Bose-Einstein Correlation Effect in a Boson-Antiboson System

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Abstract

It is shown that a Bose-Einstein correlation effect, similar to that observed in a pair of identical bosons when emitted in the same phase space region, will also occur in the C=+1 sub-sample of spinless boson-antiboson pairs. This has been verified experimentally in a recent OPAL study of Bose-Einstein correlation using inclusive $Z^0$ hadronic decays into $K_S^0K_S^0$ pairs. The expectation of this effect in higher order Bose-Einstein correlations of many bosons and antibosons systems, such as the $K^0K^0\bar{K}^0$ state, is shown. The possible impact of this boson-antiboson correlation on the coupled channel analyses carried for the determination of the $f_0(975) \rightarrow K\bar{K}$ branching ratio is investigated.

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1 Introduction

In reactions between particles which lead to multi-hadronic final states the interference between two identical bosons, the so called Bose-Einstein Correlation (BEC), is well known. These correlations lead to an enhancement of the number of identical bosons over that of non-identical bosons when the two particles are close to each other in phase space. Experimentally this effect, also known as the GGLP effect, was for the first time observed in particle physics by Goldhaber et al., [1] in like-sign charged pions produced in \( pp \) annihilations at \( \sqrt{s} = 2.1 \) GeV. In addition to the quantum mechanical aspect of the BEC, these correlations can also be used to estimate the dimension of the emitting source of the identical bosons [2]. Some recent reviews which summarize the underlying BEC theoretical aspects and the experimental results are given, for example, in references [3], [4] and [5].

In this paper we will show that under certain conditions a Bose-Einstein Correlation like enhancement is also to be expected in a boson-anti boson system like the \( \eta^0 \eta^0 \) pair.

In section 2 we will describe briefly the Bose-Einstein enhancement effect in identical bosons pairs. The current experimental knowledge concerning the BEC effect in like-sign charged pions and kaons is summarized in section 3. The results from a recent OPAL study of the correlation in the \( K_S^0 K_S^0 \) system are described in some details in the same section. In section 4 we will show that in a sample of spinless boson-antiboson system, which is a mixture of \( C = +1 \) and \( C = -1 \) states, the \( C = +1 \) part behaves like identical bosons in that it produces a BEC like low mass enhancement. The possible effect of this low mass enhancement on the determination of the \( K \bar{K} \) decay branching ratios of the problematic scalar mesons \( f_0(975) \) and \( a_0(980) \) will also be described. Next we discuss in section 5 the expectation of high order Bose-Einstein correlations involving many boson and antiboson system. Finally the summary and conclusions are presented in section 6.

2 Bose-Einstein Correlation in a Nutshell

To illustrate the underlying origin of the Bose-Einstein correlation, we will here follow an often used simplified description of the effect as that e.g. given in Ref. [4]. To this end one can consider for example an \( e^+e^- \) (or \( p\bar{p} \)) reaction leading to a multi-pion final state that includes at least one pair of like-sign charged pions. These identical pions may be envisaged to be emitted incoherently from an interaction volume as shown in Fig. 1. In this figure the pair of identical pions, \( \pi_1 \) and \( \pi_2 \), emitted from the two points \( r_A \) and \( r_B \), are detected by two counters placed at the \( r_1 \) and \( r_2 \) positions. Due to the identical nature of the two bosons, there exist two amplitudes which contribute to the same final state (see Fig. 1). The probability \( P_{1,2} \) for the final state shown in Fig. 1, assuming the production amplitude to be independent of \( r \) and uncorrelated, is given by:

\[
P_{1,2} = \int d^4r_A d^4r_B |A_{1,2}|^2 \rho(r_A) \rho(r_B) = 1 + |\rho(q)|^2 \equiv C_2(q)
\]
with \[ A_{1,2} = \frac{1}{\sqrt{2}} \left[ e^{i p_1 (r_1 - r_A)} e^{i p_2 (r_2 - r_B)} + e^{i p_1 (r_1 - r_B)} e^{i p_2 (r_2 - r_A)} \right] \] (2)

Here \( q \) is the four momentum difference between the two identical bosons, \( q = p_1 - p_2 \).

At the limit of \( q = 0 \) the correlation function reaches the value \( C_2(0) = 2 \) whereas at large \( |q| \) it is equal to \( C_2(q) = 1 \). For convenience we will further on denote \( C_2 \) by \( C \). In practice, the correlation function is measured by dividing a two particle correlation of identical particles \( \sigma(p_1, p_2) \) with a reference distribution \( \sigma_0(p_1, p_2) \) which should be, as far as possible, similar to that of \( \sigma(p_1, p_2) \) in all its details but void of the Bose-Einstein effect. Using these two distributions one defines the correlation function \( C(p_1, p_2) \) as

\[ C(p_1, p_2) \equiv C(q) = \frac{\sigma(p_1, p_2)}{\sigma_0(p_1, p_2)} \] (3)

