On the dynamics of collective motion in heavy-ion reactions

V. de la Mota[1], F. Sébille[1], F. Haddad[2], M. Farine[3], P. Schuck[4]

1) SUBATECH, Université de Nantes-IN_2P_3, EMN. LPSTA
F-44072 Nantes Cedex 03. France.

2) Cyclotron Institute, Texas A&M University.
College Station, Texas 77843. USA

3) Ecole Navale, Lanvéoc Poulmic. 29240 Bresy Marine. France
Naples, Ist. Univ. Navale

4) Institut des Sciences Nucléaires, F-38026 Grenoble Cedex. France
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Abstract

Collective transverse motion in heavy-ion reactions is studied in the frame of the Landau-Vlasov model, for a wide range of beam energies. Nuclear mean-field is given by different parametrizations of local and non-local effective forces which correctly reproduce cold nuclear matter properties. It is shown that in the case of non-local forces, they must also be constrained to reproduce basic experimental data concerning the real part of the optical potential, in the energy range of interest. Symmetric colliding systems at 400 MeV of incident energy per nucleon are simulated and compared with experimental data. In this range of energy, the calculated flow is shown to be sensitive to the incompressibility modulus, and its numerical value is discussed.

The characterization of the nuclear equation of state (EOS) has been, since the last decade, one of the main objectives of intermediate energy heavy-ion collisions. In this domain, current experimental facilities offer the opportunity to investigate the properties of nuclear systems in, either the vicinity of their ground state, or very far from them. Informations on the properties of nuclear matter at different degrees of excitation were then expected to be found.

Among the different attempts to track down the nuclear EOS, special attention has been devoted to the study of the flow observable:

$$F = \left(\frac{d < p_x >}{dy}\right)_{y=0} \tag{1}$$

in a wide range of beam energies [1], from both experimental and theoretical points of view. In this last case, the Landau-Vlasov model [2] has proven to be suitable to describe heavy-ion reactions at such energies. Indeed, our semiclassical transport model describes the evolution of the one-body distribution function through a kinetic equation. The properties of the self-consistent mean-field appearing in this equation are fundamental in the analysis of the calculated flow observables. In particular, its interplay with the nucleon-nucleon cross-section was shown to play an essential role in the build up of the flow [3, 4].

Nevertheless, the determination of the EOS at intermediate/high energy heavy-ion reactions seems to be more difficult than expected. This difficulty may be related to the fact that a genuine connection between a dynamical observable, like the flow, and the static properties of matter, like the EOS, is not direct.

Let us make a rapid summary of some of our main theoretical results in this field. In a first time, flow calculations were performed with a simplifyed density-dependent effective force, the so-called Zamick force, whose Hartree-Fock potential in phase-space reads:

$$V(\rho(\vec{r})) = a\frac{\rho}{\rho_0} + b\left(\frac{\rho}{\rho_0}\right)^{\gamma+1} \tag{2}$$

Here, ρ_0 is the saturation density of nuclear matter and a, b, and γ are parameters extracted from fundamental properties of nuclear matter. To investigate the influence on the dynamics of the incompressibility modulus K_∞, two versions of this force have been used: the soft and the stiff ones, their parameters being reported in Table I.

TABLE I. parameters of the Zamick interactions

Zamick forces	a	b	γ	K_∞	m^*/m
soft	-356.	303.	$\frac{1}{6}$	200MeV	1
stiff	-123.	70.	1.	380 MeV	1

A clear dependence of the flow with K_∞ was found for incident energies up to 200 MeV/n, as illustrated in Fig.1 for the Nb+Nb system. Nevertheless the use of a more realistic effective force, like the non-local Gogny D1-G1 interaction, introduced an ambiguity: potentials having either a strong density dependence or a non-local behavior are able to produce very similar flows.

