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New model of solid-liquid transition in a strong magnetic field

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New model of solid-liquid transition in a strong magnetic field

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Trial functions for the Wigner crystal which undergoes a continuous transition to the incompressible liquid when approaching the point $\nu = 1/(2m+1)$ from either direction are suggested. This model may explain the reentrance-phase effect observed recently in a number of works.

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The quantum Hall effect at certain fractional values of the filling factor ν (FQHE) manifests the existence of a set of incompressible states of a many-electron system. This phenomenon was explained by Laughlin[1] who suggested a variational model of an incompressible quantum liquid at $\nu = \nu_m = 1/(2m+1)$. The states with ν less or larger than ν_m were generated with the use of point-like excitations (quasiholes or quasielectrons) existing on the incompressible background. Later, this theory was generalized for filling factors $\nu = p/(2m+1)$ [2-4]. Historically, first attempts to explain FQHE employed the Wigner crystal of electrons which is clearly the ground-state of an electron system with repulsion at smallest ν. After it was realized that the crystal is always pinned even by a weakest disorder and, hence, cannot provide finite quantized values of σ_{xy} [5], the general belief is that the Wigner crystal has no relation to FQHE states. Apart from this, existing models of a crystal state[6-9] demonstrate a continuous dependence of the correlation energy on ν, so that this state is everywhere compressible. According to the model of a weakly anharmonic crystal[8,9], the crystal has lower energy than the interpolated energy of the Laughlin liquid, in the case of Coulomb repulsion, at $\nu < 1/5.6$.

Recent experiments on GaAs structures discovered remarkable fact which is generally interpreted as a melting transition: The electron system behaves like a solid on both sides of Hall plateau at $\nu = 1/5$ [10-13]. A similar *reentrance* effect is observed in GaAs hole channel at $\nu = 1/3$ [14] and in Si devices at some values $\nu > 1$ [15]. Conventional interpretation of this effect is based on the concept of an "accidental" first-order transition: The crystal term which happens to be, at $\nu = \nu_m$, somewhat higher than the incompressible liquid energy, crosses both quasihole and quasielectron branches very closely to their starting point ν_m. According to this idea, were the form of the interaction potential slightly

different (e.g. due to a different channel width), this double transition would not take place. The most striking feature of cited experiments is the sharp change of crystal parameters (threshold voltage, melting temperature, noise magnitude) with ν approaching ν_m [10,11]. This strongly suggests the existence of a second-order rather than a first-order transition.

In this letter, I propose a variational model for a Wigner crystal which experiences a *continuous* (second-order) transition to the incompressible liquid at $\nu = \nu_m$. The order parameter of such a state (the magnitude of the density oscillations) vanishes gradually when approaching ν_m from either direction. The phase consists from two parts: "left phase" ($\nu < \nu_m$) and "right phase" ($\nu > \nu_m$) which are determined by two different wave functions. Each of this states cannot be compressed (expanded) in order to cross the limiting point $\nu = \nu_m$, so that the total dependence of the correlation energy on ν must exhibit a cusp. Thus, two crystalline phases represent a possible alternative for the quasihole and quasielectron states, respectively. Based on the idea of cooperative ring exchange, the existence of incompressible points in the Wigner crystal phase was supposed by Kivelson *et al.* [16].

In what follows I restrict myself to the first Landau level. The model wave function for the "left" crystalline phase ($\nu < \nu_m$) is given by

$$\Psi_{m,\nu}\{z\} = e^{-\frac{1}{4}\Sigma_i |z_i|^2} \det\left[\exp\left(\frac{1-2m\nu}{2} z_i R_k^*\right)\right] \prod_{i<j}(z_i - z_j)^{2m} , \qquad (1)$$

where z_i are complex coordinates of electrons, points R_i form a triangular lattice with a density $\nu/2\pi$, and the determinant provides antisymmetrization (all coordinates here and below are given in units of the magnetic length). At $m = 0$, the right-hand side of Eq. (1) coincides with the well-known function suggested by Maki and Zotos (MZ)[7].

