Can one extract source radii from transport theories?

J. Aichelin

SUBATECH

Laboratoire de Physique Subatomique et des Technologies Associées

UMR Universitè de Nantes, IN2P3/CNRS, Ecole des Mines de Nantes

4, rue Alfred Kastler F-44070 Nantes Cedex 03, France.

Abstract

To know the space time evolution of a heavy ion reaction is of great interest, especially in cases where the measured spectra do not allow to ascertain the underlying reaction mechanism. In recent times it became popular to believe that the comparison of Hanbury-Brown Twiss correlation functions obtained from classical or semiclassical transport theories, like Boltzmann Uehling Uhlenbeck (BUU), Quantum Molecular Dynamics (QMD), VENUS or ARC, with experiments may provide this insight. It is the purpose of this article to show that this is not the case. None of these transport theories provides a reliable time evolution of those quantities which are mandatory for a correct calculation of the correlation function. The reason for

this failure is different for the different transport theories.

Typeset using REVTEX

I. INTRODUCTION

It is a common problem in heavy ion reactions between 25 MeV/N and 200 GeV/N that the single particle spectra do not allow to ascertain the underlying reaction mechanism. To mention only two examples: At low energies despite of many years of efforts the fragmentation of nuclei into many intermediate mass fragments remains still a process whose origin is heavily debated. At high energies it turned out to be very difficult to rule out a hadronic scenario which may produce the same spectra as those proposed as a signal for the creation of a quark gluon plasma.

In such a situation a search for experimental information beyond the single particle spectra is obvious. Most valuable would be an information on the spatial structure of the reaction. It would allow to calculate key quantities like densities or energy densities. This information is, however, hard to obtain.

The only promising method proposed up to now is based on the interferometry of identical particles. The interference of the amplitudes of two indistinguishable processes gives rise to a correlation function which in principle allows to extract the radius of the emitting source. This approach has been very successfully applied by Hanbury-Brown and Twiss [1] in astronomy to determine the angular radius of stars by measuring the spatial correlations between two photons. Later Goldhaber [2] and Kopylov and Podgoretsky [3] advanced its application in particle physics by showing that measurable momentum space correlations may contain information on the size of the emitting source.

In the ideal case of a large, randomly emitting source of known shape this method is indeed very powerful and the experimental results can be directly related to the source radius of the emitting object. In particle and heavy ion physics the situation is, however,

much more difficult. There we encounter quite a number of problems. The size of the emitting sources is of the order of the radius of a nucleus and therefore not small as compared to the size of the wave function of the emitted particles. The signal may be spoilt by the final state interaction between the emitted particles or by to the long range Coulomb force of the source. The emission time point cannot be defined unambiguously. The decay of resonances into identical particles or correlations between the momenta of the emitted particles and the coordinates of the emission point may pretend a wrong size of the source. For a discussion of these problems we refer to ref. [5] , [6]. Recently is has been discussed that the HBT correlation function observed for an expanding source yields a much more difficult connection between the space time structure of the emitting source and the correlation function as compared to a static source [7].

Due to these problems the measured correlation function in heavy ion collisions cannot be directly related to the parameters of the emitting source even if its form were known. In this situation there are two possibilities. Either one assumes the form of the source and uses the measured correlation function to fix the source parameters. Unfortunately this procedure renders this parameter useless for the determination of key quantities like the density. It cannot be used for more than a comparison of different experiments. Or one tries to describe the reaction in its entity. This turns out to be a quite complicated procedure but became very popular recently. In this approach one follows the time evolution of the system with help of one of the standard transport models. Unfortunately none of them propagates (anti)symmetrized wave functions but at most a direct product wave function. Since the HBT effect is based on the (anti)symmetrization of the wave function of identical particles the transport model themselves cannot predict the correlation function. Rather one assumes that each particle "freezes out" at some time point. The freeze out time is

different for each particle and cannot be unambiguously defined. At high energies where the potential interaction is weak usually the time of the last binary collision is considered as freeze out time. The freeze out times are widely distributed. The freeze out time, the particle momentum and position form the input for the subsequent calculation of the Hanbury-Brown and Twiss (HBT) correlation function which is then compared with experiment.

Agreement is usually interpreted as a sign that the underlying transport model gives a realistic space time evolution of the different particles. Since these transport codes provide not only the momentum space coordinates of the particles but also that of the coordinate space they can then be used to calculate the time evolution of the key quantities like the energy density or the density.

The weak point in this procedure is the transition between the transport model and the subsequent program which calculates the correlation function. Does the transport model provide the correct time evolution of those quantities which are essential for the calculation of the correlation function?

