Acounting for Multiparticle Bose-Einstein Correlations in Classical Transport Models

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Abstract

We add the multiparticle Bose-Einstein correlations to classical simulations of ultra-relativistic heavy-ion collisions and calculate their influence on various observables such as multiplicity distributions, single-particle spectra and two-particle correlation functions. We demonstrate the method using simulations of ultra-relativistic heavy-ion collisions within a parton-string model for different systems of colliding nuclei at initial energy 200 AGeV.

The multiple particle production in ultra-relativistic heavy-ion and hadronic collisions is studied to understand the production mechanism and to reach finally the conclusions about properties of the nuclear matter under extreme conditions. An important part of these studies is the investigation of particle correlations. The experimental observations indicate substantial contribution of the Bose-Einstein (BE) correlations, which enhance the probability to find two or more identical bosons at low relative momentum. These correlations arise as an interference effect due to the symmetrization requirement on the production amplitude of identical bosons.

The transport models based on the parton, classical string or hydrodynamical approaches are widely used to predict the particle production in ultra-relativistic heavy-ion collisions, to determine the acceptance of detectors and to compare the results obtained in different experiments under different kinematical conditions. They assume that the produced hadrons are on mass-shell, point-like and can be precisely localized in both coordinate and momentum spaces and predict the coordinates and momenta of all particles at all times. Being classical approaches, interference effects are beyond their scope. Since the BE correlations arise due to the interference of the amplitudes of indistinguishable processes, they cannot be taken into account in these models directly.

Since nature includes BE correlations and since they influence the spectra in a non-negligible way, it is necessary to include these correlations into the simulations of the reactions, not only to improve the predictive power of these models but also

to verify the information content of the measured spectra on the source of particle emission. This requires the consideration of how to connect a classical theory with the BE correlations.

There are several attempts to include the two-particle BE effects in the simulation of multi-particle production based on classical transport models. These attempts address mainly the two-boson interferometry (for a recent review see [1]). In the present letter we extend these approaches to the case of multi-particle BE effects and develop a practical approach allowing to calculate their influence on various observables such as multiplicity distributions, single-particle spectra and two-particle correlation functions.

The theoretical considerations are accompanied by the numerical calculations using the parton-string model [2]. This model contains most of the features of the standard Monte-Carlo event generators used to describe the ultra-relativistic heavy-ion collisions and allows to perform calculations in modest computer time. The aim of calculations within this particular model is not to make a comparison with the experimental data and not to improve the model, but to demonstrate practical possibility and necessity of the inclusion of BE and final state interaction effects and to test the developed numerical code, which can then be used within other dynamical models.

We start with the process in which, besides the system described by quantum numbers $\{\alpha\}$, only two identical spinless bosons with the 4-momenta $p_1 = (E_1, \mathbf{p}_1)$ and $p_2 = (E_2, \mathbf{p}_2)$ are produced. To introduce the space-time characteristics of the particle production, we will follow Kopylov and Podgoretsky (see, e.g., [3, 4]) and assume that particles are emitted by one-particle sources ($A \to 1, B \to 2$) which are considered as classical so they can be treated by parameters and not by amplitudes. Thus the 4-coordinates of the source centers x_A and x_B and other source characteristics in the model can be considered as a part of the quantum numbers $\{\alpha\} \equiv \{x_A, x_B, \alpha'\}$.

Usually the space-time extent of the sources is assumed much smaller than that of the production region so the 4-coordinates of the source centers x_A and x_B can be identified with those of the particle emission points x_1 and x_2. In other words, the momentum dependence of the production amplitude is assumed unimportant when varying the 4-momenta p_1 and p_2 by the amount characteristic for the interference effect, determined by the inverse size of the production region. Under such a *smoothness* assumption Kopylov and Podgoretsky arrived at the production probability [3]:

$$P_2^c(\mathbf{p}_1, \mathbf{p}_2) \doteq \int d^4x_A d^4x_B D_2(p_1, x_1; p_2, x_2)[1 + \cos(q_{12}x_{12})], \tag{1}$$

where $q_{12} = p_1 - p_2$, $x_{12} = x_1 - x_2$ and $D_2(p_1, x_1; p_2, x_2)$ represents the classical emission probability. The production probability in the case when the effect of BE statistics is absent is just $P_2(p_1, p_2) = \int d^4x_1 d^4x_2 D_2(p_1, x_1; p_2, x_2)$. Thus, in the case when the *smoothness* condition is valid, we can identify the emission function $D_2(p_1, x_1; p_2, x_2)$ with the distribution of the phase-space points $\{k_i; r_i\} \equiv \{\omega_i, \vec{k}_i; t_i, \vec{r}_i\}, \omega_i = \sqrt{\mathbf{k}_i^2 + m_i^2}$, of emitted bosons at their freeze-out times t_i simulated within a classical transport model.