These correlations have been observed in like-sign charged pion-pairs over a wide energy range and for many initial state reactions leading to multi-pion final states. A common parameterization of the correlation function, used in many experiments, is the one given by the expression:

\[ C(Q) = \left[ 1 + \lambda e^{-Q^2 R^2} \right] \] (4)

where \( Q \) is related to the center of mass energy squared \( s_2 \) of the two identical bosons of mass \( m_B \), by:

\[ Q = Q_2 = \sqrt{-q^2} = \sqrt{s_2 - 4m_B^2} \] (5)

The parameter \( \lambda \) measures the fraction of incoherent emission which is usually referred to as the "Chaoticity" or strength of the BEC effect. The radius, \( R \) in GeV\(^{-1}\) (or \( R_0 = \hbar c R \) in fm), of Eq. 4 is related to the boson emitter dimension.
3 The Experimental Situation

In addition to the former BEC studies in like-sign charged pions, recently the $\pi^+\pi^\pm$ correlations have also been studied at high energy $e^+e^-$ annihilations by three experiments at LEP, on and around the $Z^0$ mass. In these studies OPAL has measured the radius of the pion emitting source to be $(0.928 \pm 0.019 \pm 0.150)$ fm [6] whereas ALEPH reported $(0.65 \pm 0.04 \pm 0.16)$ fm [7] and DELPHI obtained $(0.62 \pm 0.04 \pm 0.20)$ fm [8]. These values for the radius $R_o$, as well as the those found for the Chaoticity $\lambda$, are very similar to those obtained at lower $e^+e^-$ energies using the variable $Q$ [5]. Higher values for $R_o$ are obtained when the BEC are studies in terms of the two variables $q_t$ and $q_o$ proposed by Kopylov and Podgoretskii [9]. These are defined as follows: if $q = p_1 - p_2 = (q_o, q_i)$ then $q_t$ denotes the component of $q$ perpendicular to $p_1 + p_2$, where $p_1, p_2$ and $q$ are the three momenta vectors.

Unlike the ample information that does exist on the BEC in pion pairs, very few BEC studies have been carried out with charged and neutral kaon-pairs [10, 11, 12, 13]. Those studies which have also given an estimate for the emitter dimension $R_o$, are summarized in Table 1 together with the recent results of OPAL [14]. In that paper, the OPAL collaboration has reported on a study of $K^0_S K^0_S$ correlation in some $5 \times 10^5$ $Z^0$ multi-hadron decays from which about 6000 events have been found to contain two or more $K^0_S$ decaying into $\pi^+\pi^-$. In order to search for a low mass enhancement in the $Q$ distribution of the $K^0_S K^0_S$ data, it was necessary to choose a reference sample, $\sigma_0(p_1, p_2)$, which should simulate the experimental data in all its features apart from the BEC effect. In the studies of the BEC like-sign charged pions one natural choice for the reference sample is the correlations obtained from the $\pi^+\pi^-$ sample of the very same data. Such natural reference samples do not exist for the BEC study in a $K^0_S K^0_S$ data sample. In particular the use of $K^+ K^-$ data as a reference sample suffers from two drawbacks. The first is the strong presence near the threshold of the $\phi(1020) \rightarrow K^+ K^-$ decay. The second is connected to the difficulties, in particular at high energies, to separate the charged kaons from protons (anti-protons) and the much more abundant charged pions. For these reasons, a detailed Monte-Carlo generated sample was chosen as a reference sample after verifying that it describes the experimental data in all its details. The resulting correlation function $C(Q)$ obtained by OPAL is shown in Fig. 3a. Two important features are present. The first, there are indications for the decay of resonances into $K^0_S K^0_S$ pairs in the mass range 1.2 to 1.6 GeV. The other prominent feature of the $C(Q)$ distribution is the presence of a low mass enhancement similar in shape and strength to that obtained for the BEC of like-sign charged pion pairs in the very same experiment [6].

4 Physics Interpretation

The interpretation of the low mass enhancement seen in the $K^\pm K^\pm$ studies, listed in Table 1, is clearly identical to that deduced in the $\pi^\pm\pi^\pm$ analyses. In both cases one deals with identical boson pairs having an overall charge of $\pm 2$ which cannot be the decay products of known mesons and therefore are automatically attributed to the Bose-Einstein Correlations. This interpretation for the enhancement seen in the $K^0_S K^0_S$
Table 1: Results for \( \lambda \) and \( R_o \) obtained from BEC studies of like-sign charged kaons and \( K^0 \) pairs using the Goldhaber variable \( Q \) and the variables \( q_t \) and \( q_o \) of Kopylov and Podgoretskii [9]. The errors given are statistical only.

correlation studies is not so evident since their origin may also be the \( K^0 \bar{K}^0 \) pairs, i.e. from non-identical bosons where a priori no Bose-Einstein Correlation enhancement should be present.