It is the purpose of this article to show that this is not the case. In order to understand the necessary failure (and the evidently accidental agreement with some data) one has to understand in detail the derivation of the different transport models from the fundamental quantal equations as well as the derivation of the equation which is employed to determine the correlation function. We will perform this investigation for the three types of present day simulation programs: The Quantum Molecular Dynamics approach (QMD) [9], BUU type models like Boltzmann Ühling Uhlenbeck (BUU) [10]- [11], Vlasov Ühling Uhlenbeck (VUU) [12] or Landau Vlasov (LV) [13] and cascade models. The later class includes also the high energy simulation programs like VENUS [14] and ARC [15].

This failure is independent of the relativistic or nonrelativistic nature of the approach. It also does neither depend on the time between the emissions of the two identical particles nor on the presence of resonances. The only demand on the programs is the definition of the emission time point but it is of no interest how the system has evolved to this point. Essential, however, is the information the programs provide at that time point. This information is quite different for the three types of programs mentioned above and hence the reason for the failure is specific to each of the different transport models. In two cases (QMD and BUU) this procedure implies systematic errors which renders the approach useless for its original purpose: The discrimination between different reaction mechanisms which yield the same single particle spectra. For the high energy simulation programs the correlation function is completely artificial.

For clarity we limit our formalism to the simplest form possible by assuming that we are dealing with two bosons which are simultaneously emitted and can be treated nonrelativistically. For this simple case the formalism is very transparent. More realistic but also more complicated scenarios [7] may add additional problems but do not overcome the problems discussed here.

II. THE CORRELATION FUNCTION

We start with the derivation of the correlation function which relates the freeze out points with the measurable two body correlation function. We assume that a source, which is considered as classical, emits simultaneously two identical bosons with momenta $\vec{p}_1$ and $\vec{p}_2$. The differential two body cross section reads then as follows [8] ($\hbar, c = 1$):

$$\frac{d^2\sigma}{d\vec{p}_1 d\vec{p}_2} = |T_S(\vec{p}_1, \vec{p}_2, \alpha, \beta)|^2 \tag{1}$$

α and β characterize the emitting source. The symmetrized production amplitude T_S is given by

$$T_S(\vec{p}_1, \vec{p}_2, \alpha, \beta) = \frac{1}{\sqrt{2}}(T(\vec{p}_1, \vec{p}_2, \alpha, \beta) + T(\vec{p}_2, \vec{p}_1, \alpha, \beta)). \tag{2}$$

where $T(\vec{p}_1, \vec{p}_2, \alpha, \beta))$ is the Fourier transform of the wave function

$$T(\vec{p}_1, \vec{p}_2, \alpha, \beta)) = \int \frac{d^3x_1 d^3x_2}{(2\pi)^3} e^{-i(\vec{p}_1\vec{x}_1 + \vec{p}_2\vec{x}_2)} < \vec{x}_1, \vec{x}_2 | \psi(\alpha, \beta) > . \tag{3}$$

Introducing the Wigner density of the two body density matrix $\rho_2 = |\psi_2 >< \psi_2|$

$$D(\vec{x}_1, \vec{p}_1, \vec{x}_2, \vec{p}_2) = \frac{1}{(2\pi)^6} \int \prod_{i=1,2} d^3y_i e^{i\vec{p}_i\vec{y}_i} < \vec{x}_1 - \vec{y}_1/2, \vec{x}_2 - \vec{y}_2/2 | \psi_2 >< \psi_2 | \vec{x}_1 + \vec{y}_1/2, \vec{x}_2 + \vec{y}_2/2 > \tag{4}$$

we can express the cross section as a function of the two particle Wigner density

$$\frac{d^2\sigma}{d\vec{p}_1 d\vec{p}_2} = \int d^3x_1 d^3x_2 [D(\vec{x}_1, \vec{p}_1, \vec{x}_2, \vec{p}_2) + D(\vec{x}_1, \frac{\vec{p}_1 + \vec{p}_2}{2}, \vec{x}_2, \frac{\vec{p}_1 + \vec{p}_2}{2}) cos(\vec{p}_1 - \vec{p}_2)(\vec{x}_1 - \vec{x}_2)]. \tag{5}$$

Hence for the calculation of this cross section the transport theories have to provide the two body Wigner density. Unfortunately most of them do not permit to calculate this quantity. Therefore one has introduced an approximation, called smoothness assumption:

$$D(\vec{x}_1, \frac{\vec{p}_1 + \vec{p}_2}{2}, \vec{x}_2, \frac{\vec{p}_1 + \vec{p}_2}{2}) \approx D(\vec{x}_1, \vec{p}_1, \vec{x}_2, \vec{p}_2). \tag{6}$$

We will discuss the limits of its validity which turns out to be crucial in the course of the article. Employing the smoothness assumption the two body cross section reads as

$$\frac{d^2\sigma}{d\vec{p}_1 d\vec{p}_2} = \int d^3x_1 d^3x_2 D(\vec{x}_1, \vec{p}_1, \vec{x}_2, \vec{p}_2)(1 + cos(\vec{p}_1 - \vec{p}_2)(\vec{x}_1 - \vec{x}_2)). \tag{7}$$

This is the standard expression for the two particle cross section employed in numerous publications to relate the measured cross section with the radius of the emitting source.

