# New possibilities of particle interferometry

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**Abstract**

A quantum approach to the particle correlations at small relative velocities in the presence of a strong Coulomb field is developed in the *adiabatic* approximation, allowing for a reliable determination of the space-time characteristics of particle production in heavy-ion reactions. The method allowing to measure which sort of particles was emitted earlier and which later at time scales as small as $10^{-22}$ s is suggested.

# 1 Introduction

The particle interferometry is here understood as a measurement of the space-time characteristics of the production process with the help of particle correlations at small relative velocities. It is well-known (see, e.g., [1-3]) that these correlations appear due to the effects of quantum statistics (QS) and final state interactions (FSI). Usually, only the mutual interaction of two particles with a small relative velocity is taken into account. This is however questionable in the case of heavy-ion reactions when the particles are produced in a strong Coulomb field of residual nuclei. Sometimes the effect of the nucleus Coulomb field is taken into account by a classical shift of the particle momenta. Since this shift is typically several tens MeV/c, such a procedure strongly influences correlations of particles with different charge-to-mass ratio. Thus in ref. [4] it was argued that np correlations can be completely destroyed due to a strong Coulomb repulsion of the protons from the emitting nucleus. The "Coulomb shift" procedure however does not take into account that both the shift of particle momenta and the correlation effect develop simultaneously and cannot be separated in a simple way. At characteristic emission times $\tau$ of several hundreds fm/c or higher this complicated 3-body problem can be solved in the classical approximation. In fact, the classical trajectory calculations show that at $\tau > \frac{|a|}{\gamma v}$, where $|a|$, $\gamma$ and $v$ are Bohr radius, Lorentz factor and velocity of the particle pair, the effect of residual nucleus on particle correlations is of minor importance despite the one-particle spectra are substantially affected [5]. To extend the theoretical description to lower values of $\tau$, we develop here a quantum approach in the *adiabatic* approximation, assuming the relative motion of the two particles much slower than their motion with respect to the Coulomb center.

Since the pioneering papers of Kopylov and Podgoretsky it is well-known that the study of directional dependence of particle correlations can be used to extract the information on the form of the production region and the emission time. We show here that not only the magnitude of the mean difference of particle emission times can be determined but that even the sign of this difference can be measured.

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In Sections 2 and 3 we briefly review the basics of interferometry of noninteracting and interacting particles. In Section 4 we develop the theory of particle correlations in a strong Coulomb field of the emitter. In Section 5 we suggest the method allowing one to measure the difference between mean emission times of the particles of various types.

# 2 Correlations of noninteracting identical particles

Let us start with the usual assumption of sufficiently small density of the produced particles in the momentum space, such that the correlation of two particles with a small relative velocity is influenced by the effects of their mutual QS and FSI only. Define the correlation function $R(p_1, p_2)$ of the two particles as the ratio of their differential production cross section to the one which would be observed in the case of absence of the effect of QS and FSI. Introducing the normalized probability $W_S(x_1, p_1; x_2, p_2)$ of the emission of two noninteracting particles, with total spin $S$ and 4-momenta $p_1$ and $p_2$, by the one-particle sources decaying at the space-time points $x_1 = \{t_1, \vec{r}_1\}$ and $x_2 = \{t_2, \vec{r}_2\}$, we get the well-known result of Kopylov and Podgoretsky (see, e.g., a review [1] and ref. [3]):

$$\begin{aligned} R(p_1, p_2) &= \sum_S \int d^4x_1\, d^4x_2 W_S(x_1, p; x_2, p) \left| \frac{e^{ip_1x_1 + ip_2x_2} + (-1)^S e^{ip_2x_1 + ip_1x_2}}{\sqrt{2}} \right|^2 \\ &= 1 + \sum_S (-1)^S \rho_S(p, p) \langle \cos(qx) \rangle_S. \end{aligned} \tag{1}$$

Here $p \equiv \frac{1}{2}P = \frac{1}{2}(p_1 + p_2)$, $q \equiv \{q_0, \vec{q}\} = p_1 - p_2$, $x \equiv \{t, \vec{r}\} = x_1 - x_2$, the mean $\langle f \rangle_S = \int d^4x_1\, d^4x_2 W_S f / \int d^4x_1\, d^4x_2 W_S$ is a function of $p_1$ and $p_2$, and

$$\rho_S(p_1, p_2) = \int d^4x_1\, d^4x_2 W_S(x_1, p_1; x_2, p_2), \quad \sum_S \rho_S(p_1, p_2) = 1, \tag{2}$$

describes the population of total spin-$S$ states, e.g., $\rho_S = (2S+1)/[(2s_1+1)(2s_2+1)]$ for unpolarized particles with spins $s_1$ and $s_2$.

It should be noted that Eq. (1) is obtained under the natural assumption of inessential momentum dependence of the emission probability $W_S(x_1, p_1; x_2, p_2)$ when varying the components of 4-momenta $p_1$ and $p_2$ by the amount $|\Delta p_\alpha| < \langle |x_\alpha| \rangle^{-1}$ characteristic for the interference effect: the smoothness assumption. In other words, the components of the mean space-time distance between particle sources $\langle |x_\alpha| \rangle$ are assumed much larger than those of the space-time extent of the sources $\sim \langle |p_\alpha| \rangle^{-1}$.

