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Licentiate Thesis

Matter and Damping Effects in Neutrino Mixing and Oscillations

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Abstract

This thesis is devoted to the study of neutrino physics in general and the study of neutrino mixing and oscillations in particular. In the standard model of particle physics, neutrinos are massless, and as a result, they do not mix or oscillate. However, many experimental results now seem to give evidence for neutrino oscillations, and thus, the standard model has to be extended in order to incorporate neutrino masses and mixing among different neutrino flavors.

When neutrinos propagate through matter, the neutrino mixing, and thus, also the neutrino oscillations, may be significantly altered. While the matter effects may be easily studied in a framework with only two neutrino flavors and constant matter density, we know that there exists (at least) three neutrino flavors and that the matter density of the Universe is far from constant. This thesis includes studies of three-flavor effects and a solution to the two-flavor neutrino oscillation problem in matter with an arbitrary density profile.

Furthermore, there have historically been attempts to describe the neutrino flavor transitions by other effects than neutrino oscillations. Even if these effects now seem to be disfavored as the leading mechanism, they may still give small corrections to the neutrino oscillation formulas. These effects may lead to erroneous determination of the fundamental neutrino oscillation parameters and are also studied in this thesis in form of damping factors.

Key words: Neutrino oscillations, neutrino mixing, matter effects, damping effects, the day-night effect, solar neutrinos, exact solutions, three-flavor effects, large density, unitarity violation, neutrino decay, neutrino decoherence, neutrino absorption.

Sammanfattning

Denna avhandling behandlar neutrinofysik i allmänhet samt blandningen och oscillationerna mellan olika neutrinosmaker i synnerhet. I partikelfysikens standardmodell kan neutrinerna inte ha någon massa. Ett resultat av detta är att de olika neutrinosmakerna inte blandas med varandra och därför kan inte heller några neutrinooscillationer förekomma. Experimentella resultat verkar dock visa på att neutrinooscillationer faktiskt förekommer och standardmodellen måste därför utökas på något sätt för att även kunna behandla massiva neutriner med blandning mellan olika neutrinosmaker.

När neutriner rör sig genom materia kan blandningen mellan neutrinosmakerna, och därigenom också neutrinooscillationerna, påverkas. Dessa materieeffekter kan lätt studeras i ett scenario med två neutrinosmaker och konstant materiedensitet. Vi vet dock att det existerar (minst) tre neutrinosmaker och att materiedensiteten i universum är långt ifrån konstant. Denna avhandling innehåller studier av både tresmakseffekter och en exakt lösning till neutrinooscillationer mellan två neutrinosmaker i materia med godtycklig densitetsprofil.

Vidare har det historiskt funnits försök att beskriva övergångar mellan olika neutrinosmaker med andra mekanismer än neutrinooscillationer. Även om neutrinooscillationer nu verkar vara en bättre beskrivning av smakövergångarna så kan dessa andra mekanismer fortfarande leda till att neutrinooscillationsformlerna förändras något. Dessa mekanismer kan därför leda till felaktiga bestämningar av de fundamentala neutrinooscillationsparametrarna och studeras i denna avhandling i form av dämpningsfaktorer.

Nyckelord: Neutrinooscillationer, neutrinoblandning, materieeffekter, dämpningseffekter, dag-natt-effekten, solneutriner, exakta lösningar, tresmakseffekter, hög materiedensitet, unitaritetsbrott, neutrinosönderfall, neutrinodekoherens, neutrinoabsorption.

Preface

This thesis is a composition of four scientific papers which are the result of my research at the Department of Physics during the period June 2003 to February 2005. The thesis is divided into two separate Parts. The first Part is an introduction to the subject of neutrino oscillations and mixing, which is intended to put the scientific papers into context. It includes a short review of the history of neutrino physics as well as a review of the standard model of particle physics and how it has to be altered in order to allow for massive neutrinos. The second Part of this thesis consists of the four scientific papers listed below.

List of papers

My research has resulted in the following scientific papers:

1. *Mattias Blennow*, Tommy Ohlsson, and Håkan Snellman
   Day-night effect in solar neutrino oscillations with three flavors
   Phys. Rev. D **69**, 073006 (2004).
   `hep-ph/0311098`

2. *Mattias Blennow* and Tommy Ohlsson
   Exact series solution to the two flavor neutrino oscillation problem in matter
   J. Math. Phys. **45**, 4053 (2004).
   `hep-ph/0405033`

3. *Mattias Blennow* and Tommy Ohlsson
   Effective neutrino mixing and oscillations in dense matter
   Phys. Lett. B **609**, 330 (2005).
   `hep-ph/0409061`

4. *Mattias Blennow*, Tommy Ohlsson, and Walter Winter
   Damping signatures in future reactor and accelerator neutrino oscillation experiments
   JHEP (submitted).
   `hep-ph/0502147`

The thesis author's contribution to the papers

In all papers, I was involved in the scientific work as well as in the actual writing. I am also the corresponding author of all the papers.

1. The paper was based on my M.Sc. thesis with some improvements. I performed the analytic and numeric calculations. I also constructed the figures and did most of the writing.

2. I had the idea of setting up a real non-linear differential equation for the neutrino oscillation probabilities rather than a linear differential equation for the neutrino oscillation amplitudes. I performed the expansion of the solution and implemented the result numerically to test the convergence of the solution. I also did most of the writing and constructed the figures.

3. The idea to use degenerate perturbation theory for large matter potentials was mine. I performed the analytic and numeric calculations and constructed the figures. I did most of the work on the discussion of effective two-flavor cases and I also did most of the writing.

4. The work was divided equally among the authors. In Sec. 3, I did all the calculations and wrote most of the discussion. I also did much of the writing in Sec. 2 and constructed Fig. 1.

Acknowledgements

First and foremost, I would like to thank my supervisor Tommy Ohlsson who has been involved in the scientific work of all the Papers included in this thesis. In addition, I would like to thank him for the interesting discussions we have had on physics in general, teaching, and subjects totally unrelated to physics whatsoever.

Very special thanks are also due to Håkan Snellman who was the supervisor of my M.Sc. thesis and introduced me to the field of neutrino physics. His wisdom and enthusiasm has been a great source of inspiration ever since I took the relativity course during my third year as an undergraduate student.

I would also like to thank Walter Winter from the Institute for Advanced Study, Princeton, USA, for our collaboration in the research leading to Paper 4 of this thesis.

Furthermore, I am very grateful to the Royal Institute of Technology (KTH) itself for providing the economical means necessary for my studies, and thus, giving me the opportunity to be involved in the research of physics.

All the lecturers with whom I have worked as a teaching assistant in the courses of Relativity theory (Håkan Snellman and Jouko Mickelsson) and Mathematical methods of physics (Edwin Langmann) should also receive special thanks. Without them, there would have been no course to teach and I have enjoyed every minute of standing by the black board trying to explain physics to my students. My students should also receive thanks for coming to my problem sessions wanting to learn the subjects that I teach (and, in some cases, succeeding).

In addition, I would like to thank my fellow Ph.D. students Martin Hallnäs and Tomas Hällgren for their great company both on and off working hours. Such thanks are also due to Martin Blom (mostly off working hours) who finally did finish his M.Sc. thesis.

Finally, I want to thank all the members of my family for their great support. My parents Mats and Elisabeth for raising me and making me become who I am. My sister Malin for always being my friend throughout our childhood and beyond. My brother Victor, whom I can no longer defeat in a friendly wrestling game since he started doing karate. My grandparents Bertil and Gunvor Syk for always taking good care of all their grandchildren. Last, but certainly not least, I want to thank my grandmother Märta for making me feel as liked and beloved as one can ever become.[1]

Mattias Blennow, March 25, 2005

[1] In Swedish: Sist, men absolut inte minst, vill jag tacka min farmor Märta för att hon får mig att känna mig så omtyckt och älskad som någon kan vara.

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To My Grandparents
Till mina mor- och farföräldrar

Part I

Introduction and background material

Chapter 1

Introduction

Ever since the beginning of human civilization, man has tried to describe and understand Nature. In very ancient times, this was mainly done by referring to the will of one or more gods. With Sir Isaac Newton [1], science, as we know it today, was born.

Physics is a branch of science in which the fundamental behavior of Nature is studied. In particular, the study of elementary particle physics is concerned with exploring the very building blocks of which nuclei, atoms, molecules and - ultimately - the Universe, consist. It is important to note that the intent of physics is to describe how Nature behaves rather than to explain it in some deeper meaning. The concepts introduced, such as particles and fields, are in no way assumed to be real entities, but rather intended to describe what we can observe as accurately as possible.

A very successful theory in elementary particle physics is the so-called standard model. This model has made extremely good predictions, which is what signifies a good theory, and contains all the particles that we know of today (and some others which have not yet been seen). However, as will be discussed, there are indications that the standard model may not tell the whole story.

This thesis deals with one very interesting type of particle, the neutrino. At present time, we know of three different types of neutrinos, the electron neutrino ν_e, the muon neutrino ν_μ and the tau neutrino ν_τ, each associated with the corresponding charged lepton (e, μ and τ, respectively). For a very long time, the neutrinos were believed to be massless. However, in recent times, there have been experimental progress which indicates that the neutrinos actually have non-zero, albeit small, masses. The fact that the neutrino does have a mass is in direct contradiction to the standard model, and thus, the standard model needs to be revised or extended to incorporate neutrino masses and neutrino physics seem to be a probe of the physics beyond the standard model.

1.1 Outline of the thesis

The outline of this thesis is as follows. In Chapter 2, the history of neutrino physics is briefly reviewed. This is followed, in Chapter 3, by an introduction to the standard model and its extensions, as well as the incorporation of neutrino masses and mixing into the theory. Chapter 4 is addressing the subject of neutrino oscillations, with which all the Papers included at the end of this thesis are concerned. The experimental evidence for neutrino oscillations are discussed in Chapter 5. Finally, in Chapter 6, the introductory part of the thesis is summarized and the most important conclusions of the Papers of the second part are given.

Chapter 2

History of neutrinos

The history of neutrino physics is very interesting and many brilliant physicists, including many Nobel prize laureates, are involved in the story.

The early 20th century was an era of great achievements in physics. It witnessed the birth of the theories of relativity and quantum mechanics (in fact, it is now 100 years since Albert Einstein[1] published seminal papers [2, 3] for both of these theories) but still, a lot of progress has been made since that time. For instance, our knowledge of elementary particles has changed drastically. In 1930, the neutron had not yet been observed. It was this year that the German physicist Wolfgang Pauli[2] postulated the existence of the neutrino to describe some obvious discrepancies in β-decays. Before the neutrino was introduced, β-decay was thought to be a two-body decay, a nucleus decayed into another nucleus and an electron. However, experiments indicated that the energy spectrum of the beta radiation electrons was continuous rather than the peaked, single energy spectrum expected in a two-body decay. In addition, there seemed to be a violation of the conservation of angular momentum which could not be described by physics at that time. It had even been suggested that the conservation laws only should hold statistically.

Pauli first introduced the neutrino in a letter to a gathering of physicists in Tübingen, since he did not attend the meeting in person (he was going to a ball). The particle Pauli postulated was a particle with a very small mass, spin 1/2, and no electric charge, called a "neutron". The name might have stuck if it were not for the fact that two years later, in 1932, James Chadwick[3] experimentally discovered the nucleon that we today know as the neutron [4]. Instead, the Italian physicist

[1] Nobel prize laureate in 1921 "for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect".

[2] Nobel prize laureate in 1945 "for the discovery of the Exclusion Principle, also called the Pauli Principle".

[3] Nobel prize laureate in 1935 "for the discovery of the neutron".

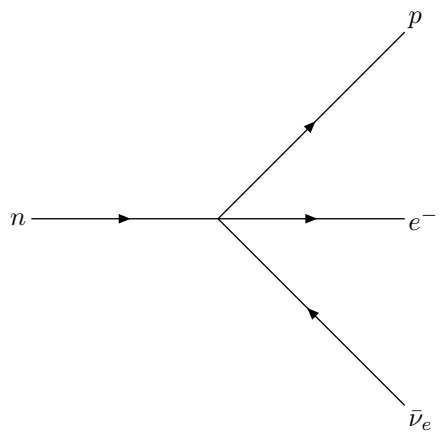

Figure 2.1. Fermi's theory of β-decay as it would be described in terms of a Feynman diagram.

Enrico Fermi[4] gave the name "neutrino" to Pauli's postulated particle in 1933, the suffix "-ino" meaning "small" in Italian. The following year, Fermi presented his theory of β-decay [5, 6], in which the neutrino had a fundamental role.

Fermi's theory of β-decay was the foundation for the Glashow–Weinberg–Salam[5] electroweak model [7–9], which unifies the electromagnetic and weak interactions in one common framework. The theory of Fermi was a low-energy effective theory with an interaction term which in today's language of Feynman[6] diagrams would describe a vertex with one incoming neutron, one outgoing proton, one outgoing electron, and one outgoing anti-neutrino, see Fig. 2.1. In 1934, Hans Albrecht Bethe[7] and Rudolf Peierls used Fermi's theory to make the first predictions for the neutrino interaction cross-sections [10]. The predicted cross-sections were very small, which made many physicists doubt that neutrinos would ever be observed by experiments. However, in 1946, Bruno Pontecorvo suggested that anti-neutrinos may be detected through the inverse β-decay of Chlorine [11]:

$$\bar{\nu}_e + {}^{37}\mathrm{Cl} \longrightarrow {}^{37}\mathrm{Ar} + e^+.$$

[4]Nobel prize laureate in 1938 "for his demonstrations of the existence of new radioactive elements produced by neutron irradiation, and for his related discovery of nuclear reactions brought about by slow neutrons".

[5]The model was named after physicists Sheldon Lee Glashow, Steven Weinberg and Abdus Salam - all Nobel prize laureates in 1979 "for their contributions to the theory of the unified weak and electromagnetic interaction between elementary particles, including, inter alia, the prediction of the weak neutral current".