### 4.1 Bose-Einstein Like Phenomenon

To understand the nature of the low mass enhancement seen in the \( Q \) distribution of the \( K^0 \bar{K}^0 \) system, we will at first ignore contributions from resonances decay. In that case, the \( K^0 \bar{K}^0 \) enhancement seen in the OPAL experiment is left with two possible sources, \( K^0 \bar{K}^0 (K^0 \bar{K}^0) \) and the \( K^0 \bar{K}^0 \) pairs. The first are pairs of identical bosons where a BEC enhancement should be expected. However these pairs of identical bosons should only account, at low \( Q \) values, for about 25% of the \( K^0 \bar{K}^0 \) sample investigated by OPAL [14] and therefore it is unlikely that they are the only source for the observed large threshold enhancement. Moreover, the \( K^0 \bar{K}^0 \) low mass BEC enhancement, seen by Cooper et al. [13], utilized a pure \( K^0 \bar{K}^0 \) sample obtained from the annihilation process \( pp \to K^0 \bar{K}^0 \pi^+ \pi^- \).

Neglecting CP violation \(^1\) we will show in the following that a Bose-Einstein like threshold enhancement is nevertheless expected in the \( Q(K^0, K^0) \) distribution although its origin is a pure \( K^0 \bar{K}^0 \) system [15].

A boson antiboson \((B \bar{B})\) pair is an eigenstate of the charge conjugation operator \( C \). In the absence of outside constraints, the density, \( \rho \), of the \((B \bar{B})\) state with the eigenvalue \( C = +1 \) is equal to that having \( C = -1 \) which means:

\[
\left\langle B \bar{B} C=+1 \right\rangle \rho \left| B \bar{B} C=+1 \right\rangle = \left\langle B \bar{B} C=-1 \right\rangle \rho \left| B \bar{B} C=-1 \right\rangle = 1/2 \quad (6)
\]

and

\[
\left\langle B \bar{B} C=+1 \right\rangle \rho \left| B \bar{B} C=+1 \right\rangle = 0
\]

We further can write the probability amplitude for a given charge conjugation eigenvalue \( C \) as follows:

\[
\frac{1}{\sqrt{2}} \left| B; \bar{B} \right\rangle_{C=\pm 1} = \frac{1}{2} \left| B(p); \bar{B}(-p) \right\rangle \pm \frac{1}{2} \left| \bar{B}(p); B(-p) \right\rangle \quad (7)
\]

\(^1\)The inclusion of CP violation effect in the following discussion is straightforward but with negligible consequences at the current experimental precision level.
where \( p \) is the three momentum vector defined in the \( B\bar{B} \) center of mass system. In the limit of \( Q = 0 \), where the BEC should be maximal, \( p = 0 \) and Eq. 7 reads

\[
\frac{1}{\sqrt{2}} |B; \bar{B}\rangle_{C=\pm 1} = \frac{1}{2} |B(0); \bar{B}(0)\rangle \pm \frac{1}{2} |\bar{B}(0); B(0)\rangle \tag{8}
\]

This means that, at \( Q = 0 \), the probability amplitude for the \( C = -1 \) state (odd \( \ell \) values) is zero whereas that of the state \( C = +1 \) (even \( \ell \) values) is maximal. It is here important to note that these equations do hold for any spinless boson-antiboson pair such the \( K^0\bar{K}^0 \), the \( K^+K^- \) and the \( \pi^+\pi^- \) systems\(^2\). What does distinguish the \( K^0\bar{K}^0 \) from the other spinless boson-antiboson pairs is the simplicity with which one is able to project out the \( C = +1 \) or the \( C = -1 \) parts of the probability amplitude.

As is well known, the \( K^0 \) and the \( \bar{K}^0 \) mesons are described in terms of the two CP eigenstates, \( K^0_S \) with \( \text{CP} = +1 \) and \( K^0_L \) with \( \text{CP} = -1 \). This being the case it then follows that, when the \( K^0\bar{K}^0 \) pair is detected through their \( K^0_S \) and \( K^0_L \) decays, a definite eigenvalue \( C \) of the \( K^0\bar{K}^0 \) system can be selected. Thus, as \( Q \) approaches zero, an enhancement should be observed in the probability to detect \( K^0_S K^0_S \) pairs and/or \( K^0_L K^0_L \) pairs (\( C = +1 \)), whereas a decrease should be seen in the probability to find \( K^0_S K^0_L \) pairs (\( C = -1 \)). Since both Copper et al. \[13\] and OPAL \[14\] restricted their investigations to the \( K^0_S K^0_S \) pairs, which pick out the \( C = +1 \) state, a BEC enhancement in the low \( Q \) region is expected. This \( C(Q) \) dependence on \( Q \), for the \( \lambda = 1 \) value, is illustrated schematically in Fig. 2. As seen, when \( Q \) approaches the 0 value, \( C(Q) \) splits into two branches. The first rises up to the value two and the other decreases to zero in such a way that their sum remains constant and equal to one for all \( Q \) values. This means, that if all the decay modes of the \( K^0\bar{K}^0 \) pairs are detected and used simultaneously in the same correlation analysis, then according to Eqs. 7 and 8, no BEC effect will be observed at \( Q = 0 \). This however should not come as a surprise if one recalls that the \( K^0\bar{K}^0 \) system is at the very end not composed of identical bosons.