The approximate Eq. (1) allows a simple interpretation. The second factor in the integral is just square of the amplitude corresponding to the emission of the two identical bosons at the points x_1 and x_2:

$$1 + \cos(q_{12}x_{12}) = \mid \frac{1}{\sqrt{2!}}[\exp(ip_1x_1 + ip_2x_2) + \exp(ip_2x_1 + ip_1x_2)] \mid^2 . \tag{2}$$

The finite source sizes still reveal themselves in the momentum dependence of the emission probability $D_2(p_1, x_1; p_2, x_2)$.

It should be noted that under the *smoothness* assumption the physically motivated Kopylov-Podgoretsky approach coincides with a more formal one based on Wigner functions [5] or space-time density matrix [6], the classical emission probability $D_2(p_1, x_1; p_2, x_2)$ being identified with a generalized two-particle Wigner density. The result of Pratt [5] then follows from Eq. (1) in the case of independent production of the two particles when the two-particle emission function reduces to the product of single-particle Wigner densities:

$$D_2(p_1, x_1; p_2, x_2) = D(p_1, x_1)D(p_2, x_2). \tag{3}$$

Let us define the correlation function as a ratio of the production probabilities $P_2^c(\mathbf{p}_1, \mathbf{p}_2)$ and $P_2(\mathbf{p}_1, \mathbf{p}_2)$ with the effect of BE statistics respectively switched on and switched off:

$$C_2(\mathbf{p}_1, \mathbf{p}_2) \doteq \frac{\int d^4x_1 d^4x_2 D_2(p_1, x_1; p_2, x_2)[1 + \cos(q_{12}x_{12})]}{\int d^4x_1 d^4x_2 D_2(p_1, x_1; p_2, x_2)}. \tag{4}$$

Note that, in experiment, the reference probability $P_2(\mathbf{p}_1, \mathbf{p}_2)$ in the denominator of Eq. (4) is usually constructed with the help of particles from different events of similar type, neglecting possible dynamical correlations. From a classical simulation the correlation function can be calculated, in given bins of $\mathbf{p}_1$ and $\mathbf{p}_2$, as

$$C_2(\mathbf{p}_1, \mathbf{p}_2) = 1 + \langle \cos[(k_{i_l} - k_{j_l})(r_{i_l} - r_{j_l})] \rangle_{\mathbf{p}_1, \mathbf{p}_2} , \tag{5}$$

where the averaging should be done over all phase-space points satisfying the conditions $\mathbf{k}_{i_l} \in \{\mathbf{p}_1 - \Delta\mathbf{p}/2, \mathbf{p}_1 + \Delta\mathbf{p}/2\}$, $\mathbf{k}_{j_l} \in \{\mathbf{p}_2 - \Delta\mathbf{p}/2, \mathbf{p}_2 + \Delta\mathbf{p}/2\}$.

Besides the 6-dimensional correlation function defined in Eq. (4), we introduce the correlation function integrated over some of the kinematic variables characterizing the two-particle system. For example, the one-dimensional correlation function in terms of the variable $q_{inv} = \sqrt{-q_{12}^2} = \sqrt{m_{12}^2 - 4m^2}$ is calculated from a classical simulation as

$$C_2(q_{inv}) = 1 + \frac{1}{\Delta N(q_{inv})} \sum_{l=1}^{\Delta N(q_{inv})} \cos[(k_{i_l} - k_{j_l})(r_{i_l} - r_{j_l})], \tag{6}$$

where $\Delta N(q_{inv})$ is the number of the simulated pairs in a given bin of q_{inv}.

In the case of independent production of the particles, when Eq. (3) is valid, we can write the correlation function in the form

$$C_2(\mathbf{p}_1, \mathbf{p}_2) = 1 + \mid F_{12} \mid^2, \tag{7}$$

where

$$F_{12} \equiv F(p_{12}, q_{12}) \doteq \frac{\int d^4x D(p_{12}, x) \exp(iq_{12}x)}{\int d^4x D(p_{12}, x)}. \tag{8}$$

From a classical simulation, this function can be precomputed as $F_{ij} = \langle \exp(iq_{ij}r_{k_l}) \rangle_{\mathbf{p}_{ij}}$, where the averaging is to be done over all phase-space points satisfying the condition $\mathbf{k}_{k_l} \in \{\mathbf{p}_{ij} - \Delta\mathbf{p}/2, \mathbf{p}_{ij} + \Delta\mathbf{p}/2\}$. To take into account the possible $x - p$ correlation, in practical calculations it is convenient to introduce the bin in $\mathbf{p}_{ij}$ in terms of rapidity $y_{ij} = \frac{1}{2}(y_i + y_j)$ and transverse momentum $\mathbf{p}_{tij} = \frac{1}{2}(\mathbf{p}_{ti} + \mathbf{p}_{tj})$. The corresponding widths Δy and $\Delta\mathbf{p}_t$ can be found empirically by choosing sufficiently large starting values and then decreasing them till the function F_{ij} saturates.

