matsubara operators over the time interval $[0, \beta_0]$. Let us divide this time interval into M small intervals $[\tau_{m-1}, \tau_m]$, $m = 1, 2 \ldots M-1$, where $\tau_m = m\Delta\tau$ and $\Delta\tau = \beta_0/M$. We imply that $\Delta\tau$ must be smaller than the characteristic time of the low-energy dynamics, but much larger than ω_c^{-1}. Then, the chronologically ordered exponent in Eq. (3.22) can be factorized as

$$T_\tau\left\{e^{-\hat{F}_1-\hat{F}_2^0}\right\} = \prod_{m=1}^{M} T_\tau\left\{e^{-\hat{F}_1^{(m)}-\hat{F}_2^{(m)}}\right\}, \tag{B1}$$

where operators $\hat{F}_1^{(m)}$ and $\hat{F}_2^{(m)}$ have the form

$$\hat{F}_1^{(m)} = \int d^3\xi\,\phi(\xi)\left(\hat{\rho}_N(\xi) - n_e^N\right)\theta\left([\xi^0-\tau_{m-1}][\tau_m-\xi^0]\right),$$

$$\hat{F}_2^{(m)} = \int d^3\xi_1\int d^3\xi_2\left[\bar{\Psi}_N(\xi_1^+)\Psi_N(\xi_2) - 2G_N(\xi_2,\xi_1)\right]\times \phi(\xi_1)\tilde{G}_0(\xi_1,\xi_2)\phi(\xi_2)\theta\left([\xi_1^0-\tau_{m-1}][\tau_m-\xi_1^0]\right). \tag{B2}$$

Definitions (B2) differ from the corresponding definitions (3.11c) and (3.11d) by the constraint on the time domain of integration. We do not need to impose an additional constraint on the time component of ξ_2 in the second of Eqs. (B2) because $\tilde{G}_0$ decays rapidly at $|\xi_1^0 - \xi_2^0| > 1/\omega_c$, see Eq. (3.16).

Now, we substitute Eq. (B1) into Eq. (3.22) and perform the integration over the field ϕ. It yields

$$\hat{\Lambda}(\beta_0) = e^{-\beta_0(\Omega_0+\Delta\Omega+\hat{H}_0^N)}\prod_{m=1}^{M}\left\langle T_\tau\left\{e^{-\hat{F}_1^{(m)}-\hat{F}_2^{(m)}}\right\}\right\rangle, \tag{B3}$$

where average $\langle\mathcal{F}\rangle$ of an arbitrary functional $\mathcal{F}\{\phi\}$ over the field ϕ is defined as

$$\langle\mathcal{F}\rangle = e^{\beta_0\Delta\Omega}\int\mathcal{D}\phi\,\mathcal{F}\{\phi\}\times \exp\left(\frac{1}{2}\int d^3\xi_1\int d^3\xi_2\,\phi(\xi_1)\phi(\xi_2)S(\xi_1-\xi_2)\right). \tag{B4}$$

The normalization factor $e^{\beta_0\Delta\Omega}$ describes the fluctuations of field ϕ around the saddle point:

$$e^{-\beta_0\Delta\Omega} = \int\mathcal{D}\phi\exp\left(\frac{1}{2}\int d^3\xi\int d^3\xi'\phi(\xi)\phi(\xi')S(\xi-\xi')\right). \tag{B5}$$

An explicit expression for $\Delta\Omega$ is presented in Subsection B 1.

Because $\Delta\tau$ is small, we can expand every factor in Eq. (B3) up to the first order in $\Delta\tau$:

$$\left\langle T_\tau\left\{e^{-\hat{F}_1^{(n)}-\hat{F}_2^{(n)}}\right\}\right\rangle \approx 1 - \Delta\tau\hat{H}_I\left\{\bar{\Psi}_N(\tau_n),\Psi_N(\tau_n)\right\}. \tag{B6}$$

This relation defines the operator $\hat{H}_I$. Let us notice that this operator actually does not depend on τ_n because it contains the same number of creation and annihilation operators taken at the same moment of time. Having this in mind, we substitute Eq. (B6) into Eq. (B3) and obtain

$$\hat{\Lambda}(\beta_0) = e^{-\beta_0(\Omega_0+\Delta\Omega+\hat{H}_0^N)}\left(1 - \frac{\beta_0}{M}\hat{H}_I\right)^M. \tag{B7}$$

