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Geometrical Scaling in Two-Body and Quasi-Two-Body Hadronic Processes*)

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Abstract

Through a survey of data on forward differential cross section of various two-body and quasi-two-body hadronic processes it is pointed out that there exists a kind of geometrical scaling, which is refered to a regularity of the impact parameter amplitude $g^{\alpha}_{\Delta\lambda}(s, b)$, where b is the impact parameter, $\Delta\lambda$ is the magnitude of the s-channel total helicity flip and α refers to a certain reaction. The geometrical scaling which we mean has three contents; (i) $g^{\alpha}_{0}(s, b) \propto g^{\beta}_{0}(s, (b^{\beta}_{d}/b^{\alpha}_{d})\, b)$ for the component which consists of s-channel background or equivalently Pomeron exchange; (ii) $g^{\alpha}_{\Delta\lambda}(s, b) \propto g^{\beta}_{\Delta\lambda}(s, (b^{\beta}_{r}/b^{\alpha}_{r})\, b)$ for the component which consists of s-channel resonances and is dual to the same ordinary "Regge" exchange; (iii) $b^{\alpha}_{d}/b^{\beta}_{d} = b^{\alpha}_{r}/b^{\beta}_{r}$. Here, quantities b_d and b_r are characteristic radii of the Pomeron exchange and of the ordinary "Regge" exchange, respectively. In particular, the forward differential, elastic and total cross sections of pp, $\bar{p}p$ and $K^{\pm}p$ elastic processes are systematically understood by the scaling $b^{NN}_{d}/b^{KN}_{d} = b^{NN}_{r}/b^{KN}_{r} = 1.23$.

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§1. Introduction

Several phenomenological models[1)~4)] based on a geometrical picture of two-component duality[5)] for two-body and quasi-two-body scattering of hadrons have yielded good results on a simple systematic description of forward phenomena, such as dip-bump structure of elastic and inelastic differential cross section (DCS) and elastic polarization. Fundamental assumptions of these models are the followings. (i) A zero structure of the imaginary part of the effective resonance[2)] amplitude with s-channel total helicity flip $\Delta\lambda$ which is dual to ordinary "Regge"*) exchange is given by a Bessel-like function "$J_{\Delta\lambda}(b_r\sqrt{-t})$" in accordance with the assumption that this amplitude is dominated by the most peripheral partial waves with $l \approx k b_r$ where k is the c.m. momentum and b_r is the impact parameter of the peripheral effective resonance. (ii) The real part of the above amplitude is related to its imaginary part by a Regge-like (rotating) phase which is expected from resonance-Regge duality.**) (iii) There exists s-channel background which is dual to Pomeron exchange, of which amplitude is dominated by low partial waves with $b < b_r$. In

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*) In general, a t-channel exchange dual to the peripheral resonance may be a combination of Regge pole and other possible associated singularities like Regge cuts. For instance, in the Reggeized absorption model[15)] the peripherality of the scattering amplitude is mainly owing to a combined effect of ordinary Regge pole and Regge cut associated by Pomeron exchange.

**) However, the real part of the ordinary "Regge" exchange amplitude with $\Delta\lambda = 0$ is not given explicitly in Harari's dual absorption model.[1)]

addition, this amplitude is predominantly imaginary and approximately conserves s-channel helicity.[6] Although the dip-bump structure of DCS and the oscillatory pattern of the polarization in the forward direction are fairly well understood by the above three features with essentially one parameter b_r,[1)~4)] it is necessary for a description of detailed t-dependence of these forward phenomena to have a knowledge about a more precise form of the impact parameter amplitude (Fourier-Bessel transform of the scattering amplitude).

The purpose of this paper is to investigate a regularity of the geometrical structure of the impact parameter amplitude for various two-body and quasi-two-body processes through a survey of data on forward DCS. As a result, it is pointed out that there is a kind of geometrical scaling.*)

A content of our geometrical scaling is mentioned in §2 and the experimental evidences are enumerated in §3. Concluding remarks are given in §4.

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*) In general, the geometrical scaling stated here is different from that of Ref. 7, which is refered to the proportionality of the slope parameter of DCS and the total cross section of the elastic processes. In this connection, see Eq.(10). On the other hand, F. Takagi has recently discussed another type of geometrical scaling which he called "the geometrical scaling of the first kind" in connection with a generalization of geometrical duality.[18] He called the geometrical scaling discussed in this paper "the geometrical scaling of the second kind."