In conclusion, when studying the \( K^0_S K^0_S \) pairs, as in the OPAL work, a BEC-like enhancement should be observed, similar to that of like-sign charged pions and charged kaons, irrespective of their origin, be it identical \( K^0\bar{K}^0 \) (\( K^+\bar{K}^- \)) boson-pair or \( K^0\bar{K}^0 \) boson-antiboson system.

Under the assumption that the low \( Q \) enhancement shown in Fig. 3 is entirely due to the BEC effect, OPAL \[14\] has fitted the expression

\[
C(Q) = N(1 + f(Q) \lambda e^{-Q^2R^2})(1 + \delta Q) \tag{9}
\]

to their data where \( N \) is a normalization factor. In this expression, which is related to Eq. 4, the term \( \delta Q \) is added in order to account for long range correlations and the function \( f(Q) \) is introduced to take care of the experimental background and its \( Q \) dependence. In practice \( f(Q) \) was found in the OPAL experiment to be independent of \( Q \) and equal to 0.79.

The fitting procedure was repeated for four, \((\lambda, R_o, \delta, N)\), three \((\lambda, R_o, \delta)\) and two \((\lambda, R_o)\) free parameters. The results of the three fits are presented in Table 2 and the

\(^2\)The \( \pi^0\pi^0 \) pair is a pure \( C = +1 \) state.
Bose–Einstein Correlation in the $K^0\bar{K}^0$ System

Figure 2: A schematic behavior of the correlation function $C(Q)$ for the $K^0\bar{K}^0$ system. At low $Q$ values the $C=+1$ probability amplitude reaches the value 2 at the limit of $Q=0$ whereas the $C=-1$ probability amplitude reaches the value 0. The sum of the $C=+1$ and $C=-1$ remains flat.

2-parameter fit values are also compared in Fig. 3a with the data. Only moderate correlation exists between both parameters as is illustrated in Fig. 3b where $R_o$ versus $\lambda$ is plotted. The cross in the figure represents the best OPAL value and the two contours define the allowed domains with 50 and 90 % confidence level. The final $R_o$ and $\lambda$ values given by OPAL [14] are:

$$\lambda = 1.12 \pm 0.33 \pm 0.29 \quad \text{and} \quad R_o = (0.72 \pm 0.17 \pm 0.19) \text{ fm}$$

where the first error is the statistical one and the second is the systematic error.

<table>
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<th>Free Parameters</th>
<th>$\chi^2/N_D$</th>
<th>$\lambda$</th>
<th>$R_o$ (fm)</th>
<th>$\delta$ (GeV$^{-1}$)</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$, $R_o$, $\delta$, $N$</td>
<td>1.9/6</td>
<td>1.13 ± 0.47</td>
<td>0.61 ± 0.16</td>
<td>0.10± 0.11</td>
<td>0.89±0.11</td>
</tr>
<tr>
<td>$\lambda$, $R_o$, $\delta$</td>
<td>2.9/7</td>
<td>1.01 ± 0.49</td>
<td>0.71 ± 0.18</td>
<td>0.01±0.04</td>
<td>-----</td>
</tr>
<tr>
<td>$\lambda$, $R_o$</td>
<td>3.4/8</td>
<td>1.12 ± 0.33</td>
<td>0.72 ± 0.17</td>
<td>-----</td>
<td>-----</td>
</tr>
</tbody>
</table>

Table 2: Results of the $\chi^2$ fits of $C(Q)$, as defined in Eq. 9, to the $K^0_S\bar{K}^0_S$ OPAL data in the range of $Q=0$ to $Q=2.0$ GeV. The region of the $f_2(1525)$ and $f_0(1710)$ resonances, $1.1 \leq Q \leq 1.55$ GeV, has been omitted from the fit. The errors are statistical only.

Following the OPAL observations another group at LEP, the DELPHI collaboration, has repeated the BEC analysis using also their data of the $Z^0$ hadronic decay. Prelim-
inary results of that experiment [16], based on somewhat smaller $K^0\bar{K}^0$ data sample, has yielded $\lambda = 0.67 \pm 0.28 \pm 0.12$ and $R_0 = 0.50 \pm 0.14 \pm 0.17$ fm. These results are somewhat different but still within errors consistent with those reported by OPAL.

In evaluating the BEC interpretation of the $K^0\bar{K}^0$ low mass enhancement one should note the following:

a) The $R_0$ value is similar to those obtained in previous BEC analyses of like-sign charged pions.

b) Although the value of the Chaoticity parameter $\lambda$ is somewhat higher than those obtained in the analyses of like-sign charged pions, it still agrees with them within the current large errors (note that the $\lambda$ upper limit is 1).

c) The $C=+1$ condition for a BEC low mass enhancement in the $K\bar{K}$ state is equivalent to require that the system will only have even angular momentum $\ell$ values.