Depending on the available information, for actual calculations one may have to employ further approximations. For BUU type equations which propagate the one particle Wigner density only one assumes that the correlation between particles are negligible

$$D(\vec{x}_1, \vec{p}_1, \vec{x}_2, \vec{p}_2) \approx D(\vec{x}_1, \vec{p}_1) \cdot D(\vec{x}_2, \vec{p}_2) \tag{8}$$

whereas for classical cascade calculations one assumes that the quantal two body Wigner density can be replaced by the classical 2 body phase space density F_{cl}

$$D(\vec{x}_1, \vec{p}_1, \vec{x}_2, \vec{p}_2) \approx F_{cl}(\vec{x}_1, \vec{p}_1, \vec{x}_2, \vec{p}_2). \tag{9}$$

III. QUANTUM MOLECULAR DYNAMICS (QMD)

We start out with the QMD approach because it is the only one which allows to calculate the 2 body Wigner density without any approximation. Hence the validity of the smoothness assumption is not necessary to predict the correlation function. On the contrary employing eqs. 5 and 7 and comparing their results the QMD approach may serve as a test for the validity of this approximation in the situation of a heavy ion reaction and hence for the predictive power of the correlation function calculated in the framework of the other models.

The QMD model is a n body theory which simulates heavy ion reactions between 30 MeV/N and 2 GeV/N on an event by event basis. Each nucleon is represented by a coherent state of the form

$$\phi_\alpha(p_1, t) = \left(\frac{L}{2\pi}\right)^{3/4} e^{-(p_1 - p_\alpha(t))^2 L/4} \, e^{-i(p_1 x_\alpha(t))} \, e^{+ip_\alpha^2(t)t/2m} \tag{10}$$

Thus the wave function has two time dependent parameters x_α, p_α, L is fixed. As we will see this wave function serves as a test wave function for a variational principle. Hence it is an *input* of the calculation and not the result of the solution of the Schrödinger equation. It relies heavily on intuition; other test wave functions may yield a different time evolution of the system. The total n body wave function is assumed to be the direct product of n coherent states

$$\phi = \phi_\alpha(x_1, x_\alpha, p_\alpha, t)\phi_\beta(x_2, x_\beta, p_\beta, t)\cdots, \tag{11}$$

thus antisymmetrization is neglected. The initial values of the parameters are chosen in a way that the ensemble of $A_T + A_P$ nucleons gives a proper density distribution as well as a proper momentum distribution of the projectile and target nuclei. The time evolution of the system is calculated by means of a generalized variational principle: We assume that $\vec{p}_\alpha$ and $\vec{x}_\alpha$contain the essential time dependence of the n-body wave function. The Lagrange function $\mathcal{L}$ can then be written as a functional of these parameters.

$$\mathcal{L} = \left(\phi \left| i\hbar(\frac{dp_\alpha}{dt}\frac{d}{dp_\alpha} + \frac{dx_\alpha}{dt}\frac{d}{dx_\alpha}) - H \right| \phi\right). \tag{12}$$

The time evolution of the parameters is obtained by the requirement that the action

$$S = \int_{t_1}^{t_2} \mathcal{L}[\phi, \phi^*]dt \tag{13}$$

is stationary under the allowed variation of the wave function. For the wave function of eq. 11 the Lagrange function is given up to a constant by

$$\mathcal{L} = \sum_\alpha(\vec{p}_\alpha\dot{\vec{x}}_\alpha - \frac{p_\alpha^2}{2m} - \frac{1}{2}\sum_\beta V(\vec{x}_\alpha, \vec{x}_\beta)). \tag{14}$$

$V(\vec{x}_\alpha, \vec{x}_\beta)$ is the expectation value of the 2 body potential. The variation of the Lagrange function gives Euler Lagrange equations for each of the 6 parameters

$$\dot{\vec{p}}_\alpha = -\vec{\nabla}_{\vec{x}_\alpha} \sum_\beta V(\vec{x}_\alpha, \vec{x}_\beta) \tag{15}$$

and

$$\dot{\vec{x}}_\alpha = \vec{p}_\alpha / m. \tag{16}$$

With these two equations one has reduced the problem of solving a n - body Schrödinger equation to that of solving 6 n ordinary differential equations. In reality V is a parametrization of the real part of the Brückner G- matrix. The imaginary part is approximated by the measured cross section. For details we refer to ref. [9]. Hence in QMD the centroids of the Gaussians in momentum and coordinate space are the only quantities which change in time. The form of the wave function around the centroids is fixed. This is a consequence of the ansatz (eq. 11) not a result.