We started with the calculation of the one-dimensional correlation function according to Eq. (6). By choosing the space-time coordinates of the pion emission points randomly, we have checked that the $x - p$ correlation, presented in the model, slightly widens the correlation function in accordance with the decrease of the effective space-time distance between the pion production points. Comparing the correlation functions calculated according to Eqs. (6) and (7), we have found practically coinciding results. This indicates that we can neglect dynamical correlations and use the precomputed F_{ij} values for calculation of the higher order BE correlations (see below).

Let us further consider the events with $n = 3$ identical spinless bosons. Assuming uncorrelated emission and absence of final state interaction, we get the three-particle production probability in the case of switched off the effect of BE statistics as

$$P_3(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = \int d^4x_1 d^4x_2 d^4x_3 D(p_1, x_1) D(p_2, x_2) D(p_3, x_3) = P(\mathbf{p}_1) P(\mathbf{p}_2) P(\mathbf{p}_3). \tag{9}$$

Similarly to the two-particle case we define three-particle correlation function in the events with three identical particles as a ratio of the symmetrized production probability $P_3^c(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$ to the reference one, $P_3(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$. Using the *smoothness* assumption, we can write this correlation function in terms of the function F_{ij} as (see, e.g., [4])

$$C_3(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = 1 + |F_{12}|^2 + |F_{23}|^2 + |F_{31}|^2 + 2Re\{F_{12}F_{23}F_{31}\}. \tag{10}$$

Like in the two-particle case, from a classical simulation we can calculate the dependence of the three-particle correlation function on the invariant variable $q_{inv} = \sqrt{-q_{123}^2} = \sqrt{m_{123}^2 - 9m^2}$, where m_{123} is the effective mass of the system of three identical particles of mass m, as

$$C_3(q_{inv}) = \frac{1}{N(q_{inv})} \sum_{i=1}^{N(q_{inv})} C_3(\mathbf{p}_{1_i}, \mathbf{p}_{2_i}, \mathbf{p}_{3_i}). \tag{11}$$

Here $N(q_{inv})$ is the number of particle triplets in a given q_{inv} bin and $C_3(\mathbf{p}_{1_i}, \mathbf{p}_{2_i}, \mathbf{p}_{3_i})$ is defined by expression (10).

The extension of these considerations to the events with larger number of produced particles is straightforward. Thus, the n-particle correlation function in the events

with n identical spinless particles can be expressed in terms of F_{ij} as (see also [7, 8])

$$C_n(\mathbf{p}_1, \mathbf{p}_2, ..., \mathbf{p}_n) = \sum_{\sigma} \prod_{i=1}^{n} F_{i\sigma_i}. \tag{12}$$

The sum in Eq. (12) is over $n!$ possible permutations σ of the sequence $\{1, 2, 3, ..., n\}$.

So far we considered identical bosons as free particles and completely neglected their interaction. However, it is well-known (see, e.g., [9, 10]) that particle correlations at small relative velocities can be strongly affected by this interaction. As usual, we will assume the density of the produced particles in momentum space sufficiently low so that only the final state interaction in the pairs of particles with small relative velocities is to be taken into account. In the case of the production of only two identical spinless particles the effect of final state interaction reduces, under the smoothness assumption, to the substitution of the plane waves in Eq. (2) by the nonsymmetrized Bethe-Salpeter amplitudes in the continuous spectrum of the two-particle states [6, 10]. The Bethe-Salpeter amplitude can be usually substituted by the corresponding wave function $\Psi^{(+)}_{-\mathbf{k}^*}(\mathbf{r}^*), \mathbf{k}^* = \mathbf{p}_1^* = -\mathbf{p}_2^*, \mathbf{r}^* = \mathbf{x}_{12}$, leading to the *equal time* approximation [10] first used by Koonin to calculate the correlation function of two nonrelativistic protons [9].

The calculation of the n-particle correlation function, in the considered two-body approximation, requires to compute $n(n-1)$ Bethe-Salpeter amplitudes for each event simulated in a transport model. Clearly, even on the factorization assumption, this calculation is much more complicated than in the case of pure BE correlations since the integration over space-time coordinates of n production points is not reduced to the single space-time point integration like in the case of the function F_{ij}.

For neutral pions at small relative momenta $k^* = |\mathbf{k}^*| = \frac{1}{2}q_{inv}$, we are interested in, the scattered wave in the wave function $\Psi^{(+)}_{-\mathbf{k}^*}(\mathbf{r}^*)$ is dominated by s-wave. Since usually the characteristic distances r^* between the pion production points are larger than the range of their strong interaction, the scatterd wave can be substituted by the spherical wave $f(k^*)\exp(ik^*r^*)/r^*$, where $f(k^*)$ is the two-pion s-wave scattering amplitude. In such conditions, the effect of strong final state interaction on the correlation function is determined by the ratio of the scattering amplitude $f(k^*)$ to the characteristic distance between the production points in the c.m.s. of the two particles [10]. As the $\pi^0\pi^0$ scattering length is positive and rather small ($f(0) \simeq 0.1$ fm), the final state interaction slightly enhances (less than 5%) the effect of BE correlation in the system of two neutral pions.