The characteristic feature of the correlation function (1) is the presence of the interference maximum or minimum at small $|\vec{q}|$ changing to a horizontal plateau at sufficiently large $|\vec{q}|$. E.g., assuming that particles are produced independently, in spin states independent of the production points:

$$W_S(x_1, p_1; x_2, p_2) = \rho_S(p_1, p_2) w(x_1, p_1) w(x_2, p_2) \tag{3}$$

and that the normalized one-particle probability

$$w(x_i; p_i) \propto \exp\left(-\frac{\vec{r}_i^{\,2}}{2r_0^2} - \frac{t_i^2}{2\tau_0^2}\right) \tag{4}$$

effectively describes the space and time limitation of the production process, we get

$$\begin{aligned} R(p_1,p_2) &= 1+\sum_S(-1)^S\rho_S(p,p)\exp(-r_0^2\vec{q}^{\,2}-\tau_0^2q_0^2) \\ &= 1+\sum_S(-1)^S\rho_S(p,p)\exp(-4r_0^2k_T^{*\,2}-4\rho^2\gamma^2k_L^{*\,2}). \end{aligned} \tag{5}$$

The correlation function in the second of Eqs. (5) is expressed in terms of the relative momentum in the pair rest frame: $q^* = \{0, 2\vec{k}^*\}$, $\vec{k}^* = \vec{p}_1^* = -\vec{p}_2^*$; $\vec{k}_T^* = \vec{q}_T/2$ and $k_L^* = q_L/(2\gamma) = q_0/(2\gamma v)$ are the transverse and logitudinal components of the vector $\vec{k}^*$ with respect to the pair velocity $\vec{v}$, $\gamma$ is the Lorentz factor of the pair, $\rho = \sqrt{r_0^2+(v\tau_0)^2}$. We see that, due to the relation $q_0 = \vec{v}\vec{q} \equiv vq_L$, the correlation function at $v\tau_0 > r_0$ substantially depends on the orientation of the vector $\vec{k}^*$ (or $\vec{q}$) even in the case of spherically symmetric spatial form of the production region. Generally, the orientation dependence of the correlation function can be used to determine both the characteristic emission time and the form of the production region [1].

# 3 Correlations of interacting particles

Making a similar smoothness assumption as in the case of identical noninteracting particles, i.e. assuming the momentum dependence of the emission probability of noninteracting particles $W_S(x_1,p_1;x_2,p_2)$ inessential when varying the 4-momenta $p_1$ and $p_2$ by the amount characteristic for the joint effect of QS and FSI, we can prove that the effect of FSI reduces to the substitution of the plane waves in Eq. (1) by the nonsymmetrized Bethe-Salpeter amplitudes in the continuous spectrum of the two-particle states [3,6]: $e^{ip_1x_1+ip_2x_2} \to \psi^{S(+)}_{p_1p_2}(x_1,x_2)$. For the correlation function of two identical particles we then have

$$\begin{aligned} R(p_1,p_2) &= \sum_S\int d^4x_1\,d^4x_2W_S(x_1,p;x_2,p)\left|\frac{\psi^{S(+)}_{p_1p_2}(x_1,x_2)+(-1)^S\psi^{S(+)}_{p_2p_1}(x_1,x_2)}{\sqrt{2}}\right|^2 \\ &= \sum_S\rho_S(p,p)\langle\left|\frac{\psi^{S(+)}_{p_1p_2}(x)+(-1)^S\psi^{S(+)}_{p_2p_1}(x)}{\sqrt{2}}\right|^2\rangle_S. \end{aligned} \tag{6}$$

and for that of two nonidentical particles

$$\begin{aligned} R(p_1,p_2) &= \sum_S\int d^4x_1\,d^4x_2W_S(x_1,p_1;x_2,p_2)|\psi^{S(+)}_{p_1p_2}(x_1,x_2)|^2 \\ &= \sum_S\rho_S(p_1,p_2)\langle|\psi^{S(+)}_{p_1p_2}(x)|^2\rangle_S. \end{aligned} \tag{7}$$

Here the amplitude $\psi^{S(+)}_{p_1p_2}(x)$, depending only on the relative 4-coordinate $x = x_1 - x_2$, is obtained from the Bethe-Salpeter amplitude after separation of the two-particle c.m.s. motion:

$$\psi^{S(+)}_{p_1p_2}(x_1,x_2) = e^{iPX}\psi^{S(+)}_{p_1p_2}(x), \tag{8}$$

where $X = [(p_1P)x_1 + (p_2P)x_2]/P^2$ is the c.m.s. 4-coordinate.

At equal emission times $t^* = t_1^* - t_2^* = 0$ in the two-particle c.m.s. the amplitude $\psi^{S(+)}_{p_1p_2}(x)$ coincides with a stationary solution of the scattering problem $\psi^{S(+)}_{-\vec{k}^*}(\vec{r}^*)$ having

at large $r^*$ the asymptotics of superposition of the plane and diverging spherical waves: $\psi^{S(+)}_{-\vec{k}^*}(\vec{r}^*) \to e^{-i\vec{k}^*\vec{r}^*} + f^S(-\vec{k}^*)e^{ik^*r^*}/r^*$. In fact, it can be shown [3] that the amplitude $\psi^{S(+)}_{p_1p_2}(x)$ can be substituted by this solution on condition $|t^*| \ll \mu r^{*2}$, where $\mu$ is the reduced mass of the two particles. For the Gaussian space-time distribution (4) of the emission points it leads to the condition $\tau_0 \ll \mu\gamma r_0\rho$. For $\tau_0 <\sim r_0$ this condition is satisfied if particle energies $\omega_i = \sqrt{m_i^2 + \vec{p}_i^{\,2}} \gg 1/r_0$, while in the case of a large characteristic emission time, when $v\tau_0 \gg r_0$, it requires $|\vec{p}_i| \gg 1/r_0$. The *equal time* approximation is usually valid for heavy particles, like kaons or nucleons. But, even for two pions at $v = 0.3$, $r_0 = v\tau_0 = 1$ fm, the exact calculations of the correlation function show that this approximation overestimates FSI contribution by less than 20% [3].

