**Abstract:**

A general derivation of Landau parameters for an interaction with **density dependent finite range terms** is given. Particular carefulness is devoted to the inclusion of rearrangement terms. This report is part of a larger project which aims at defining a new nuclear interaction improving the well-known D1 force of Gogny et al..The improvements are mainly: severe constraints on the sum rules on a large range of density, good properties in exotic nuclei and better reliability with respect to extrapolation to high energies.

# Landau parameters for nuclear interactions with density dependent finite range terms

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## 1- Landau parameters

The Landau parameters represent the effective particle-hole interaction at Fermi surface. Because of the direct connection between the physical observables and the Landau parameters, their evaluation from the effective interaction is very useful. The parameter $F_0$ determines the incompressibility K of the system,

$$K = \frac{3\hbar^2 k_F^2}{m^*}(1+F_0) \qquad (1)$$

The parameter $F_1$ determines the effective mass (thus controling the single-particle level density at Fermi surface) through the relation,

$$\frac{m}{m^*} = 1 + \frac{1}{3}F_1 \qquad (1')$$

Moreover $F'_0$ determines the symmetry energy, $G_0$ the magnetic succeptibility and $G'_0$ the onset of pion condensation in nuclear matter.

Evaluation of Landau parameters from the effective interaction is often a lengthy and tedious operation. Landau parameters have been calculated before by Backman et al. from the Skyrme interactions and by Padjen et al. from the finite range Gogny D1 interaction.

In that report we give a detailed derivation of these parameters for an interaction with **finite range density dependent** terms [EFS95]. Density dependent term make the calculation more complicated because we have to take in account rearrangement contributions.

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The effective interaction is written in the form,

$$f(k,k') = \frac{\delta^2 E}{\delta\rho(k)\delta\rho(k')} = f^{00}(\theta) + f^{01}(\theta)_1\tau_1.\tau_2 + f^{10}(\theta)\sigma_1\sigma_2 + f^{11}(\theta)\sigma_1\sigma_2\tau_1\tau_2 \quad (2)$$

where θ is the angle between the momenta k and k' of the particles at the Fermi surface. Upper indexes refer to the spin-isospin channels. Quantities $f^{ST}(\theta)$ are generally expanded in terms of Legendre polynomials as,

$$f^{ST}(\theta) = \sum_l f_l^{ST} P_l(\cos\theta) \quad (2')$$

Landau parameters $F_l^{ST}$ are simply defined as,

$$F_l^{ST} = N_0 f_l^{ST}$$
$$N_0 = (2\pi)^3 \frac{2k_F m^*}{\hbar^2\pi^2} \quad (3)$$

$N_0$ is the state density at the Fermi surface, $m^*$ is the effective mass in nuclear matter and $k_F$ is the Fermi momentum. Special cases are,

$$F_0 = F_0^{00}$$
$$F_1 = F_1^{00}$$
$$F_0' = F_0^{01}$$
$$G_0 = F_0^{10}$$
$$G_0' = F_0^{11}$$

Landau parameters must meet Migdal stability conditions [Mi67],

$$1 + \frac{F_l^{ST}}{2l+1} > 0 \quad (4)$$

Moreover, Landau parameters are not independent: they are constrained by two sum rules [Mi67],

$$\sum_{lST} (-1)^{S+T}(2S+1)(2T+1)\frac{F_l^{ST}}{1+\frac{F_l^{ST}}{2l+1}} = 0$$
$$\sum_{lST} \frac{F_l^{ST}}{1+\frac{F_l^{ST}}{2l+1}} = 0 \quad (4')$$

instabilities due to isovector breathing modes occur at physical values of the density.For Skyrme type interaction there is no candidate able to meet that demand.

Our future interaction will have first to fulfil relations (4) and (4') on a large range of nuclear densities and to minimize (or shift to higher densities) the instabilities.

## 2- A generalization of Gogny D1 interaction

We consider an interaction of the form,

$$V_{ij} = \sum_{c=1}^{n} \left(W_c + B_c P_\sigma - H_c P_\tau - M_c P_\sigma P_\tau\right) e^{-\frac{(r_i - r_j)^2}{\mu_c^2}} \left[\frac{1}{2}\left\{\rho(\mathbf{r}_i) + \rho(\mathbf{r}_j)\right\}\right]^{\gamma_c} \tag{5}$$

The spin-orbit term is not written here because it doesn't contribute to Landau parameters.

The total energy of the system is then,

$$E = \frac{1}{2}\sum_{abcd} \left\langle ab \middle| V_{ij}\left(1 - P_r P_\sigma P_\tau\right) \middle| cd \right\rangle \rho_{ca}\rho_{db} \tag{6}$$

## 3- Landau parameters calculation

To calculate Landau parameters we have to evaluate the following quantities [GP77],

$$\begin{aligned} A_{ph,p'h'} &= \left(\varepsilon_p - \varepsilon_h\right)\delta_{pp'}\delta_{hh'} + \frac{\delta^2 E}{\delta\rho_{ph}\delta\rho_{p'h'}}\Big|\rho = \rho_0 \\ B_{ph,p'h'} &= \frac{\delta^2 E}{\delta\rho_{ph}\delta\rho_{h'p'}}\Big|\rho = \rho_0 \end{aligned} \tag{7}$$

with the condition,

$$\frac{\delta E}{\delta\rho_{ph}}\Big|\rho = \rho_0 = 0 \tag{8}$$

Matrices A and B appear in the RPA equations which are obtained in linearizing TDHF equations with respect to small variations of the density matrix around the equilibrium HF density.

Indexes h and p refer to hole and particle states respectively. Inserting the expression (6) for E in the previous set we get (neglecting the first term in $A_{ph,p'h'}$),

$$A_{ph,p'h'} = \left\langle ph' \middle| V_{ij}^{a} \middle| hp' \right\rangle + \sum_{ho} \left( \left\langle h'ho \middle| \frac{\partial V_{ij}^{a}}{\partial \rho_{hp}} \middle| p'ho \right\rangle + \left\langle pho \middle| \frac{\partial V_{ij}^{a}}{\partial \rho_{p'h'}} \middle| hho \right\rangle \right)$$
$$+ \frac{1}{2} \sum_{ho\,h'o} \left\langle hoh'o \middle| \frac{\partial^2 V_{ij}^{a}}{\partial \rho_{hp} \partial \rho_{p'h'}} \middle| hoh'o \right\rangle \tag{9}$$
$$B_{ph,p'h'} = \textit{Same} \text{ with } p' \leftrightarrow h'$$

where ,

$$V_{ij}^{a} = V_{ij}\left(1 - P_r P_\sigma P_\tau\right) \tag{10}$$

The second and the third terms are due to the density dependence of the interaction. Contributions of such terms which were sometimes neglected [BJS75] have to be included.

In nuclear matter we set $p = \left(\mathbf{k}_p, s_p, t_p\right)$, s and t refering to spin and isospin.

It will be convenient to express the antisymmetrised interaction as,

$$V_{ij}^{a} = \sum_{c=1}^{n} \left(W_c + B_c P_\sigma - H_c P_\tau - M_c P_\sigma P_\tau\right) e^{-\frac{(r_i - r_j)^2}{\mu_c^2}} \left[\frac{1}{2}\left(\rho(\mathbf{r}_i) + \rho(\mathbf{r}_j)\right)\right]^{\gamma_c} \left(1 - P_r P_\sigma P_\tau\right)$$
$$= \sum_{c=1}^{n} e^{-\frac{(r_i - r_j)^2}{\mu_c^2}} \left[\frac{1}{2}\left(\rho(\mathbf{r}_i) + \rho(\mathbf{r}_j)\right)\right]^{\gamma_c} \left(P_{ij} + P_r P'_{ij}\right) \tag{11}$$

$$P_{ij}^{c} = W_c + B_c P_\sigma - H_c P_\tau - M_c P_\sigma P_\tau$$
$$P_{ij}^{'c} = M_c + H_c P_\sigma - B_c P_\tau - W_c P_\sigma P_\tau \tag{12}$$

We split the calculation of $A_{ph,p'h'}$ or $B_{ph,p'h'}$ in three parts and get rid of index c.