For identical charged pions the effect of final state interaction is dominated by the Coulomb interaction. In the usual case, when the characteristic distance between particle production points in their c.m.s. is much smaller than the two-pion Bohr radius $a = 388$ fm, it can be approximated by the Coulomb penetration factor representing modulus squared of the Coulomb wave function at zero distance:

$$A_c(k^*) = \tfrac{2\pi}{ak^*}[\exp(\tfrac{2\pi}{ak^*}) - 1]^{-1}. \tag{13}$$

Thus, in given approximation, we have to multiply the pure BE n-particle correlation function by the product of $\frac{n(n-1)}{2}$ Coulomb penetration factors. For example, the

three-particle correlation function in the events with three identical charged pions then takes the form:

$$C_3(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = A_c(k^*_{12})A_c(k^*_{13})A_c(k^*_{23})\tilde{C}_3(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3), \tag{14}$$

where $\tilde{C}_3(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$ is the correlation function in the absence of final state interaction, given by Eq. (10). It is clear from Eq. (13) that the Coulomb repulsion strongly suppresses the correlation of two identical charged pions only in the region of very small relative momenta $k^* < \frac{2\pi}{a} = 3$ MeV/c. Thus the final state interaction in the system of identical charged pions becomes of increasing importance with increasing the effective space-time extent of the production region.

In Fig. 1, we show the two-particle (Eq. (7)) and the pure three-particle (only the last term in Eq. (10)) correlation functions of positively charged pions calculated with and without the Coulomb penetration factors for central $S + S$ collisions simulated within the parton-string model [2] at $E_{lab} = 200$ AGeV. It may be seen from Fig. 1 that the Coulomb effect in the system of three charged pions is quite strong even at $q^{(123)}_{inv}$ of several tens MeV/c.

As one can see from Eqs. (10) and (12), the higher order correlation functions consist mainly of the statistical combinations of the lower order correlations. The genuine higher order correlations are measured by the so-called particle cumulants. For example, the three-particle cumulant $K_3(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = 2Re\{F_{12}F_{23}F_{31}\}$ in Eq. (10) measures the statistical dependence of the whole 3-particle set. Any of the three particles is independent of the others if the corresponding cumulant is zero. The three- and four-particle cumulants calculated for high energy nucleus-nucleus collisions within our particular dynamical model [2] appear to be clearly non-zero for neutral pions, while for charged ones they are strongly damped by the Coulomb repulsion effects (see, e.g., Fig. 1). Thus the Coulomb repulsion may be responsible for the vanishing of the third and higher order cumulants of identical charged pions as measured in nucleus-nucleus collisions [11]. The influence of the Coulomb final state interaction will be even stronger in future heavy ion experiments at higher energies due to increasing effective space-time extent of the particle production region.

As one can see from formulae (7), (10), (12) and the definition of correlation function, the two- and three-particle BE correlations lead to distortions of the original single- and two-particle distributions. Such distortions are small in the case of interference of only two or three identical particles. However, as it was pointed out in paper [7] (see also [4] and [13]), they can become essential for the events with a large number of identical spinless particles due to factorially increasing number of correction terms.

To account for the multi-boson symmetrization effect, in papers [7, 12] a phase-space weighting procedure was used with the weights in the form of a normalized square of the sum of $n!$ plane waves, like that in Eq. (2) for the case $n = 2$. This procedure however appears not practical for large numbers n of identical bosons due to the factorially large number of the terms to be computed to calculate the weight and, due to large weight fluctuations. These fluctuations can be substantially reduced by weighting only in the momentum space. The corresponding weights can be identified

with the correlation functions $C_n(\mathbf{p}_1, \mathbf{p}_2, ..., \mathbf{p}_n)$ constructed from the precalculated functions F_{ij} (see Eqs. (7) and (10) for the cases $n = 2$ and 3, respectively). However, there is still the problem with factorially large number of the terms required to calculate the weight according to Eq. (12).

A strong reduction of the number of calculated terms can be achieved by substituting the weight method by numerical integration over momenta of one or more particles in the production probability

$$P_n^c(\mathbf{p}_1, \mathbf{p}_2, ..., \mathbf{p}_n) = C_n(\mathbf{p}_1, \mathbf{p}_2, ..., \mathbf{p}_n) P(\mathbf{p}_1) P(\mathbf{p}_2) ... P(\mathbf{p}_n). \tag{15}$$