The implemented Gogny force has been proven to reproduce many nuclear properties at low energy with K_∞=228 MeV [5]. The corresponding mean-field in phase space is given by the following expression:

$$V_q(\vec{r},\vec{p}) = [\frac{3}{4}(\gamma+2)\rho^{\gamma+1}(\vec{r}) - \frac{3}{4}\gamma\rho^{\gamma-1}(\vec{r})(\rho_n^2(\vec{r})+\rho_p^2(\vec{r})) - \frac{3}{2}\rho^{\gamma}(\vec{r})\rho_q(\vec{r})]t_3$$
$$+\sum_{i=1}^{2}\int d\vec{r}'[(W_i+\frac{B_i}{2})\rho(\vec{r}') - (H_i+\frac{M_i}{2})\rho_q(\vec{r}')]exp - \frac{(\vec{r}-r')^2}{\eta_i}$$
$$\frac{2(\sqrt{\pi}\eta_i)^3}{(2\pi)^3}\sum_{i=1}^{2}\int d\vec{k}' \quad [(\frac{W_i}{2}+B_i)f_q(\vec{r},\vec{k}')-(\frac{H_i}{2}+M_i)f(\vec{r},\vec{k}')]exp-[\frac{(\vec{k}-k')^2\eta_i^2}{4}]$$

where ρ_n and ρ_p represent neutron and proton densities, respectively, q stands for isospin degree of freedom and $\rho=\rho_n+\rho_p$. The set of parameters is given in Table II.

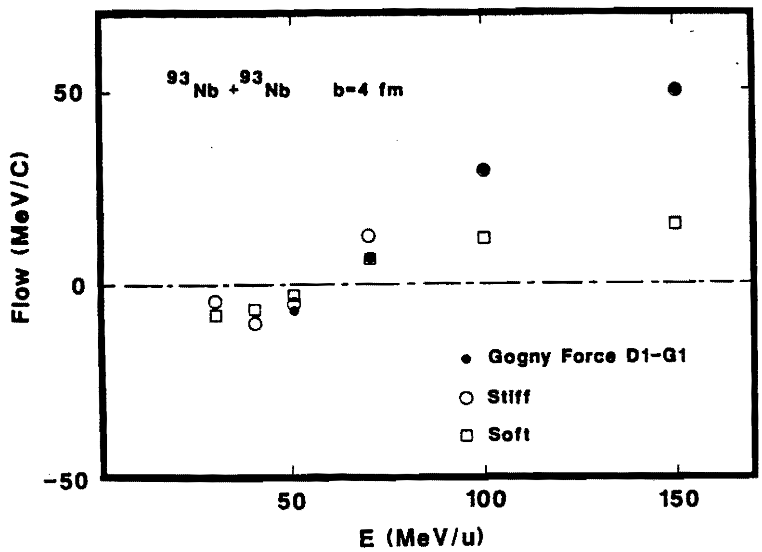


Fig.1 Collective sidewards flow as a function of the incident energy.

In Fig.1 the flow values obtained with this (soft) non-local Gogny force are also shown. For beam energies well above the energy of vanishing flow (EVF) [3] they attain the results found for the hard local one.

One way to understand this result is studying the pressure tensor Π, which is connected with the collective velocity $\vec{u}$, through the first order moment of the Landau-Vlasov equation (or momentum conservation law):

$$m\rho\ \{\frac{\partial\vec{u}}{\partial t}+(\vec{u}.\vec{\nabla})\vec{u}\}+\vec{\nabla}.\Pi = 0 \tag{3}$$

In this context the pressure tensor can be split in two contributions:

$$\Pi_{ij} = P_{ij} + P^c{}_{ij} \tag{4}$$

where P_{ij} is the compression component related with the mean field V and the potential energy density ε [3] and, defining δp_j as:

$$\delta p_j = m u_j - p_j \tag{5}$$

then, P_{ij} can be written as follows:

$$P_{ij} = m \int d^3p \, \frac{\delta p_i}{m} \frac{\delta p_j}{m} \, f(\vec{r},\vec{p}) = P^0{}_{ij} + P^{coll}{}_{ij} \tag{6}$$

this means, the sum of the pressure due to Fermi motion (P^0), and the one due to two-body collisions (P^{coll}).