First, in order to demonstrate that $\Psi_{m,\nu}$ describes a uniform crystal with the density $\nu/2\pi$, I neglect with the antisymmetrization in (1). Replacing the determinant by the product $\prod_i \exp[(1/2 - m\nu) z_i R_i^*)]$ yields

$$|\Psi_{m,\nu}|^2 = \exp(-H/2m) , \qquad (2)$$

$$H = 2m\sum_i \frac{\mathbf{z}_i^2}{2} + (2m)^2 \sum_{i<j} 2\log\frac{1}{|\mathbf{z}_i - \mathbf{z}_j|} - 2m(1-2m\nu)\sum_i \mathbf{R}_i \mathbf{z}_i , \qquad (3)$$

where vector notation for z_i and R_i is used. The effective Hamiltonian (3) describes the two-dimensional plasma on an attractive background[1] with the "external field" $(1-2m\nu)\mathbf{R}_i$ acting on ith particle. If the state has a uniform density $\nu/2\pi$, the repulsion between particles described by the second term in (3) is equivalent to the one-particle potential of the uniformly charged disc with the charge density $2m\nu/2\pi$. Hamiltonian (3) is then reduced to the effective one-particle Hamiltonian

$$V = 2m(1-2m\nu)\sum_i \left[\frac{\mathbf{z}_i^2}{2} - \mathbf{z}_i \mathbf{R}_i\right]. \qquad (4)$$

The energy (4) has, at $\nu < 1/2m$, a stable minimum at $\mathbf{z}_i = \mathbf{R}_i$. The finite "temperature" $2m$ in Eq. (2) causes fluctuations of electron positions around the lattice points $\mathbf{R}_i$. The fact that we replaced the crystal potential by a homogeneous charge when deriving (4) is not important since the average correlation corrections to the force acting on each particle at $\mathbf{z}_i = \mathbf{R}_i$ are zero because of the crystal symmetry.

The function (1) can be also written in the form

$$\Psi_{m,\nu}\{z\} = \left[e^{-\frac{m\nu}{2}\Sigma_i |z_i|^2} \prod_{i<j}(z_i - z_j)^{2m}\right] \Psi_{0,\nu'(\nu)}\left\{z\sqrt{1-2m\nu}\right\} , \qquad (5)$$

where $\nu'(\nu) = \nu/(1-2m\nu)$ and

$$\Psi_{0,\nu'}\{z\} = \det\left\{\exp\left[-\frac{1}{4}|z_i - R'_k|^2 + \frac{1}{4}(z_i R'^*_k - z_i^* R'_k)\right]\right\} \qquad (6)$$

is the MZ function with the density $\nu'/2\pi$. Thus, $\Psi_{m,\nu}$ represents formally the product of a *symmetric* Laughlin function expanded up to the density $\nu/2\pi$, and the rescaled MZ function. The demonstration of the "melting transition" to the Laughlin function given by

$$F_m\{z\} = e^{-\frac{1}{4}\Sigma_i |z_i|^2} \prod_{i<j}(z_i - z_j)^{2m+1} \qquad (7)$$

is now straightforward. Because any antisymmetric function describing the state with the density $1/2\pi$ must coincide with the function of the completely filled Landau level F_0, so does the function (6) at $\nu' = 1$: $\Psi_{0,1}\{z\} \equiv F_0\{z\}$. As a result, at $\nu = \nu_m = 1/(2m+1)$ the right-hand side of Eq. (5) crosses over to (7): $\Psi_{m,\nu_m}\{z\} \equiv F_m\{z\}$. The electron density at this point becomes constant and equal to $\nu_m/2\pi$, the further increase of the density being impossible. The same result can be also derived directly from the form (1) of $\Psi_{m,\nu}$. Note, that each exponential e^x in the determinant in (1) can be replaced, with an exponentially

high accuracy, by $N-1$-degree polynomial (truncated Taylor series), where $N >> 1$ is the particle number, provided the inequality $|x|/N < 1$ holds for all x. As easy to check, the last condition is equivalent to $\nu > \nu_m$. In this case, the determinant N by N in Eq. (1) can be reduced by linear transformations to the Vandermonde form: $\det(z_i^k) = \prod_{i<j} (z_i - z_j)$, and we again arrive to the Laughlin function.

In order to construct the wave function for the "right" phase ($\nu > \nu_m$), I use, as a heuristic rule, the replacement $z_i \to \partial/\partial z_i$ suggested by Laughlin to obtain the liquid state with a quasielectron[1]. Namely, I extract the function F_m from the right-hand side of Eq. (1) as a factor, and replace all z_i in the remaining part by $\partial/\partial z_i$ multiplied by some constants. The result has a form

$$\Phi_{m,\nu}\{z\} = e^{-\frac{1}{4}\Sigma_i |z_i|^2} \det\left[\exp\left(B_k \frac{\partial}{\partial z_i}\right)\right] P_m\{z\}, \tag{8}$$

where $P_m\{z\}$ is a homogeneous symmetric polynomial determined by the equation

$$\prod_{i<j}\left(\frac{\partial}{\partial z_i} - \frac{\partial}{\partial z_j}\right) P_m\{z\} = \prod_{i<j}(z_i - z_j)^{2m+1}. \tag{9}$$

The complex numbers B_i are to be defined from the condition that electrons in the state given by Eq. (8) are close to the points R_i of the lattice with the density $\nu/2\pi$. The fact that the replacement $z_i \to \partial/\partial z_i$ yields a crystal at $\nu > \nu_m$ is not obvious and will be verified below.