I- $\left\langle ph' \middle| V_{ij}^{a} \middle| hp' \right\rangle$

I-a Spatial part (see appendix A)

$$\left\langle k_p k_{h'} \middle| V_{ij}^a \middle\| k_h k_{p'} \right\rangle = G_\mu\left(\left|\mathbf{k}_p - \mathbf{k}_h\right|\right)$$
$$\left\langle k_p k_{p'} \middle| V_{ij}^a \middle\| k_h k_{h'} \right\rangle = G_\mu\left(\left|\mathbf{k}_p - \mathbf{k}_h\right|\right) \tag{13}$$

*and,*

$$\mathbf{k}_p + \mathbf{k}_{h'} = \mathbf{k}_h + \mathbf{k}_{p'}$$
$$\mathbf{k}_p + \mathbf{k}_{p'} = \mathbf{k}_h + \mathbf{k}_{h'}$$

$$\left\langle k_p k_{h'} \middle| V_{ij}^a P_r \middle\| k_h k_{p'} \right\rangle = G_\mu\left(\left|\mathbf{k}_p - \mathbf{k}_{p'}\right|\right)$$
$$\left\langle k_p k_{p'} \middle| V_{ij}^a P_r \middle\| k_h k_{h'} \right\rangle = G_\mu\left(\left|\mathbf{k}_p - \mathbf{k}_{h'}\right|\right)$$

*and,* (14)

$$\mathbf{k}_p + \mathbf{k}_{h'} = \mathbf{k}_h + \mathbf{k}_{p'}$$
$$\mathbf{k}_p + \mathbf{k}_{p'} = \mathbf{k}_h + \mathbf{k}_{h'}$$

*with,*

$$G_\mu(q) = \rho_0{}^\gamma \left(\frac{\mu}{2\sqrt{\pi}}\right)^3 e^{-\frac{\mu^2 q^2}{4}}$$

$\rho_0$ *is the* saturation density of infinite nuclear matter.

In the limits,

$\mathbf{q} = \mathbf{k}_p - \mathbf{k}_h$ *and* $\mathbf{q}' = \mathbf{k}_{p'} - \mathbf{k}_{h'} \to 0$

*we* have the following results,

$$\mathrm{G}_\mu\left(\left|\mathbf{k}_p - \mathbf{k}_h\right|\right) \to \mathrm{G}_\mu(0) \tag{15}$$
$$\mathrm{G}_\mu\left(\left|\mathbf{k}_p - \mathbf{k}_{p'}\right|\right) \to \mathrm{G}_\mu\left(\left|\mathbf{k}_h - \mathbf{k}_{h'}\right|\right)$$
$$\mathrm{G}_\mu\left(\left|\mathbf{k}_p - \mathbf{k}_{h'}\right|\right) \to \mathrm{G}_\mu\left(\left|\mathbf{k}_h - \mathbf{k}_{h'}\right|\right)$$

### I-b Spin-isospin part

$$
\begin{aligned}
P_{ij} &= W + B\left(\frac{1+\sigma_i\sigma_j}{2}\right) - H\left(\frac{1+\tau_i\tau_j}{2}\right) - M\left(\frac{1+\sigma_i\sigma_j}{2}\right)\left(\frac{1+\tau_i\tau_j}{2}\right) \\
&= \left(W + \frac{B-H}{2} - \frac{M}{4}\right) + \left(\frac{B}{2} - \frac{M}{4}\right)\sigma_i\sigma_j - \left(\frac{H}{2} + \frac{M}{4}\right)\tau_i\tau_j - \frac{M}{4}\sigma_i\sigma_j\tau_i\tau_j \\
P'_{ij} &= M + HP_\sigma - BP_\tau - WP_\sigma P_\tau \\
&= \left(M + \frac{H-B}{2} - \frac{W}{4}\right) + \left(\frac{H}{2} - \frac{W}{4}\right)\sigma_i\sigma_j - \left(\frac{B}{2} + \frac{W}{4}\right)\tau_i\tau_j - \frac{W}{4}\sigma_i\sigma_j\tau_i\tau_j
\end{aligned}
\tag{16}
$$

The contribution in nuclear matter is then,

$$
\boxed{
\begin{aligned}
&\left[\left(W + \frac{B-H}{2} - \frac{M}{4}\right) + \left(\frac{B}{2} - \frac{M}{4}\right)\sigma_i\sigma_j - \left(\frac{H}{2} + \frac{M}{4}\right)\tau_i\tau_j - \frac{M}{4}\sigma_i\sigma_j\tau_i\tau_j\right] G_\mu(0) \\
&+\left[\left(M + \frac{H-B}{2} - \frac{W}{4}\right) + \left(\frac{H}{2} - \frac{W}{4}\right)\sigma_i\sigma_j - \left(\frac{B}{2} + \frac{W}{4}\right)\tau_i\tau_j - \frac{W}{4}\sigma_i\sigma_j\tau_i\tau_j\right] G_\mu\left(\left|\mathbf{k}_F - \mathbf{k}'_F\right|\right)
\end{aligned}
}
\tag{CT1}
$$

with $G_\mu\left(\left|\mathbf{k}_F - \mathbf{k}'_F\right|\right) = \rho_0\left(\frac{\mu}{2\sqrt{\pi}}\right)^3 e^{\frac{\mu^2}{2}k_F^2(1-\cos\theta)}$.

This term will be expanded on the basis of Legendre polynomials $P_l(\cos\theta)$.

$$
\text{II-}\sum_{ho}\left(\left\langle h'ho\left|\frac{\partial V_{ij}^a}{\partial\rho_{hp}}\right|p'ho\right\rangle + \left\langle pho\left|\frac{\partial V_{ij}^a}{\partial\rho_{p'h'}}\right|hho\right\rangle\right)
$$

We first take in account the first term and set,

$$
\sum_{ho}\left\langle h'ho\left|\frac{\partial V_{ij}^a}{\partial\rho_{hp}}\right|p'ho\right\rangle = \left\langle ph'\left|V_{ij}^{D1}\right|hp'\right\rangle \tag{17}
$$

$$
\frac{\partial V_{ij}^a}{\partial\rho_{hp}} = \gamma e^{-\frac{(\mathbf{r}_i-\mathbf{r}_j)^2}{\mu_c^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_i) + \rho(\mathbf{r}_j)\right)\right]^{\gamma-1}\left(P_{ij} + P_r P'_{ij}\right)\left\{\frac{\partial\rho(\mathbf{r}_i)}{\partial\rho_{hp}} + \frac{\partial\rho(\mathbf{r}_j)}{\partial\rho_{hp}}\right\} \tag{18}
$$

$$\frac{\partial\rho(\mathbf{r}_i)}{\partial\rho_{hp}} = \sum_{\sigma\tau}\psi_h(\mathbf{r}_i,\sigma,\tau)\psi_h^*(\mathbf{r}_i,\sigma,\tau) = \sum_{\sigma\tau}\int\delta(\mathbf{r}_k-\mathbf{r}_i)\psi_h(\mathbf{r}_k,\sigma,\tau)\psi_h^*(\mathbf{r}_k,\sigma,\tau)$$
$$= \left\langle p\middle|\delta(\mathbf{r}_k-\mathbf{r}_i)\middle|h\right\rangle \tag{19}$$

Inserting that result in (19) we get,

$$\frac{\partial V_{ij}^a}{\partial\rho_{hp}} = \frac{\gamma}{2}\left\langle p\left|e^{-\frac{(\mathbf{r}_1-\mathbf{r}_2)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_1)+\rho(\mathbf{r}_2)\right)\right]^{\gamma-1}\left(\delta(\mathbf{r}_3-\mathbf{r}_1)+\delta(\mathbf{r}_3-\mathbf{r}_2)\right)\left(P_{12}+P_rP_{12}'\right)\right|h\right\rangle \tag{20}$$

Inserting in the left hand side of (17) we obtain,

$$\frac{\gamma}{2}\sum_{ho}\left\langle h'\ ho\ p\left|\begin{matrix}e^{-\frac{(\mathbf{r}_1-\mathbf{r}_2)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_1)+\rho(\mathbf{r}_2)\right)\right]^{\gamma-1}\left(\delta(\mathbf{r}_3-\mathbf{r}_1)+\delta(\mathbf{r}_3-\mathbf{r}_2)\right)\\ \left(P_{12}+P_rP_{12}'\right)\end{matrix}\right|p'\ ho\ h\right\rangle \tag{21}$$

Reindexing the states we may write,

$$\frac{\gamma}{2}\sum_{ho}\left\langle p\ h'\ ho\left|\begin{matrix}e^{-\frac{(\mathbf{r}_2-\mathbf{r}_3)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_2)+\rho(\mathbf{r}_3)\right)\right]^{\gamma-1}\left(\delta(\mathbf{r}_1-\mathbf{r}_2)+\delta(\mathbf{r}_1-\mathbf{r}_3)\right)\\ \left(P_{23}+P_rP_{23}'\right)\end{matrix}\right|h\ p'\ ho\right\rangle \tag{22}$$