The effect of FSI on the correlation function was first calculated numerically by Koonin for the case of two nonrelativistic unpolarized protons [2]. Simple analytical formulae, valid for arbitrary particles (including relativistic, polarized and nonidentical particles) produced at sufficiently large distances compared with the range of their strong interaction, were obtained in ref. [3] (see also a review [7]). Concerning the size of the effect of FSI on the correlation function, the contribution of the strong interaction is determined by the ratio $f^S/r^*$. It thus decreases with increasing space-time dimensions of the production region and particle velocities. For pions $|f^S| \ll \langle r^*\rangle$ and this contribution is of the order of 0.1 or smaller, while for nucleons at small $k^*$ usually $|f^S| > \langle r^*\rangle$ and the correlation function, being typically of the order of 1-10, is dominated by the effect of strong FSI.

In the case of charged particles the Coulomb repulsion (attraction) becomes important at $k^* \to 0$, forcing the correlation function to tend to $0\,(\infty)$. If the characteristic distance $\langle r^*\rangle$ between two particles in their c.m.s. is much smaller than their Bohr radius $|a| = 1/|\pm\mu z_1 z_2 e^2|$, where $\pm z_1 z_2 e^2$ is the product of the particle electric charges, the effects of Coulomb and strong interaction practically factorize:

$$R(p_1, p_2) \doteq A_c(k^*a)\tilde{R}(p_1, p_2), \tag{9}$$

where $\tilde{R}(p_1, p_2)$ represents the correlation function due to the strong interaction only and the function $A_c(k^*a)$ is the modulus squared of the Coulomb wave function at $r^* = 0$:

$$A_c(y) = \frac{2\pi}{y}/[\exp(\frac{2\pi}{y}) - 1]. \tag{10}$$

The function $A_c(k^*a)$ substantially deviates from unity only at small values of argument, i.e. in the region of small momenta $k^* < 2\pi/|a|$. E.g., for two charged pions (two protons) $|a| = 388\,(58)$ fm and, on condition $\langle r^*\rangle \ll |a|$, the Coulomb effects are important at $k^* < 3\,(22)$ MeV/c. It should be stressed that the separation of Coulomb effects in the Coulomb factor $A_c(k^*a)$ does not take place at $\langle r^*\rangle$ comparable or larger than the Bohr radius, e.g., in the case when particles are produced in the decays of long lived resonances $(\eta, \eta')$ or in the evaporation processes. In such a situation, with the increasing $\langle r^*\rangle$ the correlation function tends to 1 and not to the Coulomb factor $A_c(k^*a)$ [3]; it substantially deviates from 1 only for $k^* < 2\pi k_b^*$, where $k_b^* = (|a|\langle r^*\rangle/2)^{-1/2}$ is the classical boundary value.

# 4 Correlations in a strong Coulomb field

We shall now consider particle correlations in the presence of a strong Coulomb field, e.g. in heavy-ion reactions. In this case the Bohr radius of the system *particle + residual nucleus* can be rather small ($\sim$ 0.6 fm for the system *proton + nucleus* with the charge number $Z = 51$) so that the Coulomb interaction in this system can be substantial even for relativistic particles. In such a situation, instead of the two-particle Bethe-Salpeter amplitude $\psi^{S(+)}_{p_1p_2}(x_1, x_2)$, the correlation function is determined by the amplitude $\psi^{S\{\alpha\}(+)}_{p_1p_2}(x_1, x_2)$ representing the solution of a complicated multibody problem, taking into account interaction between the two particles and also their interaction with the residual system described by the quantum numbers $\{\alpha\}$. For the correlation function, defined as the ratio of the two-particle production cross section to the one in the case of absence of the effects of QS and FSI between the two particles ($\psi \to \tilde{\psi}$), instead of Eqs. (6) and (7) for identical and nonidentical particles we have

$$R(p_1, p_2) = \frac{\sum_S \rho_S(p,p)\frac{1}{2}\langle|\psi^{S\{\alpha\}(+)}_{p_1p_2}(x_1, x_2) + (-1)^S\psi^{S\{\alpha\}(+)}_{p_2p_1}(x_1, x_2)|^2\rangle_S}{\sum_S \rho_S(p,p)\frac{1}{2}\langle|\tilde{\psi}^{S\{\alpha\}(+)}_{p_1p_2}(x_1, x_2)|^2 + |\tilde{\psi}^{S\{\alpha\}(+)}_{p_2p_1}(x_1, x_2)|^2\rangle_S} \tag{11}$$

and

$$R(p_1, p_2) = \frac{\sum_S \rho_S(p_1, p_2)\langle|\psi^{S\{\alpha\}(+)}_{p_1p_2}(x_1, x_2)|^2\rangle_S}{\sum_S \rho_S(p_1, p_2)\langle|\tilde{\psi}^{S\{\alpha\}(+)}_{p_1p_2}(x_1, x_2)|^2\rangle_S}. \tag{12}$$

Here

$$\rho_S(p_1, p_2) = \sum_{\{\alpha\}} \int d^4x_1\, d^4x_2 W_{S\{\alpha\}}(x_1, p_1; x_2, p_2), \tag{13}$$

$\langle f\rangle_S = \sum_{\{\alpha\}} \int d^4x_1\, d^4x_2 W_{S\{\alpha\}} f / \sum_{\{\alpha\}} \int d^4x_1\, d^4x_2 W_{S\{\alpha\}}$ and $\sum_{\{\alpha\}}$, taking into account the conservation laws, represents summation over the descrete quantum numbers and integration over the continuous ones. Again the smoothness condition is implied, i.e. the dependence of the emission probability of noninteracting particles $W_{S\{\alpha\}}(x_1, p_1; x_2, p_2)$ on the 4-momenta $p_1$ and $p_2$ is assumed inessential when varying them by the amount characteristic for QS and FSI.

It can be shown that at $\langle r^*\rangle > |a|$, e.g., in the case of particles evaporated by a compound nucleus with the characteristic emission time $\tau > \frac{|a|}{\gamma v}$, and at $k^*$ not too close to the classical boundary $k^*_b = (|a|\langle r^*\rangle/2)^{-1/2}$, the conditions of the applicability of the classical approach are fulfiled and this problem can be solved with the help of the classical trajectory calculations [5]. To get rid of the limitations of such calculations, we develop below the quantum approach and solve the problem of the two-particle correlations in the field of a heavy Coulomb center.