§2. Geometrical scaling

Let the impact parameter amplitude with helicity flip $\Delta\lambda$ be $g_{\Delta\lambda}(s, b)$, which is related to the relevant scattering amplitude $f_{\Delta\lambda}(s, t)$ by:

$$f_{\Delta\lambda}(s, t) = \int_0^\infty b\,db\,g_{\Delta\lambda}(s, b) J_{\Delta\lambda}(b\sqrt{-t'}). \qquad (1)^{*)}$$

Consider two reactions α and β. The geometrical scaling we mean has three different contents. The first is refered to the s-channel background or Pomeron term and is represented by:

$$g^{\alpha}_{\Delta\lambda}(s, b) = K^{\alpha\beta}_{d} g^{\beta}_{\Delta\lambda}\left(s, \frac{b^{\beta}_{d}}{b^{\alpha}_{d}} b\right), \qquad (2)$$

where b_α is defined by:

$$(b_d)^2 = \left\{\int_0^\infty db\,b^2 g_{\Delta\lambda}(s, b)\right\} \Big/ \left\{\int_0^\infty db\,g_{\Delta\lambda}(s, b)\right\} \qquad (3)$$

and $K^{\alpha\beta}_{d}$ is a proportionality constant which does not depend on b but can depend on s in general. By the way, Eq.(2) holds identically if $g_{\Delta\lambda}(s, b)$ is happened to be the Gaussian distribution peaked at $b = 0$. The second is refered to the impact parameter amplitudes of reactions α and β which are <u>dual to the same t-channel ordinary "Regge" exchange</u>, and is given by:

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*) Here, the normalization of the amplitude is $\frac{d\sigma}{dt} = \frac{\pi}{k_i^2}\sum_{\Delta\lambda}|f_{\Delta\lambda}(s, t)|^2$, where k_i is the c.m. momentum in the initial state. The argument t' appeared in the Bessel function is $t - t_{\min}$. Throughout this paper the difference between t' and t is neglected.

$$g^{\alpha}_{\Delta\lambda}(s,\ b)=K^{\alpha\beta}_{r}g^{\beta}_{\Delta\lambda}(s,\ \frac{b^{\beta}_{r}}{b^{\alpha}_{r}}b),\tag{4}$$

where b_r is the quantity which specifies a oscillatory structure of the scattering amplitude through a Bessel-like function "$J_{\Delta\lambda}(b_r\sqrt{-t})$" as is already mentioned before, and $K^{\alpha\beta}_{r}$ is a proportionality constant. The last one is a relation which connects the quantities b_d and b_r :

$$b^{\alpha}_{d}/b^{\beta}_{d}=b^{\alpha}_{r}/b^{\beta}_{r}\ .\tag{5}$$

The above Eqs.(2) and (4) imply also the scaling between the relevant scattering amplitudes with respect to t-dependence. Substituting Eqs.(2) and (4) into Eq.(1), we obtain

$$f^{\alpha}_{\Delta\lambda}(s,\ t)=K^{\alpha\beta}_{d,r}\,\xi f^{\beta}_{\Delta\lambda}(s,\ \xi t),\tag{6}$$

where $\xi=(b^{\alpha}_{d}/b^{\beta}_{d})^{2}$ for the Pomeron exchange (s-channel background) amplitude and $\xi=(b^{\alpha}_{r}/b^{\beta}_{r})^{2}$ for the ordinary "Regge" exchange (the effective resonance[2]) amplitude. Therefore, the hypothesis of the geometrical scaling mentioned above can be directly compared with the experimental data. In particular, when the reactions α and β are dominated by the same quantum number exchange and the same $\Delta\lambda$, DCSs of α and β should satisfy the relation:

$$D^{\alpha}(s,\ t)\approx(K^{\alpha\beta}_{d,r})^{2}\xi^{2}D^{\beta}(s,\ \xi t),\tag{7}$$

where

$$D(s,\ t)=\frac{d\sigma}{dt}(s,\ t).\tag{8}$$

It should be noted here that, throughout this paper, we consider

the geometrical scaling between D^{α} and D^{β} (or equivalently $g^{\alpha}_{\Delta\lambda}$ and $g^{\beta}_{\Delta\lambda}$) for a common s for convenience. It may be rather an open question what is the best energy variable to be fixed. However, at such high energies considered in this paper, the differences arised from the choice of energy variables would give rise to a minor effect.

In the next section some evidences for the geometrical scaling represented by Eqs.(2), (4) and (5) are enumerated.