[6]Nobel prize laureate in 1965 together with Sin-Itiro Tomonaga and Julian Schwinger "for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles".

[7]Nobel prize laureate in 1967 "for his contributions to the theory of nuclear reactions, especially his discoveries concerning the energy production in stars".

A decade later, Clyde Cowan Jr. and Frederick Reines[8] used this inverse β-decay to detect the anti-neutrinos coming from the β-decays in a nuclear reactor [12, 13].

The existence of a second type of neutrino, the muon neutrino ν_μ, was confirmed by the Brookhaven National Laboratory in 1962 [14]. The third type of neutrino, the tau neutrino ν_τ, was not detected until the beginning of 2001 when its detection was announced by the DONUT collaboration [15]. However, its charged lepton partner, the tau, was observed in 1975 by a collaboration led by Martin L. Perl[9] [16], and therefore, the experimental detection of the tau neutrino had been anticipated for quite some time.

2.1 The history of neutrino oscillations

The idea of oscillating neutrinos was first discussed by Pontecorvo in 1957 [17, 18]. However, since only one neutrino flavor was known at that time (the ν_e), Pontecorvo discussed the oscillation between a neutrino and an anti-neutrino. The motivation for this was an assumed analogy between lepton number and strangeness, the oscillations between neutrinos and anti-neutrinos would then correspond to the oscillation between K^0's and $\bar{K}^0$'s, which had been observed.

When the muon neutrino had been discovered, the mixing of two massive neutrinos was discussed in work done by Maki, Nakagawa, and Sakata [19] as well as Nakagawa *et al.* [20]. The idea of neutrino oscillations between two different neutrino flavors was first discussed by Pontecorvo in 1967. Two years later, Pontecorvo and Vladimir Gribov published a phenomenological theory for the oscillations between ν_e and ν_μ [21]. However, the oscillation length presented by Pontecorvo and Gribov differs with a factor of two as compared to the correct oscillation length. The correct oscillation length was presented by Harald Fritzsch and Peter Minkowski in 1976 [22]. The oscillations among three different neutrino flavors was first studied in detail by Samoil Bilenky in 1987 [23].

A most interesting part of the theory of neutrino oscillations is the matter effect which appears when neutrinos travel through matter and is caused by coherent forward scattering of the neutrinos. This was first discussed by Lincoln Wolfenstein in 1978 [24] and elaborated on by Stanislav Mikheyev and Alexei Smirnov in the 1980's [25, 26].

The first evidence for neutrino oscillations was published in 1998 by the Super-Kamiokande collaboration [27]. The Super-Kamiokande experiment (a larger version of the Kamiokande experiment, which was originally built to measure proton decay), detected a suppression in the flux of muon neutrinos produced by cosmic rays hitting the atmosphere. Since then, there have been many experiments which seem to give evidence for oscillations of neutrinos produced by accelerators [28, 29],

[8] Nobel prize laureate in 1995 "for the detection of the neutrino" (Cowan died in 1974).

[9] Nobel prize laureate in 1995 "for the discovery of the tau lepton" (Perl and Reines were awarded half the prize each).

neutrinos coming from the Sun [30–37], and neutrinos produced by β-decays in nuclear reactors [38, 39].

Chapter 3

The standard model and beyond

3.1 Unification of physics

In the spirit of James Clerk Maxwell's description of the electricity and magnetism in a unified theory [40], it has been a dream of many physicists to describe all of Nature's interactions in one unified theory. The next large step towards the unification of physics was the unification of the electromagnetic and weak interactions, this was done by the Glashow–Weinberg–Salam (GWS) electroweak model [7–9], which includes a lot of concepts that have become important in modern physics such as non-Abelian gauge theories and spontaneous symmetry breaking. Today, the standard model (SM) of particle physics describes also the strong interaction and has a particle content which includes all observed (and some unobserved) particles. In this section, the GWS electroweak model and the SM will be briefly introduced and the shortcomings of the SM will be discussed.

3.1.1 The GWS electroweak model

The GWS electroweak model is a non-Abelian gauge theory [41] based on the gauge group $SU(2)\otimes U(1)$. The gauge symmetry is spontaneously broken by the introduction of a complex Higgs doublet with a non-zero vacuum expectation value (vev) and the symmetry that remains after this is the $U(1)$ symmetry of the electromagnetic interactions. Let us start by introducing the field content of the model. Except for the gauge fields $W^{i,\mu}$ ($i = 1, 2, 3$, or $\boldsymbol{W}^\mu$ in vector notation) and B^μ,

corresponding to the $SU(2)$ and $U(1)$ symmetries, respectively, we introduce the left- and right-handed lepton fields

$$L_\ell \equiv \begin{pmatrix} \nu_{\ell L} \\ \ell_L \end{pmatrix} \quad \text{and} \quad R_\ell \equiv \ell_R.$$

The left-handed fields L_ℓ form a $SU(2)$ doublet, while the right-handed field R_ℓ is a $SU(2)$ singlet. In general, we may introduce many different generations of leptons similar to the one described above, this generalization is straightforward, we simply denote each generation by a generation index (*i.e.*, e, μ, τ).

We can now write down the Lagrangian density $\mathcal{L}_0$ for the GWS electroweak model before the spontaneous symmetry breaking, it is given by

$$\mathcal{L}_0 \equiv -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}\boldsymbol{G}^{\mu\nu}\cdot\boldsymbol{G}_{\mu\nu} + \mathrm{i}\bar{L}_\ell\gamma^\mu\mathcal{D}_\mu L_\ell + \mathrm{i}\bar{R}_\ell\gamma^\mu\mathcal{D}_\mu R_\ell, \tag{3.1}$$

where $F^{\mu\nu} \equiv \partial^\mu B^\nu - \partial^\nu B^\mu$ is the field strength tensor of the $U(1)$ gauge field, $\boldsymbol{G}^{\mu\nu} \equiv \partial^\mu \boldsymbol{W}^\nu - \partial^\nu \boldsymbol{W}^\mu + g\boldsymbol{W}^\mu \times \boldsymbol{W}^\nu$ is the field strength tensor of the $SU(2)$ gauge fields, $\mathcal{D}_\mu \equiv \partial_\mu - \mathrm{i}g'(Y/2)B_\mu - \mathrm{i}g\boldsymbol{\tau}\cdot\boldsymbol{W}_\mu$ is the covariant derivative, g and g' are the coupling constants for the two symmetry groups, respectively, Y is the weak hypercharge operator, and $\boldsymbol{\tau}$ is a vector containing the $SU(2)$ generators.

If we introduce the currents

$$j'^\mu \equiv \bar{L}_\ell\gamma^\mu\frac{Y}{2}L_\ell + \bar{R}_\ell\gamma^\mu\frac{Y}{2}R_\ell \quad \text{and} \quad j^{i\mu} \equiv \bar{L}_\ell\gamma^\mu\tau^i L_\ell,$$

then the interaction part of the Lagrangian density is given by

$$\mathcal{L}_{0,\mathrm{int}} = g\boldsymbol{j}^\mu\cdot\boldsymbol{W}_\mu + g'j'^\mu B_\mu.$$

When we break the gauge symmetry, we want to break it in such a way that the remaining symmetry is the $U(1)$ symmetry of the electromagnetic interaction with the charge operator Q given by the weak analogue of the Gell-Mann–Nishijima relation [42, 43]

$$Q = I_3 + \frac{Y}{2}, \tag{3.2}$$

where I_3 is the third component of the isospin operator, holds. This is done by introducing the linear combinations

$$W_\mu \equiv \frac{W^1_\mu - \mathrm{i}W^2_\mu}{\sqrt{2}}, \quad W^\dagger_\mu \equiv \frac{W^1_\mu + \mathrm{i}W^2_\mu}{\sqrt{2}},$$

$$Z_\mu \equiv \cos(\theta_W)W^3_\mu - \sin(\theta_W)B_\mu, \quad \text{and} \quad A_\mu \equiv \sin(\theta_W)W^3_\mu + \cos(\theta_W)B_\mu,$$

where θ_W is the so-called Weinberg angle. We require that when the gauge symmetry is broken, A_μ is the gauge field of the remaining $U(1)$ symmetry. The current associated with the gauge field A_μ is then given by

$$ej^\mu_{\mathrm{EM}} = g\sin(\theta_W)j^{3\mu} + g'\cos(\theta_W)j'^\mu,$$

where e is the electromagnetic coupling constant, and should be identified with the electromagnetic current. However, for Eq. (3.2) to hold, we must have the relation

$$j^{\mu}_{\mathrm{EM}} = j^{3\mu} + j'^{\mu} = \sum_{\ell} \bar{\ell}\gamma^{\mu}\ell.$$

This relation results in $e = g\sin(\theta_W) = g'\cos(\theta_W)$ and the current associated with Z_μ becomes

$$j^{0\mu} = \sum_{\ell}[\bar{\nu}_{\ell L}\gamma^{\mu}\nu_{\ell L} - \bar{\ell}_L\gamma^{\mu}\ell_L + 2\sin^2(\theta_W)\bar{\ell}\gamma^{\mu}\ell],$$

while the currents associated to the gauge fields W and $W^\dagger$ are

$$j^{+\mu} = \sum_{\ell}\bar{\nu}_{\ell L}\gamma^{\mu}\ell_L \quad \text{and} \quad j^{-\mu} = \sum_{\ell}\bar{\ell}_L\gamma^{\mu}\nu_{\ell L},$$

respectively. The Lagrangian density of the electromagnetic interaction is then written on the well-known form

$$\mathcal{L}_{\mathrm{EM}} = e j^{\mu}_{\mathrm{EM}} A_\mu,$$

while the Lagrangian density of the remaining weak interaction is

$$\mathcal{L}_{\mathrm{weak}} = \frac{g}{\sqrt{2}}\left(j^{+\mu}W_\mu + j^{-\mu}W_\mu\right) + \frac{g}{2\cos(\theta_W)} j^{0\mu} Z_\mu.$$

Spontaneous symmetry breaking and the Higgs mechanism

So far, we have only considered the Lagrangian density of Eq. (3.1). What is apparently lacking in this Lagrangian density is the appearance of mass terms, *i.e.*, all of the fields in our theory are so far massless. In the GWS electroweak model, masses for the weak gauge bosons are provided by the Higgs mechanism [44–47] and masses for the charged leptons by Yukawa couplings to the Higgs field.

The Higgs mechanism introduces a new complex doublet field $\Psi \equiv (\psi^+ \ \psi^0)^T$ with hypercharge $Y = 1$ along with an addition

$$\mathcal{L}_{\mathrm{Higgs}} \equiv (\mathcal{D}_\mu\Psi)^\dagger(\mathcal{D}^\mu\Psi) + \mu^2\Psi^\dagger\Psi - \lambda(\Psi^\dagger\Psi)^2$$

to the Lagrangian density. The potential part of this Lagrangian density has the shape shown in Fig. 3.1 and the ground (vacuum) state obviously corresponds to a non-zero value of the Higgs field. The minimum of the Higgs potential is $-\mu^4/(4\lambda^2)$

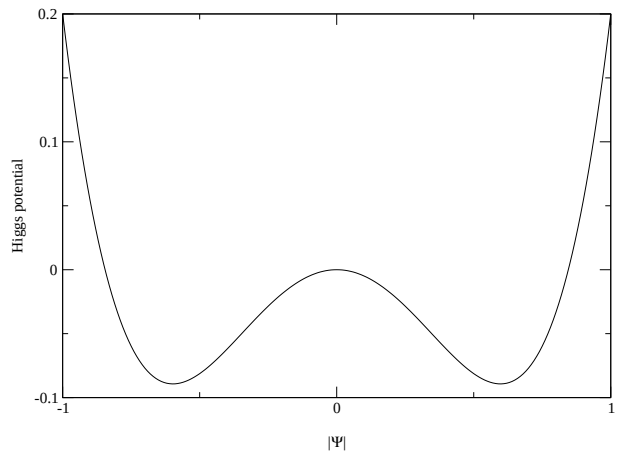

Figure 3.1. The shape of the Higgs potential as a function of $|\Psi|$ for $\mu^2 = 1/2$ and $\lambda = 0.7$.

and is obtained for $|\Psi| = \sqrt{\mu^2/2\lambda}$. Clearly, there is some freedom in choosing the vacuum state (*i.e.*, the vacuum is degenerate), we define the vacuum to be

$$\Psi_0 \equiv \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix},$$

where $v \equiv \sqrt{\mu^2/\lambda}$. With the total GWS Lagrangian density $\mathcal{L}_{\text{GWS}} = \mathcal{L}_0 + \mathcal{L}_{\text{Higgs}}$ and this non-zero vev of the Higgs fields, the Lagrangian density will include mass terms for the weak gauge bosons, resulting in the masses

$$m_W = \frac{gv}{2}, \quad m_Z = \frac{gv}{2\cos(\theta_W)}, \quad \text{and} \quad m_A = m_\gamma = 0.$$