We thus have to compute $K_2(\mathbf{p}_1) = \left\langle |F_{i_1 j_1}|^2 \right\rangle_{\mathbf{p}_1}$, $K_2 = \left\langle |F_{i_1 j_1}|^2 \right\rangle$ and corresponding average values of the higher order cumulants. We should also perform combinatorics to define multiples of these averaged cumulants. The last performance can be fulfilled up to all orders. Thus the statistical weight $\omega(n) = \int P_n^c(\mathbf{p}_1, \mathbf{p}_2, ..., \mathbf{p}_n) \prod_{i=1}^{n} \frac{d^3\mathbf{p}_i}{E_i}$ of an event with n identical spinless particles can be calculated with the help of the recurrence relation (see also [4, 13]):

$$\omega(n) = \sum_{j=0}^{n-1} C_j^{n-1} K_{j+1} \omega(n-j-1), \tag{16}$$

with C_j^{n-1} being the number of combinations of j elements out of $n-1$, $K_1 = 1$, $\omega(0) = \omega(1) = 1$ and $\omega(n) = 0$ for $n < 0$. One can check that $\omega(n) = n!$ in the case of 'coherent' particle emission (e.g., the emission from a very small space-time region) when $K_{j+1} = j!$.

However, for realistic models used to predict particle production in ultra-relativistic heavy-ion collisions, the numerical averaging of the cumulants of all orders is a difficult task (see, e.g., [13]). As a first approximation, representing a lower estimate of the effect of Bose symmetrization, we neglect higher than the second order cumulants in Eq. (12). We will call it the second order cumulant approximation or simply the two-particle approximation. We recall that there is no experimental evidence for significantly nonzero values of the higher than second order cumulants in nucleus-nucleus collisions at high energies. In this approximation, the recurrence relation (16) for the statistical weight $\omega(n_i)$ of an event $i = 1, 2, ..., N_{evt}$ with n_i identical spinless particles in final state reduces to:

$$\omega(n_i) = \omega(n_i - 1) + (n_i - 1) K_2 \omega(n_i - 2). \tag{17}$$

It should be stressed that the account of the third or still higher order cumulants is straightforward and has to be done to check the convergence of such a lowest order cumulant expansion. A fast convergence is expected for the processes, such as heavy-ion collisions, characterized by a large space-time extent of the pion production region. In this paper, we limit ourselves to the second order cumulant approximation which appears to be sufficient for our purpose: to demonstrate a strong influence of the multi-particle BE effects.

The knowledge of the statistical weights for events with different number n of produced identical particles gives us a possibility to compute the effect of BE statistics on the multiplicity distribution obtained in a classical simulation [13]. The BE corrected multiplicity distribution can be calculated as

$$W^c(n) = \sum_{i=1}^{N_{evt}} \delta(n - n_i)\omega(n_i) / \sum_{i=1}^{N_{evt}} \omega(n_i). \tag{18}$$

In the two-particle approximation, the single-particle probability for the events with n identical spinless bosons, normalized in the same way as the non-symmetrized single-particle probability $P(\mathbf{p}_1)$, takes the form:

$$P_n^c(\mathbf{p}_1) = \frac{P(\mathbf{p}_1)}{\omega(n)}[\omega(n-1) + (n-1)K_2(\mathbf{p}_1)\omega(n-2)], \tag{19}$$

leading to Eq. (17) after the integration over the momentum $\mathbf{p}_1$.

For the two-particle correlation function in the events with n identical particles, we have in given approximation:

$$\begin{aligned} C_n(\mathbf{p}_1, \mathbf{p}_2) = \omega(n-2)(1 + |F_{12}|^2) + (n-2)\omega(n-3)(K_2(\mathbf{p}_1) + K_2(\mathbf{p}_2)) + \\ +(n-2)(n-3)\omega(n-4)K_2(\mathbf{p}_1)K_2(\mathbf{p}_2). \end{aligned} \tag{20}$$

One can check that Eq. (20) leads to Eq. (19) after multiplying it by $P(\mathbf{p}_2)$ and integrating over the momentum and $\mathbf{p}_2$.

Our calculations, based on simulations of central $O + O$ and $S + S$ collisions within the parton-string model [2] at $E_{lab} = 200$ AGeV, confirm that the effect of BE statistics for the events with a large number n of identical bosons leads to substantial distortions of the multiplicity distributions, the single-particle spectra and the two-boson correlation functions. Thus the mean neutral pion multiplicity of 44.7 simulated for central $O + O$ collisions is increased, due to the effect of BE statistics ($K_2 = 0.0056$), by about 21 units.

In Fig. 2 we show the neutral pion transverse momentum distributions $\frac{1}{N}\frac{dN}{dp_t}$ and rapidity distributions $\frac{1}{N}\frac{dN}{dy}$ obtained from the events generated with fixed neutral pion multiplicity $n = 100$ in $S + S$ collisions. In Fig. 2 we also draw the ratios of BE corrected and uncorrected single-particle distributions. We can see that the BE symmetrization enhances production of the pions with low p_t and small y. We have also found that this effect becomes more pronounced with increasing pion multiplicity.

We should note here that in several papers (see, for example, review [4] and recent paper [13]) a possible enormous influence of BE statistics on the single-pion spectra and multiplicity distributions was argued and, in particular, the idea of pion laser, i.e. production of pions collimated in narrow and monochromatic jets, was discussed.