We consider the emitting nucleus N sufficiently heavy so that the recoil effects can be neglected and the c.m. of the system *particle + nucleus* can be identified with the rest frame of the nucleus, which we situate at the origin. Neglecting further the change of the nucleus electric charge during the process of particle emission, we approximate it by an effective charge $Ze$. Finally, neglecting the interaction of the particles 1 and 2 with other emitted particles, the quantum numbers $\{\alpha\}$ of the *residual system* are reduced to the effective charge number $Z$.

Let us start with the hypothetical case of particles which interact with the charge $Ze$ but their mutual interaction is "switched off". In such a situation we can treat the systems (1, N) and (2, N) independently. It follows from the results presented in previous

Section that the interaction with the Coulomb center just leads to the substitution of the spatial parts of the plane waves $e^{ip_ix_i}$ by the usual Coulomb wave functions:

$$\begin{aligned} e^{-i\vec{p}_i\vec{r}_i} &\rightarrow e^{-i\vec{p}_i\vec{r}_i}\Phi^{z_iZ}_{\vec{p}_i}(\vec{r}_i), \\ \Phi^{z_iZ}_{\vec{p}_i}(\vec{r}_i) &= e^{i\delta_i}\sqrt{A_c(|\vec{p}_i|a_i)}F[\frac{-i}{|\vec{p}_i|a_i}, 1, i(\vec{p}_i\vec{r}_i + |\vec{p}_i|r_i)], \end{aligned} \tag{14}$$

where $|a_i| = 1/(\omega_i z_i Z e^2)$ is the Bohr radius of the system $(i, \mathrm{N})$ generalized to the relativistic case by the substitution $m_i \rightarrow \omega_i$, $\delta_i = \arg\Gamma[1 + i/(|\vec{p}_i|a_i)]$ is the Coulomb s-wave shift, $A_c(|\vec{p}_i|a_i) = |\Phi^{z_iZ}_{\vec{p}_i}(0)|^2$ is the Coulomb penetration factor given in Eq. (10) and $F(\alpha, 1, z)$ is the confluent hypergeometrical function ($F \rightarrow 1$ at $r_i \ll |a_i|$). Thus

$$\tilde{\psi}^{SZ(+)}_{p_1p_2}(x_1, x_2) = e^{ip_1x_1+ip_2x_2}\Phi^{z_1Z}_{\vec{p}_1}(\vec{r}_1)\Phi^{z_2Z}_{\vec{p}_2}(\vec{r}_2) = e^{iPX}e^{-i\vec{k}^*\vec{r}^*}\Phi^{z_1Z}_{\vec{p}_1}(\vec{r}_1)\Phi^{z_2Z}_{\vec{p}_2}(\vec{r}_2). \tag{15}$$

Note that a small contribution of spin-dependent electromagnetic forces is neglected in Eq. (14) so that $\tilde{\psi}^{SZ} \equiv \tilde{\psi}^{Z}$ is independent of the total spin $S$ of the particle pair. We can thus put $\sum_S \rho_S\langle|\tilde{\psi}^{SZ}|^2\rangle_S = \langle|\tilde{\psi}^{Z}|^2\rangle$ in the denominators of Eqs. (11) and (12).

It will be convenient in the following to separate the time and the spatial parts of the two-particle c.m.s. 4-coordinate $X = \{X_0, \vec{R}\}$ and rewrite the amplitude in Eq. (15) in the form

$$\tilde{\psi}^{SZ(+)}_{p_1p_2}(x_1, x_2) = e^{iP_0X_0}e^{-i\vec{k}^*\vec{r}^*}\tilde{\Phi}^{Z}_{\vec{p}_1\vec{p}_2}(\vec{r}_1, \vec{r}_2), \tag{16}$$

where

$$\tilde{\Phi}^{Z}_{\vec{p}_1\vec{p}_2}(\vec{r}_1, \vec{r}_2) = e^{-i\vec{P}\vec{R}}\Phi^{z_1Z}_{\vec{p}_1}(\vec{r}_1)\Phi^{z_2Z}_{\vec{p}_2}(\vec{r}_2). \tag{17}$$

Let us now adiabatically "switch on" the interaction between particles 1 and 2, i.e. consider relative motion of the two particles at characteristic distances much slower compared with their motion with respect to the Coulomb center. It is clear that in such a case the plane wave $e^{-i\vec{k}^*\vec{r}^*}$ will be substituted by the Bethe-Salpeter amplitude $\psi^{S(+)}_{p_1p_2}(x)$ describing the relative motion of isolated interacting particles and that the function $\tilde{\Phi}^{Z}_{\vec{p}_1\vec{p}_2}(\vec{r}_1, \vec{r}_2)$ will be only slightly modified. Generally, we can write

$$\psi^{SZ(+)}_{p_1p_2}(x_1, x_2) = e^{iP_0X_0}\psi^{S(+)}_{p_1p_2}(x)\Phi^{Z}_{\vec{p}_1\vec{p}_2}(x_1, x_2), \tag{18}$$

where the function $\Phi^{Z}_{\vec{p}_1\vec{p}_2}(x_1, x_2)$ can be substituted by $\tilde{\Phi}^{Z}_{\vec{p}_1\vec{p}_2}(\vec{r}_1, \vec{r}_2)$ on certain conditions which will be considered below. After this substitution we get our main result:

$$\psi^{SZ(+)}_{p_1p_2}(x_1, x_2) = e^{iPX}\psi^{S(+)}_{p_1p_2}(x)\Phi^{z_1Z}_{\vec{p}_1}(\vec{r}_1)\Phi^{z_2Z}_{\vec{p}_2}(\vec{r}_2). \tag{19}$$

We will call it *adiabatic* approximation using a close analogy with the quantum theory of the motion of electrons and nuclei in a molecule. In the adiabatic approximation the fast motion of electrons in a molecule is separated from the slow motion of nuclei, leading to the corresponding factorization of the total wave function. In our problem, the fast motion is the motion of the two considered particles with respect to the Coulomb center, and the slow one is their relative motion.