The charged lepton masses

The introduction of the Higgs field and the Higgs Lagrangian density solved the problem of the masses of the weak gauge bosons. However, we still have to incorporate the masses of the charged leptons into our theory. This is done by introducing Yukawa couplings between the fermion fields and the Higgs field of the form

$$\mathcal{L}_{\text{Yuk},\ell} \equiv -G_\ell(\bar{L}_\ell \Psi R_\ell + \bar{R}_\ell \Psi^\dagger L_\ell),$$

where G_ℓ is a dimensionless coupling constant (known as the Yukawa coupling, the constant is different for different lepton generations ℓ), into the Lagrangian density. Due to the non-zero vev of the Higgs field, this will introduce the terms

$$-G_\ell \frac{v}{\sqrt{2}}(\bar{\ell}_L \ell_R + \bar{\ell}_R \ell_L)$$

Fermions		
Name	*SU(2) doublets*	*SU(2) singlets*
Leptons	$(\nu_{eL}\ e_L)^T$, $(\nu_{\mu L}\ \mu_L)^T$, $(\nu_{\tau L}\ \tau_L)^T$	e_R, μ_R, τ_R
Quarks	$(u_L\ d'_L)^T$, $(c_L\ s'_L)^T$, $(t_L\ b'_L)^T$	u_R, d_R, c_R, s_R, t_R, b_R
Bosons		
Name	*SU(3) ⊗ SU(2) representation*	*Fields*
$U(1)$ boson	$\mathbf{1} \otimes \mathbf{1}$	B^μ
$SU(2)$ bosons	$\mathbf{1} \otimes \mathbf{3}$	$W^{i\mu}$, $(i = 1, 2, 3)$
Gluons	$\mathbf{8} \otimes \mathbf{1}$	$G^{i\mu}$, $(i = 1, 2, \ldots, 8)$
Higgs	$\mathbf{1} \otimes \mathbf{2}$	$(\psi^+\ \psi^0)^T$

Table 3.1. The field content of the SM. The left-handed quark fields transform as the **3** representation of the $SU(3)$ gauge group, while the right-handed quark fields transform as the $\mathbf{3}^*$ representation. The primed left-handed quark fields are linear combinations of the left-handed quark fields that diagonalize the quark mass terms, they are related by the CKM matrix [51,52]. The $U(1)$ and $SU(2)$ bosons are related to the photon and the weak gauge bosons as described in Sec. 3.1.1.

into the Lagrangian density of the theory. This term is equivalent to a mass term for the charged lepton ℓ with the mass given by

$$m_\ell = \frac{G_\ell v}{\sqrt{2}}.$$

3.1.2 The standard model

The SM of particle physics is builds upon the GWS electroweak model, but also includes the strong interaction [48–50]. It has been a very successful theory and even if it seems that the SM may not tell the whole story, it is still a very useful approximation in many cases. The gauge theory which is the foundation of the SM is built upon the gauge group $SU(3) \otimes SU(2) \otimes U(1)$, where the group $SU(2) \otimes U(1)$ is the gauge group from the GWS electroweak model and the $SU(3)$ group is the gauge group describing the strong interaction among quarks. The $SU(2) \otimes U(1)$ symmetry is broken in the same way as in the GWS electroweak model, and so, there is a Higgs doublet also in the SM. The SM includes three generations of fermions, where each generation consists of two quarks (one with electric charge $+2/3$ and one with electric charge $-1/3$) and two leptons (a charged lepton and a neutrino). The total field content of the SM is presented in Tab. 3.1.

One important thing to note about the SM is that it does not include any right-handed neutrino fields, and thus, cannot incorporate neutrino masses in the same manner as it incorporates masses for the charged leptons. In a naive approach, one might think that neutrino masses could arise from higher order loop corrections in the quantized theory. However, this would break the accidental $B - L$ symmetry of the SM, and thus, cannot happen. There are many suggestions for extensions of

the SM where the neutrinos do have mass, all of those models include the addition of a right-handed neutrino field in one way or another.

3.2 Massive neutrinos and neutrino mixing

3.2.1 Dirac masses

There are a number of ways of extending the SM in such a way that the neutrinos become massive. The most obvious of these ways is the simple introduction of a right-handed neutrino field $\nu_{\ell R}$ which transforms as a singlet under the $SU(2)$ gauge group. Neutrino masses can then be introduced into the theory in a manner similar to charged lepton masses by the introduction of the Yukawa coupling

$$\mathcal{L}_{\mathrm{Yuk},\nu_\ell} \equiv -G_{\nu_\ell}[\overline{\nu^c_{\ell R}}(\Psi^c)^\dagger L_\ell + \bar{L}_\ell \Psi^c \nu^c_{\ell R}],$$

where c denotes the charge conjugate and G_{ν_ℓ} is the Yukawa coupling constant, into the Lagrangian density. With this Yukawa coupling, the mass of the neutrino becomes

$$m_{\nu_\ell} = \frac{G_{\nu_\ell} v}{\sqrt{2}}.$$

The drawback of this approach is that, since the charged leptons are far heavier than the neutrinos, $G_{\nu_\ell} \ll G_\ell$, *i.e.*, for this approach to work, the Yukawa couplings of the neutrinos must be much weaker than the corresponding couplings for the charged leptons. This difference in the Yukawa coupling constants is quite unaesthetic. For instance, if our theory was the effective theory of some more general theory, then we would expect the Yukawa couplings to be of the same order of magnitude unless there is some fine-tuning involved.

3.2.2 Majorana masses

Since neutrinos are neutral in terms of all quantum numbers that change under charge conjugation, the neutrinos may be Majorana particles [53], *i.e.*, the charge conjugate particle is equal to the anti-particle up to a phase factor. This allows for the introduction of a so-called Majorana mass term into the Lagrangian density, this term is of the form

$$\mathcal{L}_{\mathrm{M},\nu_{\ell L}} \equiv -\frac{1}{2} m_{\nu_\ell}(\overline{\nu^c_{\ell L}}\nu_{\ell L} + \bar{\nu}_{\ell L}\nu^c_{\ell L}),$$

where m_{ν_ℓ} is the Majorana mass of the neutrino ν_ℓ. The introduction of a Majorana mass seems to be a nice way of evading the introduction of an extra right-handed neutrino field, the only fields needed are the ones corresponding to the neutrinos and anti-neutrinos.

Unfortunately, a Majorana mass term of this type cannot be invariant under the $SU(2)$ gauge group of the electroweak interaction. This clearly causes us some trouble.

3.2.3 Dirac–Majorana mass and the see-saw mechanism

There is a third way of introducing neutrino masses. This involves using both Dirac and Majorana mass terms and gives rise to what is known as the see-saw mechanism [22,54–58], which could account for the smallness of the neutrino masses.

Instead of introducing the Majorana mass term for the left-handed neutrinos, we introduce right-handed neutrinos (as in the case of a Dirac mass term) and add the Majorana mass term[1]

$$\mathcal{L}_{\mathrm{M},\nu_R} \equiv -\frac{1}{2}M(\bar{\nu}_R\nu_R^c + \overline{\nu_R^c}\nu_R),$$

where M is the Majorana mass of the right-handed neutrino, to the Lagrangian density. It is important to note that while the Majorana mass term for the left-handed neutrino is not invariant under the $SU(2)$ gauge group, the above term is. This is due to the fact that the right-handed neutrinos are $SU(2)$ singlets. Thus, the introduction of this term is not as controversial as the introduction of a Majorana mass term for the left-handed neutrino. The introduction of the Majorana mass term for the right-handed neutrino does not prevent us from also introducing Dirac mass terms. Thus, masses of this type are known as Dirac–Majorana masses.

Introducing the Majorana mass term along with a Dirac mass term as described above, we will have a total mass term of the type

$$\mathcal{L}_{\mathrm{DM}} = -\frac{1}{2}\begin{pmatrix} \overline{\nu_L^c} & \bar{\nu}_R \end{pmatrix} \underbrace{\begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix}}_{\mathcal{M}} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + \mathrm{h.c.},$$

where m_D is the Dirac mass. If $M \gg m_D$, then the Majorana fields ν_L and ν_R are also the approximate mass eigenfields of this mass term. The masses of the two fields will be given by the roots to the eigenvalue equation of the matrix $\mathcal{M}$, *i.e.*,

$$m_i'(m_i' - M) - m_D^2 = 0.$$

The solutions to this equation are

$$m_i' = \frac{M}{2} \pm \sqrt{\frac{M^2}{4} + m_D^2} = \frac{M}{2}\left(1 \pm \sqrt{1 + \frac{4m_D^2}{M^2}}\right) \simeq \frac{M}{2}\left[1 \pm \left(1 + \frac{2m_D^2}{M^2}\right)\right],$$

which gives

$$m_R' \simeq M \quad \text{and} \quad m_L' \simeq -\frac{m_D^2}{M}.$$

Since physical masses are always positive, we take $m_i = \eta_i m_i'$, where $\eta_i = \pm 1$, for the physical mass,[2] and thus, we obtain the masses

$$m_R \simeq M \quad \text{and} \quad m_L \simeq \frac{m_D^2}{M}.$$

[1] For simplicity, we will only consider the Dirac–Majorana mass in the case of one lepton generation.

[2] The factor of ± 1 will place conditions on the CP phases of the Majorana fields.

Assuming that m_D is of the same order of magnitude as the charged lepton masses (*i.e.*, requiring the Yukawa coupling constant of the neutrinos to be of the same order as the Yukawa coupling constant of the charged leptons) and that the Majorana mass M is of a magnitude corresponding to some higher physical scale (*e.g.*, the GUT or Planck scales), then the left-handed neutrino masses will be naturally suppressed from the mass scale of the charged leptons by a factor of m_D/M. However, as the matrix $\mathcal{M}$ is not exactly diagonalized by the fields ν_L and ν_R^c, this will imply a small mixing between the left- and right-handed fields analogous to the mixing of neutrino flavor eigenstates (see below). Since the right-handed neutrinos do not participate in the weak interaction, this would then correspond to mixing with a sterile neutrino. It should also be noted that the see-saw mechanism is not only utilized to account for the small neutrino masses, it may also be the mechanism behind the observed asymmetry between matter and anti-matter in the Universe through leptogenesis requiring the existence of the heavy right-handed neutrinos [59].

3.2.4 Neutrino mixing

In general, with more than one neutrino flavor, there is essentially no reason why the neutrino flavor eigenfields (the fields participating in the weak interaction along with the corresponding charged lepton) should also be the fields which diagonalize the mass terms. In general, a Dirac mass term including n neutrino flavors can be written as

$$\mathcal{L}_D = -\bar{\nu}_R \mathcal{M}_D \nu_L + \text{h.c.},$$

where $\nu_A = (\nu_{eA}\ \nu_{\mu A}\ \ldots)^T$ $(A = L, R)$ and $\mathcal{M}_D$ is a complex $n \times n$ matrix. As all complex matrices, the matrix $\mathcal{M}_D$ can be diagonalized by a bi-unitary transformation, *i.e.*,

$$M_D = V^\dagger \mathcal{M}_D U$$

is a diagonal matrix for some unitary $n \times n$ matrices V and U. It follows that

$$\mathcal{M}_D = V M_D U^\dagger$$

and that the Dirac mass term can be written as

$$\mathcal{L}_D = -\bar{\nu}_{mR} M_D \nu_{mL} + \text{h.c.},$$

where we have introduced the mass eigenfields

$$\nu_{mL} \equiv U^\dagger \nu_L \quad \text{and} \quad \nu_{mR} \equiv V^\dagger \nu_R.$$

Since the matrix M_D is diagonal, the fields in ν_{mA} $(A = L, R)$ correspond to neutrinos with definite mass. The matrix U, relating the left-handed mass and flavor eigenfields is known as the leptonic mixing matrix[3].

[3] The leptonic mixing matrix is also known as the Maki–Nakagawa–Sakata (MNS) matrix [19]. It is the leptonic equivalent of the Cabibbo–Kobayashi–Maskawa (CKM) matrix [51, 52] in the quark sector.

In the case of a Majorana mass term for n neutrino flavors, the addition to the Lagrangian density is

$$\mathcal{L}_M = -\frac{1}{2}\overline{\nu_L^c}\mathcal{M}_M\nu_L + \text{h.c.},$$

where $\nu_L = (\nu_{eL}\ \nu_{\mu L}\ \ldots)^T$ and $\mathcal{M}_M$ is the Majorana mass matrix. In this case, the Majorana properties of the neutrino fields can be used to show that (see, *e.g.*, Ref. [60])

$$\overline{\nu_L^c}\mathcal{M}_M\nu_L = \overline{\nu_L^c}\mathcal{M}_M^T\nu_L,$$

i.e., the Majorana mass matrix is symmetric. It follows that the Majorana mass matrix can be diagonalized as

$$M_M = U^T\mathcal{M}_M U,$$

where U is some unitary $n\times n$ matrix. The Majorana mass term can now be written on the form

$$\mathcal{L}_M = -\frac{1}{2}\overline{\nu_{mL}^c}M_M\nu_{mL} + \text{h.c.},$$

where $\nu_{mL} \equiv U^\dagger\nu_L$ is the left-handed component of a massive Majorana field. Again, U is called the leptonic mixing matrix.

Number of mixing parameters

In general, any unitary $n \times n$ matrix can be parameterized by n^2 real parameters, $n(n-1)/2$ mixing angles and $n(n+1)/2$ complex phases. However, in physics, we are only interested in what is observable and for the leptonic mixing matrix U, the total Lagrangian density does not change if we absorb some of the phases of U into the fields [61, 62]. For any Dirac field ψ, we can absorb a constant phase $\exp(\mathrm{i}\phi)$ by instead of ψ using the redefined field

$$\psi' \equiv \psi\exp(\mathrm{i}\phi),$$

By using such redefinitions of the charged lepton fields, we may absorb n complex phases from the leptonic mixing matrix. This leaves us with $n(n-1)$ mixing parameters (where $n(n-1)/2$ are complex phases). In the case of Majorana neutrinos, this is all we can do, since Majorana fields cannot absorb complex phases in the same way as Dirac fields. On the other hand, in the case of Dirac neutrinos, also the neutrino fields can absorb complex phases. At first sight, one may think that it would be possible to absorb another n phases into the neutrino fields. However, one of these phases just corresponds to an overall phase which may just as well be absorbed into the charged lepton fields. Therefore, we can only absorb another $n-1$ phases into the neutrino fields, leaving us with a total of $(n-1)^2$ parameters (of which $(n-1)(n-2)/2$ are complex phases) for the leptonic mixing matrix in the case of Dirac neutrinos. The number of mixing parameters for different n are given in Tab. 3.2. Clearly, in the one-flavor case, there are no mixing parameters

Neutrino flavors	*Mixing angles*	*Complex phases*	*Total*
1	0	0 [0]	0 [0]
2	1	0 [1]	1 [2]
3	3	1 [3]	4 [6]
4	6	3 [6]	9 [12]
$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	$\frac{n(n-1)}{2}$	$\frac{(n-1)(n-2)}{2}$ $[\frac{n(n-1)}{2}]$	$(n-1)^2$ $[n(n-1)]$

Table 3.2. The number of mixing parameters as a function of the number of neutrino flavors. The numbers within the brackets are for the case of Majorana neutrinos. It should be noted that the extra Majorana phases do not contribute to the neutrino oscillation probabilities, see Ch. 4.