We then have,

$$V_{12}^{D1} = \frac{\gamma}{2}\sum_{ho}\left\langle ho\left|e^{-\frac{(\mathbf{r}_2-\mathbf{r}_3)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_2)+\rho(\mathbf{r}_3)\right)\right]^{\gamma-1}\left(\delta(\mathbf{r}_1-\mathbf{r}_2)+\delta(\mathbf{r}_1-\mathbf{r}_3)\right)\left(P_{23}+P_rP_{23}'\right)\right|ho\right\rangle \tag{23}$$

II-a Direct term

$$V_{12}^{D1,1} = \frac{\gamma}{2}\sum_{ho}\left\langle ho\left| e^{\frac{(\mathbf{r}_2-\mathbf{r}_3)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_2)+\rho(\mathbf{r}_3)\right)\right]^{\gamma-1}\left(\delta(\mathbf{r}_1-\mathbf{r}_2)+\delta(\mathbf{r}_1-\mathbf{r}_3)\right)(P_{23})\right| ho\right\rangle$$

$$= \frac{\gamma}{2}\sum_{s_3t_3}\int d^3k\int d^3r_3\left|\begin{array}{l}\left[\frac{1}{2}\left(\rho(\mathbf{r}_2)+\rho(\mathbf{r}_3)\right)\right]^{\gamma-1}\frac{e^{-i\mathbf{k}\mathbf{r}_3}}{(2\pi)^{3/2}}e^{\frac{(\mathbf{r}_2-\mathbf{r}_3)^2}{\mu^2}} \\ \left(\delta(\mathbf{r}_1-\mathbf{r}_2)+\delta(\mathbf{r}_1-\mathbf{r}_3)\right)\frac{e^{i\mathbf{k}\mathbf{r}_3}}{(2\pi)^{3/2}}\langle s_3t_3|(P_{23})\end{array}\right. |s_3t_3\rangle \tag{24}$$

$$\sum_{s_3t_3}\langle s_3t_3|(P_{23})|s_3t_3\rangle = \sum_{s_3t_3}\left[\begin{array}{l}W+\frac{B}{2}\left(1+\vec{\sigma}_2\langle s_3|\vec{\sigma}_3|s_3\rangle\right)-\frac{H}{2}\left(1+\vec{\tau}_2\langle t_3|\vec{\tau}_3 t_3\rangle\right) \\ -\frac{M}{4}\left(1+\vec{\sigma}_2\langle s_3|\vec{\sigma}_3|s_3\rangle\right)\left(1+\vec{\tau}_2\langle t_3|\vec{\tau}_3|t_3\rangle\right)\end{array}\right]$$

$$= \sum_{s_3t_3}\left[\begin{array}{l}\left(W+\frac{B}{2}-\frac{H}{2}-\frac{M}{4}\right)+\left(\frac{B}{2}-\frac{M}{4}\right)\vec{\sigma}_2\langle s_3|\vec{\sigma}_3|s_3\rangle-\left(\frac{H}{2}+\frac{M}{4}\right)\vec{\tau}_2\langle t_3|\vec{\tau}_3|t_3\rangle \\ -\frac{M}{4}\vec{\sigma}_2\langle s_3|\vec{\sigma}_3|s_3\rangle\vec{\tau}_2\langle t_3|\vec{\tau}_3|t_3\rangle\end{array}\right]$$

$$= 4\left(W+\frac{B}{2}-\frac{H}{2}-\frac{M}{4}\right)$$

$$V_{12}^{D1,1} = \frac{\gamma}{2}4\left(W+\frac{B}{2}-\frac{H}{2}-\frac{M}{4}\right)\frac{\rho_0}{4}\left|\begin{array}{l}\int d^3r_3\left[\frac{1}{2}\left(\rho(\mathbf{r}_2)+\rho(\mathbf{r}_3)\right)\right]^{\gamma-1}e^{\frac{(\mathbf{r}_2-\mathbf{r}_3)^2}{\mu^2}} \\ \left(\delta(\mathbf{r}_1-\mathbf{r}_2)+\delta(\mathbf{r}_1-\mathbf{r}_3)\right)\end{array}\right.$$

$$= \frac{\gamma}{2}\rho_0\left(W+\frac{B}{2}-\frac{H}{2}-\frac{M}{4}\right)\left\{\begin{array}{l}\left[\frac{1}{2}\left(\rho(\mathbf{r}_2)+\rho(\mathbf{r}_1)\right)\right]^{\gamma-1}e^{\frac{(\mathbf{r}_2-\mathbf{r}_1)^2}{\mu^2}} \\ +\delta(\mathbf{r}_1-\mathbf{r}_2)\int d^3r_3\left[\frac{1}{2}\left(\rho(\mathbf{r}_2)+\rho(\mathbf{r}_3)\right)\right]^{\gamma-1}e^{\frac{(\mathbf{r}_2-\mathbf{r}_3)^2}{\mu^2}}\end{array}\right\} \tag{25}$$

The contribution for that term is then,

$$\langle ph'|V_{12}^{D1,1}|hp'\rangle = \frac{\gamma}{2}\left(W+\frac{B}{2}-\frac{H}{2}-\frac{M}{4}\right)\left\{\begin{array}{l}G_\mu(\mathbf{k}_p-\mathbf{k}_h) \\ +\rho_0^{\gamma-1}\langle ph'|\delta(\mathbf{r}_1-\mathbf{r}_2)\int d^3r_3 e^{\frac{(\mathbf{r}_2-\mathbf{r}_3)^2}{\mu^2}}|hp'\rangle\end{array}\right\} \tag{26}$$

which gives,

$$\boxed{\langle ph'|V_{12}^{D1,1}|hp'\rangle = \frac{\gamma}{2}\left(W+\frac{B}{2}-\frac{H}{2}-\frac{M}{4}\right)\left(G_\mu\left(\mathbf{k}_p-\mathbf{k}_h\right)+G_\mu(0)\right)} \tag{27}$$

In the case of nuclear matter and $\mathbf{q}=\mathbf{k}_p-\mathbf{k}_h \rightarrow 0$ we have,

,

$$\boxed{\langle ph'|V_{12}^{D1,1}|hp'\rangle = \gamma\left(W+\frac{B}{2}-\frac{H}{2}-\frac{M}{4}\right)G_\mu(0)} \tag{CT2}$$

II-b Exchange term

$$V_{12}^{D1,2} = \frac{\gamma}{2}\sum_{ho}\left\langle ho\left|e^{\frac{(\mathbf{r}_2-\mathbf{r}_3)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_2)+\rho(\mathbf{r}_3)\right)\right]^{\gamma-1}\left(\delta(\mathbf{r}_1-\mathbf{r}_2)+\delta(\mathbf{r}_1-\mathbf{r}_3)\right)\left(P_r P'_{23}\right)\right|ho\right\rangle$$

$$= \frac{\gamma}{2}\sum_{s_3t_3}\int d^3k\int d^3r_3\left|\begin{array}{l}\left[\frac{1}{2}\left(\rho(\mathbf{r}_2)+\rho(\mathbf{r}_3)\right)\right]^{\gamma-1}\frac{e^{-i\mathbf{k}\mathbf{r}_3}}{(2\pi)^{3/2}}e^{\frac{(\mathbf{r}_2-\mathbf{r}_3)^2}{\mu^2}} \\ \left(\delta(\mathbf{r}_1-\mathbf{r}_2)+\delta(\mathbf{r}_1-\mathbf{r}_3)\right)P_r\frac{e^{i\mathbf{k}\mathbf{r}_3}}{(2\pi)^{3/2}}\langle s_3t_3|(P'_{23})\end{array}\right|s_3t_3\rangle \tag{28}$$

$$\sum_{s_3t_3}\left\langle s_3t_3\left|\left(P'_{23}\right)\right|s_3t_3\right\rangle = \sum_{s_3t_3}\left[\begin{array}{l}M+\frac{H}{2}\left(1+\vec{\sigma}_2\langle s_3|\vec{\sigma}_3|s_3\rangle\right)-\frac{B}{2}\left(1+\vec{\tau}_2\langle t_3|\vec{\tau}_3 t_3\rangle\right) \\ -\frac{W}{4}\left(1+\vec{\sigma}_2\langle s_3|\vec{\sigma}_3|s_3\rangle\right)\left(1+\vec{\tau}_2\langle t_3|\vec{\tau}_3|t_3\rangle\right)\end{array}\right]$$