The use of the test wave function eq.(10) allows to calculate the Wigner density of a pair of particles

$$D_{QMD}(\vec{x}_1, \vec{p}_1, \vec{x}_2, \vec{p}_2) = \frac{1}{\pi^6} e^{-(\vec{P}-\vec{K}(t))^2 L/4 - (\vec{X}-\vec{R}(t))^2 4/L} e^{-(\vec{p}-\vec{k}(t))^2 L - (\vec{x}-\vec{r}(t))^2/L} \tag{17}$$

where we have used the definitions

$$\begin{aligned}
\vec{P} &= \vec{p}_1 + \vec{p}_2; \vec{K} = \vec{p}_\alpha + \vec{p}_\beta \\
\vec{X} &= \frac{\vec{x}_1 + \vec{x}_2}{2} \ ; \vec{R} = \tfrac{\vec{x}_\alpha + \vec{x}_\beta}{2} \\
\vec{p} &= \frac{\vec{p}_1 - \vec{p}_2}{2} \ ; \vec{k} = \tfrac{\vec{p}_\alpha - \vec{p}_\beta}{2} \\
\vec{x} &= \vec{x}_1 + \vec{x}_2; \vec{r} = \vec{x}_\alpha + \vec{x}_\beta.
\end{aligned} \tag{18}$$

The Wigner density $D(\vec{x}_1, \frac{\vec{p}_1+\vec{p}_2}{2}, \vec{x}_2, \frac{\vec{p}_1+\vec{p}_2}{2})$ is obtained by replacing $\vec{p}_1$ and $\vec{p}_2$ by $\frac{\vec{p}_1+\vec{p}_2}{2}$.

Inserting this Wigner density in eq. 5 one obtains after integration over the pair's center of mass momentum

$$\int \frac{d^2\sigma}{d\vec{p}_1 d\vec{p}_2} d^3P = (\frac{L}{\pi})^{3/2}(e^{-(\vec{p}-\vec{k}(t))^2 L} + e^{-\vec{p}^2 L - \vec{k}(t)^2 L} \cos 2\vec{p}\vec{r}) \tag{19}$$

where t is the common freeze out time. This is the cross section, as a function of the relative momentum, produced by two particles emitted from freeze out points with a relative distance of $\vec{r}$ and with a relative momentum of $\vec{k}$.

In order to understand the relation between the distribution of the freeze out points and the observed cross section we average the cross section over a given freeze out point distribution S(k,r)

$$S(k,r) = (\frac{B}{A\pi^2})^{3/2} e^{-k^2 B - r^2/A}. \tag{20}$$

For such a distribution one obtains

$$\begin{aligned} \int \frac{d^2\tilde{\sigma}}{d\vec{p}_1 d\vec{p}_2} d^3P &= \int d^3k d^3r S(k,r) \int \frac{d^2\sigma}{d\vec{p}_1 d\vec{p}_2} d^3P \\ &= (\frac{BL}{\pi(L+B)})^{3/2}(e^{-p^2 \frac{LB}{(L+B)}} + e^{-p^2(L+A)}). \end{aligned} \tag{21}$$

With help of the measurable HBT correlation function

$$C(p) = \frac{\int \frac{d^2 d\tilde{\sigma}}{d\vec{p}_1 d\vec{p}_2} d^3P}{\int \frac{d\sigma}{d\vec{p}_1} \frac{d\sigma}{d\vec{p}_2} d^3P} = \frac{\int \frac{d^2\tilde{\sigma}}{d\vec{p}_1 d\vec{p}_2} d^3P}{(\frac{BL}{\pi(L+B)})^{3/2} e^{-p^2 \frac{LB}{(L+B)}}}. = 1 + e^{-(L+A-\frac{LB}{L+B})p^2} \tag{22}$$

with $\frac{d\sigma}{d\vec{p}_1} = \int d^3x_1 D(\vec{x}_1, \vec{p}_1)$ we can relate the mean square relative momentum $< p^2 >= \int d^3p C(p) p^2 / \int d^3p C(p)$ with A, the source size in coordinate space.

$$\frac{1}{< p^2 >} = 2/3(A + \frac{L^2}{(L+B)}) \tag{23}$$

The transverse energy distribution of the heavy ion reactions of interest shows temperatures of the order of 50 - 100 MeV, corresponding to B in between 0.4 and 0.8 fm^2. In the standard versions of QMD resp. IQMD the parameter L has the value 4.33 and 8.66 fm^2,

respectively. At normal nuclear matter density A is about $A_{syst}^{2/3}$ in unities of $[fm^2]$ where A_{syst} is the mass number of the emitting system.

From the above equation one sees that the correlation function does not measure directly the mean square radius of the distribution of the freeze out points but about that of the density distribution of the emitted particles. This has already been observed by Podgoretzky [4]. For large systems ($L << A$) both can be identified, in the case of nuclear reactions they differ by about 25%.