§3. Experimental evidences

(a) Comparison of K^+p and pp elastic DCS

We assume that the Pomeron exchange amplitude is predominantly imaginary and the ordinary "Regge" exchange (or equivalently the effective resonance) amplitude is almost real in this case because K^+p and pp channels are exotic. Then, DCS of these processes may be approximately given by Pomeron exchange alone for relatively small $|t|$ region. Thus, these processes may provide a good test for the geometrical scaling Eq.(2). From Eq. (7) the following relation is derived:

$$\frac{D^{K^+p}(s,\ t)}{D^{K^+p}(s,\ 0)} = \frac{D^{pp}(s,\ (b_d^{KN}/b_d^{NN})^2 t)}{D^{pp}(s,\ 0)} \quad . \tag{9}$$

The experimental values of both sides of Eq.(9) are plotted in Fig. 1 by taking $(b_d^{NN}/b_d^{KN})^2 = (1.23)^2$. Data are taken from Ref. 8. The nice fits of these DCSs for $t \gtrsim -1\ (\mathrm{GeV/c})^2$ seem to verify directly the geometrical scaling for the Pomeron exchange.

Fig. 1

(b) $\frac{d\sigma}{dt}(s, t=0)$, $\sigma_{el}(s)$ and $\sigma_{tot}(s)$ of K^+p and pp elastic processes

With the same approximation as is done in (a), the following relations for the DCS at $t=0$ $(\frac{d\sigma}{dt}(s, t=0) = D(s, 0))$, the elastic cross section ($\sigma_{el}(s)$), and the total cross section ($\sigma_{tot}(s)$) of these processes are easily derived from Eq.(2) or Eqs.(6) and (7):

$$\left.\begin{aligned} & D^{pp}(s, 0) = K_d^2 (b_d^{NN}/b_d^{KN})^4 D^{K^+p}(s, 0) \\ & \sigma_{el}^{pp}(s) = K_d^2 (b_d^{NN}/b_d^{KN})^2 \sigma_{el}^{K^+p}(s) \\ & \text{and} \\ & \sigma_{tot}^{pp}(s) = K_d (b_d^{NN}/b_d^{KN})^2 \sigma_{tot}^{K^+p}(s). \end{aligned}\right\} \tag{10}$$

From Eq.(10) the value b_d^{NN}/b_d^{KN} can be estimated by two ways:

$$b_d^{NN}/b_d^{KN} = [\{D^{pp}(s, 0)/\sigma_{el}^{pp}(s)\}/\{D^{K^+p}(s, 0)/\sigma_{el}^{K^+p}(s)\}]^{\frac{1}{2}} \tag{11}$$

and

$$b_d^{NN}/b_d^{KN} = [\{(\sigma_{tot}^{pp}(s))^2/\sigma_{el}^{pp}(s)\}/\{(\sigma_{tot}^{K^+p}(s))^2/\sigma_{el}^{K^+p}\}]^{\frac{1}{2}}. \tag{12}$$

It is found using the data at $p_{lab} \simeq 6 \sim 15$ GeV/c[8] that b_d^{NN}/b_d^{KN} is almost energy independent and is about 1.24 from Eq.(11) and 1.21 from Eq.(12). These values of b_d ratio are consistent with the value obtained in (a). The proportionality constant K_d is given in term of $\sigma_{el}(s)$ and $\sigma_{tot}(s)$ by:

$$K_d = \{\sigma_{el}^{pp}(s)/\sigma_{tot}^{pp}(s)\}/\{\sigma_{el}^{K^+p}(s)/\sigma_{tot}^{K^+p}(s)\}. \tag{13}$$

Substituting the experimental data[8] into the right hand side of Eq.(13), we found that K_d is about 1.51 and is almost energy independent.

(c) The scaling of b_d and b_r

Since the Pomeron exchange dominates elastic processes, the value of b_d^2 can be estimated directly from the slope parameter of elastic DCS. On the other hand, the value of b_r is determined by the positions of the cross-over zero and the dip or the change of slope of elastic DCS.[1)~4)] In Table 1, the values of b_d^2 and b_r estimated in this way are shown, where the values of the slope parameter are taken from the summary given in Ref. 7 (the references of data are also listed there) and those of b_r are taken from Ref. 4. The relation between b_d and b_r of these elastic processes seems to be consistent with the geometrical scaling Eq.(5).