Note that the symmetrized amplitude $[\psi^{SZ(+)}_{p_1p_2}(x_1, x_2) + (-1)^S\psi^{SZ(+)}_{p_2p_1}(x_1, x_2)]/\sqrt{2}$, determining the correlation function of two identical particles, containes combination of the two-particle amplitudes $[\psi^{S(+)}_{p_1p_2}(x) - (-1)^S\psi^{S(+)}_{p_2p_1}(x)]/\sqrt{(2)}$ corresponding to an isolated pair

with forbidden symmetry. In particular, the production amplitude of two identical nucleons in triplet state $S = 1$ in the Coulomb field depends on otherwise forbidden s-wave interaction in the isolated pair.

We will discuss the applicability conditions of the *adiabatic* approximation in the nonrelativistic limit but, as the Coulomb wave functions with the modified Bohr radius satisfy the relativistic wave equation, this analysis can be generalized also for the case of relativistic particles. First we derive equations for the functions $\Phi^Z$ and $\tilde{\Phi}^Z$. In the nonrelativistic limit we have

$$\vec{R} = \frac{(m_1\vec{r}_1 + m_2\vec{r}_2)}{(m_1+m_2)}, \quad \vec{r}^* = \vec{r} - \vec{v}t, \quad \vec{v} = \frac{\vec{P}}{m_1+m_2}, \quad \vec{k}^* = \vec{k} = \frac{m_2\vec{p}_1 - m_1\vec{p}_2}{m_1+m_2}. \tag{20}$$

Neglecting eventual weak dependence of the function $\Phi^Z$ on the emission times, i.e. assuming $\Phi^Z_{\vec{p}_1\vec{p}_2}(x_1,x_2) = \Phi^Z_{\vec{p}_1\vec{p}_2}(\vec{r}_1,\vec{r}_2)$, we can put $t^* = 0$ and substitute the amplitude $\psi^{S(+)}_{p_1p_2}(x)$ by $\psi^{S(+)}_{-\vec{k}}(\vec{r}^*)$. The amplitude $\psi^{SZ(+)}_{p_1p_2}(x_1,x_2)$ then satisfies Schrödinger equation

$$[\hat{H}_0 + V_1(r_1) + V_2(r_2) + V^S(\vec{r}^*) - E_{12}]\psi^{SZ(+)}_{p_1p_2}(x_1,x_2) = 0. \tag{21}$$

Here $V_i(r_i) = (a_i m_i r_i)^{-1}$ is the potential between particle i and the Coulomb center, $V^S(\vec{r}^*)$ is the interaction potential of the two particles,

$$E_{12} = \frac{1}{2}\vec{p}_1{}^2/m_1 + \frac{1}{2}\vec{p}_2{}^2/m_2 \equiv \frac{1}{2}\vec{P}^2/(m_1+m_2) + \frac{1}{2}\vec{k}^2/\mu \tag{22}$$

is their kinetic energy and

$$\hat{H}_0 = -\frac{1}{2}\vec{\nabla}_1^2/m_1 - \frac{1}{2}\vec{\nabla}_2^2/m_2 \equiv -\frac{1}{2}\vec{\nabla}_R^2/(m_1+m_2) - \frac{1}{2}\vec{\nabla}_r^2/\mu \tag{23}$$

is the free Hamiltonian. Using now the Schrödinger equation

$$[-\frac{1}{2}(\vec{\nabla}_r^2 + \vec{k}^2)/\mu + V^S(\vec{r}^*)]\psi^{S(+)}_{-\vec{k}}(\vec{r}^*) = 0 \tag{24}$$

and the relation

$$\vec{\nabla}_r^2(\psi\cdot\Phi) = \Phi\cdot\vec{\nabla}_r^2\psi + \psi\cdot[\vec{\nabla}_r^2 + 2(\vec{\nabla}_r\ln\psi)\cdot\vec{\nabla}_r]\Phi, \tag{25}$$

and also taking into account that $\vec{\nabla}_r = \vec{\nabla}_{r^*}$, we get the equation for the function $\Phi^Z$:

$$[\hat{H}_0 + V_1(r_1) + V_2(r_2) + \hat{V} - E_{12}]e^{-i\vec{k}\vec{r}^*}\Phi^Z_{\vec{p}_1\vec{p}_2}(\vec{r}_1,\vec{r}_2) = 0, \tag{26}$$

where

$$\hat{V} = -\frac{1}{\mu}[\vec{\nabla}_r\ln\psi^{S(+)}_{-\vec{k}}(\vec{r}^*) + i\vec{k}]\cdot(\vec{\nabla}_r + i\vec{k}). \tag{27}$$

Clearly, the function $\tilde{\Phi}^Z$ satisfies similar equation with $\hat{V} = 0$. Thus the equations for functions $\Phi^Z$ and $\tilde{\Phi}^Z$ differ only in the term $\hat{V}$ containing gradient $\vec{\nabla}_r$. In the *adiabatic* approximation one can neglect this term considering it as a small perturbation in the continuous spectrum. Noting that the functions $\Phi^{z_i Z}_{\vec{p}_i}(\vec{r}_i)$ defined in Eq. (14) depend on $\vec{r}_i$ only through the variable $\rho_i = \vec{p}_i\vec{r}_i + |\vec{p}_i|r_i$, we can write the perturbation in the form:

$$\hat{V}e^{-i\vec{k}\vec{r}^*}\tilde{\Phi}^Z_{\vec{p}_1\vec{p}_2}(\vec{r}_1,\vec{r}_2) = -\frac{1}{\mu}[\vec{\nabla}_r\ln\psi^{S(+)}_{-\vec{k}}(\vec{r}^*) + i\vec{k}]\cdot$$
$$[\vec{\nabla}_r\rho_1\frac{d}{d\rho_1}\ln\Phi^{z_1 Z}_{\vec{p}_1}(\vec{r}_1) + \vec{\nabla}_r\rho_2\frac{d}{d\rho_2}\ln\Phi^{z_2 Z}_{\vec{p}_2}(\vec{r}_2)]e^{-i\vec{k}\vec{r}^*}\tilde{\Phi}^Z_{\vec{p}_1\vec{p}_2}(\vec{r}_1,\vec{r}_2). \tag{28}$$