In Fig. 3 we present a ratio of the renormalized two-particle correlation functions $C_n(q_{inv})/\omega(n)$ and $C_2(q_{inv})/\omega(2)$. It is calculated for the events generated with fixed neutral pion multiplicities $n = 30$ and 60 in $O + O$ collisions. We can see that the multi-boson BE correlations distort the two-body correlation function in such a way that it becomes lower and wider. This is in qualitative agreement with the results of papers [7, 12, 13].

Concerning the two-pion interferometry, we note that the experimental correlation function is calculated as a ratio of the measured two-particle distribution to the reference one, the latter being usually also influenced by multi-pion BE correlations (the so-called residual correlations [7, 12]). For correct comparison of the theoretical and experimental correlation functions, in the case when the event mixing technique is used to construct the reference distribution, one has to take into account the change of the single-particle spectra according to Eq. (19).

Concerning the Coulomb effects in a multi-pion system, they should somewhat weaken the considered effects of the multi-particle BE correlations, depending on the effective space-time extent of the production region of charged pions. Though their detailed investigation is beyond the scope of present paper, it is certainly of a large practical interest. Here we only mention that such an investigation would require consideration of the pions of both signs (for $\pi^+\pi^-$ pairs there will be Coulomb attraction corresponding to the negative Bohr radius $a = -388$ fm in Eq. (13)) and, eventually, of other charged particles; the total production probability, in the absence of dynamical correlations, being just a product of the production probabilities of various particle species.

In conclusion, we would like to stress that large numbers of identical bosons are produced in ultra-relativistic heavy-ion collisions and that their production can be strongly affected by the requirements of quantum statistics. However, these requirements are usually neglected in classical simulations within transport models (event generators) which are widely used for description of the multi-particle production.

We have presented here the theoretical basis allowing to calculate the effect of the multi-particle BE correlations on various observables within a classical transport model. We have also discussed the inclusion of the effect of Coulomb and strong final state interaction in the produced multi-particle system.

We have developed the computer code BOSE which can be coupled with a classical transport model to account for the effects of BE statistics and final state interaction and thus to allow a reliable comparison of the model with experimental data. Coupling this code with the parton-string model, we have demonstrated the importance of the multi-particle BE effects on multiplicity distributions, single-pion spectra and two-pion correlation functions.

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Figure 1: The two-particle (Eq. (7)) and the pure three-particle (only the last term in Eq. (10)) correlation functions of positively charged pions calculated with (full circles) and without (open circles) the Coulomb penetration factors (left) and their ratios (right) for central $S + S$ collisions simulated within the parton-string model at $E_{lab} = 200$ AGeV.

Figure 2: Transverse momentum and rapidity distributions of neutral pions (left) and their ratios (right) calculated in the parton-string model for $S + S$ collisions with the fixed π^0 multiplicity $n = 100$ at $E_{lab} = 200$ AGeV. The distributions with and without inclusion of BE correlations are shown by full and open circles, respectively.

Figure 3: The ratio of the renormalized correlation functions of two neutral pions $C_n(q_{inv})/\omega(n)$ and $C_2(q_{inv})/\omega(2)$ calculated in the parton-string model for central $O + O$ collisions with fixed π^0 multiplicities $n = 30$ and 60 at $E_{lab} = 200$ AGeV.