$$= \sum_{s_3t_3}\left[\begin{array}{l}\left(M+\frac{H}{2}-\frac{B}{2}-\frac{W}{4}\right)+\left(\frac{H}{2}-\frac{W}{4}\right)\vec{\sigma}_2\langle s_3|\vec{\sigma}_3|s_3\rangle-\left(\frac{B}{2}+\frac{W}{4}\right)\vec{\tau}_2\langle t_3|\vec{\tau}_3|t_3\rangle \\ -\frac{W}{4}\vec{\sigma}_2\langle s_3|\vec{\sigma}_3|s_3\rangle\vec{\tau}_2\langle t_3|\vec{\tau}_3|t_3\rangle\end{array}\right]$$

$$= 4\left(M+\frac{H}{2}-\frac{B}{2}-\frac{W}{4}\right)$$

$$\langle ph'|V_{12}^{D1,2}|hp'\rangle = \langle p\ h'\ ho\,|V_{12}^{D1,2}|h\,p'\ ho\,\rangle$$

$$= \frac{\gamma}{2}\frac{1}{(2\pi)^6}\int\frac{d^3k_3}{(2\pi)^3}\int d^3r_1\int d^3r_2\int d^3r_3 e^{i\mathbf{k}_h\mathbf{r}_1}e^{i\mathbf{k}_{p'}\mathbf{r}_2}e^{i\mathbf{k}_3\mathbf{r}_3}e^{-\frac{(\mathbf{r}_2-\mathbf{r}_3)^2}{\mu^2}}$$

$$\left[\frac{1}{2}\left(\rho(\mathbf{r}_2)+\rho(\mathbf{r}_3)\right)\right]^{\gamma-1}\left(\delta(\mathbf{r}_1-\mathbf{r}_2)+\delta(\mathbf{r}_1-\mathbf{r}_3)\right)e^{-i\mathbf{k}_p\mathbf{r}_1}e^{-i\mathbf{k}_{h'}\mathbf{r}_2}e^{-i\mathbf{k}_3\mathbf{r}_3} \tag{29}$$

$$= \frac{\gamma}{2}\frac{1}{(2\pi)^6}\int\frac{d^3k_3}{(2\pi)^3}\int d^3r_1\int d^3r_2\int d^3r_3 e^{i(\mathbf{k}_h-\mathbf{k}_p)\mathbf{r}_1}e^{i(\mathbf{k}_{p'}-\mathbf{k}_3)\mathbf{r}_2}e^{i(\mathbf{k}_3-\mathbf{k}_{h'})\mathbf{r}_3}e^{-\frac{(\mathbf{r}_2-\mathbf{r}_3)^2}{\mu^2}}$$

$$\left[\frac{1}{2}\left(\rho(\mathbf{r}_2)+\rho(\mathbf{r}_3)\right)\right]^{\gamma-1}\left(\delta(\mathbf{r}_1-\mathbf{r}_2)+\delta(\mathbf{r}_1-\mathbf{r}_3)\right)$$

- term with $\delta(\mathbf{r}_1-\mathbf{r}_2)$

$$\frac{\gamma}{2}\frac{1}{(2\pi)^6}\int\frac{d^3k_3}{(2\pi)^3}\int d^3r_1\int d^3r_2\int d^3r_3 e^{i(\mathbf{k}_h-\mathbf{k}_p)\mathbf{r}_1}e^{i(\mathbf{k}_{p'}-\mathbf{k}_3)\mathbf{r}_2}e^{i(\mathbf{k}_3-\mathbf{k}_{h'})\mathbf{r}_3}e^{-\frac{(\mathbf{r}_2-\mathbf{r}_3)^2}{\mu^2}}$$

$$\left[\frac{1}{2}\left(\rho(\mathbf{r}_2)+\rho(\mathbf{r}_3)\right)\right]^{\gamma-1}\left(\delta(\mathbf{r}_1-\mathbf{r}_2)\right)$$

$$= \frac{\gamma}{2}\frac{1}{(2\pi)^6}\int\frac{d^3k_3}{(2\pi)^3}\int d^3r_1\int d^3r_3 e^{i(\mathbf{k}_h-\mathbf{k}_p+\mathbf{k}_{p'}-\mathbf{k}_3)\mathbf{r}_1}e^{i(\mathbf{k}_3-\mathbf{k}_{h'})\mathbf{r}_3}e^{-\frac{(\mathbf{r}_1-\mathbf{r}_3)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_1)+\rho(\mathbf{r}_3)\right)\right]^{\gamma-1} \tag{30}$$

$$= \frac{\gamma}{2}\frac{1}{(2\pi)^6}\int\frac{d^3k_3}{(2\pi)^3}\int d^3r_1\int d^3r_3 e^{i(-\mathbf{k}_3+\mathbf{k}_{h'})(\mathbf{r}_1-\mathbf{r}_3)}e^{-\frac{(\mathbf{r}_1-\mathbf{r}_3)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_1)+\rho(\mathbf{r}_3)\right)\right]^{\gamma-1}$$

$$= \frac{\gamma}{2}\frac{1}{(2\pi)^6}\int d^3r_1\int d^3r_3 e^{i(\mathbf{k}_{h'})(\mathbf{r}_1-\mathbf{r}_3)}e^{-\frac{(\mathbf{r}_1-\mathbf{r}_3)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_1)+\rho(\mathbf{r}_3)\right)\right]^{\gamma-1}\int\frac{d^3k_3}{(2\pi)^3}e^{i(-\mathbf{k}_3)(\mathbf{r}_1-\mathbf{r}_3)}$$

$$\int\frac{d^3k_3}{(2\pi)^3}e^{i(-\mathbf{k}_3)(\mathbf{r}_1-\mathbf{r}_3)} = \frac{1}{(2\pi)^3}2\pi\int_0^\pi\sin(\theta)d\theta\int_0^{k_F}k^2dk e^{-i\mathbf{k}(\mathbf{r}_1-\mathbf{r}_3)}$$

$$= \frac{1}{(2\pi)^3}2\pi\int_0^{k_F}k^2dk\int_0^\pi\sin(\theta)d\theta e^{-ikr\cos(\theta)} \tag{31}$$

$$= \frac{1}{(2\pi)^3}\frac{2\pi}{r}\int_0^{k_F}kdk\sin(kr) = \frac{1}{(2\pi)^3}\frac{4\pi}{r}k_F^2 j_1(k_F r) = \frac{k_F^3}{2\pi^2}\frac{j_1(k_F r)}{k_F r} = \frac{3\rho_0}{4}\frac{j_1(k_F r)}{k_F r}$$

Inserting in the previous expression we get,

$$= \frac{\gamma}{2}\frac{1}{(2\pi)^6}\int d^3r_1\int d^3r_3 e^{i(\mathbf{k}_{h'})(\mathbf{r}_1-\mathbf{r}_3)}e^{-\frac{(\mathbf{r}_1-\mathbf{r}_3)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_1)+\rho(\mathbf{r}_3)\right)\right]^{\gamma-1}\frac{3\rho_0}{4}\frac{j_1(k_F r)}{k_F r}$$

$$= \frac{3}{4}\frac{\gamma}{2}\rho_0^\gamma\frac{1}{(2\pi)^3}\int d^3r e^{ik_F r\cos\theta}e^{-\frac{r^2}{\mu^2}}\frac{j_1(k_F r)}{k_F r}$$

$$= \frac{3}{4}\frac{\gamma}{2}\rho_0^\gamma\frac{1}{(2\pi)^2}\int_0^\infty dr r^2 e^{-\frac{r^2}{\mu^2}}\frac{j_1(k_F r)}{k_F r}\int_0^\pi d\theta\sin\theta e^{ik_F r\cos\theta}$$

$$= \gamma\frac{3}{4}\rho_0^\gamma\frac{1}{(2\pi)^2}\int_0^\infty dr r^2 e^{-\frac{r^2}{\mu^2}}\frac{j_1(k_F r)}{k_F r}j_0(k_F r) \tag{32}$$

The final result is then,

$$\gamma\frac{3}{4}\rho_0^\gamma\frac{1}{(2\pi)^2}\int_0^\infty dr r^2 e^{-\frac{r^2}{\mu^2}}\frac{j_1(k_F r)}{k_F r}j_0(k_F r)$$