(as expected). However, one very interesting feature of the number of mixing parameters is that there are no complex phases in the case of two-flavor neutrino mixing. Thus, for any two-flavor mixing of neutrinos, the mixing matrix can be made real (even if this is true only for the case of Dirac neutrinos, only the Dirac phases will influence the neutrino oscillation probabilities, see Ch. 4). It should also be noted that the total number of mixing parameters grows very quickly ($\sim n^2$) as the number of neutrino flavors n increases, making the cases of many-flavor mixing harder to study in detail.

Chapter 4

Neutrino oscillations

As was discussed in Ch. 2, there are now many experiments [27–39] which indicate that neutrinos oscillate among different neutrino flavors. In this chapter, the background material to neutrino oscillations in both vacuum and matter, with which this thesis is concerned (Papers 1–3), is presented. Historically, there have also been other attempts (see, *e.g.*, Refs. [63–84]) to describe neutrino flavor transitions with mechanisms other than neutrino oscillations. Even if these scenarios now seem experimentally disfavored, they may still contribute with small corrections to the neutrino oscillation formulas. This is also discussed in this chapter and is what the final Paper (Paper 4) of this thesis is concerned with.

4.1 Neutrinos oscillations in vacuum

For ultra-relativistic neutrinos, a neutrino flavor eigenstate (the state produced in weak interactions) is given by [85]

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle \,,$$

where U is the leptonic mixing matrix and $|\nu_i\rangle$ is a neutrino mass eigenstate (*i.e.*, the state corresponding to one quanta of the quantized mass eigenfield ν_i, see Sec. 3.2.4). In the above equation and in what follows, Latin indices (*i.e.*, i, j, k, ...) will denote indices belonging to the mass eigenstate basis (with the possible values $1, 2, \ldots, n$) and Greek indices (*i.e.*, α, β, γ, ...) will denote indices belonging to the flavor eigenstate basis (with the possible values e, μ, τ, ...). The corresponding relation for anti-neutrinos is

$$|\bar{\nu}_\alpha\rangle = \sum_i U_{\alpha i} |\bar{\nu}_i\rangle \,,$$

i.e., the only difference is in the complex conjugation of the mixing matrix elements. The time-evolution of the neutrino flavor eigenstate is given by

$$|\nu_\alpha(t)\rangle = \exp(-\mathrm{i}Ht)\,|\nu_\alpha\rangle\,,$$

where H is the Hamiltonian operator. We assume that the flavor eigenstate is produced with a definite momentum, *i.e.*, that all the mass eigenstates in the superposition have the same momentum p. The mass eigenstates are also eigenstates of the Hamiltonian operator with energy eigenvalues

$$E_i = \sqrt{p^2 + m_i^2}.$$

For ultra-relativistic neutrinos, this energy can be Taylor expanded to

$$E_i = p\sqrt{1 + \frac{m_i^2}{p^2}} \simeq p + \frac{m_i^2}{2p}.$$

It follows that the neutrino flavor eigenstate at time t is given by

$$|\nu_\alpha(t)\rangle = \sum_i U_{\alpha i}^* e^{-\mathrm{i}\left(p+\frac{m_i^2}{2p}\right)t}\,|\nu_i\rangle\,.$$

The probability amplitude for detecting the neutrino in the flavor eigenstate $|\nu_\beta\rangle$ at time t is then given by

$$A_{\alpha\beta}(t) = \langle\nu_\beta|\nu_\alpha(t)\rangle = \sum_i U_{\beta i} U_{\alpha i}^* e^{-\mathrm{i}E_i t}$$

and the corresponding probability is therefore

$$P_{\alpha\beta}(t) = |A_{\alpha\beta}(t)|^2.$$

We note that $\exp(-\mathrm{i}pt)$ is a common phase factor of all terms in the probability amplitude, and thus, will not influence the final probability. Because the neutrinos we are discussing here are ultra-relativistic, it is common practice to put $t = L$ and use the length L travelled by the neutrino as a parameter instead of the time t.

There are a lot of ways of writing the neutrino oscillation probability $P_{\alpha\beta}(L)$. By just squaring the modulus of the probability amplitude, we obtain the expression

$$P_{\alpha\beta}(L) = \sum_i \sum_j J_{\alpha\beta}^{ij} \exp\left(-\mathrm{i}\frac{\Delta m_{ij}^2}{2p}L\right), \tag{4.1a}$$

where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ is the mass squared difference between the mass eigenstates $|\nu_i\rangle$ and $|\nu_j\rangle$ and we have introduced the quantity

$$J_{\alpha\beta}^{ij} \equiv U_{\beta i} U_{\alpha i}^* U_{\beta j}^* U_{\alpha j}.$$

For simplicity, it is also common to make the approximation $p \simeq E$ in the denominator of the argument of the exponential of Eq. (4.1a) and write

$$\Delta_{ij} \equiv \frac{\Delta m_{ij}^2}{4E} L.$$

In this case Eq. (4.1a) can be written as

$$P_{\alpha\beta} = \sum_i \sum_j J_{\alpha\beta}^{ij} \exp\left(-\mathrm{i}2\Delta_{ij}\right). \tag{4.1b}$$

By making use of the definitions of the sine and cosine functions in terms of the real and imaginary parts of $\exp(\mathrm{i}x)$, we find the form

$$P_{\alpha\beta} = \sum_i J_{\alpha\beta}^{ii} + 2\sum_{i<j} \mathrm{Re}\left(J_{\alpha\beta}^{ij}\right) \cos\left(2\Delta_{ij}\right) - 2\sum_{i<j} \mathrm{Im}\left(J_{\alpha\beta}^{ij}\right) \sin\left(2\Delta_{ij}\right) \tag{4.1c}$$

for the neutrino oscillation probability. Using simple trigonometry, the neutrino oscillation probability may also be written as

$$P_{\alpha\beta} = \sum_i J_{\alpha\beta}^{ii} + 2\sum_{i<j} \left|J_{\alpha\beta}^{ij}\right| \cos\left(2\Delta_{ij} + \arg J_{\alpha\beta}^{ij}\right). \tag{4.1d}$$

If we instead study the anti-neutrino oscillation probabilities, we have to make the substitution $U_{\alpha i} \to U_{\alpha i}^*$ according to what was stated in the beginning of this chapter. Thus, the anti-neutrino oscillation probabilities are given by the same expressions as the neutrino survival probabilities with the substitution $J_{\alpha\beta}^{ij} \to J_{\bar{\alpha}\bar{\beta}}^{ij} = J_{\alpha\beta}^{ij*}$. From Eqs. (4.1), we notice that

$$\mathrm{Re}\left(J_{\bar{\alpha}\bar{\beta}}^{ij}\right) = \mathrm{Re}\left(J_{\alpha\beta}^{ij}\right) \quad \text{and} \quad \mathrm{Im}\left(J_{\bar{\alpha}\bar{\beta}}^{ij}\right) = -\mathrm{Im}\left(J_{\alpha\beta}^{ij}\right)$$

as well as

$$\left|J_{\bar{\alpha}\bar{\beta}}^{ij}\right| = \left|J_{\alpha\beta}^{ij}\right| \quad \text{and} \quad \arg J_{\bar{\alpha}\bar{\beta}}^{ij} = -\arg J_{\alpha\beta}^{ij}$$

implies that the neutrino and anti-neutrino oscillation probabilities differ (*i.e.*, there is *CP* violation in neutrino oscillations) only if there are complex entries in the leptonic mixing matrix. It should also be noted that the survival probabilities of neutrinos and the corresponding anti-neutrinos are always equal $P_{\alpha\alpha} = P_{\bar{\alpha}\bar{\alpha}}$, since $J_{\alpha\alpha}^{ij}$ is real.

In the same fashion, we notice that $J_{\alpha\beta}^{ij} = J_{\beta\alpha}^{ij*}$, and thus, the time-reversed probability $P_{\beta\alpha}$ is given by

$$P_{\beta\alpha} = P_{\bar{\alpha}\bar{\beta}} \tag{4.2}$$

and T violation only occurs if there are complex entries in the leptonic mixing matrix. It should be noted that Eq. (4.2) is only valid for neutrino oscillations in vacuum. When matter effects come into play, there are other mechanisms than the

complex conjugation of the elements of the leptonic mixing matrix that can give rise to CP or T violation. The need for a complex entry in the leptonic mixing matrix for CP or T violation to occur implies that, since there are no complex phases in a scenario with only two neutrino flavors, there must be (at least) three neutrino flavors for these effects to occur. In fact, if we have only two neutrino flavors, say ν_e and ν_x, then the unitarity relations

$$P_{ee} + P_{ex} = 1 \quad \text{and} \quad P_{ee} + P_{xe} = 1$$

automatically implies that $P_{ex} = P_{xe}$. This relation is clearly true even for the case of neutrino oscillations in matter as long as no neutrinos are lost[1]. For a recent review on CP and T violation in neutrino oscillations, see Ref. [86].

Majorana phases and neutrino oscillations

As was discussed in Sec. 3.2.4, there may be additional complex phases in the leptonic mixing matrix if neutrinos are Majorana particles. However, as we will now show, these phases do not influence the neutrino oscillation probabilities [61, 62].

The extra Majorana phases are added into the leptonic mixing matrix by the disability of absorbing complex phases into the Majorana neutrino fields. It follows that the elements of the leptonic mixing matrix in the case of Majorana neutrinos can be written as

$$U_{\alpha i} = U'_{\alpha i} e^{\mathrm{i}\varphi_i},$$

where U' is a leptonic mixing matrix in the case of Dirac neutrinos and φ_i is independent of α. It then immediately follows that

$$J^{ij}_{\alpha\beta} = U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} = U'_{\beta i} U'^*_{\alpha i} U'^*_{\beta j} U'_{\alpha j} = J'^{ij}_{\alpha\beta},$$

which is independent of the Majorana phases. Thus, the neutrino oscillation probabilities do not depend on the Majorana phases. This is also true in the case of neutrino oscillations in matter (see, *e.g.*, Ref. [87]).

4.1.1 The two-flavor scenario

In the simplest case of neutrino oscillations imaginable, we have only two neutrino flavors. In this case, the leptonic mixing matrix will be real and unitary, *i.e.*, it will fulfill the relation $U^T U = \mathbf{1}_2$, where $\mathbf{1}_2$ is the 2×2 unit matrix. This is the relation defining the group of orthogonal 2×2 matrices (rotations in $\mathbb{R}^2$), and thus, the leptonic mixing matrix can be parameterized as

$$U = \begin{pmatrix} c & s \\ -s & c \end{pmatrix},$$

where $c \equiv \cos(\theta)$, $s \equiv \sin(\theta)$, and θ is a real parameter known as the leptonic mixing angle.

[1] A loss of neutrinos may be due to some decay-like damping signature (see Paper 4).

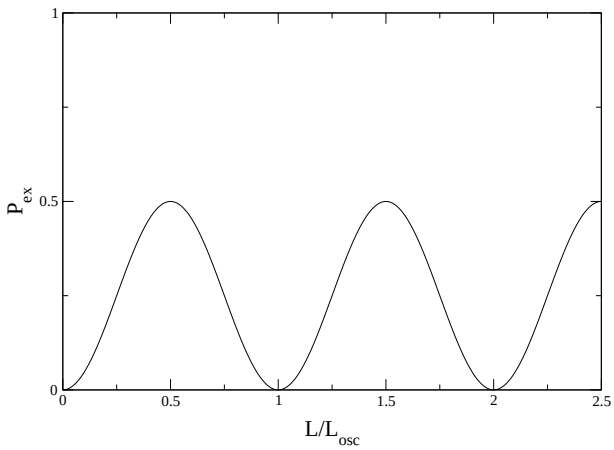

Figure 4.1. The neutrino oscillation probability P_{ex} as a function of $L/L_{\rm osc}$. The leptonic mixing angle has been put to $\theta = \pi/8$, *i.e.*, $\sin^2(2\theta) = 0.5$.

If we name the two neutrino flavors involved in the oscillations ν_e and ν_x, then Eqs. (4.1) will now have the form

$$P_{ex} = P_{xe} = \sin^2(2\theta)\sin^2(\Delta) \tag{4.3a}$$

and

$$P_{ee} = P_{xx} = 1 - \sin^2(2\theta)\sin^2(\Delta), \tag{4.3b}$$

where $\Delta \equiv \Delta_{21}$ is the oscillation phase and $\sin^2(2\theta)$ is the oscillation amplitude. The oscillation phase Δ can be rewritten as

$$\Delta = \pi \frac{L}{L_{\rm osc}}, \quad \text{where} \quad L_{\rm osc} \equiv \frac{4\pi E}{\Delta m^2_{21}}$$

is the distance over which a full period of oscillation takes place (known as the oscillation length). The neutrino oscillation probability P_{ex} is illustrated in Fig. 4.1.

It should be noted that while the two-flavor case serves as a very simple example of neutrino oscillations and, as we will show in Sec. 4.1.2, can be applicable in some special cases[2], the existence of three neutrino flavors often makes it necessary to use a full three-flavor scenario to accurately describe neutrino oscillations [88]. The main advantages of using a two-flavor scenario is the large reduction of the number of fundamental neutrino oscillation parameters (there are only two in the two-flavor case, the leptonic mixing angle θ and the mass squared difference Δm^2_{21}).