Table 1

(d) The scaling of $K^{\pm}p$, pp and $\bar{p}p$ elastic DCS

Now we consider the K^-p and $\bar{p}p$ elastic scattering in addition to K^+p and pp scattering. In the former (K^-p and $\bar{p}p$) reactions, the ordinary "Regge" exchange amplitudes is considered to be complex in contrast to the case of (exotic) K^+p and pp reactions. Therefore, the DCSs for these four processes may be written as:[9)]

$$\left.\begin{aligned}
&D^{K^+p}(s,t) \simeq [P^{KN}(s,t)]^2, \\
&D^{K^-p}(s,t) \simeq [P^{KN}(s,t)]^2 + 2P^{KN}(s,t)\cdot R^{KN}(s,t), \\
&D^{pp}(s,t) \simeq [P^{NN}(s,t)]^2, \\
&D^{\bar{p}p}(s,t) \simeq [P^{NN}(s,t)]^2 + 2P^{NN}(s,t)\cdot R^{NN}(s,t),
\end{aligned}\right\} \qquad (14)$$

where $R(s, t)$ is the imaginary part of the ordinary Regge exchange amplitude with $\Delta\lambda = 0$ of K^-p and $\bar{p}p$ elastic scattering. The geometrical scaling of Eqs.(2), (4), and (5) requires that

$$\left.\begin{aligned} &P^{KN}(s, t) = (K_d\xi)^{-1} P^{NN}(s, \xi^{-1}t), \\ &R^{KN}(s, t) = (K_r\xi)^{-1} R^{NN}(s, \xi^{-1}t), \\ &\text{and} \\ &\xi = (b_d^{NN}/b_d^{KN})^2 = (b_r^{NN}/b_r^{KN})^2, \end{aligned}\right\} \tag{15}$$

where K_d is the same quantity as K_d appeared in (b). Finally, $D^{K^+p}(s, t)$ and $D^{K^-p}(s, t)$ can be represented by $D^{pp}(s, t)$ and $D^{\bar{p}p}(s, t)$ using eqs.(14) and (15) as:

$$D^{K^+p}(s, t) = AD^{pp}(s, \xi^{-1}t), \tag{16}$$

and

$$D^{K^-p}(s, t) = AD^{pp}(s, \xi^{-1}t) + B[D^{\bar{p}p}(s, \xi^{-1}t) - D^{pp}(s, \xi^{-1}t)], \tag{17}$$

where

$$A = (K_d\xi)^{-2} \quad \text{and} \quad B = (K_d K_r \xi^2)^{-1}. \tag{18}$$

Using the best fit value obtained by assuming the functional form of $e^{\alpha+\beta t+\gamma t^2}$ for $D^{\bar{p}p}(s, t)$ and $D^{pp}(s, t)$ in the right hand sides of Eqs.(16) and (17), the experimental data of $D^{K^+p}(s, t)$ and $D^{K^-p}(s, t)$ are well reproduced by taking $A = 0.224$, $B = 0.166$ and $\sqrt{\xi} = 1.23$ as is shown in Fig. 2. The value of A is to be compared with o.192 which is calculated from Eq.(18) with $\sqrt{\xi} = 1.23$ and $K_d = 1.51$ obtained in (b). Now, the ratio of the total cross section difference is predicted to be $\{\sigma_{tot}^{\bar{p}p}(s) - \sigma_{tot}^{pp}(s)\}/\{\sigma_{tot}^{K^-p}(s) -$

$\sigma_{tot}^{K^+p}(s)|=K_r\xi=\frac{\sqrt{A}}{B}=2.85$, which is to be compared with the experimental value of 3.0 ± 0.3 obtained in Ref. 10. The consistency of Eqs.(16) and (17) verifies the scaling for the imaginary part of the helicity nonflip amplitude of these processes.