So far OPAL has not carried out a partial wave analysis of the $K^0\bar{K}^0$ system at low $Q$ values, but due to the proximity of the enhancement to threshold it is not unreasonable to expect that it is strongly dominated by the $s$-wave.

4.2 Resonance Decay Interpretation

The isoscalar meson $f_0(975)$ and the nearby lying isovector $a_0(980)$, both of which are below but near to the $K\bar{K}$ threshold, pose to this very day a puzzle as they do not fit in a simple way to the SU(3) nonets scheme built out of $q\bar{q}$ states. Their mass values are too low, their width too narrow and their apparent strong coupling to the $K\bar{K}$ channel is difficult to understand if they are the light quark members of the scalar nonet [17, 18]. In fact, it is currently believed that the $p$-wave $q\bar{q}$ mesons that should be assigned to the $J^{PC} = 0^{++}$ scalar nonet are the $a_0(1320), K_0(1430), f_0(1400)$ and the $f_0(1590)$ resonances [17].

This being the case, several attempts have been made to define the nature of the $f_0(975)$ resonance in terms of structures other than that of a $q\bar{q}$ state. Among those considered are also a $q\bar{q}q\bar{q}$ structure and a $K\bar{K}$ molecule [17, 19]. This molecule-like proposal is to a large extent motivated by the proximity of the $f_0(975)$ to the $K\bar{K}$ threshold and its relative large branching ratio of

$$BR(f_0(975) \to K\bar{K}) = (21.9 \pm 2.4)\%,$$

deduced from the coupled channel analyses [18].

The $f_0(975)$ in its decay to $\pi^+\pi^-$ is well established and has recently also been seen in the multi-hadron decays of the $Z^0$ [20]. Since the $K\bar{K}$ threshold is above the $f_0(975)$ mass, its decay into kaon pairs cannot cover the full Breit-Wigner width but should be seen as a threshold enhancement in reactions having a kaon pair in their final state. Experimentally the use of reactions such as $\pi^- p \to n K^+ K^-$ are disfavored since the $K^+ K^-$ system is strongly dominated by the presence of the vector meson $\phi(1020)$ which is very near the $K\bar{K}$ threshold.

For this reason the $K^0_S K^0_S$ system of the reaction $\pi^- p \to n K^0\bar{K}^0 \to n K^0_S K^0_S$, has been utilized in various coupled channel analyses to determine the scalar $f_0(975)$ decay branching ratio to $K\bar{K}$ [18, 21]. In these analyses the possibility that a BEC like effect could contribute to the low $K^0_S K^0_S$ mass region has at that time not explicitly been considered.
Figure 3: a) The OPAL measured correlation function $C(Q)$ of the $K^0 \bar{K}^0$ pairs in the range $0.0 \leq Q \leq 2.0$ GeV. The error bars represent the combined statistical uncertainty of the data and Monte-Carlo samples. The solid line represents the best fit to the data using Eq. 9 with a 2-parameter $\chi^2$ fit. The region of the $f_2'(1525)$ and the $f_0(1710)$ has been omitted from the fit. b) $\lambda$ versus $R_o$ obtained in a 2-parameter $\chi^2$ fit of $C(Q)$ to the OPAL data. The cross represents the best values and the contours around it represent the allowed regions for a 50% and 90% confidence level using the statistical errors only.
As mentioned before, in a spinless boson-antiboson system the $C = +1$ state corresponds to an even angular momentum $\ell$ value and the $C = -1$ corresponds to an odd $\ell$ value. This being the case, Lipkin [22] has pointed out that the correlation function $C(Q)$ for a $K^0\bar{K}^0 \rightarrow K_S^0K_S^0$ case can be written as:

$$\frac{2P_{K_S^0K_S^0}(Q)}{P_{K^0\bar{K}^0}(Q)} = \frac{\sum_{\text{even}} \int d\xi <f_\ell(Q,\xi)|f_\ell(Q,\xi)>}{\sum_{\text{odd}} \int d\xi <f_\ell(Q,\xi)|f_\ell(Q,\xi)>} \rightarrow 1$$

Here $\xi$ represents all possible variables of the $BB$ system in addition to $Q$. At high $Q$ values the ratio should be less than 1 due to the fact that at the denominator odd and even waves are summed up whereas at the nominator only the even ones. As $Q$ decreases to zero, the number of partial waves decreases until near threshold only the $s$-wave is present and the ratio reaches from below the value one.