[2]Historically, two-flavor cases have been and are still beeing applied by many experimental collaborations.

Parameter	*Best-fit*	*3σ confidence*
Δm^2_{21} [10^{-5} eV^2]	8.1	7.2-9.1
$\lvert\Delta m^2_{31}\rvert$ [10^{-3} eV^2]	2.2	1.4-3.3
θ_{12} [°]	33.2	28.6-38.1
θ_{23} [°]	45.0	35.7-53.1
θ_{13} [°]	0.0	≤ 12.5

Table 4.1. The best-fit values along with the 3σ confidence intervals for the fundamental three-flavor neutrino oscillation parameters. There are no bounds for the complex phase δ. The data have been taken from Ref. [90].

4.1.2 The three-flavor scenario

In the case of neutrino oscillations among three different neutrino flavors, the leptonic mixing matrix U is parameterized by three mixing angles and one complex phase. The standard parameterization of the leptonic mixing matrix is [89]

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-\mathrm{i}\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\mathrm{i}\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{\mathrm{i}\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{\mathrm{i}\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\mathrm{i}\delta} & c_{23}c_{13} \end{pmatrix}, \tag{4.4}$$

where $c_{ij} \equiv \cos(\theta_{ij})$, $s_{ij} \equiv \sin(\theta_{ij})$, θ_{ij} are leptonic mixing angles, and δ is the complex phase. In addition to the four parameters of the leptonic mixing matrix, there are two independent mass squared differences Δm^2_{21} and Δm^2_{31} (from the definition of the mass squared differences follows that $\Delta m^2_{32} = \Delta m^2_{31} - \Delta m^2_{21}$), this gives a total of six fundamental neutrino oscillation parameters in the case of three-flavor neutrino oscillations. Most of the fundamental parameters are more or less accurately determined by global fits to the results of the neutrino oscillation experiments (see, *e.g.*, Ref. [90] for which the results are presented in Tab. 4.1).

While three-flavor neutrino oscillation formulas cannot be written on a form as simple as the corresponding two-flavor formulas, there are two special cases for which the three-flavor formulas essentially have the same structure as the two-flavor formulas, *i.e.*, a neutrino survival probability is of the form

$$P_{\alpha\alpha} = 1 - A\sin^2(\Delta)$$

and a neutrino transition probability is of the form

$$P_{\alpha\beta} = A\sin^2(\Delta).$$

These cases are:

1. $\boldsymbol{U_{\alpha i} = 0}$ **:** If any element of the leptonic mixing matrix, say U_{e3}, is equal to zero, then $J^{ij}_{\alpha\beta} = 0$ if α or β is equal to e and i or j is equal to 3. In this case, we have

$$P_{ee} = 1 - 4J^{12}_{ee}\sin^2(\Delta_{21}) = 1 - 4|U_{e1}|^2|U_{e2}|^2\sin^2(\Delta_{21})$$

and

$$P_{e\alpha} = -4J^{12}_{e\alpha}\sin^2(\Delta_{12}) = -4U^*_{e1}U_{e2}U_{\alpha 1}U^*_{\alpha 2}\sin^2(\Delta_{12}),$$

where $\alpha \neq e$. The generalization of this to any of the leptonic mixing matrix elements being zero is straightforward, the reason to have $U_{e3} = 0$ for the example is that this element is known to be small [*cf.*, Eq. (4.4) and Tab. 4.1]. It should also be noted that, unless there is another zero in the same column or row as the first one, there will be neutrino oscillation probabilities which are not of the two-flavor form, *i.e.*, $P_{\alpha\beta}$ where $\alpha, \beta \neq e$ in our example. However, if two entries of the same row or column in the leptonic mixing matrix is zero, then it follows that there is only mixing between two of the three neutrino flavors and all neutrino oscillation probabilities are of the two-flavor form.

2. $\boldsymbol{\Delta m^2_{ij} = 0}$ **:** If any of the mass squared differences are equal to zero, then the other two mass squared differences will be equal up to a sign and it trivially follows that all neutrino oscillation probabilities are of the two-flavor form.

In the above cases, the neutrino oscillation probabilities are of exact two-flavor form. However, there are other cases for which the neutrino oscillation probabilities are simplified as compared to the full three-flavor formulas. For example, if $\Delta_{31} \simeq \Delta_{32} \gg 1$, then experiments will not be able to resolve the fast oscillations involving the third mass eigenstate and the terms $\sin^2(\Delta_{3i})$ will average out to 1/2. In this case, the ν_e survival probability will be given by

$$P_{ee} = c^4_{13}P^{2\mathrm{f}}_{ee} + s^4_{13},$$

where $P^{2\mathrm{f}}_{ee}$ is the ν_e survival probability in a two-flavor scenario with $\theta = \theta_{12}$ and $\Delta = \Delta_{21}$.[3] In addition, it is also possible to make Taylor expansions of the neutrino oscillation probabilities in the small parameters $\alpha \equiv \Delta m^2_{21}/\Delta m^2_{31}$ (the ratio between the small and large mass squared differences) and s_{13} to make the expressions somewhat less cumbersome (see, *e.g.*, Ref. [91]).

4.2 Neutrino oscillations in matter

As was first discussed by Wolfenstein [24] and later by Mikheyev and Smirnov [25, 26], the presence of matter may strongly affect the neutrino oscillation probabilities. In general, for low-energy neutrinos (giving center-of-mass energies lower than the masses of the weak gauge bosons) the cross-sections for neutrino absorption in matter are very small, since they are proportional to the square of the Fermi coupling constant G_F. However, the neutrino oscillation probabilities may be affected by coherent forward scattering, where the incoming and outgoing states only differ by a phase factor (the momenta will be the same). In this case, there

[3] The exact same formula will arise in the case of a decoherence-like effect where there is complete decoherence between the third mass eigenstate and the other two mass eigenstates, see Paper 4 for details.

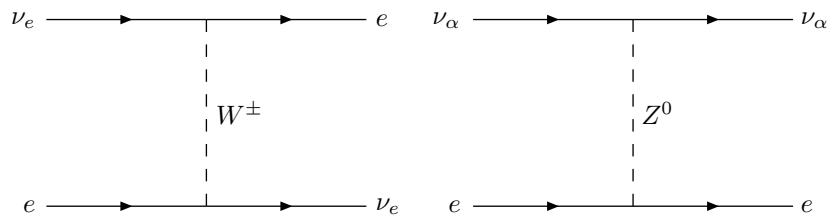

Figure 4.2. Tree level Feynman diagrams for CC (left) and NC (right) scattering of neutrinos on electrons. The CC scattering is only allowed for electron neutrinos ν_e, while the NC scattering can occur for any type of neutrino flavor.

will be interference between the leading order contribution (the free propagation) and the first order contribution in G_F, and thus, the effects will be of order G_F instead of G_F^2. It should be noted that G_F multiplied with the number density of matter[4] is still a very low energy scale. However, the scale set by the mass squared differences and the neutrino energy [$\sim \Delta m_{ij}^2/(2E)$] is also a very low energy scale and if the scale given by G_F and the matter density is of the same order or larger, then neutrino oscillations will be affected in a significant way.

4.2.1 Coherent forward scattering

As was stated above, coherent forward scattering on electrons is the main mechanism behind matter effects on neutrino oscillations. Since neutrinos are weakly interacting, they may interact either through charged-current (CC) or neutral-current (NC) interactions. For the coherent forward scattering on electrons, only the electron neutrinos can have a CC contribution. This is simply due to the fact that the incoming and outgoing particles of an interaction must be the same. The Feynman diagrams for coherent forward scattering by CC and NC interactions are given in Fig. 4.2.

At low center-of-mass energies, the effective contribution to the Lagrangian density of the CC scattering of neutrinos on electrons is given by

$$\mathcal{L}_{\mathrm{CC}} = -\frac{G_F}{\sqrt{2}}[\bar{e}\gamma^\mu(1-\gamma^5)\nu_e][\bar{\nu}_e\gamma_\mu(1-\gamma^5)e],$$

which, by means of a Fierz transformation, can be rewritten as

$$\mathcal{L}_{\mathrm{CC}} = -\frac{G_F}{\sqrt{2}}[\bar{e}\gamma^\mu(1-\gamma^5)e][\bar{\nu}_e\gamma_\mu(1-\gamma^5)\nu_e].$$

Assuming coherent forward scattering and that the electrons in the medium in which the neutrinos propagate are unpolarized with zero average momenta, this interaction term results in an effective contribution H_{CC} to the Hamiltonian operator

[4] More precisely, the number density of electrons, see below.

given by

$$H_{\rm CC}\left|\nu_\alpha\right\rangle = \delta_{e\alpha} V_{\rm CC}\left|\nu_\alpha\right\rangle,$$

where $V_{\rm CC} \equiv \sqrt{2} G_F n_e$ and n_e is the electron number density.

When treating the NC scattering in a similar fashion, we also obtain an effective contribution to the Hamiltonian operator, this contribution is given by

$$H_{\rm NC}\left|\nu_\alpha\right\rangle = V_{\rm NC}\left|\nu_\alpha\right\rangle,$$

where $V_{\rm NC} \equiv -G_F n_n/\sqrt{2}$ and n_n is the neutron number density[5]. Apparently, the NC scattering contribution is the same for all neutrino flavors. However, if taking loop corrections into account, then the contributions among different flavors will be somewhat different due to the different masses of the charged leptons.

4.2.2 Neutrino oscillation probabilities in matter

An interesting feature of the coherent forward scattering contribution to the Hamiltonian operator is that, while the Hamiltonian operator in vacuum is diagonal in the basis of mass eigenstates $\{|\nu_i\rangle\}$, the effective contribution from the interaction with matter is diagonal in the basis of flavor eigenstates $\{|\nu_\alpha\rangle\}$. The total Hamiltonian $H = H_{\rm vac} + H_{\rm CC} + H_{\rm NC}$, where $H_{\rm vac}$ is the vacuum Hamiltonian, can be written in matrix form in either the basis of flavor eigenstates as $(H_f)_{\alpha\beta} \equiv \langle\nu_\alpha\left|H\right|\nu_\beta\rangle$ or in the basis of mass eigenstates as $(H_m)_{ij} \equiv \langle\nu_i\left|H\right|\nu_j\rangle$. The two bases will be related by the leptonic mixing matrix as

$$H_f = U H_m U^\dagger \quad \text{and} \quad H_m = U^\dagger H_f U,$$

respectively. The vacuum Hamiltonian in the basis of mass eigenstates is given by

$$H_{{\rm vac},m} \simeq p\mathbf{1}_n + \frac{1}{2p}\begin{pmatrix} m_1^2 & 0 & 0 & \cdots \\ 0 & m_2^2 & 0 & \cdots \\ 0 & 0 & m_3^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

where $\mathbf{1}_n$ is the $n \times n$ unit matrix, while the effective part coming from coherent forward scattering in matter has the form

$$H_{{\rm CC},f} + H_{{\rm NC},f} = V_{\rm NC}\mathbf{1}_n + V_{\rm CC}\begin{pmatrix} 1 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

in the basis of flavor eigenstates. The fact that the NC contribution is proportional to the unit matrix means that it will only contribute with a common phase factor

[5] Neutrinos do not only NC scatter on electrons, but also on protons and neutrons. In an electrically neutral medium, the electron and proton contributions to $V_{\rm NC}$ will cancel, leaving only the neutron contribution.

to all neutrino states (in analogy with the term $p\mathbf{1}_n$ of the vacuum Hamiltonian), and thus, the NC contribution will not affect the final neutrino oscillation probabilities. It is therefore common practice to leave out both the $p\mathbf{1}_n$ term of the vacuum Hamiltonian and the $V_{\mathrm{NC}}\mathbf{1}_n$ term of the effective matter addition to the Hamiltonian[6].

We now introduce yet another basis for the neutrino states, which is useful when studying neutrino oscillations in matter, namely the matter eigenstate basis $\{|\tilde{\nu}_i\rangle\}$. This basis is defined by demanding that it diagonalizes the full Hamiltonian operator in matter, *i.e.*,

$$(\tilde{H}_m)_{ij} = \langle \tilde{\nu}_i \,|H|\, \tilde{\nu}_j\rangle = 0 \quad \text{if} \quad i \neq j.$$

As in the case of the mass eigenstate basis, the neutrino flavor states will be related to the matter eigenstates by some unitary transformation[7] $\tilde{U}$ as

$$|\nu_\alpha\rangle = \sum_i \tilde{U}^*_{\alpha i}\,|\tilde{\nu}_i\rangle$$

and the Hamiltonian operator in the different bases will be related by the same unitary transformation as

$$H_f = \tilde{U}\tilde{H}_m\tilde{U}^\dagger \quad \text{and} \quad \tilde{H}_m = \tilde{U}^\dagger H_f \tilde{U}.$$

The unitary matrix $\tilde{U}$ is known as the (effective) leptonic mixing matrix in matter and can be parameterized in the same manner as the leptonic mixing matrix, the effective mixing parameters are then denoted by tildes ($\tilde{\theta}_{ij}$ and $\tilde{\delta}$). In addition, the differences between the eigenvalues of the Hamiltonian in matter may not be the same as the corresponding differences in vacuum. Thus, we define the effective mass squared differences in matter to be

$$\Delta\tilde{m}^2_{ij} \equiv 2E\Delta E_{ij},$$

where E is the total neutrino energy and ΔE_{ij} is the difference in energy of the ith and jth neutrino matter eigenstates. With these definitions, the matter eigenstates will play exactly the same role in neutrino oscillations in matter as the mass eigenstates do in the study of neutrino oscillations in vacuum. If neutrinos propagate

[6]Sometimes it is also useful to subtract $[m_1^2/(2p)]\mathbf{1}_n$ or some other quantity proportional to the unit matrix from the total Hamiltonian. Due to the same reasons as described, this will give an overall phase contribution which does not affect the neutrino oscillation probabilities.