- term with $\delta(\mathbf{r}_1-\mathbf{r}_3)$

We get the same result as in the preceding term and finally,

$$\boxed{\langle ph'|V_{12}^{D1,2}|hp'\rangle = 4\left[M+\frac{H}{2}-\frac{B}{2}-\frac{W}{4}\right]\gamma\frac{3}{2}\rho_0^\gamma\frac{1}{(2\pi)^2}\int_0^\infty dr r^2 e^{-\frac{r^2}{\mu^2}}\frac{j_1(k_F r)}{k_F r}j_0(k_F r)} \tag{CT3}$$

The second term in (A),

$$\left\langle pho\left|\frac{\partial V_{ij}^a}{\partial\rho_{p'h'}}\right|hho\right\rangle = \left\langle ph'\left|V_{ij}^{D2} = V_{ij}^{D2,1}+V_{ij}^{D2,2}\right|hp'\right\rangle \tag{33}$$

gives the same contributions as (17),

$$\boxed{\langle ph'|V_{12}^{D2,1}|hp'\rangle = \gamma\left(W+\frac{B}{2}-\frac{H}{2}-\frac{M}{4}\right)G_\mu(0)} \tag{CT4}$$

$$\boxed{\langle p\,h'\,|V_{12}^{D2,2}|h\,p'\,\rangle = 4\left(M+\frac{H}{2}-\frac{B}{2}-\frac{W}{4}\right)\frac{1}{2\pi^2}\frac{3\gamma}{8}\rho_0^\gamma\int_0^\infty r^2 dr e^{-\frac{(r)^2}{\mu^2}}\left(\frac{j_1(k_F r)}{k_F r}\right)} \tag{CT5}$$

III- Finally we set,

$$\frac{1}{2}\sum_{ho\,h'o}\left\langle hoh'o\left|\frac{\partial^2 V_{ij}^a}{\partial\rho_{hp}\partial\rho_{p'h'}}\right|hoh'o\right\rangle=\left\langle ph'\left|V_{12}^{D3}\right|hp'\right\rangle$$

We have seen that,

$$\frac{\partial V_{12}^a}{\partial\rho_{hp}}=\frac{\gamma}{2}\left\langle p\left|e^{-\frac{(\mathbf{r}_1-\mathbf{r}_2)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_1)+\rho(\mathbf{r}_2)\right)\right]^{\gamma-1}\left(\delta(\mathbf{r}_3-\mathbf{r}_1)+\delta(\mathbf{r}_3-\mathbf{r}_2)\right)\left(P_{12}+P_rP_{12}'\right)\right|h\right\rangle \tag{34}$$

deriving once more we obtain,

$$\frac{\partial^2 V_{12}^a}{\partial\rho_{hp}\partial\rho_{p'h'}}=\frac{\gamma(\gamma-1)}{4}\left\langle p\left|\begin{array}{l}e^{-\frac{(\mathbf{r}_1-\mathbf{r}_2)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_1)+\rho(\mathbf{r}_2)\right)\right]^{\gamma-2}\left[\frac{\partial\rho(\mathbf{r}_1)}{\partial\rho_{p'h'}}+\frac{\partial\rho(\mathbf{r}_2)}{\partial\rho_{p'h'}}\right]\\ \left(\delta(\mathbf{r}_3-\mathbf{r}_1)+\delta(\mathbf{r}_3-\mathbf{r}_2)\right)\left(P_{12}+P_rP_{12}'\right)\end{array}\right|h\right\rangle \tag{35}$$

We have previously seen that,

$$\frac{\partial\rho(\mathbf{r}_i)}{\partial\rho_{p'h'}}=\left\langle h'\left|\delta(\mathbf{r}_4-\mathbf{r}_i)\right|p'\right\rangle$$

Inserting in the previous expression,

$$\frac{\partial^2 V_{12}^a}{\partial\rho_{hp}\partial\rho_{p'h'}}=\frac{\gamma(\gamma-1)}{4}\left\langle ph'\left|\begin{array}{l}e^{-\frac{(\mathbf{r}_1-\mathbf{r}_2)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_1)+\rho(\mathbf{r}_2)\right)\right]^{\gamma-2}\left[\delta(\mathbf{r}_4-\mathbf{r}_1)+\delta(\mathbf{r}_4-\mathbf{r}_2)\right]\\ \left(\delta(\mathbf{r}_3-\mathbf{r}_1)+\delta(\mathbf{r}_3-\mathbf{r}_2)\right)\left(P_{12}+P_rP_{12}'\right)\end{array}\right|hp'\right\rangle \tag{36}$$

Reindexing the states,

$$\left\langle ph'\middle|V_{12}^{D3}\middle|hp'\right\rangle = \frac{1}{8}\gamma(\gamma-1)\sum_{ho\,h'o}\left\langle ph'\,hoh'o\left|\begin{array}{l} e^{-\frac{(\mathbf{r}_3-\mathbf{r}_4)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_3)+\rho(\mathbf{r}_4)\right)\right]^{\gamma-2} \\ \left[\left\{\delta(\mathbf{r}_2-\mathbf{r}_3)+\delta(\mathbf{r}_2-\mathbf{r}_4)\right\}\right] \\ \left[\left\{\delta(\mathbf{r}_1-\mathbf{r}_3)+\delta(\mathbf{r}_1-\mathbf{r}_4)\right\}\right] \\ (P_{34}+P_rP'_{34}) \end{array}\right| hp'\,hoh'o\right\rangle \tag{37}$$

We have the following expressions,

$$V_{12}^{D3,1} = \frac{1}{8}\gamma(\gamma-1)\sum_{ho\,h'o}\left\langle hoh'o\left|\begin{array}{l} e^{-\frac{(\mathbf{r}_3-\mathbf{r}_4)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_3)+\rho(\mathbf{r}_4)\right)\right]^{\gamma-2}\delta(\mathbf{r}_2-\mathbf{r}_3) \\ (\delta(\mathbf{r}_1-\mathbf{r}_3))(P_{34}+P_rP'_{34}) \end{array}\right| hoh'o\right\rangle$$

$$V_{12}^{D3,2} = \frac{1}{8}\gamma(\gamma-1)\sum_{ho\,h'o}\left\langle hoh'o\left|\begin{array}{l} e^{-\frac{(\mathbf{r}_3-\mathbf{r}_4)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_3)+\rho(\mathbf{r}_4)\right)\right]^{\gamma-2}\delta(\mathbf{r}_2-\mathbf{r}_3) \\ \delta(\mathbf{r}_1-\mathbf{r}_4)(P_{34}+P_rP'_{34}) \end{array}\right| hoh'o\right\rangle$$

$$V_{12}^{D3,3} = \frac{1}{8}\gamma(\gamma-1)\sum_{ho\,h'o}\left\langle hoh'o\left|\begin{array}{l} e^{-\frac{(\mathbf{r}_3-\mathbf{r}_4)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_3)+\rho(\mathbf{r}_4)\right)\right]^{\gamma-2}\delta(\mathbf{r}_2-\mathbf{r}_4) \\ \delta(\mathbf{r}_1-\mathbf{r}_3)(P_{34}+P_rP'_{34}) \end{array}\right| hoh'o\right\rangle \tag{38}$$

$$V_{12}^{D3,4} = \frac{1}{8}\gamma(\gamma-1)\sum_{ho\,h'o}\left\langle hoh'o\left|\begin{array}{l} e^{-\frac{(\mathbf{r}_3-\mathbf{r}_4)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_3)+\rho(\mathbf{r}_4)\right)\right]^{\gamma-2}\delta(\mathbf{r}_2-\mathbf{r}_4) \\ \delta(\mathbf{r}_1-\mathbf{r}_4)(P_{34}+P_rP'_{34}) \end{array}\right| hoh'o\right\rangle$$

III-a Direct terms

$$V_{12}^{D3,1} = \frac{1}{8}\gamma(\gamma-1)\int_0^{k_F}\frac{d^3k}{(2\pi)^3}\int_0^{k_F}\frac{d^3k'}{(2\pi)^3}\sum_{s_3t_3}\sum_{s_4t_4}\int d^3r_3\int d^3r_4\, e^{-\frac{(\mathbf{r}_3-\mathbf{r}_4)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_3)+\rho(\mathbf{r}_4)\right)\right]^{\gamma-2} \tag{39}$$
$$\delta(\mathbf{r}_2-\mathbf{r}_3)(\delta(\mathbf{r}_1-\mathbf{r}_3))\langle s_3t_3s_4t_4|P_{34}|s_3t_3s_4t_4\rangle$$

$$V_{12}^{D3,1} = \frac{1}{8}\gamma(\gamma-1)\delta(\mathbf{r}_1-\mathbf{r}_2)\int_0^{k_F}\frac{d^3k}{(2\pi)^3}\int_0^{k_F}\frac{d^3k'}{(2\pi)^3}\sum_{s_3t_3}\sum_{s_4t_4}\int d^3r_4 e^{-\frac{(\mathbf{r}_1-\mathbf{r}_4)^2}{\mu^2}}\left[\frac{1}{2}(\rho(\mathbf{r}_1)+\rho(\mathbf{r}_4))\right]^{\gamma-2} \langle s_3t_3s_4t_4|P_{34}|s_3t_3s_4t_4\rangle \tag{40}$$