This would not be troublesome if QMD treats the time evolution of the wave function in a proper way. However, this is not the case. The form of this wave function is based on intuition and the width is kept constant. It is adjusted to give the right density profile of the initial nuclei. Its time evolution has not been considered as important for the single particle observables and appears in all other results of the approach only as part of the expectation value of the potential $< V >=< \phi|V|\phi >$.

The modification of the relation $\frac{1}{<p^2>} = 2/3A_{true}$, valid for infinite systems, due to the finite width of the wave function is on the one side not negligible but its calculation is on the other side beyond the scope of the QMD approach. Assuming the difference between $2/3(A_{true} + \frac{L^2}{(L+B)})$ and $2/3A_{true}$ as a measure of the systematic error induced by the finite width on the cross section we arrive at a systematic error for the density of about 40%.

The other way round the rms value for the relative momentum of the correlation function calculated with the freeze out point distribution provided by QMD has a systematic error of about 25 % due to the lack of knowledge of the detailed form of the wave function.

Is this uncertainty important? Certainly, because the range of possible sizes of the emitting source in nuclear physics is rather limited. At the end of the reaction we cannot expect a system with a density higher than normal nuclear matter density. If the system is

sufficiently excited to disassemble at the end of the reaction the 40 mb cross section brings collisions to an end when the system passes a density of about half normal nuclear matter density. After that the particles move freely. Hence the systematic error on the density is of the same order as the possible variation of the density.

In reality the situation is worse because there are strong correlations between the emission point and the momentum of the emitted particle, known under the name hot spot resp. participants and spectators at low and intermediate energies. At high energies we observe strong correlations between the space time rapidity and the rapidity of the emitted particles. All these correlations reduce the size of the emitting source "seen" by the Hanbury-Brown correlation function and hence enhance the correction due to the finite size of the wave function.

In conclusion we see that the HBT correlation function calculated with the freeze out point distribution function provided by QMD depends in a non negligible way on the form of the QMD wave function. As said, the QMD wave function is a test wave function based on intuition and is not solution of the Schrödinger equation. Results which depend on its detailed form can therefore not be predicted. Consequently, in the QMD approach the HBT correlation functions cannot be determined sufficiently precise to be of value for solving the relevant physical questions.

IV. ONE BODY TRANSPORT THEORIES

In order to derive the equation for the time evolution of the one–body Wigner density of a particle moving in a selfconsistent potential $V(\vec{x})$ we start from that for the one body density matrix $\rho_1 = |\psi_1 >< \psi_1|$

$$\dot{\rho}_1 = -i[H, \rho_1] \tag{24}$$

Applying to this equation the Wigner transformation

$$O_W(\vec{x}, \vec{p}) = \frac{1}{(2\pi)^3} \int d^3y e^{i\vec{p}\vec{y}} < \vec{x} - \frac{\vec{y}}{2} | O | \vec{x} + \frac{\vec{y}}{2} > \tag{25}$$

one obtains the differential equation

$$(\frac{\partial}{\partial t} + \frac{\vec{p}_1}{m} \nabla_{x_1}) D(\vec{x}_1, \vec{p}_1, t) = \int d^3p_1' K_1(\vec{p}_1 - \vec{p}_1', \vec{x}_1) D(\vec{x}_1, \vec{p}_1, t) \tag{26}$$

D being the Wigner of the one body density operator and K_1 is defined as

$$K_1(\vec{p}_1 - \vec{p}_1', \vec{x}) = \frac{1}{i\hbar} \int \frac{d^3y}{(2\pi\hbar)^3} e^{-i(\vec{p}_1 - \vec{p}_1')\vec{y}/\hbar} (V(\vec{x} + \vec{y}/2) - V(\vec{x} - \vec{y}/2)). \tag{27}$$

We have restored $\hbar$ here for reasons which will soon become obvious. One can expand the integration kernel around x_1

$$K_1(\vec{p}_1 - \vec{p}_1', \vec{x}_1) = \frac{2}{\hbar} \sin(\frac{\hbar \nabla_{x_1} \nabla_{p_1}}{2}) V(\vec{x}_1) \delta(\vec{p}_1 - \vec{p}_1'). \tag{28}$$

We see that K_1 can be viewed as a series with the expansion coefficient $\hbar \nabla_{x_1} \nabla_{p_1}$. Hence in the limit that the expansion can be terminated after the first term the Schrödinger equation in its Wigner representation is equivalent to the classical Vlasov equation :

$$(\frac{\partial}{\partial t} + \frac{\vec{p}_1}{m} \nabla_{x_1}) D(\vec{x}_1, \vec{p}_1, t) = (\nabla_{x_1} V(\vec{x}_1)) \nabla_{p_1} D(\vec{x}_1, \vec{p}_1, t) \tag{29}$$