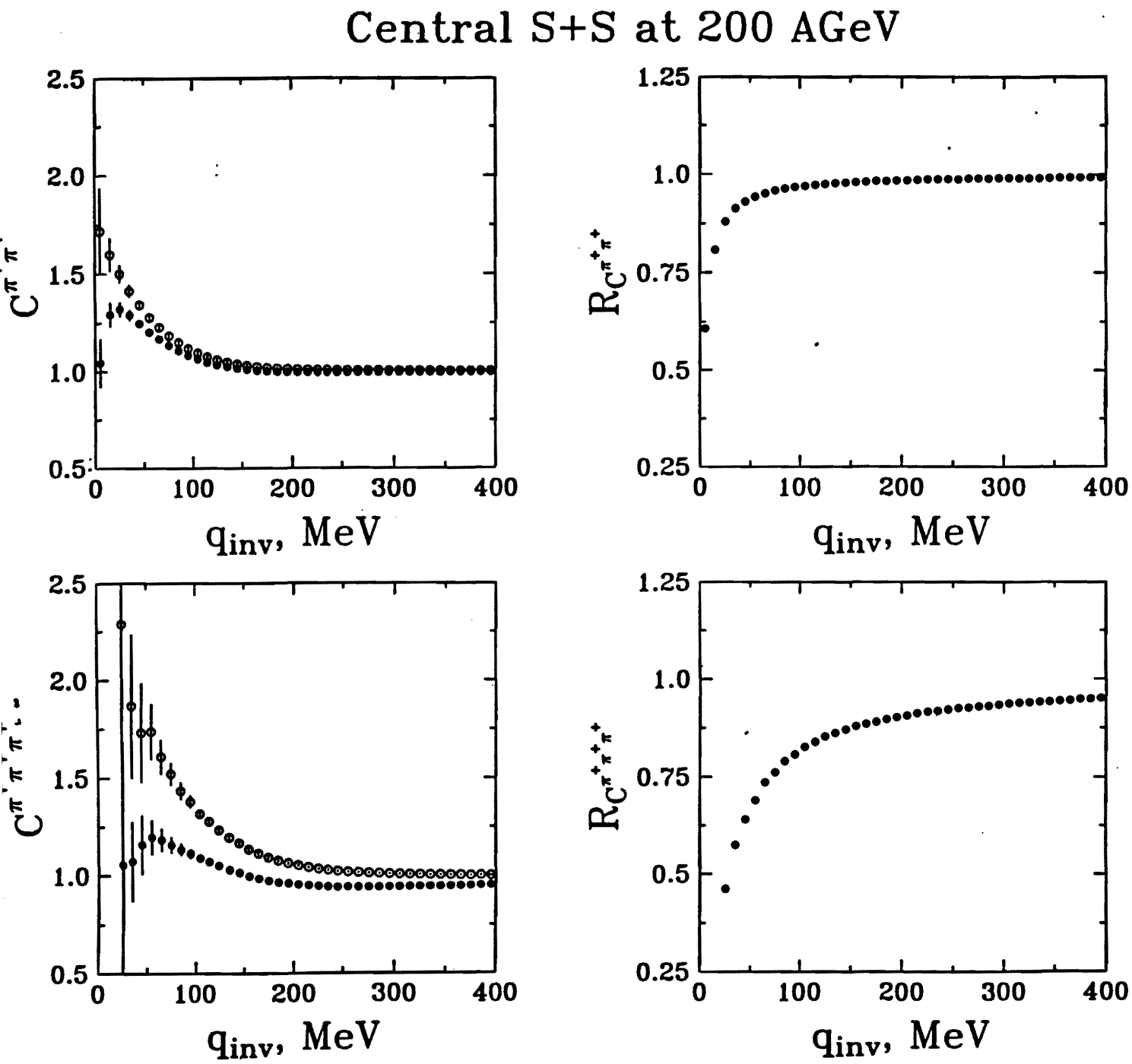
Central S+S at 200 AGeV
2.5
2.0
1.5
1.0
0.5
0
100
200
300
400
q_{inv}, MeV
1.25
1.0
0.75
0.5
0.25
$R_{C^{\pi^+\pi^+}}$

Fig. 1

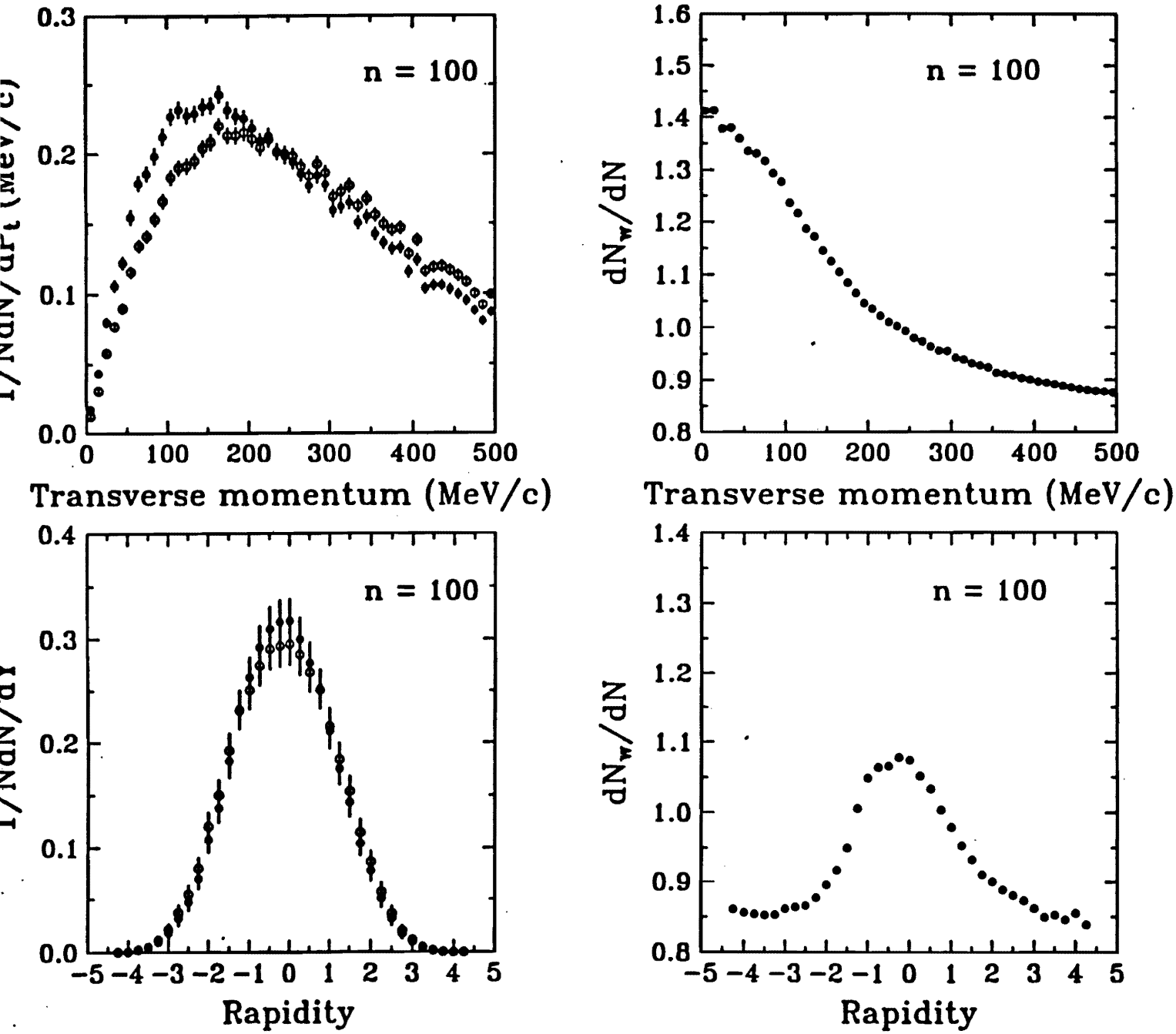
n = 100
1/NdN/dP$_t$ (MeV/c)
Transverse momentum (MeV/c)
n = 100
dN$_w$/dN
Transverse momentum (MeV/c)
n = 100
1/NdN/dY
Rapidity
n = 100
dN$_w$/dN
Rapidity