[7]Clearly, also the mass eigenstates will be related to the matter eigenstates by some unitary transformation. However, this transformation is not as interesting, since it is flavor eigenstates that are measured in experiments.

through matter of constant density, then the neutrino oscillation probabilities will be given by

$$P_{\alpha\beta} = \sum_i \sum_j \tilde{J}^{ij}_{\alpha\beta} \exp\left(-\mathrm{i}\frac{\Delta\tilde{m}^2_{ij}}{2p}L\right),$$

where

$$\tilde{J}^{ij}_{\alpha\beta} \equiv \tilde{U}_{\beta i}\tilde{U}^*_{\alpha i}\tilde{U}^*_{\beta j}\tilde{U}_{\alpha j}.$$

Neutrino oscillations in matter with varying density are more difficult to treat, since the matter eigenstates and the effective leptonic mixing matrix in matter will vary along the path of propagation. In that case, there will be transitions among the various neutrino matter eigenstates. In particular, if we study the time evolution of the probability amplitude $\phi_i = \langle\tilde{\nu}_i|\nu(t)\rangle$ of a neutrino with the initial state $|\nu(0)\rangle$, then we obtain

$$\mathrm{i}\frac{\mathrm{d}\phi_i}{\mathrm{d}t} = \mathrm{i}\left(\frac{\mathrm{d}\langle\tilde{\nu}_i|}{\mathrm{d}t}|\nu(t)\rangle + \langle\tilde{\nu}_i|\frac{\mathrm{d}|\nu(t)\rangle}{\mathrm{d}t}\right) = \left(-\mathrm{i}\tilde{U}^*_{\alpha i}\frac{\mathrm{d}\tilde{U}_{\alpha j}}{\mathrm{d}t} + \tilde{H}_{ij}\right)\phi_j,$$

or, in matrix notation,

$$\mathrm{i}\frac{\mathrm{d}\boldsymbol{\phi}}{\mathrm{d}t} = \left(-\mathrm{i}\tilde{U}^\dagger\frac{\mathrm{d}\tilde{U}}{\mathrm{d}t} + \tilde{H}\right)\boldsymbol{\phi}, \tag{4.5}$$

where $\boldsymbol{\phi} \equiv (\phi_1\ \phi_2\ \ldots)^T$. In this equation, the term $\tilde{H}$ is diagonal by definition and the first term is a matrix with off-diagonal entries leading to transitions among different matter eigenstates.

Oscillations of anti-neutrinos in matter

When treating anti-neutrino oscillations in matter, the effective potential V_{CC} will change sign, since the matter still contains electrons and not positrons. Thus, the diagonalization of the total Hamiltonian will be quite different from the diagonalization of the total Hamiltonian in the neutrino case. In particular, the effective anti-neutrino matter eigenstates are not related to the anti-neutrino flavor eigenstates by the adjoint of the leptonic mixing matrix in matter. Instead, we will have to diagonalize the anti-neutrino mixing matrix separately with a change of sign in the effective potential V_{CC}. From this follows that there may be *CP* violation in the neutrino oscillation probabilities in matter without any complex phases in the leptonic mixing matrix U.

Sterile neutrinos

While discussing neutrino oscillations in matter, we should also note how the neutrino oscillations are affected if there are sterile neutrinos[8] involved. These sterile

[8]Neutrinos which do not participate in the weak interaction, and thus, only interact through gravitation or by mixing with the active neutrino flavors. A sterile neutrino mixing with the active flavors could be the mechanism behind the LSND anomaly [92] which will be tested by the MiniBooNE experiment [93].

neutrinos may be of some unknown origin or be the right-handed Majorana neutrino fields from the see-saw mechanism.

The important thing to note is that, while the active flavors ν_e, ν_μ, and ν_τ also coherently forward scatter via NC interactions, sterile neutrinos do not coherently forward scatter at all. Thus, the sterile neutrinos do not obtain an effective NC potential $V_{\rm NC}$ as the active neutrinos do. The result of this is that the effective addition to the Hamiltonian in matter will be given by

$$H_{{\rm CC},f} + H_{{\rm NC},f} = \mathrm{diag}(\underbrace{V_{\rm CC} + V_{\rm NC},\ V_{\rm NC},\ \ldots, V_{\rm NC}}_{n \text{ entries}}, \underbrace{0,\ \ldots,\ 0}_{m \text{ entries}}),$$

if there are n active and m sterile neutrino flavors. In the remainder of this thesis, we will only be concerned with the oscillations of active neutrino flavors.

4.2.3 The two-flavor scenario

As for the two-flavor neutrino oscillations in vacuum, the two-flavor effective leptonic mixing matrix in matter is an orthogonal matrix, which is parameterized by one real parameter only, it is given by

$$\tilde{U} = \begin{pmatrix} \tilde{c} & \tilde{s} \\ -\tilde{s} & \tilde{c} \end{pmatrix},$$

where $\tilde{c} \equiv \cos(\tilde{\theta})$, $\tilde{s} \equiv \sin(\tilde{\theta})$, and $\tilde{\theta}$ is the leptonic mixing angle in matter. The effective Hamiltonian operator is of the form

$$H_f = \frac{1}{2}\left[\frac{\Delta m^2}{2E}\begin{pmatrix} -\cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{pmatrix} + V_{\rm CC}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right],$$

where we have subtracted $\mathrm{tr}\,(H)\,\mathbf{1}_2/2$ from the Hamiltonian given in Sec. 4.2.2, which does not change the neutrino oscillation probabilities as stated in that section.

The explicit diagonalization of the effective Hamiltonian gives the relation

$$\tan(2\tilde{\theta}) = \frac{\sin(2\theta)}{\cos(2\theta) - Q},$$

where $Q \equiv 2EV_{\rm CC}/\Delta m^2$, or, if we make use of the trigonometric relation $\sin^2(x) = \tan^2(x)/[1 + \tan^2(x)]$,

$$\sin^2(2\tilde{\theta}) = \frac{\sin^2(2\theta)}{[\cos(2\theta) - Q]^2 + \sin^2(2\theta)}.$$

From the difference between the eigenvalues of the Hamiltonian, we also obtain the effective mass squared difference

$$\Delta\tilde{m}^2 = \Delta m^2\sqrt{[\cos(2\theta) - Q]^2 + \sin^2(2\theta)}.$$

It is apparent that when $V_{\rm CC} \to 0$, then $Q \to 0$, $\tilde{\theta} \to \theta$, and $\Delta\tilde{m}^2 \to \Delta m^2$, which means that we regain the two-flavor vacuum parameters in this limit in accordance with what should be expected.

The two-flavor neutrino oscillation probabilities in matter of constant electron number density is given by

$$P_{ex} = P_{xe} = \sin^2(2\tilde{\theta})\sin^2(\tilde{\Delta})$$

and

$$P_{ee} = P_{xx} = 1 - \sin^2(2\tilde{\theta})\sin^2(\tilde{\Delta}),$$

where $\tilde{\Delta} = \Delta\tilde{m}^2 L/(4E)$. One apparent feature of the above formulas is that when

$$Q = \cos(2\theta),$$

then $\sin^2(2\tilde{\theta}) = 1$ and the neutrino oscillations have the largest possible amplitude leading to the neutrino transition probabilities P_{ex} and P_{xe} being equal to unity for some values of the neutrino path length L, even if the leptonic mixing angle θ is small. This resonance phenomenon is known as the Mikheyev–Smirnov–Wolfenstein (MSW) resonance [24–26]. For the resonance condition, we notice that if we choose θ such that $\cos(2\theta) > 0$ (this corresponds to the ordering of the mass eigenstates), then the resonance condition can be fulfilled for neutrinos ($V_{\rm CC} > 0$) only if $\Delta m^2 > 0$ and for anti-neutrinos ($V_{\rm CC} < 0$) only if $\Delta m^2 < 0$.

It is of particular interest to note that for matter effects on neutrino oscillations to exist in a two-flavor scenario, one of the two neutrino flavors involved must be the electron neutrino ν_e. In a two-flavor neutrino oscillation framework with ν_μ and ν_τ, there would be no matter effects, since none of these neutrino flavors undergo coherent forward scattering via CC interactions and the contribution from NC interactions are equal.

Two-flavor oscillations in matter of varying density

Clearly, in most applications where neutrinos propagate through matter (*i.e.*, neutrinos propagating in the Earth and/or in the Sun), the matter will be of varying density, it is therefore important to study neutrino propagation in such matter. For the evolution of the matter eigenstate components ϕ_i, Eq. (4.5) becomes

$$\mathrm{i}\frac{\mathrm{d}\boldsymbol{\phi}}{\mathrm{d}t} = \underbrace{\begin{pmatrix} 0 & -\mathrm{i}\dot{\tilde{\theta}} \\ \mathrm{i}\dot{\tilde{\theta}} & \frac{\Delta\tilde{m}^2}{2E} \end{pmatrix}}_{\tilde{H}'} \boldsymbol{\phi}$$

with $\boldsymbol{\phi} = (\phi_1\ \phi_2)^T$. Since the matrix $\tilde{H}'$ is time dependent, this differential equation cannot be solved by simple exponentiation. Instead, one often relies on approximate solutions to this differential equation. For instance, if $\gamma = |\Delta\tilde{m}^2/(4E\dot{\tilde{\theta}})| \gg 1$, then one can essentially disregard the transitions between the different matter eigenstates [94–101]. This type of scenario is called adiabatic and occurs when the

matter density is only slowly varying, the parameter γ is known as the adiabaticity parameter. In the case of adiabatic neutrino evolution, the matter eigenstate components simply evolve as

$$\phi_1(t) = \phi_1(0) \quad \text{and} \quad \phi_2(t) = \phi_2(0)\exp[-\mathrm{i}\Gamma(t)],$$

where the phase factor $\Gamma(t)$ is given by

$$\Gamma(t) = \int_0^t \frac{\Delta\tilde{m}^2}{2E}\mathrm{d}t.$$

The initial values $\phi_i(0)$ are given by $\phi_i(0) = \langle\tilde{\nu}_i|\nu(0)\rangle$, and thus, if the initial neutrino state is $|\nu_e\rangle$, then

$$\phi_1(0) = \tilde{c}(0) \quad \text{and} \quad \phi_2(0) = \tilde{s}(0).$$

The neutrino transition probability P_{ex} will then be given by

$$\begin{aligned} P_{ex} &= |\langle\nu_x|\nu_e(t)\rangle|^2 = \left|\sum_i \langle\nu_x|\tilde{\nu}_i\rangle\langle\tilde{\nu}_i|\nu_e(t)\rangle\right|^2 = \left|\begin{pmatrix} -\tilde{s}(t) & \tilde{c}(t) \end{pmatrix}\begin{pmatrix} \phi_1(t) \\ \phi_2(t) \end{pmatrix}\right|^2 \\ &= \frac{1}{2}\left\{1 - \cos[2\tilde{\theta}(t)]\cos[2\tilde{\theta}(0)] - \sin[2\tilde{\theta}(t)]\sin[2\tilde{\theta}(0)]\cos[\Gamma(t)]\right\}. \end{aligned}$$

In many applications (such as in the treatment of neutrinos from the Sun), the production or detection region is larger than the oscillation length of the neutrinos. For these cases, the $\cos[\Gamma(t)]$ term will effectively average out to zero. This is also the case if we have any damping-like signature (see Paper 4) between the two matter eigenstates, *e.g.*, some decoherence between the wave packets of different matter eigenstates[9] or some energy averaging in the detector. If we produce electron neutrinos in a region with an electron density which is well above the MSW resonance and let them propagate adiabatically to vacuum, then $\cos[2\tilde{\theta}(0)] \simeq -1$ and $\cos[2\tilde{\theta}(t)] = \cos(2\theta)$, which gives the neutrino transition probability

$$P_{ex} = \frac{1}{2}[1 + \cos(2\theta)] = \cos^2(\theta).$$

Thus, for small θ, the neutrino transition probability will be large, this is known as the MSW effect [24–26]. It should be noted that the smaller the leptonic mixing angle θ, the slower we are allowed to change the matter density and still have the adiabaticity condition fulfilled (see, *e.g.*, Ref. [87]). The adiabatic propagation of neutrinos in matter can be used, *e.g.*, in treating the oscillation of solar neutrinos, this is discussed in Paper 1. Another approximate solution to problem of neutrino

[9] Since the matter eigenstates have the same momentum, but different masses, they will propagate at different velocities. After some time, the wave packets of different matter eigenstates will no longer overlap, and thus, cannot create the interference term containing $\Gamma(t)$.

oscillations in matter can be given when the matter in which the neutrinos propagate is not very dense, *i.e.*, when we are far below the MSW resonance, this was studied in Refs. [102, 103].

The two-flavor neutrino evolution in matter can also be solved exactly, although in a more cumbersome way. In Paper 2 of this thesis, an exact series solution to this problem is presented for matter of arbitrary density. Essentially, this is done by rewriting the neutrino evolution as a second order non-linear differential equation for the neutrino oscillation probability and expanding the matter density along the neutrino path in orthogonal polynomials.

4.2.4 The three-flavor scenario

In the same way as it is harder to treat three-flavor neutrino oscillations in vacuum than two-flavor neutrino oscillations in vacuum, it is harder to treat the matter effects on neutrino oscillations in a three-flavor scenario.

In the case of three-flavor neutrino oscillations in matter, there is no simple mapping between the fundamental neutrino oscillation parameters as there is in the two-flavor case (see, *e.g.*, Ref. [104]). However, we can still introduce the leptonic mixing angles in matter $\tilde{\theta}_{ij}$ and the effective mass squared differences in matter $\Delta\tilde{m}^2_{ij}$ and have the same kind of parameterization of the leptonic mixing matrix in matter as for the leptonic mixing matrix in vacuum.