Fig. 2

(e) $K^-p \to \pi^-\Sigma^+$ and $\pi^+p \to K^+\Sigma^+$ processes

These processes are related to each other by $s-u$ crossing (line reversed process). In the Regge pole model with exact exchange degenerate K^* and K^{**} Regge trajectory the DCSs of these processes should be equal to each other. Experimentally this equality seems to be broken even at $p_{lab} \sim 15$ GeV/c. From the view-point of geometrical model this breaking may be due to the difference of b_r. One way to estimate the value of b_r is to use the empirical effective trajectory as has been done in Ref. 4. The effective trajectories of $K^-p \to \pi^-\Sigma^+$ and $\pi^+p \to K^+\Sigma^+$ given in Ref. 11 are $\alpha_{eff}(t) \approx 0.3 + 0.4t$ and $0.6 + 1.1t$, respectively. Identifying the t value at $\alpha_{eff}(t) = 0$ to the first zero of $J_1(b_r\sqrt{-t})/b_r\sqrt{-t}$, one can obtain $(b_r^{K^-p\to\pi^-\Sigma^+}/b_r^{\pi^+p\to K^+\Sigma^+})^2 \approx 8/11$. According to the geometrical scaling of Eq.(4) this ratio should correspond to that of the slope parameters of these DCSs. Experimentally, the approximate values of the slope parameters[12] are 8 and 9.6 $(\text{GeV/c})^{-2}$ for $K^-p \to \pi^-\Sigma^+$ at $p_{lab} = 16$ GeV/c and $\pi^+p \to K^+\Sigma^+$ at $p_{lab} = 14$ GeV/c, respectively, and thus consistent with the prediction of the geometrical scaling.

(f) Six charge exchange processes

We consider the following three pairs of charge exchange processes.

(A) $\pi^- p \to \pi^0 n$, $\pi^+ p \to \pi^0 \Delta^{++}$ (ρ exchange)

(B) $K^- p \to \bar{K}^0 n$, $K^+ p \to \bar{K}^0 \Delta^{++}$ (ρ-A_2 exchange)

(C) $\pi^- p \to \eta n$, $\pi^+ p \to \eta \Delta^{++}$ (A_2 exchange)

It has been remarked that t-dependence of forward DCS of each couples of reactions (A), (B), and (C) are very similar to each other near $p_{lab} \sim 4$ GeV/c.[13] This fact can be interpreted from the viewpoint of the geometrical scaling that the values of b_r for each couples of reactions are approximately equal to each other. In fact the dips of $\pi^- p \to \pi^0 n$ and $\pi^+ p \to \pi^0 \Delta^{++}$ DCS are found near $t = -0.55$ (GeV/c)^2 and the dips of $\pi^- p \to \eta n$ and $\pi^+ p \to \eta \Delta^{++}$ DCS are observed near $t = -1.5$ GeV/c.[14),*)]

§4. Concluding remarks

We have discussed the phenomenological evidence for the possible existence of the geometrical scaling of the impact parameter amplitudes. In particular, the forward DCSs of $K^{\pm}p$, pp and $\bar{p}p$ elastic processes can be understood systematically by the scaling relation $b_d^{NN}/b_d^{KN} = b_r^{NN}/b_r^{KN} = 1.23$. More precise

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*) In our previous paper,[4] the value of b_r of $\pi^- p \to \eta n$ process was estimated to be 0.7~0.8 fm by examining the effective trajectory. This value is smaller than that of other inelastic processes.[4] Now, the dip of $\pi^- p \to \eta n$ (at $t \simeq -1.5$(GeV/c)^2) can be explained by taking $b_r \sim 1.1$ fm, but the explanation of the effective trajectory remains as a rather complicated open question.

measurements of DCS and polarization as well as quantitative analyses based on the hypothesis of the geometrical scaling will provide an extensive test of the empirical regularity of the geometrical structure of the hadronic processes. For instance, precise measurements of $|D^{\bar{p}p}(s,t)-D^{pp}(s,t)|$, $|D^{K^-p}(s,t)-D^{K^+p}(s,t)|$, and $|D^{\pi^-p}(s,t)-D^{\pi^+p}(s,t)|$ are useful to verify directly the scaling of $R(s,t)$ discussed in (d).

Although the geometrical scaling represented by Eq.(4) refers to the impact parameter amplitudes which are dual to the "same" "Regge" exchange, this kind of scaling may be extended to that between the vector and the tensor Regge exchanges with respect to their imaginary parts for the case where the exchange degeneracy is expected to hold.

The scaling of b_d to b_r ratio may be qualitatively understood from the view-point of the strong absorption models[15] with the additional assumption of the geometrical scaling of the Pomeron exchange amplitude, since in these models the peripherality of Im $g_{d\lambda}(s,t)$ distributed around $b \sim b_r$ is the result of the strong absorption effect due to the Pomeron exchange which has the interaction radius of $b \sim b_d$. Recently it has been pointed out by Zarmi[16] that b_d has different values for πN, $\bar{K}N$ and $\bar{N}N$ scattering while b_r is around 1 fm commonly for the above three processes. This point of view is different from that of the scaling of b_d to b_r ratio and probably that of the strong absorption models. Of course, we believe that the view-point of the geometrical scaling discussed in this paper is more preferable to that of Zarmi, and it is necessary to get more precise experimental data for a decisive

test.