This means that even if the $s$-wave contribution to the $K_S^0K_S^0$ state remains constant and independent of $Q$, BEC like enhancement will occur. As a consequence, partial wave analysis of the $K_S^0K_S^0$ system as a function of $Q$ should be useful to separate the BEC enhancement from the contribution of an $s$-wave resonance like the $f_0(975)$. At the same time it will be very instructive to demonstrate that indeed in a $\textit{bona fide}$ BEC enhancement, like the one observed in the $\pi^\pm\pi^\pm$ systems, the $s$-wave contribution does not increase as $Q \rightarrow 0$. In this connection it will also be interesting to compare the $Q$ dependence of the $s$-wave of the $\pi^\pm\pi^\pm$ system with that of the $\pi^\pm\pi^0$ system. Both are in a pure $I = 2$ state when $\ell$ is even, and in particular when $\ell = 0$, apart from the fact that the first system is composed of identical bosons whereas the second not.

5 Higher Order Bose-Einstein Correlations

Identical Pions

In recent years the BEC effect has also been studied in systems of more than two identical pions [23, 24], often referred to as Higher Order BEC. To study these, the simplest variable to use is the one which is the natural extension of the $Q_2 = Q$ variable applied to a pair of identical bosons. Namely,

$$Q_n^2 = s_n - (n \cdot m)^2 = M_n^2 - (n \cdot m)^2$$

where $n$ is the number of identical bosons (the $nth$ order) used in the correlation and $m$ the mass of the boson. This variable, $Q_n^2$, is then related to the order two variable $Q_2^2$ by

$$Q_n^2 = Q_{1,2}^2 + \ldots + Q_{(n-1),n}^2$$

The expected BEC enhancement is $C_n(0) = n!$ for $\lambda = 1$. These higher order BEC have been observed by the UA1-MINIMUM BIAS, (UA1MB), collaboration in like-charged pions up to the 5th order in $\bar{p}p$ annihilations at $\sqrt{s} = 630$ and 900 GeV [24]. The BEC

\[\text{limit 1 of the BEC enhancement, rather than the value 2, is due to the fact that the denominator is normalized to the total $K^0\bar{K}^0$ state rather than to the over Q integrated value of the nominator used in the previously defined correlation function $C(Q)$}.\]
results of that experiment, using the variable $Q^2$ rather than $Q$, are shown in Fig. 4. In that experiment an estimate of the dimension of the pion emitter and its Chaoticity has been derived using the expressions for the correlation function of reference [25]. Specifically for $n = 3$, which will be of interest later on, the basic formula to be fitted, taking a Gaussian distribution for the emitter, is:

$$C_3(Q_3) = 1 + 6\lambda(1 - \lambda)e^{-\left(\frac{1}{2}RQ_3^2\right)} + 3\lambda^2(3 - 2\lambda)e^{-\left(\frac{1}{2}RQ_3^2\right)} + 2\lambda^3e^{-\left(RQ_3^2\right)}$$

A similar expression has been considered by replacing $RQ_3^2$ with $RQ_3$ assuming an exponential distribution for the source which was found by UAIMB to be a better description of the data. While the $\lambda$ parameter is more or less constant to all BEC orders, the $R$ value increases. To offset this trend, UAIMB has introduced a phenomenological factor so that the modified radius $R_n = R\sqrt{2/n(n - 1)}$ remains almost constant to all orders.

**Multi - Neutral Kaons**

Next let us consider the general case of a multi $K^0$ and $\bar{K}^0$ variable of order $n = n_1 + n_2$ where $n_1$ is equal to the number of $K^0$ and $n_2$ to the number of $\bar{K}^0$. Furthermore we will not restrict ourselves to the case where $n_1 = n_2$, and allow also that $n_1 \neq n_2$, which are not eigenstates of the charge conjugation operator $C$. The expected BEC enhancement depends on $\lambda$ and on the detected final state configuration of the variable. For the $\lambda = 1$ case, this can be derived from Eq. 12. A summary of the expected values of the correlation function at $Q = 0$ is given in Table 3 for the $n = 2$ and $3$ cases. In that table, $\{K^0\}$ stands for the leptonic decays, $K^0 \rightarrow \mu(e)\pi\nu$, which do not isolate a particular CP eigenvalue.

For $n = 2$, all possible final states of a pair of identical $K^0\bar{K}^0$ (or their charge conjugate state) will show at $Q = 0$ a BEC enhancement of $C_2(0) = 2$. In the case of a $K^0\bar{K}^0$ pair the behavior of the BEC does depend, as discussed in some details earlier, on the mode of detection. Further to note is the case where the $K^0(\bar{K}^0)$ detection occurs via the leptonic decays. Since through this detection mode one sums up both the $C = +1$ and $C = -1$ states, no enhancement should be observed, i.e., $C_2(0) = 1$. This will also be the case when one adds in the BEC analysis all the four possible $K^0_{3L}\bar{K}^0_{3L}$ pairs.