Fig. 2

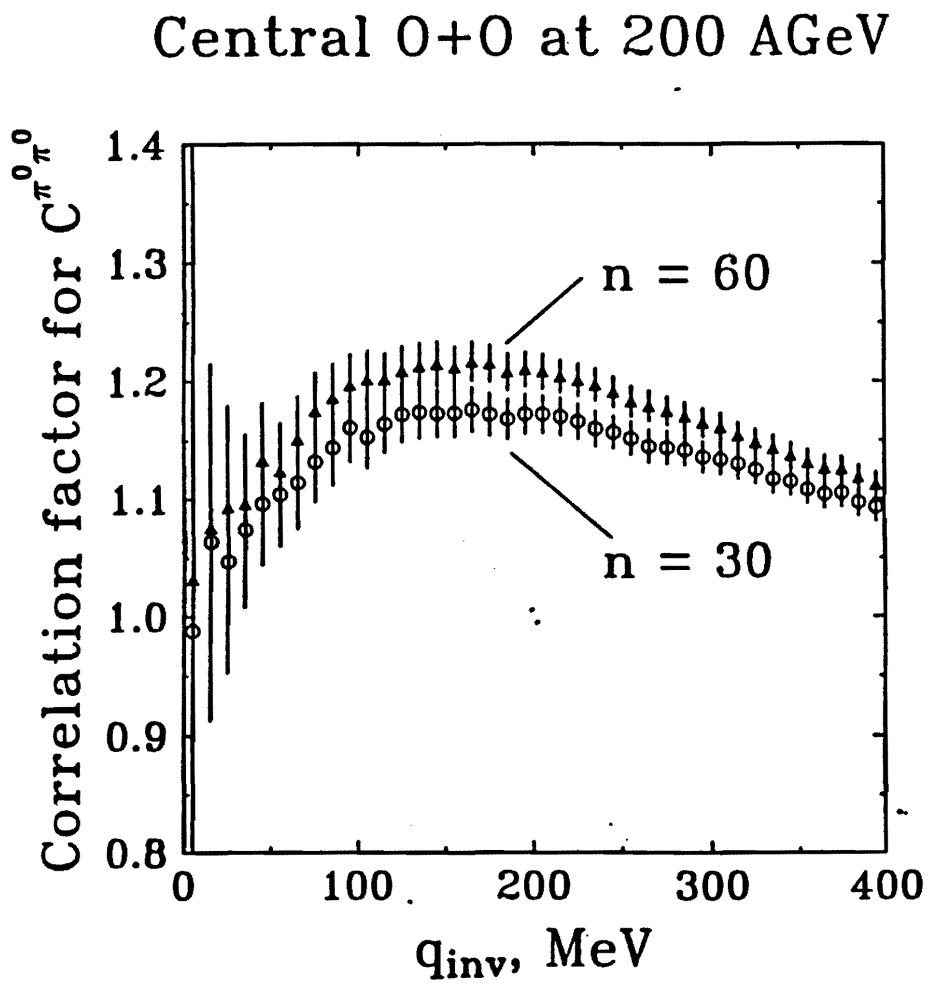
Central 0+0 at 200 AGeV
Correlation factor for $C^{\pi^0\pi^0}$
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0
100
200
300
400
q_{inv}, MeV
n = 60
n = 30

Fig. 3