The possible existence of the geometrical scaling pointed out in this paper suggests that the impact parameter plays an important role for a simple systematic description of hadronic processes at high energies.

After this work was completed we received a preprint by S. Otsuki.[17] He pointed out and discussed the scaling of b_d to b_r ratio on the basis of a composite model. Their value of b_d ratio is similar to that obtained in this paper in the case of K^+p and pp elastic scattering.

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References

1) H. Harari, Ann. of Phys. 63 (1971), 432; Phys. Rev. Letters 26 (1971), 1400.

2) M. Imachi, S. Otsuki and F. Toyoda, Prog. Theor. Phys. 43 (1970), 1105.
M. Imachi, T. Oroguchi, S. Otsuki and F. Toyoda, Prog. Theor. Phys. 44 (1970), 1110; 45 (1971), 1849.
K. Ghoroku, M. Imachi, S. Otsuki and F. Toyoda, preprints, KYUSHU-71-HE-9 and -10, December 1971.

3) Y. Kohsaka and F. Takagi, Prog. Theor. Phys., 47 (1972) 925; Tohoku Univ. preprint TU/71/79, August 1971.

4) Y. Kohsaka and F. Takagi, Nuovo Cim. Letters 2 (1971), 965.

5) H. Harari, Phys. Rev. Letters 20 (1968), 1395.
P.G.O. Freund, Phys. Rev. Letters 20 (1968), 235.

6) V. Barger and R.J.N. Phillips, Phys. Letters 26B (1968), 730.
G. Höhler and R. Strauss, Z. Physik, 232 (1970), 205.
F.J. Gilman, J. Pumplin, A. Schwimmer and L. Stodolsky, Phys. Letters 31B (1970), 387.

7) V. Barger and D. Cline, Nucl. Phys. B23 (1970), 227.

8) K.J. Foley, S.L. Lindenbaum, W.A. Love, S. Ozaki, J.J. Russell and L.C.L. Yuan, Phys. Rev. Letters 11 (1963), 425; Phys. Rev. Letters 11 (1963), 503.

9) M. Davier and H. Harari, Phys. Letters 35B (1971), 239.

10) S.P. Denisov, S.V. Donskov, Yu.P. Gorin, A.I. Petrukhin,

Yu.D. Prokoshkin, D.A. Stoyanova, J.V. Allaby and G. Giacomelli, Phys. Letters 36B (1971), 415.

11) A.C. Irving, A.D. Martin and C. Michael, Nucl. Phys. B32 (1971), 1,

12) K.Wu Lai and J. Louie, Nucl. Phys. B19 (1970), 205.

13) R.D. Mathews, Nucl. Phys. B11 (1969), 339.

14) D.F. Grether and G. Gidal, Phys. Rev. Letters 26 (1971), 792.

E.H. Harvey, E. Marquit, E.A. Peterson, T.G. Rhoades, H. Romer and K. Ruddick, Phys. Rev. Letters 27 (1971), 885.

15) I. Kimel and H. Miyazawa, Prog. Theor. Phys. 40 (1968),124.

M. Ross, F. S. Henyey and G.L. Kane, Nucl. Phys. B23 (1970), 269.

16) Y. Zarmi, Phys. Letters 37B (1971), 393.

17) S. Otsuki, preprint, KYUSHU-71-HE-11, December 1971.

18) F. Takagi, Tohoku Univ. preprint TU/72/87, March 1972.

Table caption

Table 1 - Approximate values of the slope parameter a and b_r. For details see (C).