One explicit feature of three-flavor neutrino oscillations in matter is that there are now two possible resonances[10]. Depending on the sign of Δm^2_{31}, both resonances may be for neutrinos ($\Delta m^2_{31} > 0$) or there may be one resonance for neutrinos and one for anti-neutrinos ($\Delta m^2_{31} < 0$).[11] The Hamiltonian for three-flavor neutrino oscillations in matter is given by

$$H_f = \begin{pmatrix} V_{\rm CC} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{1}{2E} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix} U^\dagger$$

in the flavor eigenstate basis. If we assume that $\Delta m^2_{21} \ll |\Delta m^2_{31}|$, which is in accordance with experiments (*cf.*, Tab. 4.1), then the resonances will occur for $V_{\rm CC} \sim \Delta m^2_{21}/(2E)$ and $|V_{\rm CC}| \sim |\Delta m^2_{31}|/(2E)$. Let us examine these two regions more carefully. First of all, we assume that $V_{\rm CC} \sim \Delta m^2_{21}/(2E) \ll |\Delta m^2_{31}|$. With this approximation the third mass eigenstate $|\nu_3\rangle$ will still be an approximate eigenstate to the Hamiltonian (*i.e.*, $|\tilde{\nu}_3\rangle \simeq |\nu_3\rangle$) and decouple from the other two states. There will still be interference between the third mass eigenstate and the remaining states leading to oscillations with oscillation phases of the order $|\Delta m^2_{31} L/(2E)|$, but for the moment, we concentrate on the matter eigenstates involved in the resonance.

[10] If also including the loop level diagrams for the NC coherent forward scattering, there will be three possible resonances.

[11] The mass squared difference Δm^2_{21} is known to be positive, see Tab. 4.1.

If constraining the Hamiltonian to the subspace spanned by the remaining states, then we obtain the effective two-flavor Hamiltonian operator

$$H_m^{2\mathrm{f}} = \frac{1}{2}\left[c_{13}^2 V_{\mathrm{CC}} \begin{pmatrix} \cos(2\theta_{12}) & \sin(2\theta_{12}) \\ \sin(2\theta_{12}) & -\cos(2\theta_{12}) \end{pmatrix} + \frac{\Delta m_{21}^2}{2E} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\right].$$

This two-flavor system can be solved in a way analogous to the pure two-flavor case, we obtain the resonance condition

$$c_{13}^2 Q_{21} = \cos(2\theta_{12}),$$

where we have defined $Q_{ij} \equiv 2EV_{\mathrm{CC}}/\Delta m_{ij}^2$. If the condition $V_{\mathrm{CC}} \ll |\Delta m_{31}^2|$ is fulfilled for the entire propagation of the neutrinos and the fast oscillations corresponding to the decoupled third mass eigenstate are averaged out, then we will have the familiar form

$$P_{ee} = c_{13}^4 P_{ee}^{2\mathrm{f}} + s_{13}^4,$$

where $P_{ee}^{2\mathrm{f}}$ is the two-flavor ν_e survival probability computed with $\theta = \theta_{12}$ and the effective potential $V_{\mathrm{CC}}^{2\mathrm{f}} = c_{13}^2 V_{\mathrm{CC}}$, for the ν_e survival probability.

In order to examine the second resonance, we note that if $V_{\mathrm{CC}} \sim \Delta m_{31}^2/(2E) \gg \Delta m_{21}^2/(2E)$, then we may put $\Delta m_{21}^2/(2E) \simeq 0$ in a first-order approximation. The result is that the matrix

$$V = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & e^{\mathrm{i}\delta} \end{pmatrix}$$

commutes with $H_{\mathrm{vac},m}$. Thus, if we introduce the new basis

$$|\nu_i'\rangle \equiv \sum_j V_{ij}^* |\nu_j\rangle,$$

then the Hamiltonian in this basis will be given by

$$H' = VH_mV^\dagger = VU^\dagger H_{\mathrm{CC},f} UV^\dagger + H_{\mathrm{vac},m}.$$

By explicitly writing out this Hamiltonian, we obtain

$$H' = \begin{pmatrix} V_{\mathrm{CC}} c_{13}^2 & 0 & V_{\mathrm{CC}} s_{13} c_{13} \\ 0 & 0 & 0 \\ V_{\mathrm{CC}} s_{13} c_{13} & 0 & V_{\mathrm{CC}} s_{13}^2 + \frac{\Delta m_{31}^2}{2E} \end{pmatrix}.$$

This matrix has an obvious eigenstate with zero eigenvalue, namely the state $|\nu_2'\rangle$, which decouples from the other two states in the same manner as the third mass eigenstate decoupled near the first resonance. What remains is an effective two-flavor scenario involving the states $|\nu_1'\rangle$ and $|\nu_3'\rangle$, which can be solved to yield the resonance condition

$$Q_{31} = \cos(2\theta_{13}).$$

In both of the above cases, the adiabaticity condition can be derived from the effective two-flavor cases, respectively.

As was discussed in Sec. 4.1.2, three-flavor neutrino oscillation probabilities become effective two-flavor probabilities if either $\Delta m^2_{ij} = 0$ for some i and j or if $U_{\alpha i} = 0$ for some α and i. This is also true for the case of three-flavor neutrino oscillations in matter, but using the effective matter parameters instead of the vacuum ones. Such a case is discussed in Paper 3, where the case $2E|V_{\rm CC}| \gg |\Delta m^2_{31}|$ is studied. In this case, the electron neutrino state $|\nu_e\rangle$ becomes equal to the third matter eigenstate $|\nu_3\rangle$, which means that $\tilde{U}_{e3} = 1$ and $\tilde{U}_{e1} = \tilde{U}_{e2} = 0$.

It is worthwhile to note that, in three-flavor neutrino oscillations, also the probabilities of $\nu_\mu \leftrightarrow \nu_\tau$ oscillations are affected. This is a pure three-flavor effect which, as was noted, does not appear in the two-flavor case, since neither ν_μ nor ν_τ can coherent forward scatter via CC interactions on electrons. This three-flavor effect can lead to large errors if trying to analyze $\nu_\mu \leftrightarrow \nu_\tau$ oscillations in matter using a two-flavor scenario, especially close to the high resonance at $Q_{31} \sim V_{\rm CC}$ [105].

A more detailed review on three-flavor effects in neutrino oscillations can be found in Ref. [86].

4.3 Damping effects in neutrino oscillations

As was mentioned in the beginning of this chapter, there may be corrections to the neutrino oscillation probabilities given in Eqs. (4.1) due to neutrino wave-packet decoherence [63–67], neutrino decay [68–75], or some other effect not included in the standard neutrino oscillation scenario [76–84, 106]. Generally, effects like these will affect the mass or matter eigenstates and will alter the neutrino oscillation probabilities to the form

$$P_{\alpha\beta} = \sum_i \sum_j D_{ij} J^{ij}_{\alpha\beta} \exp\left(-\mathrm{i}\frac{\Delta m^2_{ij}}{2p} L\right),$$

where

$$D_{ij} = D_{ji} = \exp\left(-\alpha_{ij} \frac{|\Delta m^2_{ij}|^\xi L^\beta}{E^\gamma}\right)$$

are factors damping the corresponding terms and α_{ij}, β, γ, and ξ are characteristic for the type of effect.

As it turns out, there are two main types of damping factors, the first of these being of the form $D_{ii} = 1$ (*i.e.*, $\xi = 0$ or $\alpha_{ii} = 0$) for all i. This type of damping is essentially an effective averaging of the oscillating terms in the neutrino oscillation probabilities, but preserves the overall probability, *i.e.*,

$$\sum_\beta P_{\alpha\beta} = \sum_\alpha P_{\alpha\beta} = 1.$$

The best example of such a damping factor is the factor arising due to the decoherence of wave-packets of different mass eigenstates, and therefore, this type of

damping is known as "decoherence-like". The second type of damping is of the form $D_{ij} = A_i A_j$, where A_i depends only on the mass (or matter, depending on if we are studying neutrino oscillations in vacuum or in matter) eigenstate $|\nu_i\rangle$. This type of damping factor appears in, for example, neutrino decay and is therefore known as "decay-like". For decay-like damping, the overall probability is generally not conserved.

Damping effects on neutrino oscillations are further studied in Paper 4 of this thesis.

Chapter 5

Neutrino oscillations in experiments

Without the means of experimental confirmation, even the most beautiful physical theory would be quite useless. In this chapter, the most important types of experiments which indicate neutrino oscillations are discussed. These include the atmospheric [27, 107], solar [30–37], and long-baseline reactor [38, 39] neutrino experiments. Clearly, neutrinos can be (and have been) detected in other types of experiments such as supernova [108, 109] and short-baseline reactor [110, 111] neutrino experiments. However, neutrino oscillations have not been detected in experiments of this type and they are therefore not included in this chapter.

5.1 Atmospheric neutrinos

As was mentioned in Ch. 2, the first evidence for neutrino oscillations [27] was published in 1998 by the Super-Kamiokande collaboration and concerned the oscillation of neutrinos produced by cosmic rays hitting the atmosphere. One of the products of the collisions of cosmic rays and particles in the atmosphere is charged pions $\pi^{\pm}$, which subsequently decay according to

$$\pi^{\pm} \longrightarrow \mu^{\pm} + \overset{(-)}{\nu}_{\mu}$$

where the $\mu^{\pm}$ then decays as

$$\mu^{\pm} \longrightarrow e^{\pm} + \overset{(-)}{\nu}_{e} + \overset{(-)}{\nu}_{\mu}.$$

At low energies, the muons always decay before hitting the Earth and the ratio R_{ν_μ/ν_e} between the number of muon and electron neutrinos is therefore equal to two at those energies. At higher energies (muon energies of about 5 GeV and higher),

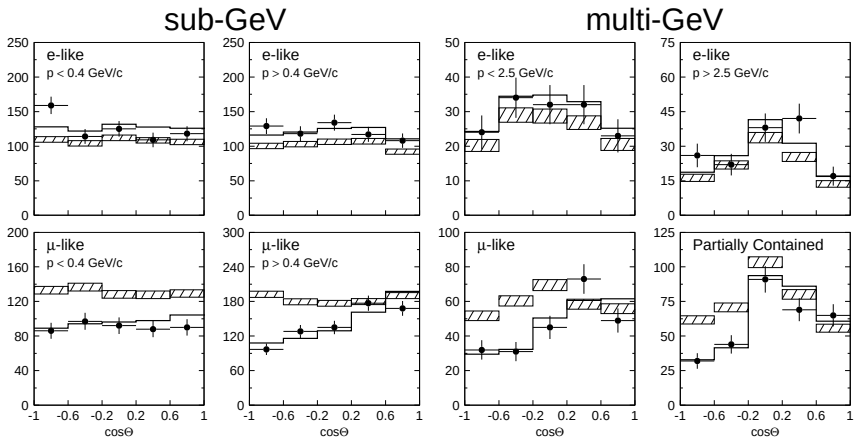

Figure 5.1. The 1998 results from the Super-Kamiokande collaboration. The hatched regions correspond to the expected results without neutrino oscillations and the solid lines to the best-fit expectancy with neutrino oscillations. There is an apparent deficit in the muon neutrino flux at longer path-lengths due to $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. Figure from Ref. [27].

the muons reach the Earth before decaying and lose energy due to interaction with the Earth matter, and thus, the ratio R_{ν_μ/ν_e} is increased at high energies.

What was done by the Super-Kamiokande collaboration in 1998 was to observe e- and μ-like events for different zenith angles (corresponding to different path-lengths for the neutrinos). The Super-Kamiokande results are given in Fig. 5.1. As can be seen from this figure, there are less muon neutrinos detected for longer path-lengths than what is expected from a scenario where there are no neutrino oscillations. This effect can be well described by neutrino oscillations where the ν_μ survival probability $P_{\mu\mu}$ is equal to unity for short path-lengths and then evolves with increasing path-lengths according to the neutrino oscillation formulas.

In more recent times, the Super-Kamiokande collaboration has also published an analysis [107] where neutrino oscillations are compared to neutrino decay and neutrino quantum decoherence as the mechanism behind neutrino flavor transitions. What was found was that among these three options, neutrino oscillations are strongly favored as the leading contribution[1].

[1] As is discussed in Paper 4, neutrino decay and neutrino quantum decoherence can still affect the neutrino oscillation probabilities by the introduction of damping factors.

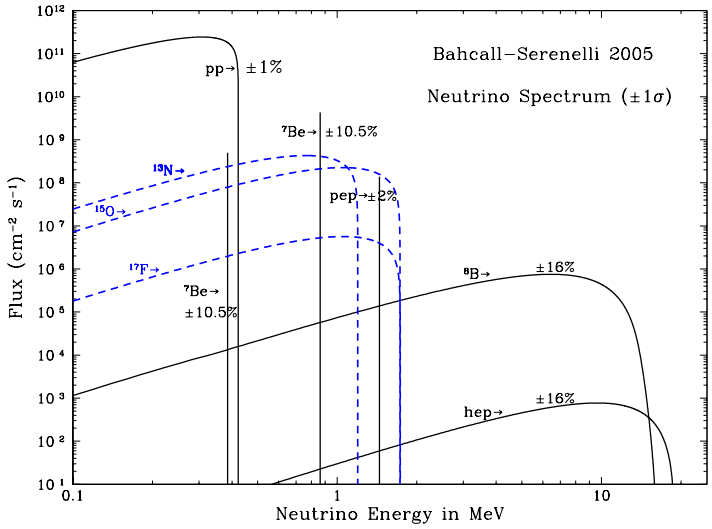

Figure 5.2. The neutrino energy spectra for different thermonuclear fusion reactions inside the Sun. Figure from Ref. [113].