It turns out that B_i should be chosen in order to form a rescaled lattice, as given by

$$B_i = \alpha R_i = [(2m+2)\nu - 1]\, R_i. \tag{10}$$

To demonstrate this, I neglect with the antisymmetrization as it was done for the "left" phase. Using the fact that the exponential of derivative is the shift operator, one gets from (8) $|\Phi_{m,\nu}|^2 = \exp(-H/(2m+2))$, where the effective Hamiltonian H is given by

$$H\{z\} = (2m+2)\left[\sum_i \frac{|z_i|^2}{2} + H_p\{w\}\right], \tag{11}$$

$$w_i = z_i + B_i, \tag{12}$$

$$H_p\{w\} = -\log |P_m\{w\}|^2.$$

If all B_i were zero ($w_i = z_i$), Hamiltonian H would describe a plasma with a many-particle interaction defined by the form of the polynomial $P_m\{w\}$. The density of the plasma $\nu_p/2\pi$ is related to the homogeneity degree $M_p(N)$ of $P_m\{w\}$ by the general formula[4]

$$\nu_p^{-1} = \lim_{N\to\infty} [2M_p(N)/N^2]$$

which yields, with the use of (9), $\nu_p = 1/(2m+2)$. Hence, if w_i have a uniform density $\nu_w/2\pi$, the interaction term $H_p\{w\}$ in Eq. (11) is equivalent to the one-particle potential of the disc with the charge density $(2m+2)\nu_w/2\pi$. The Hamiltonian (11) can be then reduced to

$$V\{\mathbf{z}\} = (2m+2)\sum_i \left[\frac{\mathbf{z}_i^2}{2} - (2m+2)\nu_w \frac{(\mathbf{z}_i + \mathbf{B}_i)^2}{2}\right].$$

which is analogous to one-particle Hamiltonian (4). The lowest-energy state of this system is given by

$$\mathbf{z}_i = \frac{(2m+2)\nu_w}{1-(2m+2)\nu_w}\mathbf{B}_i. \tag{13}$$

Thus, if one puts $\mathbf{B}_i = \alpha\mathbf{R}_i$, the system of points $\mathbf{w}_i$ will be a lattice [Eq.(12)], so that the assumption we made above about uniformity of the distribution of $\mathbf{w}_i$ is justified. Unknown α and ν_w should satisfy two self-consistency equations: $\mathbf{z}_i = \mathbf{R}_i$ [where $\mathbf{z}_i$ are given by (13)], and $\nu_w = \nu/(1+\alpha)^2$. Solving that system, one arrives to Eq. (10) for α.

Like in the case of the "left" phase, the antisymmetrization imposed by the determinant in (8) causes transition to the liquid state (7) at $\nu = \nu_m$. The properties of the polynomial part of $\Phi_{m,\nu}$ are determined by the ratio $\max(|B_i|)/\max(|z_i|) = \alpha(\nu)$. If $\alpha(\nu)$ is small enough, one can expand the determinant in powers of B_i and keep the sum of lowest-degree terms which is equal to

$$\prod_{i<j}(B_i - B_j) \prod_{i<j}\left(\frac{\partial}{\partial z_i} - \frac{\partial}{\partial z_j}\right)$$

and can be obtained by the expansion of each exponential in (8) up to $N-1$ degree of the argument. As a result, with the use of Eq. (9), one gets the function (7). Because the latter provides the electron density $\nu_m/2\pi$, the transition from $\Phi_{m,\nu}$ to F_m should take place at