Finally, we take the limit $M \to \infty$ in Eq. (B7) and substitute the result in Eq. (3.6). It yields

$$\hat{H}_{eff} = \Omega_0 + \Delta\Omega + \hat{H}_0^N + \hat{H}_I. \tag{B8}$$

Now, one has to perform the actual calculation of the average in the left hand side of Eq. (B6) in order to find the operator $\hat{H}_I$. It can be done by using perturbation theory. The leading terms in the small parameters r_s and $1/N$ are

$$\left\langle T_\tau\left\{e^{-\hat{F}_1^{(n)}-\hat{F}_2^{(n)}}\right\}\right\rangle \approx 1 - \left\langle\hat{F}_1^{(n)} + \hat{F}_2^{(n)}\right\rangle + \frac{1}{2}\left\langle T_\tau\left(\hat{F}_1^{(n)} + \hat{F}_2^{(n)}\right)^2\right\rangle. \tag{B9}$$

To find the operator $\hat{H}_I$, only terms linear in $\Delta\tau$ should be retained.

Calculation of the right-hand side of Eq. (B9) is carried out with the help of Eqs. (B2) and of the correlation functions of the field ϕ:

$$\langle\phi\rangle = 0, \tag{B10}$$

$$\langle\phi(r,\tau)\phi(0,0)\rangle = -\frac{e^2}{\kappa|r|}\delta(\tau) - \int\frac{d\omega d^2q}{(2\pi)^3}\left(\frac{2\pi e^2}{\kappa q}\right)^2\frac{\Pi(q,\omega)}{\varepsilon(q,\omega)}e^{iqr-i\omega\tau}.$$

The latter expression follows from Eq. (B4) and (3.19). In the second of Eqs. (B10), we explicitly separated a term which is singular at $\tau = 0$.

The averages that give the leading contribution to the effective Hamiltonian are

$$\left\langle \hat{F}_2^{(n)} \right\rangle = \Delta\tau \left(\mu_{ex} + \mu_c\right) \int d^2\mathbf{r}\hat{\rho}_N(\mathbf{r}), \tag{B11}$$
$$\frac{1}{2}\left\langle T_\tau \left(\hat{F}_1^{(n)}\right)^2 \right\rangle = -\Delta\tau \left(\hat{H}_{int}^{eff} + \mu_p \int d^2\mathbf{r}\hat{\rho}_N(\mathbf{r})\right),$$

where the operator $\hat{H}_{int}^{eff}$ is defined by Eq. (3.23).

In Eqs. (B11), μ_{ex} coincides with the well-known exchange correction to the chemical potential:

$$\mu_{ex} = -2\pi\ell^2 \int d^2\mathbf{r}\frac{e^2}{\kappa r}\tilde{G}_0\left(\mathbf{r}, 0; 0, \epsilon \to +0\right) P_N\left(0, \mathbf{r}\right), \tag{B12}$$

where $\tilde{G}_0$ is defined by Eq. (3.16), and μ_c, μ_p are the correlation corrections to the chemical potential found in the random phase approximation:

$$\mu_c = -\int \frac{d\omega d^2q}{(2\pi)^3}\left(\frac{2\pi e^2}{\kappa q}\right)^2 \frac{\Pi(q,\omega)}{\varepsilon(q,\omega)} \sum_{n\neq N} \frac{\omega_c(N-n)}{\omega^2 + \omega_c^2(N-n)^2}$$
$$\times(-1)^{(N-n)} L_n^{N-n}\left(\frac{q^2\ell^2}{2}\right) L_N^{n-N}\left(\frac{q^2\ell^2}{2}\right) e^{-q^2\ell^2/2}, \tag{B13}$$

$$\mu_p = \pi\ell^2 \int d^2\mathbf{r} P_N\left(0, \mathbf{r}\right) P_N\left(\mathbf{r}, 0\right)\left(U(r) - \frac{e^2}{\kappa r}\right). \tag{B14}$$

When deriving Eqs. (B12), (B13), we use Eqs. (3.14) and (3.16).

Finally, substitution of Eq. (B11) into Eq. (B9) enables us to find the operator $\hat{H}_I$. Comparing Eq. (B8) and Eq. (3.27), we obtain the expressions for the parameters of the Hamiltonian H_{eff}:

$$\Omega = \Omega_0 + \Delta\Omega, \tag{B15}$$
$$\mu^* = \mu - N\hbar\omega_c - \mu_{ex} - \mu_c - \mu_p. \tag{B16}$$

The following subsections are devoted to the explicit calculation of the parameters of the effective Hamiltonian in terms of the filling factor and interaction strength. We will assume that condition (4.18) is met.