For the spin-isospin contribution we have,

$$\begin{aligned}\sum_{s_3t_3}\sum_{s_4t_4}\langle s_3t_3s_4t_4|P_{34}|s_3t_3s_4t_4\rangle &= \sum_{s_3t_3}\sum_{s_4t_4}\left(W+\frac{B-H}{2}-\frac{M}{4}\right)+\left(\frac{B}{2}-\frac{M}{4}\right)\langle s_3|\sigma_3|s_3\rangle\langle s_4|\sigma_4|s_4\rangle \\ &\quad -\left(\frac{H}{2}+\frac{M}{4}\right)\langle t_3|\tau_3|t_3\rangle\langle t_4|\tau_4|t_4\rangle-\frac{M}{4}\langle s_3|\sigma_3|s_3\rangle\langle s_4|\sigma_4|s_4\rangle\langle t_3|\tau_3|t_3\rangle\langle t_4|\tau_4|t_4\rangle \\ &= 16\left(W+\frac{B-H}{2}-\frac{M}{4}\right)\end{aligned}$$

The contributions for the different terms are then,

$$\begin{aligned}\langle ph'|V_{12}^{D3,1}|hp'\rangle &= \frac{16}{8}\left(W+\frac{B-H}{2}-\frac{M}{4}\right)\left(\frac{\rho_0}{4}\right)^2\gamma(\gamma-1)\left\langle ph'\left|\begin{matrix}\int d^3r_4 e^{-\frac{(\mathbf{r}_1-\mathbf{r}_4)^2}{\mu^2}}\delta(\mathbf{r}_2-\mathbf{r}_1)\\ \left[\frac{1}{2}(\rho(\mathbf{r}_1)+\rho(\mathbf{r}_4))\right]^{\gamma-2}\end{matrix}\right|hp'\right\rangle \\ &= \frac{1}{8}\left(W+\frac{B-H}{2}-\frac{M}{4}\right)\gamma(\gamma-1)\rho_0^{\gamma}\left\langle ph'\left|\int d^3r_4 e^{-\frac{(\mathbf{r}_1-\mathbf{r}_4)^2}{\mu^2}}\delta(\mathbf{r}_2-\mathbf{r}_1)\right|hp'\right\rangle \\ &= \frac{1}{8}\left(W+\frac{B-H}{2}-\frac{M}{4}\right)\gamma(\gamma-1)G_\mu(0)=\langle ph'|V_{12}^{D3,4}|hp'\rangle\end{aligned} \tag{41}$$

$$\begin{aligned}\langle ph'|V_{12}^{D3,2}|hp'\rangle &= \frac{16}{8}\left(W+\frac{B-H}{2}-\frac{M}{4}\right)\left(\frac{\rho_0}{4}\right)^2\gamma(\gamma-1)\left\langle ph'\left|e^{-\frac{(\mathbf{r}_1-\mathbf{r}_2)^2}{\mu^2}}\left[\frac{1}{2}(\rho(\mathbf{r}_1)+\rho(\mathbf{r}_2))\right]^{\gamma-2}\right|hp'\right\rangle \\ &= \frac{1}{8}\left(W+\frac{B-H}{2}-\frac{M}{4}\right)\gamma(\gamma-1)G_\mu(\mathbf{k}_p-\mathbf{k}_h)=\langle ph'|V_{12}^{D3,3}|hp'\rangle\end{aligned} \tag{42}$$

In the case of nuclear matter and $\mathbf{q}=\mathbf{k}_p-\mathbf{k}_h \to 0$, we have,

$$\begin{aligned}\langle ph'|V_{12}^{D3,1}|hp'\rangle &= \frac{1}{8}\left(W+\frac{B-H}{2}-\frac{M}{4}\right)\gamma(\gamma-1)G_\mu(0)=\langle ph'|V_{12}^{D3,4}|hp'\rangle \\ \langle ph'|V_{12}^{D3,2}|hp'\rangle &= \frac{1}{8}\left(W+\frac{B-H}{2}-\frac{M}{4}\right)\gamma(\gamma-1)G_\mu(0)=\langle ph'|V_{12}^{D3,3}|hp'\rangle\end{aligned} \tag{CT6}$$

III-b Exchange terms

$$V_{12}^{D3,1}=\frac{1}{8}\gamma(\gamma-1)\sum_{ho\,h'o}\left\langle hoh'o\left|e^{-\frac{(\mathbf{r}_3-\mathbf{r}_4)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_3)+\rho(\mathbf{r}_4)\right)\right]^{\gamma-2}\delta(\mathbf{r}_2-\mathbf{r}_3)\right|\delta(\mathbf{r}_1-\mathbf{r}_3)(P_rP'_{34})\left|hoh'o\right\rangle\right.$$

$$V_{12}^{D3,2}=\frac{1}{8}\gamma(\gamma-1)\sum_{ho\,h'o}\left\langle hoh'o\left|e^{-\frac{(\mathbf{r}_3-\mathbf{r}_4)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_3)+\rho(\mathbf{r}_4)\right)\right]^{\gamma-2}\delta(\mathbf{r}_2-\mathbf{r}_3)\right|\delta(\mathbf{r}_1-\mathbf{r}_4)(P_rP'_{34})\left|hoh'o\right\rangle\right.$$

$$V_{12}^{D3,3}=\frac{1}{8}\gamma(\gamma-1)\sum_{ho\,h'o}\left\langle hoh'o\left|e^{-\frac{(\mathbf{r}_3-\mathbf{r}_4)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_3)+\rho(\mathbf{r}_4)\right)\right]^{\gamma-2}\delta(\mathbf{r}_2-\mathbf{r}_4)\right|\delta(\mathbf{r}_1-\mathbf{r}_3)(P_rP'_{34})\left|hoh'o\right\rangle\right. \tag{43}$$

$$V_{12}^{D3,4}=\frac{1}{8}\gamma(\gamma-1)\sum_{ho\,h'o}\left\langle hoh'o\left|e^{-\frac{(\mathbf{r}_3-\mathbf{r}_4)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_3)+\rho(\mathbf{r}_4)\right)\right]^{\gamma-2}\delta(\mathbf{r}_2-\mathbf{r}_4)\right|\delta(\mathbf{r}_1-\mathbf{r}_4)(P_rP'_{34})\left|hoh'o\right\rangle\right.$$

One difference with the direct term is the spin-isospin contribution,

$$\begin{aligned}\sum_{s_3t_3}\sum_{s_4t_4}\langle s_3t_3s_4t_4|P'_{34}|s_3t_3s_4t_4\rangle &= \sum_{s_3t_3}\sum_{s_4t_4}\left(M+\frac{H-B}{2}-\frac{W}{4}\right)+\left(\frac{H}{2}-\frac{W}{4}\right)\langle s_3|\sigma_3|s_3\rangle\langle s_4|\sigma_4|s_4\rangle\\ &\quad-\left(\frac{B}{2}+\frac{W}{4}\right)\langle t_3|\tau_3|t_3\rangle\langle t_4|\tau_4|t_4\rangle-\frac{W}{4}\langle s_3|\sigma_3|s_3\rangle\langle s_4|\sigma_4|s_4\rangle\langle t_3|\tau_3|t_3\rangle\langle t_4|\tau_4|t_4\rangle\\ &=16\left(M+\frac{H-B}{2}-\frac{W}{4}\right)\end{aligned}$$

$$\begin{aligned}V_{12}^{D3,1}&=\frac{1}{8}16\left(M+\frac{H-B}{2}-\frac{W}{4}\right)\gamma(\gamma-1)\delta(\mathbf{r}_1-\mathbf{r}_2)\left[\frac{1}{2}\left(\rho(\mathbf{r}_2)+\rho(\mathbf{r}_1)\right)\right]^{\gamma-2}\\ &\int d^3r_3\int d^3r_4 e^{-\frac{(\mathbf{r}_3-\mathbf{r}_4)^2}{\mu^2}}\int_0^{k_F}\frac{d^3k}{(2\pi)^3}\int_0^{k_F}\frac{d^3k'}{(2\pi)^3}e^{i(\mathbf{k}-\mathbf{k}')(\mathbf{r}_3-\mathbf{r}_4)}\end{aligned} \tag{44}$$

$$V_{12}^{D3,2} = \frac{1}{8}16\left(M + \frac{H-B}{2} - \frac{W}{4}\right)\gamma(\gamma-1)e^{-\frac{(\mathbf{r}_2-\mathbf{r}_1)^2}{\mu^2}}\left[\frac{1}{2}\left(\rho(\mathbf{r}_1)+\rho(\mathbf{r}_2)\right)\right]^{\gamma-2}$$
$$\int_0^{k_F}\frac{d^3k}{(2\pi)^3}\int_0^{k_F}\frac{d^3k'}{(2\pi)^3}e^{i(\mathbf{k}-\mathbf{k}')(\mathbf{r}_1-\mathbf{r}_2)} \tag{45}$$