The Vlasov equation describes the time evolution of the phase space density of particles which move on classical orbits specified by the Hamilton equations $\frac{\partial \vec{x}_1}{\partial t} = \frac{\vec{p}_1}{m}$ and $\frac{\partial \vec{p}_1}{\partial t} = -\nabla_{x_1} V$. As in QMD V presents the real part of the Brückner G- matrix and the imaginary part is added as a cross section is

This observation allows two approaches to solve the above equation. Either one solves the differential equation directly or one creates a swarm of test particles which are subject

to the Hamilton equations and fulfil the initial condition $D(\vec{x}_1, \vec{p}_1, t_0)$. One propagates this swarm up to a time t and then constructs the Wigner density $D(\vec{x}_1, \vec{p}_1, t)$ by coarse graining. The later solution method is call test particle method and is employed in the BUU, VUU and LV approaches.

When calculating the observables, i.e. the expectation values of operators, the transition from the first to the second method corresponds to the replacement of an analytical integration by a Monte Carlo procedure. Using the swarm of test particles the analytical solution

$$\langle O(t) \rangle = \int D(\vec{x}_1, \vec{p}_1, t) O(\vec{x}_1, \vec{p}_1)\, d^3x_1\, d^3p_1 \tag{30}$$

is replaced by the corresponding Monte Carlo type integral

$$\langle O(t) \rangle = \frac{1}{N} \sum_{i=1}^{N} O(\vec{r}_i(t), \vec{k}_i(t)) \tag{31}$$

where the $\vec{r}_i(t)$ and $\vec{k}_i(t)$ are the phase space coordinates of the N particles propagated with the Hamilton equations. As said, they are distributed like $D(\vec{x}_1, \vec{p}_1, t)$. According to the theory of Monte Carlo integration both integration procedures yield the same result in the limit of an infinite number of test particles. In practice one has to verify that the results do not depend on this number. Usually 100 test particles per physical nucleon in the system are considered as sufficient. It is very important to realize that these test particles have nothing to do with physical nucleons. They serve only as a representation of the one body Wigner density $D(\vec{x}_1, \vec{p}_1, t)$. All observables which require more than its knowledge are beyond the scope of applicability of these theories. In view of the exact result for the two particle cross section (eq.5) the possibility to extract source radii from the one body theories requires the validity of two approximations:

- The smoothness assumption is valid
- $D(\vec{x}_1, \vec{p}_1, \vec{x}_2, \vec{p}_2, t) \approx D(\vec{x}_1, \vec{p}_1, t) D(\vec{x}_2, \vec{p}_2, t)$

They are a consequence of the impossibility to create two body Wigner densities or Wigner densities of two body observables like $\vec{p}_1 + \vec{p}_2$ from the swarm of test particles defined as above.

If both approximation were valid the correlation function is given by

$$\begin{aligned} \frac{d^2\sigma}{d\vec{p}_1 d\vec{p}_2} &= \int d^3x_1 d^3x_2 D(\vec{x}_1, \vec{p}_1) D(\vec{x}_2, \vec{p}_2)(1 + cos(\vec{p}_1 - \vec{p}_2)(\vec{x}_1 - \vec{x}_2)) \\ &= \sum_{i \neq j} (1 + cos(\vec{p}_i(t_0) - \vec{p}_j(t_0))(\vec{x}_i(t_0) - \vec{x}_j(t_0))) \end{aligned} \tag{32}$$

t_0 is the (assumed common) freeze out time. The second approximation, the absence of two body correlations, is hard to control. The importance of many particle correlations for the fragment formation has been discussed in [16] but its relevance for the proton or pion emission has not yet been investigated.

The validity of the smoothness assumption cannot be verified in the framework of the one body transport theories because they do not provide the two body Wigner density. It can, however, be studied in QMD approach which allows both, the calculation of the HBT correlation function obtained with and without (eq. 22) applying the smoothness assumption. Therefore we will use the QMD approach to test this assumption.

Both approaches, BUU and QMD, are bound to reproduce the measured density and momentum space distribution of nucleons in cold nuclei. This does, however, not allow to fix the width of the QMD test wave function. For a range of values of L we can obtain (with different distributions of $\vec{x}_\alpha$ and $\vec{p}_\alpha$) the same density profile $|\phi^2|$. However, the range of L is limited. Wave functions with a width L smaller than that employed in the

QMD approach would not allow to model an almost constant density in the interior of large nuclei. Hence the employed values of L present the lower limit of the possible widths of a single particle wave function.