The two terms in Eq. (28) can be considered as distortions of the Coulomb potentials $V_1(r_1)$ and $V_2(r_2)$. The sufficient applicability conditions of the *adiabatic* approximation, guaranting approximate equality of functions $\Phi^Z$ and $\tilde{\Phi}^Z$, thus follow from the requirement

$$\frac{1}{\mu}|[\vec{\nabla}_r \ln \psi^{S(+)}_{-\vec{k}}(\vec{r}^*) + i\vec{k}]\vec{\nabla}_r \rho_i \frac{d}{d\rho_i} \ln \Phi^{z_i Z}_{\vec{p}_i}(\vec{r}_i)| \ll |V_i(\vec{r}_i)|. \tag{29}$$

Using known asymptotic formulae for the confluent hypergeometric functions in the limit of both large and small distances, and the relations $\vec{\nabla}_r \rho_i = \pm\mu \frac{|\vec{p}_i|}{m_i}(\frac{\vec{p}_i}{|\vec{p}_i|} + \frac{\vec{r}_i}{r_i})$, $i = 1, 2$, we get the following sufficient conditions in the region of small $k <\sim 1/r^*$ we are interested in:

$$1/|a| \ll |\vec{p}_i|, \qquad 1/[r^* + r^{*2}/|f^S(k)|] \ll |\vec{p}_i|. \tag{30}$$

These conditions simply reflect the fact that the Coulomb potentials $V_i(r_i)$ are distorted to the extent determined by the ratios of the momenta $1/|a|$ and $1/[r^* + r^{*2}/|f^S(k)|]$, characteristic for Coulomb and strong interaction between the two particles (at $k <\sim 1/r^*$), to the total particle momenta $|\vec{p}_i|$. As practically $|\vec{p}_i|$ is larger than the classical boundary value $p_{bi} = (|a_i|\langle r_i\rangle/2)^{-1/2}$, we can substitute it by this value to get still more sufficient conditions. In the case of two protons emitted by a typical nucleus with $Z = 51$ and $\langle r_i\rangle = 6$ fm ($p_{bi} = 154$ MeV/c, $a = 58$ fm, $f^0(0) = 7.8$ fm), these conditions are not very restrictive: $41 \gg 1$, $r^* \gg 1.1$ fm. In the case of heavier particles or fragments the first condition becomes stronger while the second one is weakened. E.g., for two deuterons ($p_{bi} = 218$ MeV/c): $32 \gg 1$, $r^* \gg 0.8$ fm.

For quantitative estimates we assume that, in the case of absence of FSI, the compound nucleus isotropically emits unpolarized particles with the energies distributed according to Maxwellian law with the temperature $T = 4$ MeV. The distribution of the 4-coordinates of the particle sources is approximated by Gaussian law in Eq. (4) with the parameter $r_0 = 3.5$ fm roughly corresponding to the mass number of the emitting nucleus $A = 120$. Requiring the same dispersion of the difference $t = t_1 - t_2$ of the emission times for the Gaussian law and the exponential decay law, the parameter $\tau_0$ can be identified with the emitter lifetime $\tau$. For evaporation processes $\tau$ is typically several hundreds fm/c (leading to $\langle r^*\rangle \simeq v\tau$ of several tens fm). For the effective charge number of the residual nucleus we put $Z = 51$, which leads to about twice as large mean kinetic energy of the emitted protons or deuterons as compared with that of neutrons. The mean velocity of pp, pd or np pairs at small values of $k^*$ is $\langle v\rangle \sim 0.15$. The above parameters roughly describe particle emission in the reaction $^{40}Ar +^{108} Ag$ at 44 MeV/nucleon [5].

Instead of the 6-dimensional correlation function $R(p_1, p_2)$ we calculate the 1-dimensional one $R^Z(k^*)$ corresponding to Eq. (11) or (12) with the nominator and denominator integrated over 1-particle spectra $d^3\sigma_i(\vec{p}_i)/d^3\vec{p}_i$. To separate pure effect of the nucleus Coulomb field on particle correlations, we compare the correlation function $R^Z(k^*)$ with the one $R^{"Z"}(k^*)$ taking into account the effect of the nucleus Coulomb field on one-particle spectra but not on particle correlations. For instance, for nonidentical particles

$$R^{"Z"}(k^*) = \frac{\sum_S \int d^3\sigma_1(\vec{p}_1)\, d^3\sigma_2(\vec{p}_2)\delta(k^* - |\vec{p}_1^*|)\rho_S(p_1, p_2)\langle|\psi^{S(+)}_{p_1 p_2}(x_1, x_2)|^2\rangle_S \langle|\Phi^{z_1 Z}_{\vec{p}_1}(\vec{r}_1{}')\Phi^{z_2 Z}_{\vec{p}_2}(\vec{r}_2{}')|^2\rangle}{\int d^3\sigma_1(\vec{p}_1)\, d^3\sigma_2(\vec{p}_2)\delta(k^* - |\vec{p}_1^*|)\langle|\Phi^{z_1 Z}_{\vec{p}_1}(\vec{r}_1{}')\Phi^{z_2 Z}_{\vec{p}_2}(\vec{r}_2{}')|^2\rangle}. \tag{31}$$

Our results confirm the classical trajectory calculations showing negligible effect of the residual nucleus on particle correlations at large emitter lifetimes $\tau > \frac{|a|}{\gamma v}$. In Figs. 1-3

we present the results of quantum calculations for pp, pd and np systems in the region of smaller lifetimes where the classical approach is not valid. Our calculations show an increasing influence of the residual nucleus with decreasing $\tau$ leading to a suppression of the correlation. Though this suppression appeares much less important than expected from the simple "Coulomb shift" procedure [4]. For example, at $\tau = 100$ (10) fm/c the np correlation function is suppressed at small relative momenta by $\sim 10$ (35)% only. A noticeable suppression at $\tau < 50$ fm/c is obtained also for pd system ($\sim 15\%$) and even for pp system ($\sim 10\%$).