In Fig.2 we represent the quantity $(\mathrm{Tr}\Pi)/\rho$ as a function of the distance along the principal axis of the density distribution in the center of mass frame. The calculations were performed with the same forces as in Fig.1, and the same system of colliding nuclei at 100 MeV per nucleon.

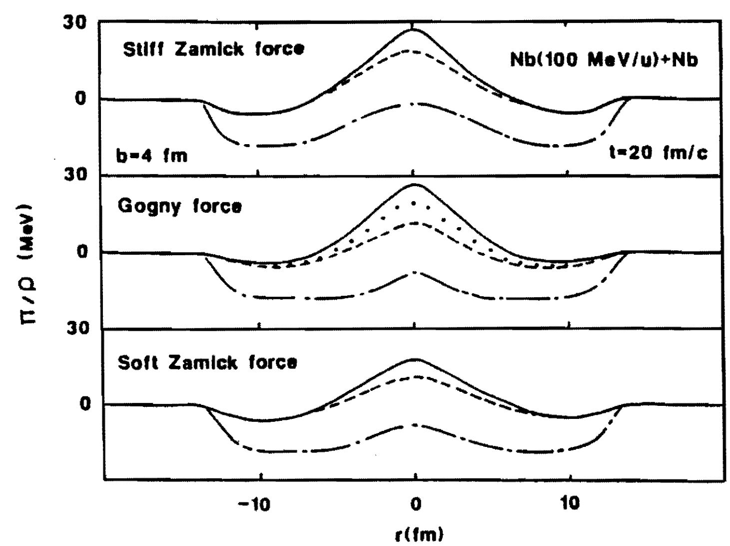


Fig.2 Trace of the pressure tensor as a function of the distance along the spatial distribution principal axis.

In dashed-dotted line we have drawn only the local compression contribution, in dashed line, the sum of the former to the cold term, generated by Fermi motion, in dotted line is the preceding quantity plus the non-local part of the Gogny contribution, finally in solid line, the total result which includes two-body collisions.

We observe that both compression and cold contributions have similar shapes for the soft local as for the Gogny interactions, whereas the stiff force is much more repulsive in the overlap region. The effect of residual interactions is clearly to increase the repulsive bump in this region, but the resulting contribution gives similar results for the three forces. Contrary to this, the p-dependent part of the mean-field generated by the Gogny force enhances the pressure around the origin, reaching values of the same order of those obtained with the stiff local force.

The results of Fig.2 have to be put in parallel with those of Fig.3 where the mean transverse momentum and the aspect ratio are plotted as a function of the time. The latter quantity is a global measure of the momentum distribution anisotropy, and then, of the degree of equilibration achieved during the course of the reaction. It is defined as the ratio between the larger (λ_3) to the smaller (λ_1) eigenvalue of the kinetic tensor:

$$T_{xy} = \sum_{part.} p_x \, p_y \tag{7}$$

When this ratio is close to unity, the momentum distribution is close to spherical symmetry and it is an indication that the system has attained a strong degree of equilibration. The results show that whatever the force is, comparable levels of equilibration are obtained. Nevertheless, the time evolution of the average transverse momentum differs considerably; the soft local force showing a late and slow building up of the flow. In the two other cases the flow is built very early during the reaction; the initial rate of increase is established when the aspect ratio has not yet taken its final value, in any case, when the two interpenetrating nuclei are still in high relative motion.