1. Calculation of $\Delta\Omega$.

Equations (B5), (3.8) and (3.19) yield

$$\frac{\Delta\Omega}{L_x L_y} = \frac{1}{2}\int \frac{d\omega d^2q}{(2\pi)^3} \ln\left(\varepsilon(q,\omega)\right). \tag{B17}$$

With the help of Eqs. (3.20), (3.21) and (3.14), one can transform Eq. (B17) into a more convenient form:

$$\Delta\Omega = \Delta\Omega_{ex} + \Delta\Omega_c, \tag{B18}$$
$$\frac{\Delta\Omega_{ex}}{L_x L_y} = -\int d^2\mathbf{r}\frac{e^2}{\kappa r}\tilde{G}_0\left(\mathbf{r}, 0; 0, \epsilon\right)\tilde{G}_0\left(0, 0; \mathbf{r}, \epsilon\right), \tag{B19}$$
$$\frac{\Delta\Omega_c}{L_x L_y} = -\frac{1}{2}\int_0^1 d\alpha \int \frac{d\omega d^2q}{(2\pi)^3}\left(\frac{2\pi e^2}{\kappa q}\right)^2 \frac{\alpha\Pi^2(q,\omega)}{1 - \alpha\frac{2\pi e^2}{\kappa q}\Pi(q,\omega)}. \tag{B20}$$

Expression (B19) corresponds to the first order exchange correction to the ground state energy, and $\epsilon \to +0$. Equation (B20) is the correlation correction to the ground state energy equivalent to the sum of ring diagrams[32].

First, we evaluate the exchange energy (B19). Using the explicit expression for the Green's function (3.16), the property of the Landau level wave functions,

$$P_n(\mathbf{r}_1, \mathbf{r}_2) = \sum_k \varphi_{n,k}^*(\mathbf{r}_2)\varphi_{n,k}(\mathbf{r}_1) = \tag{B21}$$
$$\frac{1}{2\pi\ell^2} e^{i(y_1-y_2)(x_1+x_2)/2\ell^2 - |\mathbf{r}_2-\mathbf{r}_1|^2/4\ell^2} L_n\left(\frac{|\mathbf{r}_2-\mathbf{r}_1|^2}{2\ell^2}\right),$$

and the identity for the Laguerre polynomials,

$$\sum_{i=0}^{n} L_i(x) = L_n^1(x), \tag{B22}$$

we obtain

$$\frac{\Delta\Omega_{ex}}{L_x L_y} = -\frac{e^2}{\kappa\ell\left(2\pi\ell^2\right)}\int_0^\infty dx e^{-x^2/2}\left[L_{N-1}^1\left(\frac{x^2}{2}\right)\right]^2. \tag{B23}$$

Integration in Eq. (B23) can be easily performed, which yields for $N \gg 1$:

$$\frac{\Delta\Omega_{ex}}{L_x L_y} = -\frac{e^2}{\kappa\pi^2\ell^3}\left[\frac{2}{3}\left(2N\right)^{3/2} + \frac{\ln N}{16(2N)^{1/2}} + \mathcal{O}\left(\frac{1}{N^{1/2}}\right)\right]. \tag{B24}$$

The first term in brackets coincides with the exchange energy of 2D electron gas at zero magnetic field, and the second term appears due to the confinement of the electron wave functions by the magnetic field. The logarithmical factor $\ln N \sim \ln\left(k_F R_c\right)$ arises due to the integration over relatively large spatial scales $k_F^{-1} \ll r \lesssim R_c$ in Eq. (B19). On these scales, the screened interaction potential is significantly smaller than the bare potential appearing in Eq. (B19). Therefore, it is plausible to anticipate that the correlation term of the thermodynamic potential, which accounts for the screening effect, should partially compensate the large logarithmic factor in Eq. (B24), and lead to the replacement $R_c \to a_B$ in the argument of the logarithm. The calculation of the correlation energy (B20) explicitly demonstrates this.

In zero magnetic field the contribution of the correlation energy[32] is only of the order of $\mathcal{O}(r_s^2)$. The situation changes in the magnetic field, where a contribution proportional to $r_s \ln(N r_s)$ appears. This term arises mainly from integration over the domain of wave

vectors $R_c^{-1} \ll q \ll a_B^{-1}$ in Eq. (B20), where we can use the asymptotic expression (A7d) for $\Pi(q,\omega)$. It yields with the logarithmic accuracy

$$\frac{\Delta\Omega_c}{L_x L_y} = \frac{e^2}{\kappa\ell\sqrt{2N}\,(16\pi\ell^2)} \ln(N r_s) + \mathcal{O}(r_s^2). \tag{B25}$$

Finally, with the help of Eqs. (B24) and (B25), we obtain for the thermodynamic potential (B15),

$$\Omega = \frac{L_x L_y}{\pi\ell^2}\left\{\hbar\omega_c \frac{N(N-1)}{2} - \mu N - \frac{e^2}{\pi\kappa\ell}\left[\frac{2}{3}(2N)^{3/2} + \frac{1}{16(2N)^{1/2}}\ln\left(r_s^{-1}\right)\right]\right\}. \tag{B26}$$

This result contradicts Ref. 33 where the contributions (B19) and (B20) were evaluated and a correction $\propto r_s^{3/4}$ was obtained instead of $r_s|\ln r_s|$.