# 5 Measurement of the difference of the mean emission times of various particles

Clearly, the correlation function of two nonidentical particles is sensitive not only to the magnitude but also to the sign of the difference of the emission times $t = t_1 - t_2$. Below we demonstrate this sensitivity and the way how the correlation function of two nonidentical particles can be used to estimate the difference between their mean emission times.

Let us consider sufficiently small momentum $k^*$ of the particles in their c.m.s. so that their strong interaction is dominated by s-wave and the wave function $\psi^{S(+)}_{-\vec{k}^*}(\vec{r}^*)$ in the absence of Coulomb interaction takes the form

$$\psi^{S(+)}_{-\vec{k}^*}(\vec{r}^*) = e^{-\vec{k}^*\vec{r}^*} + \phi^S_{k^*}(r^*), \tag{32}$$

where the scattered wave $\phi^S_{k^*}(r^*)$ is independent of the directions of the vectors $\vec{k}^*$ and $\vec{r}^*$. Neglecting the interaction of the two particles with the residual system and assuming that the conditions of *equal time* approximation are fulfiled, we can write the correlation function in the form [3]

$$\begin{aligned} R(p_1, p_2) &= \sum_S \rho_S \langle |\psi^{S(+)}_{-\vec{k}^*}(\vec{r}^*)|^2 \rangle_S \\ &= 1 + \sum_S \rho_S \langle |\phi^S_{k^*}(r^*)|^2 + 2\mathrm{Re}\phi^S_{k^*}(r^*) \cos \vec{k}^*\vec{r}^* - 2\mathrm{Im}\phi^S_{k^*}(r^*) \sin \vec{k}^*\vec{r}^* \rangle_S. \end{aligned} \tag{33}$$

Consider now the behaviour of the vector $\vec{r}^*$ in the limit $|vt| \gg r$. Making the Lorentz transformation from the source rest frame to c.m.s. of the two particles: $r^*_L = \gamma(r_L - vt)$, $r^*_T = r_T$, we see that, in the considered limit, the vector $\vec{r}^* \doteq -\gamma\vec{v}t$ is nearly parallel or antiparallel to the vector of pair velocity $\vec{v}$, depending on the sign of the time difference $t$. Therefore, the correlation function is sensitive to sign($t$) due to the odd term $\sim \sin \vec{k}^*\vec{r}^*$. For charged particles there arise additional odd terms with respect to the argument $\vec{k}^*\vec{r}^*$ due to the confluent hypergeometrical function $F(\alpha, 1, z) = 1 + \alpha z + \alpha(\alpha+1)(z/2!)^2 + \ldots$ modifying the plane wave in Eq. (32):

$$\psi^{S(+)}_{-\vec{k}^*}(\vec{r}^*) = e^{i\delta}\sqrt{A_c(k^*a)}\{e^{-\vec{k}^*\vec{r}^*} F[\frac{-i}{k^*a}, 1, i(\vec{k}^*\vec{r}^* + k^*r^*)] + \phi^S_{ck^*}(r^*)\}, \tag{34}$$

$\delta = \arg\Gamma[1 + i/(k^*a)]$. The sensitivity of the correlation function to sign($t$) can be also modified due to Coulomb interaction with the emitting nucleus (see Eq. (19)).

[3]The correlation function of two noninteracting particles was first introduced in ref. [3] in a symmetrized form, so that the odd terms in variable $\vec{k}^*\vec{r}^*$ were omitted.

Thus we see that the sign of the mean time difference $\langle t \rangle$ can be determined provided the sign of the scalar product $\vec{k}^*\vec{v}$ is fixed. A straightforward way is to measure the correlation functions $R_+(\vec{k}^*\vec{v} \geq 0)$ and $R_-(\vec{k}^*\vec{v} < 0)$. Depending on $\langle t \rangle$, their ratio $R_+/R_-$ should show a peak or a dip or oscillate in the region of small $k^*$ and approach 1 both at $k^* \to 0$ and $k^* \to \infty$.

As the sign of the scalar product $\vec{k}^*\vec{v}$ is approximately equal to that of the difference of particle velocities $v_1 - v_2$ (this equality is exact for particles of equal masses), the sensitivity of the correlation functions $R_+$ and $R_-$ to the sign of the difference of particle emission times has a simple explanation in terms of the classical trajectory approach. Clearly, the interaction between the particles in the case of earlier emission of faster particle will be different compared with the case of its later emission (the interaction time being longer in the latter case).

Our calculations show that the correlation functions $R_+$, $R_-$ and their ratio are substantially sensitive to $\langle t \rangle$ provided that $\langle t \rangle^2$ is comparable to or higher than the dispersions $\langle t_i^2 - \langle t_i \rangle^2 \rangle$. In Figs. 4 and 5 we present the results for pd and np systems assuming that the particles are emitted by a nucleus with the effective charge number $Z = 51$ and temperature 4 MeV according to the Gaussian time distributions of the width of 50 fm/c and $\langle t \rangle$ = -100 fm/c. We see that in both cases the ratio $R_+/R_-$ strongly deviates from unity in the region of small $k^*$ achieving maximum value of $\sim$ 2.0 (1.3) at $k^* \simeq 10$ (35) MeV/c for pd (np) system. This maximum would be replaced by a minimum in the case of reversed sign of the mean difference of the emission times $\langle t \rangle$. We can also see that the ratio $R_+/R_-$ is only slightly affected by the Coulomb field of the residual nucleus.