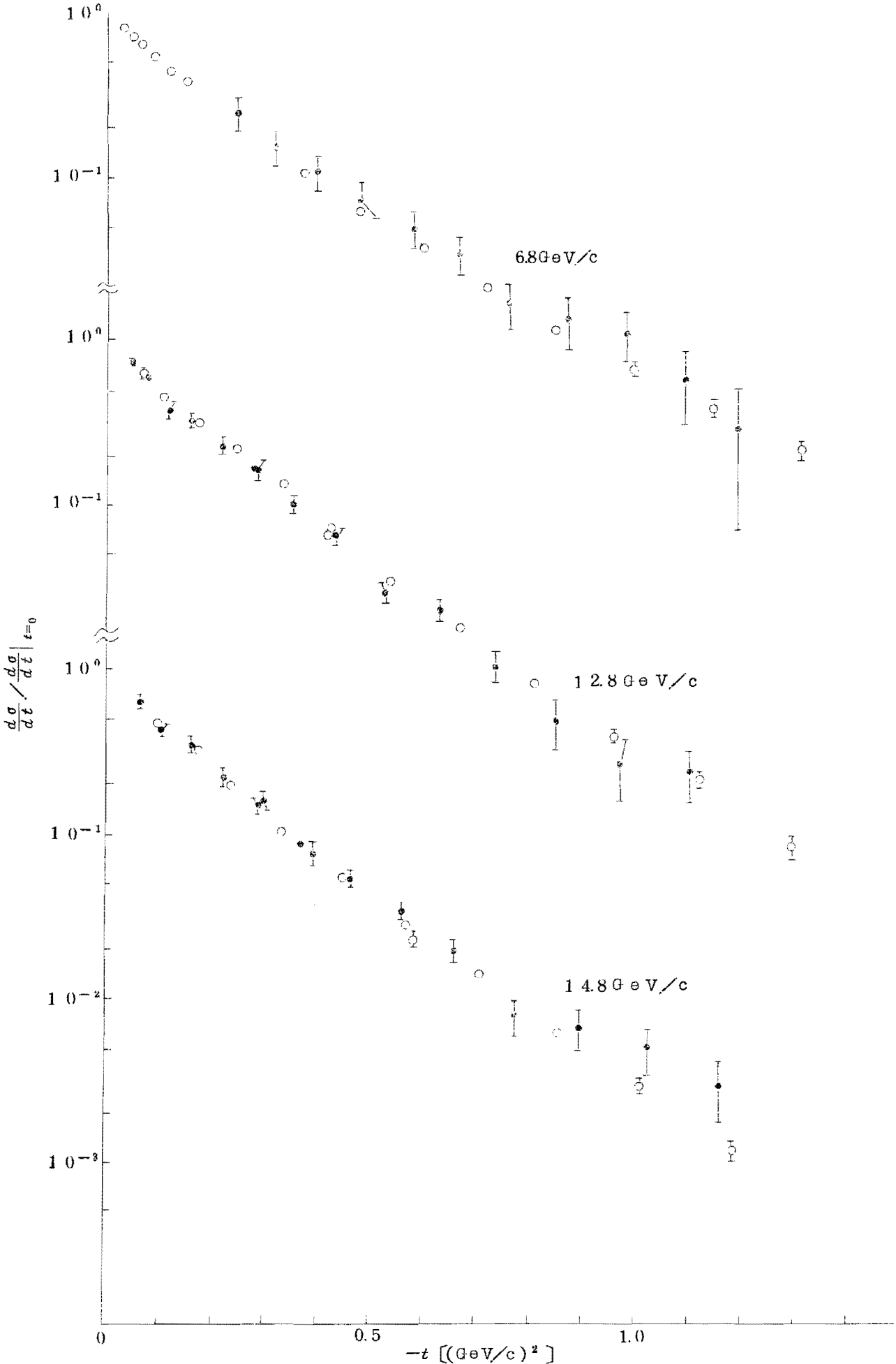
Figure captions

Fig. 1 - The left hand side of Eq.(9) (♦) and the right hand side of Eq.(9) with ($b_d^{NN}/b_d^{KN})^2 = (1.23)^2$ (♀) at p_{lab} = 6.8, 12.8 and 14.8 GeV/c. The experimental data are taken from Ref.8.

Fig. 2 - The consistency of Eq.(16) and that of Eq.(17). The experimental points Φ and Φ are refered to $D^{K^+p}(s, t)$ and $D^{K^-p}(s,t)$, respectively.[8] The solid curves are the right hand sides of Eqs.(16) and (17) calculated using the best fit value for $D^{pp}(s, t)$ and $D^{\bar{p}p}(s, t)$ obtained in Ref.8, $A = 0.224$, $B = 0.166$, and $\sqrt{\xi} = 1.23$.