Using the previously proved identity $\int\frac{d^3k_3}{(2\pi)^3}e^{i(-\mathbf{k}_3)(\mathbf{r}_1-\mathbf{r}_3)} = \frac{3\rho_0}{4}\frac{j_1(k_F r)}{k_F r}$ we get,

$$V_{12}^{D3,1} = \frac{9}{16}\frac{1}{8}16\left(M + \frac{H-B}{2} - \frac{W}{4}\right)\gamma(\gamma-1)\delta(\mathbf{r}_1-\mathbf{r}_2)[\rho_0]^{\gamma}\int d^3r e^{-\frac{(r)^2}{\mu^2}}\left(\frac{j_1(k_F r)}{k_F r}\right)^2 \tag{46}$$

$$V_{12}^{D3,2} = \frac{9}{16}\frac{1}{8}16\left(M + \frac{H-B}{2} - \frac{W}{4}\right)\gamma(\gamma-1)e^{-\frac{(\mathbf{r}_2-\mathbf{r}_1)^2}{\mu^2}}[\rho_0]^{\gamma}\left(\frac{j_1(k_F(\mathbf{r}_2-\mathbf{r}_1))}{k_F(\mathbf{r}_2-\mathbf{r}_1)}\right)^2 \tag{47}$$

$$\begin{aligned}
\langle ph'|V_{12}^{D3,1}|hp'\rangle &= \frac{9}{16}\frac{1}{8}16\left(M + \frac{H-B}{2} - \frac{W}{4}\right)\gamma(\gamma-1)[\rho_0]^{\gamma}\int d^3r e^{-\frac{(r)^2}{\mu^2}}\left(\frac{j_1(k_F r)}{k_F r}\right)^2 \\
&\quad \langle ph'|\delta(\mathbf{r}_1-\mathbf{r}_2)||hp'\rangle \\
&= \left(\frac{1}{2\pi}\right)^3\frac{9}{16}\frac{1}{8}16\left(M + \frac{H-B}{2} - \frac{W}{4}\right)\gamma(\gamma-1)[\rho_0]^{\gamma}\int d^3r e^{-\frac{(r)^2}{\mu^2}}\left(\frac{j_1(k_F r)}{k_F r}\right)^2 \\
&= \left(\frac{1}{2\pi}\right)^3\frac{9}{16}\frac{1}{8}16\left(M + \frac{H-B}{2} - \frac{W}{4}\right)\gamma(\gamma-1)[\rho_0]^{\gamma}4\pi\int_0^{\infty}drr^2 e^{-\frac{(r)^2}{\mu^2}}\left(\frac{j_1(k_F r)}{k_F r}\right)^2 \\
&= \frac{9}{16\pi^2}\left(M + \frac{H-B}{2} - \frac{W}{4}\right)\gamma(\gamma-1)[\rho_0]^{\gamma}\int_0^{\infty}drr^2 e^{-\frac{(r)^2}{\mu^2}}\left(\frac{j_1(k_F r)}{k_F r}\right)^2 \\
&= \langle ph'|V_{12}^{D3,4}|hp'\rangle
\end{aligned} \tag{48}$$

$$\begin{aligned}
\langle ph'|V_{12}^{D3,2}|hp'\rangle &= \frac{9}{8}\left(M + \frac{H-B}{2} - \frac{W}{4}\right)[\rho_0]^{\gamma}\gamma(\gamma-1)\langle ph'|e^{-\frac{(\mathbf{r}_2-\mathbf{r}_1)^2}{\mu^2}}\left(\frac{j_1(k_F(\mathbf{r}_2-\mathbf{r}_1))}{k_F(\mathbf{r}_2-\mathbf{r}_1)}\right)^2|hp'\rangle \\
&= \frac{9}{8}\left(M + \frac{H-B}{2} - \frac{W}{4}\right)[\rho_0]^{\gamma}\gamma(\gamma-1)\frac{1}{(2\pi)^3}\int d\mathbf{r}e^{i(\mathbf{k}_h-\mathbf{k}_p)\mathbf{r}}e^{-\frac{(\mathbf{r})^2}{\mu^2}}\left(\frac{j_1(k_F r)}{k_F r}\right)^2
\end{aligned} \tag{49}$$

In the case of nuclear matter and $\mathbf{q} = \mathbf{k}_p - \mathbf{k}_h \rightarrow 0$, we have,

$$\boxed{\begin{aligned}&\langle ph'|V_{12}^{D3,1}|hp'\rangle = \langle ph'|V_{12}^{D3,3}|hp'\rangle = \frac{9}{16\pi^2}\left(M + \frac{H-B}{2} - \frac{W}{4}\right)\gamma(\gamma-1)[\rho_0]^\gamma \\ &\qquad \int_0^\infty drr^2 e^{-\frac{(r)^2}{\mu^2}}\left(\frac{j_1(k_F r)}{k_F r}\right)^2 \\ &= \langle ph'|V_{12}^{D3,4}|hp'\rangle = \langle ph'|V_{12}^{D3,2}|hp'\rangle\end{aligned}} \qquad \text{(CT7)}$$

The total contribution of that finite range density dependent term is then (taking the limits $\mathbf{q} = \mathbf{k}_p - \mathbf{k}_h$ *and* $\mathbf{q}' = \mathbf{k}_{p'} - \mathbf{k}_{h'} \rightarrow 0$),

$$\begin{aligned}A_{ph,p'h'} &= \left[\left(W + \frac{B-H}{2} - \frac{M}{4}\right) + \left(\frac{B}{2} - \frac{M}{4}\right)\sigma_i\sigma_j - \left(\frac{H}{2} + \frac{M}{4}\right)\tau_i\tau_j - \frac{M}{4}\sigma_i\sigma_j\tau_i\tau_j\right]G_\mu(0) \\ &+ \\ &\left(\left(M + \frac{H-B}{2} - \frac{W}{4}\right) + \left(\frac{H}{2} - \frac{W}{4}\right)\sigma_i\sigma_j - \left(\frac{B}{2} + \frac{W}{4}\right)\tau_i\tau_j - \frac{W}{4}\sigma_i\sigma_j\tau_i\tau_j\right)G_\mu(|\mathbf{k}_F - \mathbf{k}'_F|) \qquad (50) \\ &+ \\ &+ \text{Terms arising from rearrangement}\end{aligned}$$

Rearrangement terms,

$$\boxed{\begin{aligned}&2\gamma\left(W + \frac{B}{2} - \frac{H}{2} - \frac{M}{4}\right)G_\mu(0) + \frac{3}{\pi^2}\left[M + \frac{H}{2} - \frac{B}{2} - \frac{W}{4}\right]\gamma\rho_0^\gamma \int_0^\infty r^2 dr e^{-\frac{(r)^2}{\mu^2}}\left(\frac{j_1(k_F r)}{k_F r}\right)j_0(k_F r) \\ &+\frac{1}{2}\left(W + \frac{B-H}{2} - \frac{M}{4}\right)\gamma(\gamma-1)G_\mu(0) \\ &+\frac{9}{4\pi^2}\left(M + \frac{H-B}{2} - \frac{W}{4}\right)\gamma(\gamma-1)[\rho_0]^\gamma \int_0^\infty drr^2 e^{-\frac{(r)^2}{\mu^2}}\left(\frac{j_1(k_F r)}{k_F r}\right)^2\end{aligned}} \qquad (51)$$

All theses terms are only acting in the (S=T=0) channel and for l = 0. Landau parameters are obtained by equating the expression (50) to (2). The $F_0$ Landau coefficient, which is the only one involving rearrangement terms, is the following,

$$F_0 = \frac{1}{2}\sum_c (\gamma_c+1)(\gamma_c+2)\left(W_c + \frac{B_c}{2} - \frac{H_c}{2} - \frac{M_c}{4}\right)\rho_0^{\gamma_c}\mu_c^{\,3}\pi^{3/2}$$
$$+\left(M_c + \frac{H_c - B_c}{2} - \frac{W_c}{4}\right)\left\{\begin{array}{l} 6\pi\gamma_c[\rho_0]^{\gamma_c}\int_0^\infty drr^2 e^{-\frac{(r)^2}{\mu_c^2}}\left[3(\gamma_c-1)\left(\frac{j_1(k_F r)}{k_F r}\right)^2 + 4\left(\frac{j_1(k_F r)}{k_F r}\right)j_0(k_F r)\right] \\ +\mu_c^{\,3}\pi^{3/2}\rho_0^{\gamma_c}\left(\frac{1-e^{-\mu_c^2 k_F^2}}{\mu_c^{\,2}k_F^2}\right)\end{array}\right\}$$

Other Landau parameters are not different from the D1Landau parameters and can be found in (GP77).