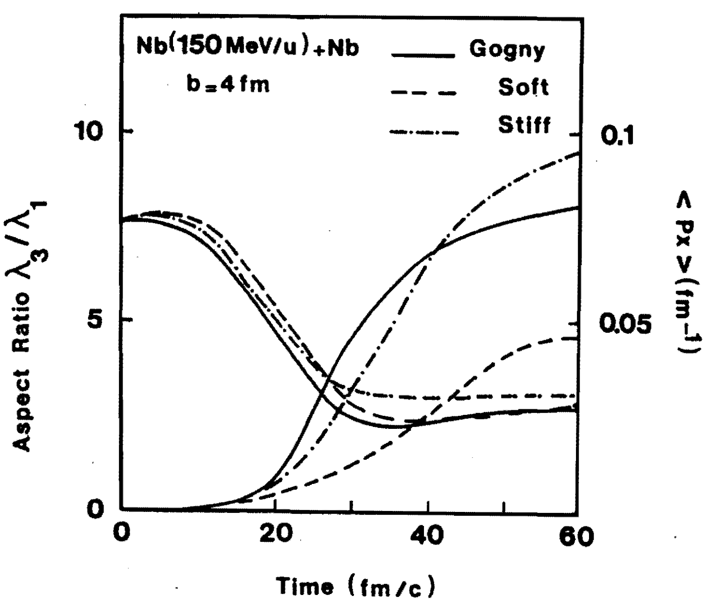


Fig.3 Time evolution of the aspect ratio and mean transverse momentum in the reaction plane.

The next point in this investigation was to know wether unambiguous EOS signatures could be extracted from collective flow when only non-local forces were considered. In a first time, a particular set of Gogny D1-type forces were constructed to reproduce different values of K_∞ and of the m^*/m ratio.

It has been checked that the different parametrizations of the force provide, for the static Landau-Vlasov solution, correct ground state properties of finite nuclei.

In Fig.4 the influence of K_∞ and of m^* on flow values is shown, for the semi-central collision of two Nb nuclei. The behavior of the yields with respect the effective mass is connected with the momentum dependence of the potential, since as this one is enhanced (lowering the m^*/m ratio) the flow is expected to increase.

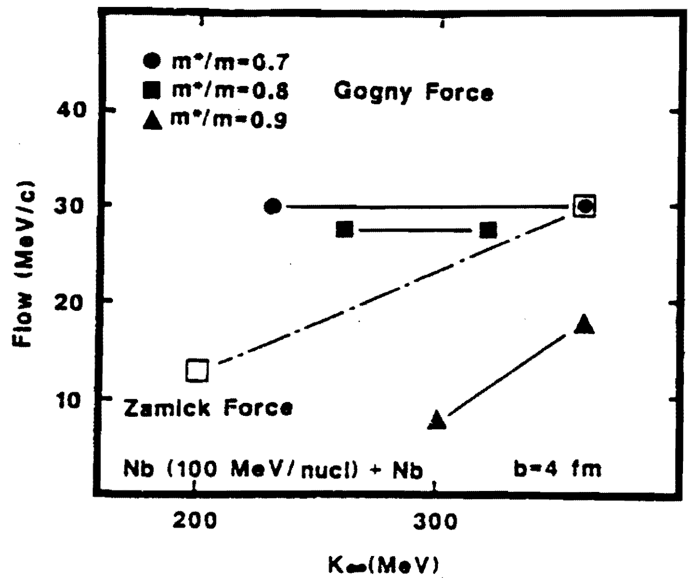


Fig.4 Flow values calculated with the Gogny force for various incompressibility moduli and average effective masses.

On the other side, the results are shown to be insensitive to K_∞ at least for low values of the effective mass.

It has been shown [4] that a reliable aproximation to the exact K_∞ can be given by:

$$K_\infty = \frac{3}{5}\frac{\hbar^2}{m}k_F{}^2\left(5\left(\frac{m}{m^*}\right)-6\right)+\frac{27}{8}t_3\gamma(\gamma+1)\rho^{\gamma+1} \tag{8}$$

According to this equation, the contribution of the density dependent part of the interaction, significantly weakens with decreasing effective-mass. Then, in the actual range of energies two effects can cooperate to wash-out K_∞ signatures. First, decreasing m^*/m ratio leads to a greater spreading of the nuclear matter and low densities. Second, the p-dependence of the mean-field acts essentially at the beginning of the reaction, when relative velocities are large, and the interaction region did not attain densities higher than the saturation value.

The preceding results brought us to explore beam energies ranging from 100 to 400 MeV pernucleon where experimental data are currently available and where the non-locality of the optical potential is expected to play a significant role.

The following calculations were performed with the set of parametrizations of the Gogny force given in Table II, and whose main static characteristics are resumed in Table III.