In the case of three identical $K^0$ (or $\bar{K}^0$) mesons, all modes of detection will manifest the same BEC enhancement, namely $C_3(0) = 6$ for $\lambda = 1$, identical to the expectation for a three like-charged pions. Of a particular interest is the situation for the 3rd order BEC of the system $K^0\bar{K}^0K^0$ or its charge conjugate state. The sub-samples detected via identical decay modes of definite CP eigenvalue, i.e. the $3K^0_S$ or $3K^0_L$ states, should have a correlation enhancement that reaches the value 6. For a mixed decay modes states, $K^0_S\bar{K}^0_{3L}$ or $K^0_S\bar{K}^0_SK^0_L$, one expects $C_3(0) = 2/3$.

So far higher order BEC of neutral kaons have not been measured. However, an estimate from the OPAL study [14] of the lower BEC order ($n = 2$), a sample of some 2 to 3 million $Z^0$ hadronic decays should yield a sufficiently large number of three $K^0_S$
Figure 4: Plots for higher order BEC of like-sign pions of Ref. 24. (a), (b) 2nd, (c), (d) 3rd, (e), (f) 4th and (g), (h) 5th order. (a), (c), (e) and (g) are uncorrected data samples; the other four plots show data corrected for the Coulomb interaction.
Table 3: BEC enhancement in two and three neutral kaon systems for $\lambda = 1$.

<table>
<thead>
<tr>
<th>$K^0 K^0$</th>
<th>$C_2(0)$</th>
<th>$K^0 \bar{K}^0$</th>
<th>$C_2(0)$</th>
<th>$K^0 K^0 K^0$</th>
<th>$C_3(0)$</th>
<th>$K^0 \bar{K}^0 K^0$</th>
<th>$C_3(0)$</th>
</tr>
</thead>
<tbody>
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<td>$K_{lep}^0 K_{lep}^0$</td>
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<td>$K_{lep}^0 K_{lep}^0$</td>
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<td>$K_{lep}^0 K_{lep}^0 K_{lep}^0$</td>
<td>6</td>
<td>$K_{lep}^0 K_{lep}^0 K_{lep}^0$</td>
<td>2</td>
</tr>
<tr>
<td>$K_S^0 K_S^0$</td>
<td>2</td>
<td>$K_S^0 K_S^0$</td>
<td>2</td>
<td>$K_S^0 K_S^0 K_S^0$</td>
<td>6</td>
<td>$K_S^0 K_S^0 K_S^0$</td>
<td>6</td>
</tr>
<tr>
<td>$K_L^0 K_L^0$</td>
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<td>$K_L^0 K_L^0$</td>
<td>2</td>
<td>$K_L^0 K_L^0 K_L^0$</td>
<td>6</td>
<td>$K_L^0 K_L^0 K_L^0$</td>
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</tr>
<tr>
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<td>$K_S^0 K_L^0 K_L^0$</td>
<td>6</td>
<td>$K_S^0 K_L^0 K_L^0$</td>
<td>2/3</td>
</tr>
<tr>
<td>$K_L^0 K_S^0$</td>
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<td>0</td>
<td>$K_S^0 K_L^0 K_L^0$</td>
<td>6</td>
<td>$K_S^0 K_L^0 K_L^0$</td>
<td>2/3</td>
</tr>
</tbody>
</table>

events for a significant 3rd order BEC analysis in view of the high value expected for $C_3(0)$. Interesting to note is that if in these events the $n = 2$ order BEC enhancement gets contribution from the $f_0(975)$ decay then the expected enhancements ratio $6/2$ for the 3rd order BEC over that of the of the lower one ($n = 2$) does not have to hold.

6 Summary and Conclusions

It has been shown that a Bose-Einstein like low mass enhancement is expected in a sample of a spinless boson-antiboson system when the $C = +1$ state is selected. This enhancement is similar to that observed in a system of identical bosons like $\pi^+\pi^-$. On the other hand, the boson-antiboson state with the eigenvalue $C = -1$ behaves like identical fermions. Since for a spinless boson-antiboson state $C = (-1)^l = (-1)^J$, the condition for the correlation enhancement and depletion can be rephrased in term of even and odd $J$. Thus the correlation function $C_2(Q)$ schematically also describes the behavior of two identical fermions, in an orbital angular momentum $\ell = 0$ state, where the odd $J = 1$ state is forbidden and the even $J = 0$ state is allowed.