$\nu = \alpha(\nu) = \nu_m$. To give some direct evidence of the existence of a critical point in ν, I calculated numerically at different ν the averaged homogeneity degree

$$M_T(\nu) = \left[\frac{\partial \log |T_m\{\lambda z\}|}{\partial \log \lambda}\right]_{\lambda = 1}$$

of the polynomial

$$T_m\{z\} = \det\left[\exp\left(B_k \frac{\partial}{\partial z_i}\right)\right] \prod_{i<j}(z_i - z_j)^{2m+1}$$

which is the result of application of operator $\prod_{i<j}(\partial/\partial z_i - \partial/\partial z_j)$ to the polynomial part of (8). If the latter coincides with the Jastrow factor in (7), $T_m\{z\}$ will be homogeneous, $M_T(\nu)$ being equal to $mN(N-1)$. If not, parameter M_T has only some averaged sense and should depend on ν (the crystal function is not a total momentum eigenstate and, hence, is not represented by a homogeneous polynomial). The dependence $M_T(\nu)$ was calculated for the number of particles N = 5-8. The corresponding configurations of points z_i, $B_i = \alpha(\nu)z_i$ were chosen to be highly symmetric (regular polygons with a point in the middle) in order to diminish the size effect. The latter was fitted very good by the formula $M_T(\nu,N) = A(\nu)2^{-N} + M_T(\nu,\infty)$ which was used for the extrapolation to $N = \infty$. The normalized dependence $M_{\rm norm}(\nu) = M_T(\nu)[mN(N-1)]^{-1}$, as shown in Fig. 1 for m = 1, demonstrates well-pronounced critical point ν=1/3.

The fact that each crystal phase suggested above coincides with one of Laughlin states at $\nu = \nu_m$ means that its energy (for $1/r$-interaction) must be lower than the energy of the correlated-crystal phase studied earlier [8,9], at least, in the vicinity of ν_1 = 1/3 and ν_2 = 1/5. It is also interesting to compare both models in the limit of very small ν, where the wave function of a harmonic crystal obtained by Lam and Girvin in Ref. 8 is asymptotically exact. Apart from the antisymmetrization, this function has a form

$$\Psi_{\rm LG}\{z\} = e^{-\frac{1}{4}\Sigma_i |\xi_i|^2 + \frac{1}{4}\Sigma_{ij}\, b(R_i - R_j)\xi_i\xi_j}\; e^{\frac{1}{4}\Sigma_i (\xi_i R_i^* - \xi_i^* R_i)}, \qquad (14)$$

where $\xi_i = z_i - R_i$ are displacements of electrons from the lattice sites. The coupling coefficients $b(R_i)$ which determine zero-point oscillation of the crystal are related to the oscillation spectrum of the classical Wigner crystal with $1/r$-interaction between electrons, as given by

$$|b(\mathbf{q})| = |b_{\rm LG}(\mathbf{q})| \equiv \frac{\omega_L(\mathbf{q}) - \omega_T(\mathbf{q})}{\omega_L(\mathbf{q}) + \omega_T(\mathbf{q})}, \qquad (15)$$

where $b(\mathbf{q}) = \Sigma_i\, b(\mathbf{R}_i)\exp(-\mathbf{q}\mathbf{R}_i)$ is the lattice Fourier transform, and $\omega_{L,T}(\mathbf{q})$ are the frequencies of the longitudinal and transverse oscillation mode, respectively[8]. The wave function of the "left" phase (5) has the same asymptotic form (14) in the region $\nu << 1$, $1/\nu - (2m+1) \gtrsim 1$, where the antisymmetrization is not important, and typical displacements of electrons ξ_i are much smaller than the lattice constant. Expanding the logarithm of $\Psi_{m,\nu}$ up to the second-order terms in ξ_i, one gets the right-hand side of Eq. (14) with $b(R_i) = 4m/R_i^2$, $R_i \neq 0$. By analogy with (15), the Fourier transform $b(\mathbf{q})$ can be related to the spectrum $\Omega_{L,T}(\mathbf{q})$ of the classical crystal with *logarithmic* repulsion between electrons:

$$|b(\mathbf{q})| = |b_m(\mathbf{q})| \equiv 2m\nu\frac{\Omega_L^2(\mathbf{q}) - \Omega_T^2(\mathbf{q})}{\Omega_L^2(\mathbf{q}) + \Omega_T^2(\mathbf{q})}.$$

Since $b_m(\mathbf{q})$, strictly speaking, differs from $b_{\rm LG}(\mathbf{q})$ at any m, neither of functions (5) is an exact wave function at small ν. The minimum of the mean-square difference $b_{\rm LG}(\mathbf{q})-b_m(\mathbf{q})$ is reached at $m \approx 0.3/\nu$ and is as small as 6%. The remaining discrepancy can be compensated by a simple interpolative trick, namely, by multiplying (non-symmetrized) function $\Psi_{m,\nu}$ by a factor of the form $\frac{1}{4}\Sigma_{ij}\, c_{m,\nu}(R_i - R_j)\xi_i\xi_j$, where $c_{m,\nu}(\mathbf{q})$ is equal to $b_{\rm LG}(\mathbf{q})-b_m(\mathbf{q})$ at $\nu << 1$ and vanishes at $\nu = \nu_m$. The introducing of a new (in addition to m) variational parameter $c_{m,\nu}(\mathbf{q})$ would enlarge the region of variational search. This emphasizes the fact that the wave function (5) is not unique (though probably the simplest) model of the crystal which "melts" to the incompressible liquid.