Employing the smoothness assumption eq. 19 reads as

$$\int \frac{d^2\sigma}{d\vec{p}_1 d\vec{p}_2} d^3P = (\frac{L}{\pi})^{3/2} e^{-(\vec{p}-\vec{k}(t))^2 L}(1+\cos 2\vec{p}\vec{r}). \tag{33}$$

Averaging over the source, eq. 20, we find

$$\int \frac{d^2\tilde{\sigma}}{d\vec{p}_1 d\vec{p}_2} d^3P = (\frac{BL}{\pi(L+B)})^{3/2} e^{-p^2 \frac{LB}{(L+B)}}(1+e^{-p^2 A}). \tag{34}$$

and calculate for the mean square relative momentum

$$< p^2 >= 3/2 \frac{1}{A_{smooth}} \tag{35}$$

Comparing for the same value of $< p^2 >$ the width for the freeze out point distribution with and without the smoothness assumption

$$\frac{A_{smooth}}{A} = \frac{< r^2_{smooth} >}{< r^2_{true} >} = 1 + \frac{L^2}{(L+B)A} \tag{36}$$

we see that the mean square radius of the extracted source differ by $\frac{L^2}{(L+B)A}$, i.e. by about 25%. Thus the smoothness assumption pretends for a given $< p^2 >$ a notable larger source size. Opposite, for the same distribution of the freeze out points the mean square momentum of the correlation function differs by the same value if calculated with and without smoothness assumption.

As said the employed values of L present the lower limit for the width of the single particle wave function. Actually it may be larger, as predicted by mean field calculations. Hence the smoothness assumption cannot be considered as reliable if one aims at a determination of densities to a precision of 40%. Since the validity of the smoothness assumption

is a necessary condition for relating source radii with the HBT correlation function in one body theories, they are probably not the right tool to address the question of the space time evolution of a nuclear system.

V. CASCADE MODELS

A third class of models employed to extract source radii by comparing experimental results with model predictions are the so called cascade models. These are classical n-body models which solve the Hamilton equations of a n-body system. They include the simulation programs at very high energies VENUS [14] and ARC [15] as well as now less frequently employed programs for heavy ion reactions at an energy of around 1 GeV/N [17]. In these models the particles do not interact via potentials but suffer two body collisions if they come sufficiently close in coordinate space. In between the collisions the particles move on straight lines. In the computer programs they are treated as classical particles with a fixed momentum and position.

One may ask how classical models can be employed to calculate a correlation function which is solely based on the interference of amplitudes and hence presents a genuine quantal effect. For an understanding we have to make a detour. In order to employ eq. 5 we have to construct the Wigner density out of the classical two body phase space density. This is of course not unique but the approach

$$F_{cl} = \delta(\vec{x}_1 - \vec{x}_\alpha)\delta(\vec{p}_1 - \vec{p}_\beta))\delta(\vec{x}_2 - \vec{x}_\beta)\delta(\vec{p}_2 - \vec{p}_\beta) \tag{37}$$

$$= \lim_{C\to\infty, D\to 0} \frac{C^3 D^3}{\pi^6} e^{-(\vec{P}-\vec{K}(t))^2 C/4 - (\vec{X}-\vec{R}(t))^2 4/D} e^{-(\vec{p}-\vec{k}(t))^2 C - (\vec{x}-\vec{r}(t))^2/D} \tag{38}$$

serves our purpose. The expression in the last line will be considered as Wigner density. Please note that in comparison to eq. 17 this Wigner density does not respect the uncer-

tainty relation. Inserting this expression in eq. 5 and performing the limit procedure we obtain without smoothness assumption

$$\int \frac{d^2\sigma}{d\vec{p}_1 d\vec{p}_2} d^3P = \delta(p - k(t)) \tag{39}$$

whereas applying the smoothness assumption we find

$$\int \frac{d^2\sigma}{d\vec{p}_1 d\vec{p}_2} d^3P = \delta(p - k(t))(1 + cos2\vec{p}\vec{x}). \tag{40}$$

Thus the smoothness assumption yields the unwanted result that it produces correlations out of nothing. One faces the somewhat surprising result that the correlation function and hence the extracted radii are artificial and are only due to the differences between the approximate and the exact formula for the correlation function. Applying the correct formula the cascade calculations yield no correlation function, as the exact result shows.

The truth of this observation can even be easily verified without any calculation. If two particles with a sharp momentum are emitted from two localized sources one can measure the momentum sufficiently precise in order to identify the source from which each particle has been emitted. Thus there are no alternative processes, hence no interference of their amplitudes and there is, as a consequence, no HBT correlation function.

If one traces back the correlation function obtained with the smoothness assumption on its physical origin then one finds that the smoothness assumption pretends a wavefunction of finite width whereas the cascade knows only point like particles. Since this width has no physical origin and is alien to the treatment of the particle transport in cascade models the created correlation function cannot contain any physics.