In Fig.5 the real part of the optical potential at normal density is reported and compared with experimental data [6]. In open squares the results obtained with the D1-G1 and G3 forces are drawn, while in open circles are those corresponding to the D1-G7 ang G8 interactions. Up to energies of about 150 MeV per nucleon the experimental values are rather well reproduced. Above that value, the forces exhibiting smaller m^* reproduce better the data than the original Gogny force. In this way, this new parametrizations of the D1-Gogny interaction are not only constrained to give good ground states properties, but also a more repulsive optical potential according with the data.

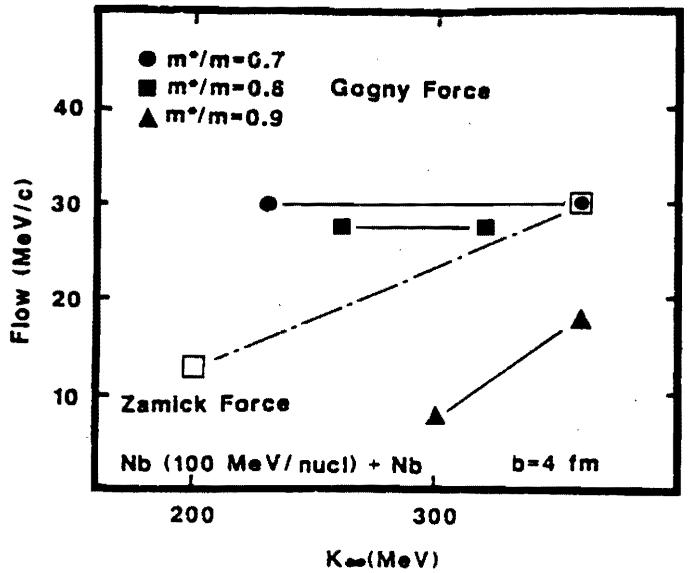
TABLE II. parameters of the different momentum interactions

forces	i	η_i	W_i	B_i	H_i	M_i	T_3	γ
D1-G1	1	0.7	-402.4	-100.	-496.2	-23.56	1350.	$\frac{1}{6}$
	2	1.2	-21.3	-11.77	37.27	-68.81	1350.	$\frac{1}{6}$
D1-G3	1	0.7	-402.4	-100.	-496.2	-23.56	1896	1.24
	2	1.2	30.96	35.4	34.56	-63.41	1896	1.24
D1-G7	1	0.7	-402.4	-70.	-496.2	-23.56	916.723	0.35
	2	1.2	61.087	-83.963	95.463	-165.197	916.723	0.35
D1-G8	1	0.7	-402.4	10.	-496.2	-23.56	1166.97.	1.2
	2	1.2	79.84	-96.53	66.77	-127.82	1166.97	1.2

TABLE III. main characteristics of the non-local interactions

forces	m^*/m	K_∞
D1-G1	0.67	228 MeV
D1-G3	0.67	360 MeV
D1-G7	0.56	228 MeV
D1-G8	0.56	300 MeV

In Fig.4 the influence of K_∞ and of m^* on flow values is shown, for the semi-central collision of two Nb nuclei. The behavior of the yields with respect the effective mass is connected with the momentum dependence of the potential, since as this one is enhanced (lowering the m^*/m ratio) the flow is expected to increase.



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Following Ref.[6] the flow parameter $F=\frac{d<p_x>/m}{dy})_{y_{cm}}$ is analyzed as a function of the energy for the Nb+Nb collision. The impact parameter was chosen equal to 4 fm in order to select the same kind of events as the experimental data [8, 9]. We have also simulated experimental filters according to the Plastic Ball and Wall detectors.

In Fig.6-a the flow parameter calculated with the D1-G1 and D1-G3 Gogny forces are shown. According to the preceding calculations , this observable is not very sensitive to K_∞ up to 200 MeV. Only at higher energies there is a splitting of their corresponding values, but in any case underestimating the experimental yields. In Fig.6-b the same trends below 200 MeV/n are found for the D1-G7 and D1-G8 forces. Nevertheless, their results above 200 MeV/n are higher than those in Fig.6-a and the soft D1-G7 parametrization fits correctly the experimental data.