Table 1

Elastic Process	Slope parameter $a \approx b_d^2$ [(GeV/c)$^{-2}$]	Approximate b_r [fm]
$\pi^- p$	7.8	1
$\pi^+ p$	7.4	
$K^- p$	7.0	1
$K^+ p$	6.0	
pp	11.5	1.3
$\bar{p}p$	9.5	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10^{-1}
10^0
10^{-1}
10^0
10^{-1}
10^{-2}
10^{-3}
6.8GeV/c
12.8GeV/c
14.8GeV/c
$\frac{d\sigma}{dt}\Big|_{t=0} / \frac{d\sigma}{dt}$
0
0.5
1.0
$-t$ [(GeV/c)^2]

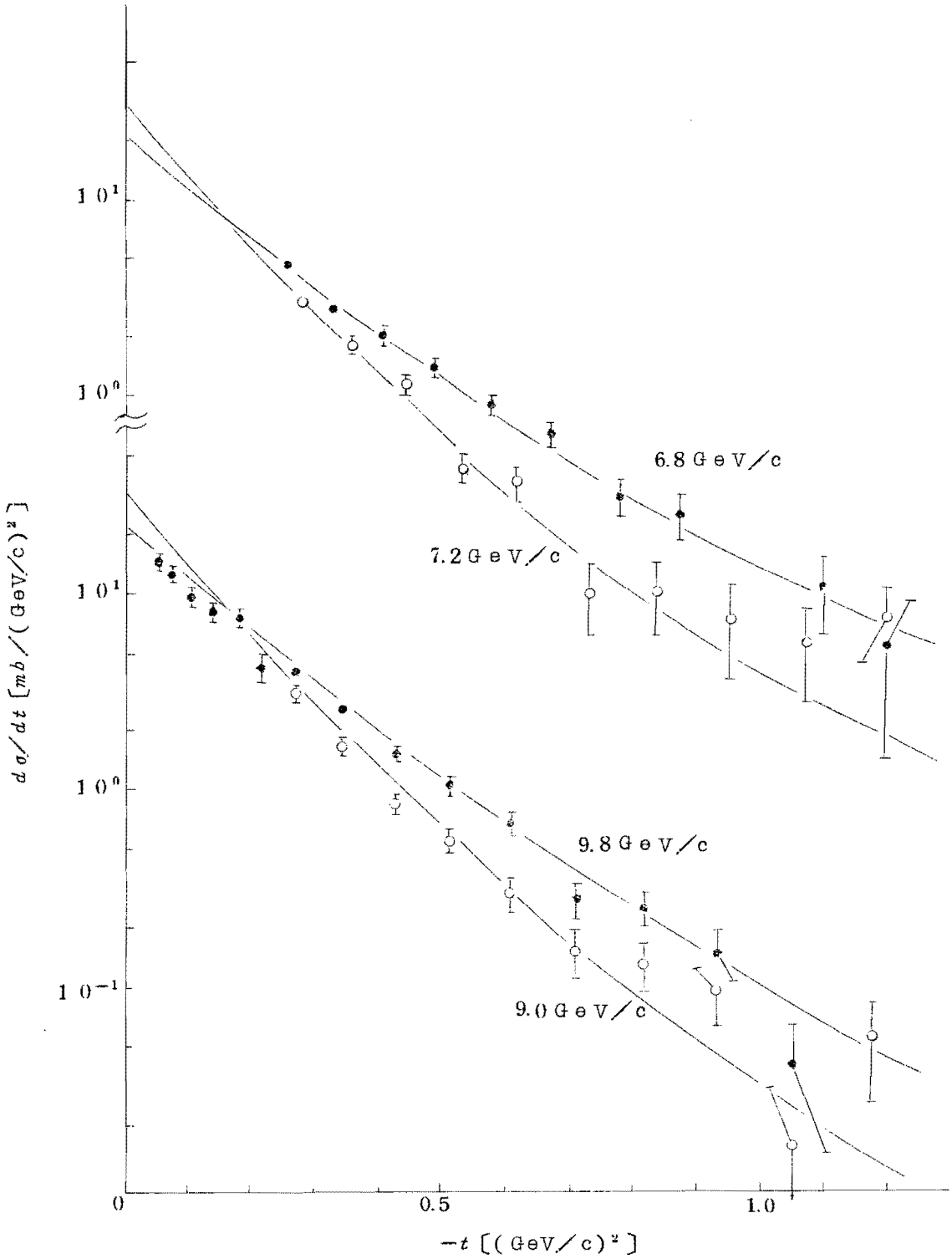
10^1
10^0
10^1
10^0
10^{-1}
6.8 GeV/c
7.2 GeV/c
9.8 GeV/c
9.0 GeV/c
0
0.5
1.0
$-t$ [(GeV/c)^2]
$d\sigma/dt$ [mb/(GeV/c)^2]

Fig. 2