VI. CONCLUSION

We have discussed the possibility to extract source radii by comparing the experimental results with the prediction of simulation programs. There is no doubt that the experimental results indeed show momentum space correlations caused by the bosonic or fermionic nature of the observed hadrons. These correlations carry information about the space time structure of the reaction. The goal to relate the observed correlation functions in momentum space with physical parameters in coordinate space like source radii, densities or energy densities is presently too ambitious.

Transport theories offer at the moment the only possibility to achieve this goal. None of these transport models takes the bosonic or fermionic nature of the hadrons into account. Whereas this may be no essential drawback for many observables it makes it impossible to calculate the correlation function in a straight forward manner. Every model requires the introduction of the (anti)symmetrization of the wave function in an ad hoc fashion in order to predict a correlation function.

We have found that for all presently existing models which can be subdivided into three classes this introduction poses problems. The reason is different for the different classes of models. Quantum molecular dynamic models, which carry along a test wave function of a fixed form for each nucleon can predict the correlation function. It turns out that it depends on the detailed form of the wave function which - being a test wave function - can not be predicted in these models. One body theories are only able to predict the correlation function if two approximations are fulfilled, which is hardly the case in the investigated reactions. Cascade models, in which classical particles are propagated, contain no correlations The quoted values of source radii are totally artificial being a consequence

of the employed approximation and not of physical origin.

Of course this raises the question how to proceed. As we have seen we are plagued with systematic errors of the order L/A. There is first of all the open question whether the wave function of a nucleon is smaller than L. Mean field calculation yield a much broader wave function. Therefore one may conjecture that the value of L in the QMD calculation is close to the lower limit. In this case the basis of the HBT approach presented here, i.e. that the particles are emitted from a **classical** source breaks down. If not, is there a possibility to construct dynamical theories which can provide a prediction for the correlation function? Either one can try to decrease L or to avoid the systematic errors.

The first suggestion implies a localization of the particles with a precision of about 1fm. This will be hardly possible. Not only because in a nuclear environment the root mean square radius of the wave function of the nucleon is considerably larger than the radius of a free nucleon but also because it implies an uncertainty of 200 MeV/c for the momentum of the nucleons which poses several severe problems for transport theories:

- How to propagate particles in semiclassical theories whose velocity uncertainty is about 0.2c is unknown.
- The sequence of collisions becomes undetermined
- The applied scattering cross section have to be modified because the scattering partners are asymptotically not in a plane wave state.

The second suggestion implies the construction of transport theories which propagates at least two particle wave functions and not parameters of the wave function. Presently such an approach is not available.

Before a solution to these problems has been found the Hanbury Brown Twiss effect is a very nice quantal effect. Its application in nuclear physics to study the space time structure remains, however, premature.

Interesting discussions with Drs. Ardouin, Erazmus, Gyulassy, Heinz, Lednitzky and Werner are gratefully acknowledged.

REFERENCES

[1] R. Hanbury Brown and R.Q. Twiss, Nature **178** (1956) 1046

[2] G. Goldhaber et al., Phys. Rev. **120** (1960) 300

[3] G. I. Kopylov and M. I. Podgoretsky, Sov. J. Nucl. Phys. **15** (1972) 219; ibid. **19** (1974) 215; G. I. Kopylov, Phys. Lett. **B50** (1974) 572.; M. I. Podgoretsky, Sov. J. Part. Nucl., **20** (1989) 266

[4] M. I. Podgoretsky, Acta Phys. Pol. **B24** (1993) 1433

[5] W. Zajc, Particle Production in Highly Excited Matter, edited by H.H. Gutbrod and J. Rafelski, Nato ASI Series 303 Plenum Press, New York, 1993

[6] S. S. Padula, M. Gyulassy and S. Gavin, Nucl. Phys. **B329** (1990) 357.

[7] U. Heinz, preprint nucl-th/9608002 and references therein

[8] S. Pratt, Phys. Rev. Lett. **53** (1984) 1219; Phys. Rev. **D33** (1986) 1314.

[9] J. Aichelin, Phys. Rep. **202**, 233 (1991).

[10] J. Aichelin and G. Bertsch. Phys. Rev. **C31**, 1730 (1985).

[11] W. Cassing, V. Metag, U. Mosel and K. Niita. Phys. Reports **188**, 361 (1990).

[12] H. Stöcker and W. Greiner. Phys. Reports **137**, 277 (1986).

[13] C. Gregoire, B. Remaud, F. Sebille, L. Vinet, and Y. Raffray, Nucl. Phys. **A465**, 317 (1987).

[14] K. Werner, Physics Reports **232** (1993) 87

[15] Pang, Y. et al., Nucl. Phys. **A544** (1992) 435c

[16] P.B. Gossiaux et al. , Phys. Rev **C 51** (1995) 3357

[17] J. Cugnon, The Nuclear equation of State, edited by H. Stoecker and W Greiner Nato ASI Series Plenum Press, New York, 1989