5.2 Solar neutrinos

According to the standard solar model (SSM) [112, 113], the solar energy comes from thermonuclear fusion reactions in the Sun's interior. The most abundant of these reactions is the fusion of two protons

$$p + p \longrightarrow {}^{2}\mathrm{H} + e^{+} + \nu_e,$$

but there are also many other reactions that produce electron neutrinos. In Fig. 5.2, the neutrino energy spectra predicted by the SSM for different reactions are shown. Because of the high density of the Sun, the only direct information from the fusion reactions in its center will be carried away by the neutrinos.

The first experiment to measure the flux of solar neutrinos was the Homestake experiment [114] in which neutrinos were detected using the reaction

$$\nu_e + {}^{37}\mathrm{Cl} \longrightarrow {}^{37}\mathrm{Ar} + e^{-}$$

and then counting the number of Argon atoms produced. The Homestake experiment did detect solar neutrinos. However, it did not detect as many as predicted by the SSM, see Fig. 5.3. There were three possible reasons for this discrepancy, known as the "solar neutrino problem", between theory and experiment:

1. The SSM could be wrong.

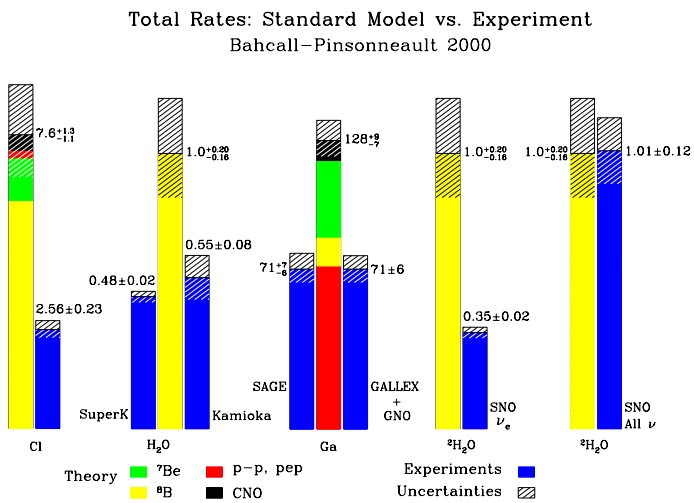

Figure 5.3. The fluxes of solar neutrinos for different neutrino experiments compared to the fluxes predicted by the SSM. Figure from Ref. [115].

2. The calculations of the neutrino cross-sections in the detector could be wrong.

3. Electron neutrinos could be lost when propagating from the center of the Sun to the detector at the Earth.

The first two reasons both seemed quite unlikely, since the SSM was in very good agreement with other observations of the Sun and there was no reason why the cross-section calculations should be wrong.

The Homestake experiment had two major drawbacks. First of all, neutrinos could not be detected in real-time, since some exposure time was needed before it was worthwhile to count the Argon atoms. Second, the reaction used has a threshold energy of about 0.9 MeV, and thus, the Homestake experiment was unable to detect the neutrinos from the fusion of two protons (the "*pp*-reaction", see Fig. 5.2), and thus, the largest part of the neutrino energy spectra was invisible to the experiment.

The second of the two drawbacks was solved by the Gallium experiments SAGE [116] and GALLEX [117], which both utilized the reaction

$$\nu_e + {}^{71}\mathrm{Ga} \longrightarrow {}^{71}\mathrm{Ge} + e^-$$

in much the same way as Homestake utilized the capture of electron neutrinos on Chlorine. The above reaction has an energy threshold of about 0.2 MeV, and thus, the Gallium experiments had a threshold low enough to detect a significant part of

the flux from the *pp*-reaction. Still, also the Gallium experiments detected a deficit in the flux of solar neutrinos compared to the prediction of the SSM.

The first drawback of the Homestake experiment, the inability of real-time detection, was first solved by the Super-Kamiokande experiment [30–32], which detects solar neutrinos by detecting the Cherenkov light in water produced by electrons which have undergone elastic scattering

$$\nu_x + e^- \longrightarrow \nu_x + e^-$$

with the neutrinos. The drawback of Super-Kamiokande having an energy threshold of about 5 MeV is compensated by the real-time detection and the information on the neutrino energy and direction of propagation[2]. In addition, the elastic scattering reaction can occur for both electron neutrinos and neutrinos of other flavors[3]. Also for the Super-Kamiokande, the event rates were lower than the rates predicted from the SSM, see Fig. 5.3.

Finally, the Sudbury Neutrino Observatory (SNO) [33–37], see Fig. 5.4, observed solar neutrinos in a fashion similar to Super-Kamiokande. However, instead of using normal water, the SNO experiment uses heavy water, which also allows the CC reaction

$$\nu_e + d \longrightarrow p + p + e^-$$

and the NC reaction

$$\nu_x + d \longrightarrow p + n + \nu_x,$$

where the NC reaction is equally sensitive to all neutrino flavors. The results of the SNO measurements are also given in Fig. 5.3. As can be seen from this figure, the NC measurements are in excellent agreement with the predictions of the SSM. Thus, the solution to the solar neutrino problem seems to be that neutrinos change their flavor on their way from the Sun to the Earth, which is what would be expected from neutrino oscillations.

5.2.1 The day-night effect

Both the Super-Kamiokande and the SNO experiments have the possibility of detecting neutrinos in real-time. Thus, both of these experiments can tell if the solar neutrinos have passed through the Earth (which happens during night) or not (which happens during day) on their way from the Sun to the detector at Earth. As was discussed in Sec. 4.2, the presence of matter may influence the neutrino oscillation probabilities. It is therefore interesting to examine the possible effect of the solar neutrinos passing through the Earth before detection and study the different event rates during night and day, this is known as the "day-night effect"

[2]Strictly speaking, it is the energy and direction of the electron after the elastic scattering that is measured. However, these quantities will be correlated to the corresponding neutrino quantities.

[3]However, the cross-section for electron neutrino scattering is about six times as large as the cross-sections for the other neutrino flavors due to the fact that there are additional CC diagrams which are only allowed for electron neutrinos.

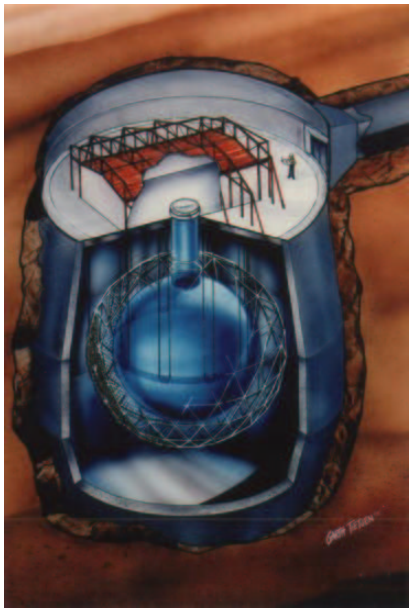

Figure 5.4. A schematic view of the SNO experiment. Courtesy of SNO.

and has been studied in the two-flavor case for a long time [118–124]. The first three-flavor treatment was made in Paper 1 using a constant Earth matter density and was generalized to an arbitrary matter density profile in Ref. [103]. The theoretical prediction is that there will be an increased flux of electron neutrinos during night. However, the experimental uncertainties in this effect are still as large as the effect itself [32, 35, 37].

5.3 Reactor neutrinos

The nuclear power plants on Earth provide a quite abundant artificial source of electron anti-neutrinos. In fact, as was mentioned in Ch. 2, the first neutrinos ever observed were reactor electron anti-neutrinos.

There have been experiments which have tried to observe oscillations of reactor neutrinos with relatively short path-lengths (*e.g.*, the CHOOZ experiment [110, 111]). These experiments have so far only been able to put an upper limit on the leptonic mixing angle θ_{13}.[4] However, deficits in the fluxes of electron anti-neutrinos from reactors have been observed in the KamLAND experiment [38, 39], which observes anti-neutrinos coming from nuclear power plants in or near Japan with an

[4] The existence of damping effects (see Paper 4) could weaken those limits.

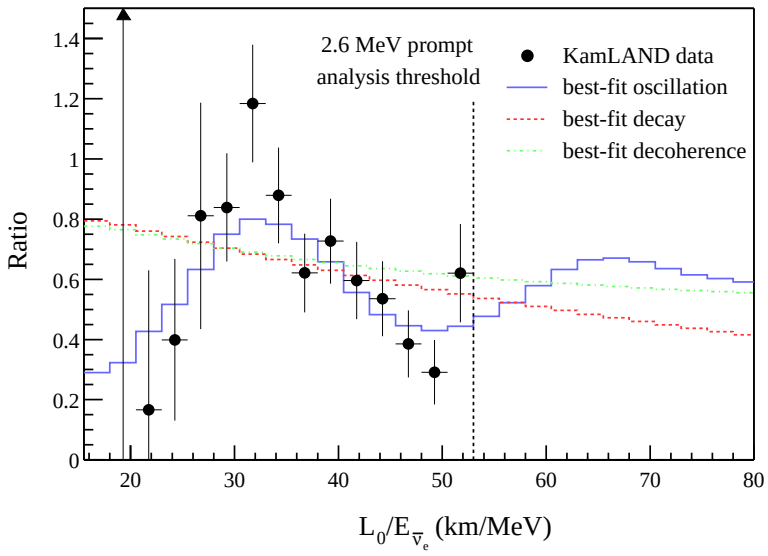

Figure 5.5. The results of the KamLAND experiment along with the prediction of the best-fit of neutrino oscillations. Also shown in this figure are the best-fit predictions if assuming that neutrino decay or neutrino quantum decoherence is the main mechanism behind the suppression of the flux of electron anti-neutrinos. Figure from Ref. [39].

average path-length of about 180 km. The results of the KamLAND experiment are shown in Fig. 5.5. As in the case of atmospheric neutrinos, the results of the KamLAND experiment has also been fit to theories of neutrino decay and neutrino quantum decoherence [39]. The result was similar to the result of the atmospheric neutrinos, with neutrino oscillations clearly favored as the description for the disappearance of reactor electron anti-neutrinos.

Chapter 6

Summary and conclusions

In this Part of the thesis, we have introduced the framework in which the Papers of Part II is set. We have introduced the SM of particle physics and discussed how it can be extended to include massive neutrinos, which are a prerequisite for neutrino oscillations to occur. The theory of neutrino oscillations in vacuum as well as in a background of matter has been briefly reviewed and we have discussed the history of neutrino physics, starting by its introduction by Wolfgang Pauli.

Throughout this introduction, it has also been mentioned where the Papers of Part II come into context. The main conclusions of the Papers are listed below:

- The three-flavor effects on the day-night effect have been computed. Essentially, the day-night difference in the ν_e survival probability scales as c_{13}^6, while the overall probability scales as c_{13}^4. Therefore, the day-night asymmetry scales essentially as c_{13}^2.

- The day-night asymmetry could possibly be used as a complementary way of setting bounds for the leptonic mixing angle θ_{13}. This was further elaborated on by Akhmedov *et al.* in Ref. [103].

- The two-flavor neutrino oscillation probability in matter has been exactly solved in terms of a series solution with a recursive relation for the coefficients. This has been done by rewriting the differential equations describing the Schrödinger equation for neutrino oscillations in matter as a second order non-linear differential equation for the neutrino oscillation probability.

- The effective two-flavor case which arises in matter with very large densities ($2VE \gg \Delta m_{ij}^2$) has been studied in detail. The accuracy of the approximation of infinitely dense matter ($2VE/\Delta m_{ij}^2 \to \infty$) has also been examined both numerically and analytically.

- The concept of general damping signatures altering the neutrino oscillation formulas has been introduced along with examples of scenarios where they

occur. The damping signatures have been divided into two main classes, decoherence- and decay-like signatures.

- The alteration of the neutrino oscillation probabilities due to damping signatures have been carefully examined in both the two- and three-flavor cases.
- The effect of damping signatures on the determination of the fundamental neutrino oscillation parameters has been discussed. We have given an example where damping signatures may lead to an erroneous determination of the leptonic mixing angle θ_{13} and examined how one can distinguish among different types of damping signatures.

In addition, more detailed conclusions are listed separately at the end of each Paper.

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Part II

Scientific papers

1

Paper 1

Mattias Blennow, Tommy Ohlsson, and Håkan Snellman

Day-night effect in solar neutrino oscillations with three flavors

Physical Review D **69**, 073006 (2004).

2

Paper 2

Mattias Blennow and Tommy Ohlsson

Exact series solution to the two flavor neutrino oscillation problem in matter

Journal of Mathematical Physics **45**, 4053 (2004).

Paper 3

Mattias Blennow and Tommy Ohlsson

Effective neutrino mixing and oscillations in dense matter

Physics Letters B **609**, 330 (2005).

3

Paper 4

Mattias Blennow, Tommy Ohlsson, and Walter Winter

Damping signatures in future reactor and accelerator neutrino oscillation experiments

Journal of High Energy Physics (submitted).

4

Matter and Damping Effects in Neutrino Mixing and Oscillations

Mattias Blennow

Errata

The following typos have been found in the thesis:

Page ii, first sentence "Akademisk avhandling for avlagande av teknologie licentiatexamen (TeknL) inom amnesomradet teoretisk fysik." should be replaced by "Akademisk avhandling for avlaggande av teknologie licentiatexamen (TeknL) inom amnesomradet teoretisk fysik."

Page v, List of papers, Paper 4 "JHEP (submitted)" should be replaced by "JHEP (to be published)"

Page 7, second paragraph "It was this year that the German physicist Wolfgang Pauli ..." should be replaced by "It was this year that the Austrian physicist Wolfgang Pauli ..."

Page 21, title of section 4.1 "Neutrinos oscillations in vacuum" should be replaced by "Neutrino oscillations in vacuum"

Page in front of Paper 4 "Journal of High Energy Physics (submitted)" should be replaced by "Journal of High Energy Physics (to be published)"