## 4- Conclusion

We presented a complete and detailed derivation of Landau parameters for an interaction involving density dependent finite range terms. Complications arise from the rearrangement terms which cannot be neglected in order to fulfil sum rules. Previous calculations of Landau parameters for finite range density dependent interactions were done for the simplified Seyler-Blanchard interaction with no exchange terms [RD92]. These parameters are currently used to constraint our future interaction to fulfil the sum rules and to avoid instabilities in nuclear matter.

## REFERENCES

**BJS75** Backman,S.O., Jackson, A.D. and Speth, J., Phys. Lett. 56B (1975)209
**EFS95** Von Eiff, D., Farine, M. and Schuck, P., Berger, J.F., Déchargé, J., to be submitted
**EFS96** Von Eiff, D., Farine, M. and Schuck, in progress
**GP77** Gogny, D. and Padjen, R., Nucl. Phys. A293 (1977) 365
**Mi67** Migdal, A.B., Theory of finite Fermi systems and application to atomic nuclei (Interscience, NY, 1967)
**RD92** Rudra, N., De, J.N., Nucl. Phys. A545 (1992) 608

## Appendix A

$$\left\langle k_p k_{h'} \middle| V_{ij}^a \middle\| k_h k_{p'} \right\rangle = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \left\langle \mathbf{k}_p \mathbf{k}_{h'} \middle| V_{ij}^a \middle| \mathbf{r}_1 \mathbf{r}_2 \right\rangle \left\langle \mathbf{r}_1 \mathbf{r}_2 \middle| \mathbf{k}_h \mathbf{k}_{p'} \right\rangle$$

$$\left\langle \mathbf{r}_1 \mathbf{r}_2 \middle| \mathbf{k}_h \mathbf{k}_{p'} \right\rangle = \left\langle \mathbf{r}_1 \middle| \mathbf{k}_h \right\rangle \left\langle \mathbf{r}_2 \middle| \mathbf{k}_{p'} \right\rangle = \frac{1}{(2\pi)^3} e^{i\mathbf{k}_h \mathbf{r}_1} e^{i\mathbf{k}_{p'} \mathbf{r}_2}$$

$$\left\langle \mathbf{k}_p \mathbf{k}_{h'} \middle| V_{ij}^a \middle| \mathbf{r}_1 \mathbf{r}_2 \right\rangle = e^{-\frac{(\mathbf{r}_1 - \mathbf{r}_2)^2}{\mu_c^2}} \left[ \frac{1}{2} \left( \rho(\mathbf{r}_1) + \rho(\mathbf{r}_2) \right) \right]^\gamma \frac{1}{(2\pi)^3} e^{-i\mathbf{k}_p \mathbf{r}_1} e^{-i\mathbf{k}_{h'} \mathbf{r}_2}$$

$$\left\langle k_p k_{h'} \middle| V_{ij}^a \middle\| k_h k_{p'} \right\rangle = \frac{1}{(2\pi)^6} [\rho_0]^\gamma \int d\mathbf{r}_1 \int d\mathbf{r}_2 e^{i(\mathbf{k}_h - \mathbf{k}_p)\mathbf{r}_1} e^{i(\mathbf{k}_{p'} - \mathbf{k}_{h'})\mathbf{r}_2} e^{-\frac{(\mathbf{r}_1 - \mathbf{r}_2)^2}{\mu_c^2}}$$

$$= \frac{1}{(2\pi)^6} [\rho_0]^\gamma \int d\mathbf{r}_1 \int d\mathbf{r}_2 e^{i(\mathbf{k}_h - \mathbf{k}_p)(\mathbf{r}_1 - \mathbf{r}_2)} e^{-\frac{(\mathbf{r}_1 - \mathbf{r}_2)^2}{\mu_c^2}}$$

$$= \frac{1}{(2\pi)^3} [\rho_0]^\gamma \int d\mathbf{r} e^{i(\mathbf{k}_h - \mathbf{k}_p)\mathbf{r}} e^{-\frac{(\mathbf{r})^2}{\mu_c^2}} = \frac{1}{(2\pi)^3} [\rho_0]^\gamma (\mu\pi)^{3/2} e^{-\frac{\mu^2 (\mathbf{k}_h - \mathbf{k}_p)^2}{4}}$$

$$= [\rho_0]^\gamma \left( \frac{\mu}{2\sqrt{\pi}} \right)^3 e^{-\frac{\mu^2 (\mathbf{k}_h - \mathbf{k}_p)^2}{4}}$$

$$\left\langle k_p k_{h'} \middle| V_{ij}^a P_r \middle\| k_h k_{p'} \right\rangle = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \left\langle \mathbf{k}_p \mathbf{k}_{h'} \middle| V_{ij}^a P_r \middle| \mathbf{r}_1 \mathbf{r}_2 \right\rangle \left\langle \mathbf{r}_1 \mathbf{r}_2 \middle| \mathbf{k}_h \mathbf{k}_{p'} \right\rangle$$

$$\left\langle \mathbf{k}_p \mathbf{k}_{h'} \middle| V_{ij}^a P_r \middle| \mathbf{r}_1 \mathbf{r}_2 \right\rangle = e^{-\frac{(\mathbf{r}_1 - \mathbf{r}_2)^2}{\mu_c^2}} \left[ \frac{1}{2} \left( \rho(\mathbf{r}_1) + \rho(\mathbf{r}_2) \right) \right]^\gamma \frac{1}{(2\pi)^3} e^{-i\mathbf{k}_p \mathbf{r}_2} e^{-i\mathbf{k}_{h'} \mathbf{r}_1}$$

$$\left\langle k_p k_{h'} \middle| V_{ij}^a P_r \middle\| k_h k_{p'} \right\rangle = \frac{1}{(2\pi)^6} [\rho_0]^\gamma \int d\mathbf{r}_1 \int d\mathbf{r}_2 e^{i(\mathbf{k}_h - \mathbf{k}_{h'})\mathbf{r}_1} e^{i(\mathbf{k}_{p'} - \mathbf{k}_p)\mathbf{r}_2} e^{-\frac{(\mathbf{r}_1 - \mathbf{r}_2)^2}{\mu_c^2}}$$

$$= \frac{1}{(2\pi)^6} [\rho_0]^\gamma \int d\mathbf{r}_1 \int d\mathbf{r}_2 e^{i(\mathbf{k}_p - \mathbf{k}_{p'})(\mathbf{r}_1 - \mathbf{r}_2)} e^{-\frac{(\mathbf{r}_1 - \mathbf{r}_2)^2}{\mu_c^2}}$$

$$= \frac{1}{(2\pi)^3} [\rho_0]^\gamma \int d\mathbf{r} e^{i(\mathbf{k}_p - \mathbf{k}_{p'})\mathbf{r}} e^{-\frac{(\mathbf{r})^2}{\mu_c^2}} = \frac{1}{(2\pi)^3} [\rho_0]^\gamma (\mu\pi)^{3/2} e^{-\frac{\mu^2 (\mathbf{k}_p - \mathbf{k}_{p'})^2}{4}}$$

$$= [\rho_0]^\gamma \left( \frac{\mu}{2\sqrt{\pi}} \right)^3 e^{-\frac{\mu^2 (\mathbf{k}_p - \mathbf{k}_{p'})^2}{4}}$$