2. Calculation of μ^*.

We start from the calculation of the exchange correction to the chemical potential. With the help of Eqs. (3.16), (B21) and (B22) we transform Eq. (B12) to the form

$$\mu_{ex} = -\frac{e^2}{\kappa\ell}\int_0^\infty dx e^{-x^2/2} L_{N-1}^1\left(\frac{x^2}{2}\right) L_N\left(\frac{x^2}{2}\right). \tag{B27}$$

After integration in Eq. (B27) we obtain for $N \gg 1$ with logarithmic accuracy:

$$\mu_{ex} = -\frac{e^2}{\kappa\ell}\left(\frac{2}{\pi}(2N)^{1/2} - \frac{1}{2\pi(2N)^{1/2}}\ln N\right). \tag{B28}$$

The correlation shift of the chemical potential, Eq. (B14), is calculated with the help of Eqs. (B21) and (3.25). With the logarithmic accuracy, the calculation gives

$$\mu_p = -\frac{e^2}{\kappa\ell}\frac{1}{2\pi(2N)^{1/2}}\ln(N r_s). \tag{B29}$$

It can be checked by an explicit calculation that the other correlation correction to the chemical potential, μ_c, see Eq. (B13), contains an additional small factor in comparison with Eq. (B29): $\mu_c \simeq \mu_p/N$, and, therefore, it can be neglected. Finally, with the logarithmic accuracy, we have for μ^* appearing in Eq. (B16) the result:

$$\mu^* = \mu - N\hbar\omega_c + \frac{e^2}{\kappa\ell}\left(\frac{2}{\pi}(2N)^{1/2} - \frac{\ln(r_s^{-1})}{2\pi(2N)^{1/2}}\right). \tag{B30}$$

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FIGURES

FIG. 1. The renormalized pair interaction $U(r)$ given by Eq. (3.25) as the function of the inter-electron distance (solid line). Dashed line is the bare Coulomb potential. The asymptotic behavior of the renormalized potential is given by Eqs. (2.5) and (2.6)

FIG. 2. The thermodynamic density of states as a function of the filling factor of the partially filled Landau level in the domain $0 \leq \nu \leq 1$. The asymptotic behavior of the thermodynamic density of states is given by Eqs. (4.15), (4.17), and (4.21)

FIG. 3. (a): The spectral density as a function of energy. (b): The dependence of the characteristic energy scales on filling factor of the partially filled Landau level $\Delta\nu$. The asymptotic behavior of the width of the pseudogap, Δ_g is given by Eq. (5.12). The width of the peak in the one-electron spectral density Δ_p is determined by the variation of the interstitial energy and it is smaller than $\hbar\omega_c/2N$ at all filling factors.