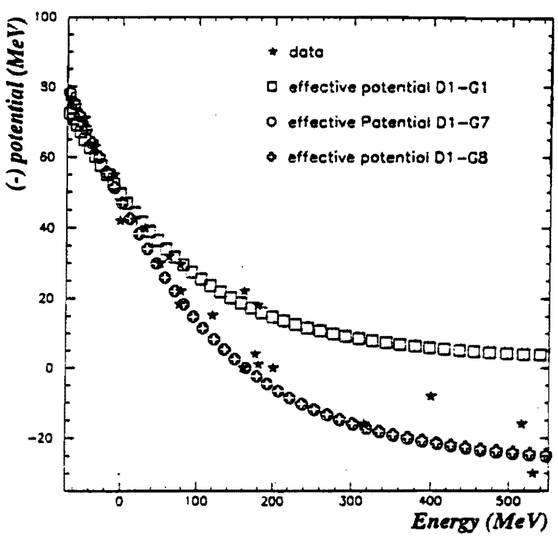

Fig.5 Real part othe optical potential at saturation density as a function of the energy compared with data. from Ref.[6].

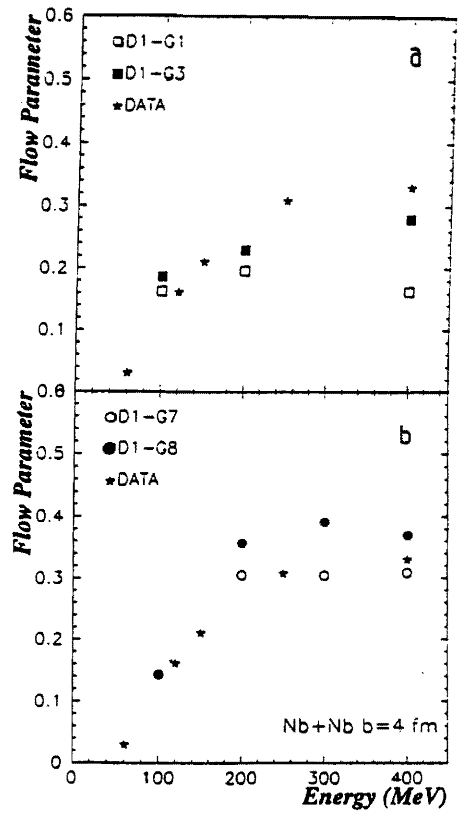

Fig.6 Flow parameter as a function of incident energy with Gogny forces. Experimental data are extracted from Refs.[8,9].

Let us mention that in Ref.[9] the effects of density and momentum dependence of the optical potential on the flow, at similar energies, was analyzed. Our results confirm their ealier findings, there is however some quantitative differences possibly due to the different incompressibilities and implemented forces.

Summarizing, this contribution was a brief compilation of recent results concerning transverse collective motion in heavy-ion reactions, within the framework of the Landau-Vlasov model. The objective of this investigation was to shed light on the nuclear equation of state. Different parametrizations of local and non-local effective interactions were used to construct the self-consistent mean field, all of them constrained to reproduce standard static properties of nuclei. Up to 200 MeV per nucleon the data could be equally fitted by either the hard local potential or any D1-type (non-local) Gogny force. The role of the non-locality in the collision dynamics could be analyzed, for instance, through the evolution of the pressure tensor in the bulk. In that range of energies no significant sensitivity of the transverse collective flow with the incompressivity modulus K_∞ was found, for non-local forces. It was necessary to go up in bombarding energy to notice differences in the calulated flow with the various Gogny forces here utilized (D1-G1,G3 and D1-G7,G8). Among them, the D1-G7 interaction (K_∞=228 MeV) seemed to provide a better agreement with the experiment and it has been shown to give the correct asymptotic behavior of the optical potential. In contrast, the usual D1-G1, whose reliability was proven at low energy, failed to describe flow data due to its too weak repulsive behavior. This stressed the fact that the implemented force must necessarily be also constrained to correctly reproduce the optical potential in the current energy domain.

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