The system $K^0 \bar{K}^0$ is an ideal one for the experimental study of the BEC effect as both the $K_S^0$ and $K_L^0$ are eigenstates of CP. Due to the long lifetime of the $K_L^0$ meson the studies of the BEC effect were so far restricted to the $C = +1$ sample of $K_S^0 K_S^0$ pairs [12, 13, 14]. In the OPAL study a $K_S^0 K_S^0$ low mass enhancement has been seen [14] and further analyzed under the assumption that it is entirely due to the BEC. In that analysis the values for the emitter dimension and the Chaoticity parameter obtained were found to be very similar to those found in the same experiment for the like-sign charged pions. At present it is impossible to determine experimentally what fraction of the low mass enhancement seen by OPAL originates from decay of the $f_0(975)$ and/or the $a_0(980)$ resonances. In any case it is unlikely that the $K^0 \bar{K^0}$ mass enhancement seen by OPAL is entirely due to the decay of the $f_0(975)$ meson as there is no evidence in the same experiment for the $f_0(975)$ decay into the $K^+ K^-$ channel [26]. To further clear this point, it may be of use to compare some features, like the Feynman
scaling variable $x_f$ distribution, of the $f_0(975)$ in its decay to $\pi^+\pi^-$ to the $K^0_S K^0_S$ low mass enhancement. Furthermore a partial wave analysis of the OPAL observed $K^0_S K^0_S$ system may also be useful to clarify the nature of the low mass enhancement.

A BEC like effect can also be experimentally studied in other neutral boson-antiboson pairs like of the $B^0\bar{B}^0$ system, a meson-antimeson pair constructed from the fifth $b\bar{b}$ quark [15]. On the other hand in the case of a charged boson-antiboson pair, like the $\pi^+\pi^-$ system, there is not in general a simple way to project out a state with a single $C$ eigenvalue. There may be however some special cases where it will be possible to take advantage of outside constraints. For example, one may consider the decay $J/\psi(3097) \rightarrow K^+K^-\pi^+\pi^-$ with a branching ratio of $(9.0 \pm 0.3) \times 10^{-3}$ [18]. In this decay the $\pi^+\pi^-$ system is a mixture of $C = +1$ and $C = -1$ states. Part of this decay (about 10%) proceeds via the intermediate process of $J/\psi(3097) \rightarrow \phi\pi^+\pi^-$ so that the $\pi^+\pi^-$ is in a $C = +1$ state. To investigate the $\pi^+\pi^-$ BEC like effect in this $\phi\pi^+\pi^-$ final state, one can then use the $J/\psi$ decay final state $K^+K^-\pi^+\pi^-$ as a reference sample by using only those events with values of $M(K^+K^-)$ outside but near to the $\phi$ mass value of 1020 MeV.

The presence of BEC like effect in the $K^0_S K^0_S$ system may introduce some uncertainty to the value of the decay branching ratio $f_0(975) \rightarrow K\bar{K}$ as obtained from coupled channel analyses. In some analyses the low $K^0_S K^0_S$ mass enhancement seen in the reaction $\pi^-p \rightarrow n K^0_S K^0_S$ was assumed to originate entirely from the $f_0(975)$ decay ignoring the possibility of a BEC contribution. A more reliable determination of the $f \rightarrow K^0\bar{K}^0$ is coming from the partial wave analyses of the $K^0_S K^0_S$ where an enhancement in the s-wave is seen at threshold. This method will in particular be reliable if one could experimentally show that the genuine BEC enhancement seen in identical bosons like the $\pi^\pm\pi^\pm$ is not connected to an enhancement in the s-wave as $Q$ approaches zero.

As mentioned above, at high energy reactions the experiments are restricted in their BEC studies to the $K^0_S K^0_S$ system and could not study the $K^0_L K^0_L$ pairs due to their long lifetime. Of a particular interest is to study the system $K^0_L K^0_S$ where one should observe a depletion of events at low $Q$ values (see Fig. 2). Such a study may be realized with the $\bar{p}p$ low energy LEAR collider at CERN and in the near future with the "$\phi$-factory" $DA\Phi NE$, now under construction in the Italian National Laboratory in Frascati. This "factory", which is designed to reach $\sqrt{s}$ values up to about 2 GeV, will allow to study for example the reaction $e^+e^- \rightarrow \pi^+\pi^-K^0\bar{K}^0$ which has an approximate cross section of 1 nb. Due to the relative low energy of the $K^0$ mesons, it will be possible to identify the $K^0_L K^0_S$ pairs with a high efficiency using the large KLOE detector earmarked for $DA\Phi NE$. Furthermore, in these BEC studies at $DA\Phi NE$ one can use an ideal reference sample namely, the sum of all the $K^0\bar{K}^0$ decay modes, $K^0_S K^0_S$, $K^0_L K^0_L$, $K^0_S K^0_L$ and $K^0_L K^0_S$. The drawback of these studies at low energy is the presence of low mass resonances which will have to be accounted for.

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References


[18] Particle Data Group, Phys. Rev. D45 (1992) part II. See also in this review the "Note on Non-qq Mesons" in section VII.192.

[19] For a recent discussion on the $K\bar{K}$ molecule interpretation of the $f_0(975)$ resonance, see D. Morgan and M.R. Pennington, Phys. Lett. B258 (1991) 444.


[21] The first attempt to estimate the branching ratio of $a_0(980)$ to $K\bar{K}$ via a coupled channel analysis has been carried out by S. M. Flatté, Phys. Lett. B63 (1976) 224.


[26] OPAL Collaboration, P.D. Acton et al., C56 (1992) 521.