The "left" and "right" crystal phases with a given m will manifest itself as a reentrance melting transition at the point ν_m, if their correlation energies are less than the energies of corresponding quasiparticle states and of crystal phases with other m. Direct calculation of the crystal energy is very difficult at the moment because of a crucial role of antisymmetrization. Indeed, the latter is responsible for the "melting" of the crystal at $\nu = \nu_m$ analogous to the melting of any Wigner crystal phase at ν = 1. All known models of the crystal[6-8] were studied in the interval ν < 1/2 where exchange effects play a minor role. The accurate treatment of the case when the exchange dominates crystal properties is a subject of a future work. Now, in principle, two following scenarios of a reentrance transition are possible: (i) Absolute slopes of both crystal terms at $\nu = \nu_m$ (chemical potential) are less than that of quasiparticle terms. In this case, the Wigner crystal crosses directly to Laughlin liquid by the second-order transition in ν from either direction. Plateau in σ_{xy} should disappear with the disorder decreased. (ii) The quasihole term is lower than the crystalline

term in the direct vicinity of ν_m, both terms intersecting at a small distance from ν_m. Then a first-order transition between the Wigner crystal and the Hall crystal of holes[17] takes place in the region of the critical behavior of the former. Plateau in the linear Hall conductivity remains finite in the limit of small disorder, where the width of plateau in ν is determined by the width of the Hall crystal region[17].

In conclusion, I note that the wave functions $\Psi_{m,\nu}$ and $\Phi_{m,\nu}$ can be easily generalized for hierarchic points $\nu = p/(2m+1)$ using the antisymmetric polynomials suggested by Macdonald *et al.*[4] for incompressible liquid at these points. One has only to substitute the right-hand side in Eq. (9) with a polynomial from Ref. 4 and the last term in (1) with the same polynomial divided by $\prod_{i<j} (z_i - z_j)$.

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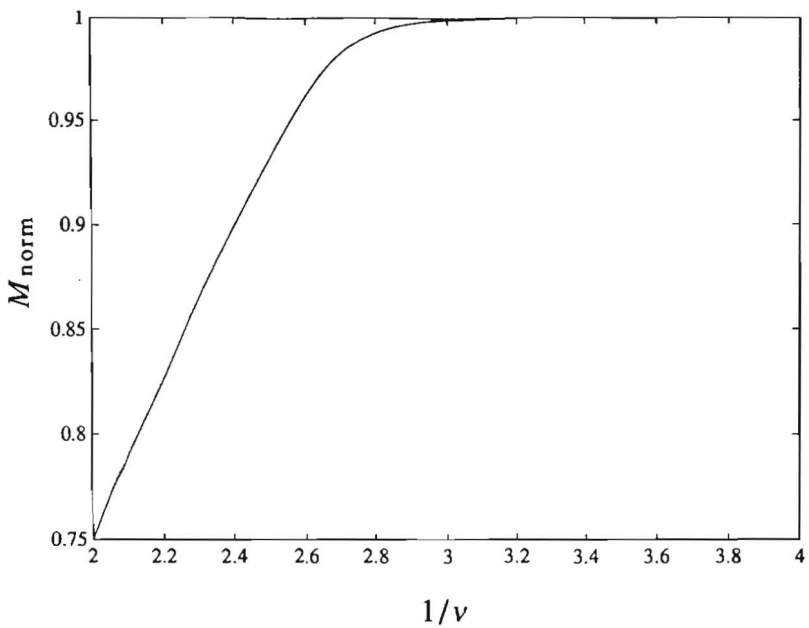

Fig. 1. Normalized "average" homogeneity degree M_{norm} of the polynomial part of the crystal wave function $\Phi_{1,\nu}$ at different values of ν. The curve demonstrates a sharp transition at $\nu = 1/3$ from inhomogeneous polynomial (crystal) to a homogeneous one (incompressible liquid).