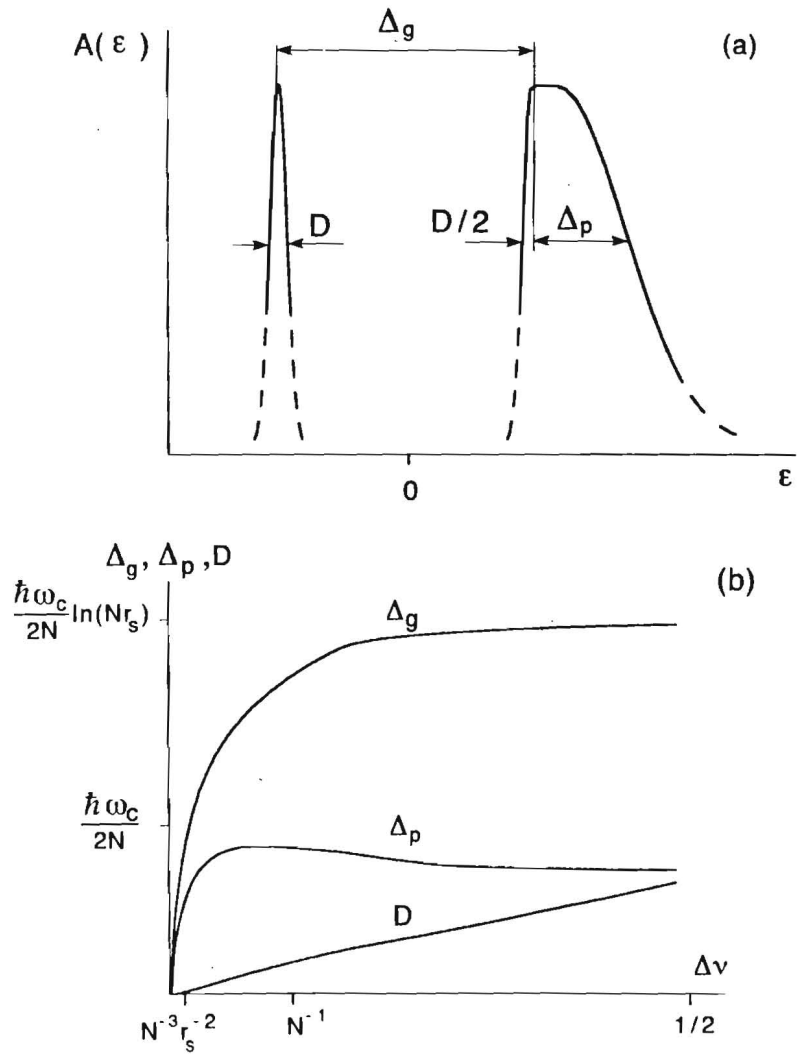

Fig. 3

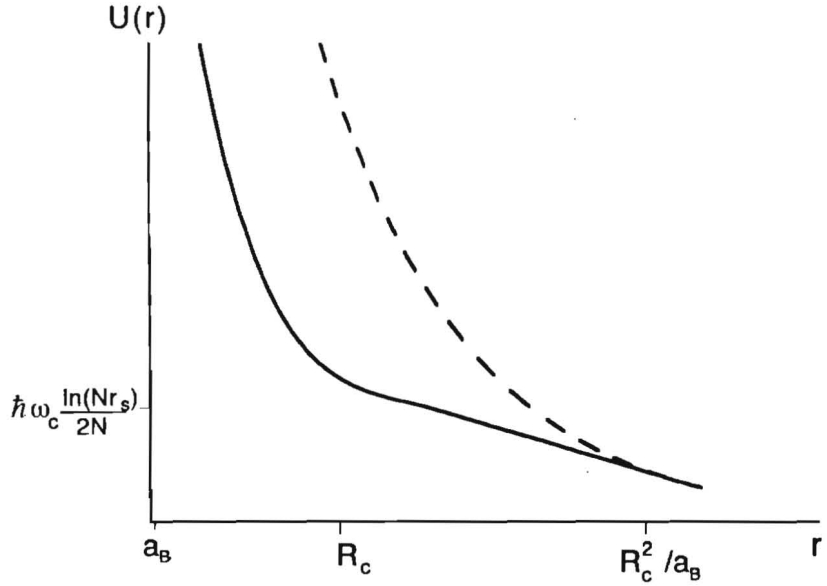
U(r)
$\hbar\omega_c \frac{\ln(Nr_s)}{2N}$
a_B
R_c
R_c^2/a_B
r

Fig. 1

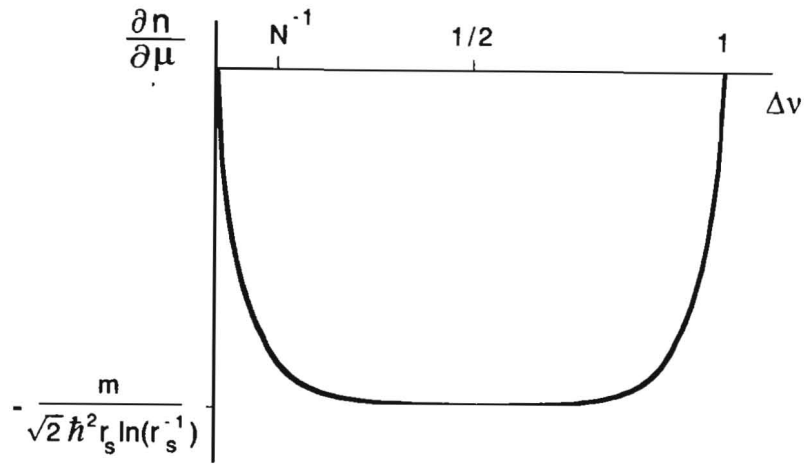
$\frac{\partial n}{\partial \mu}$
N^{-1}
1/2
1
$\Delta\nu$
$-\frac{m}{\sqrt{2}\hbar^2 r_s \ln(r_s^{-1})}$

Fig. 2