# 6 Conclusions

In the *adiabatic* approximation we have developed quantum approach to the correlations of particles produced with a small relative velocity in the field of a heavy Coulomb center. We have demonstrated that, contrary to some expectations, substantial correlations of particles with different charge-to-mass ratio survive even in a strong Coulomb field of residual nuclei in heavy-ion reactions. In these processes, the quantum *adiabatic* approach to the particle correlations at small relative velocities can be used for a quantitative study of the space-time picture of particle production.

It is well known that the directional dependence of two-particle correlations can be used to estimate the form of the production region and the magnitude of the mean difference of the times of particle emission. We have shown that also the sign of this difference can be measured. Thus a new possibility is open to determine which sort of particles (e.g., protons or neutrons, pions or kaons) was produced earlier and which later. For this we suggest to study the correlation functions of two nonidentical particles separately for the angles between the relative velocity $\vec{k}^*/\mu$ and the total pair velocity $\vec{v}$ in the emitter rest frame less and greater than 90°.


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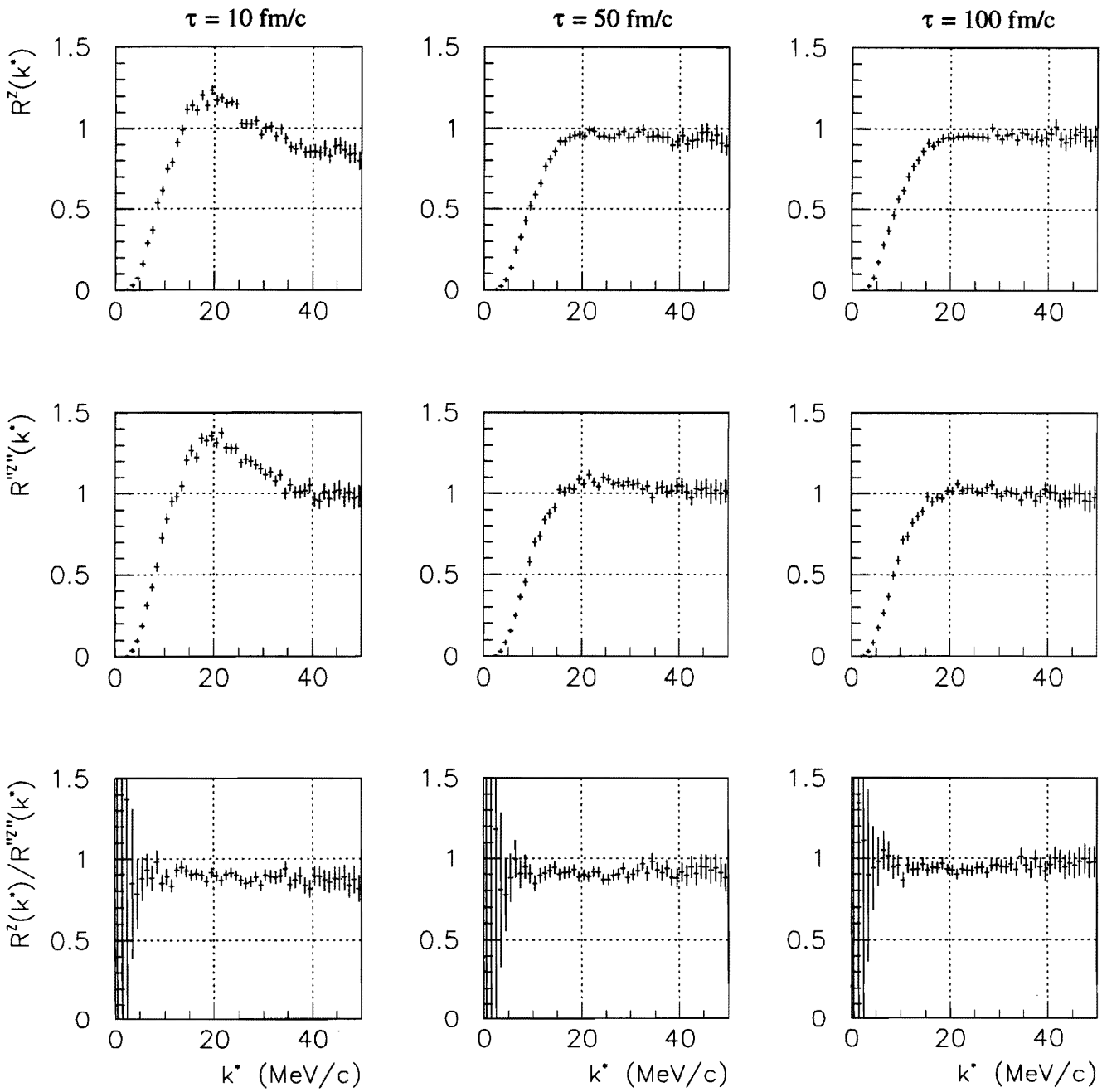


Figure 1: The pp correlation functions $R^Z$ and $R^{"Z"}$ (only one-particle spectra are influenced by the nucleus charge) and their ratio calculated for the effective charge number of the emitting nucleus $Z = 51$. The particles are assumed to be unpolarized and emitted isotropically according to Maxwellian law with the temperature 4 MeV. The distribution of space-time coordinates of the particle sources is approximated by Gaussian law in Eq. (4) with $r_0 = 3.5$ fm, $\tau_0 \equiv \tau = 10,\ 50$ and 100 fm/c.

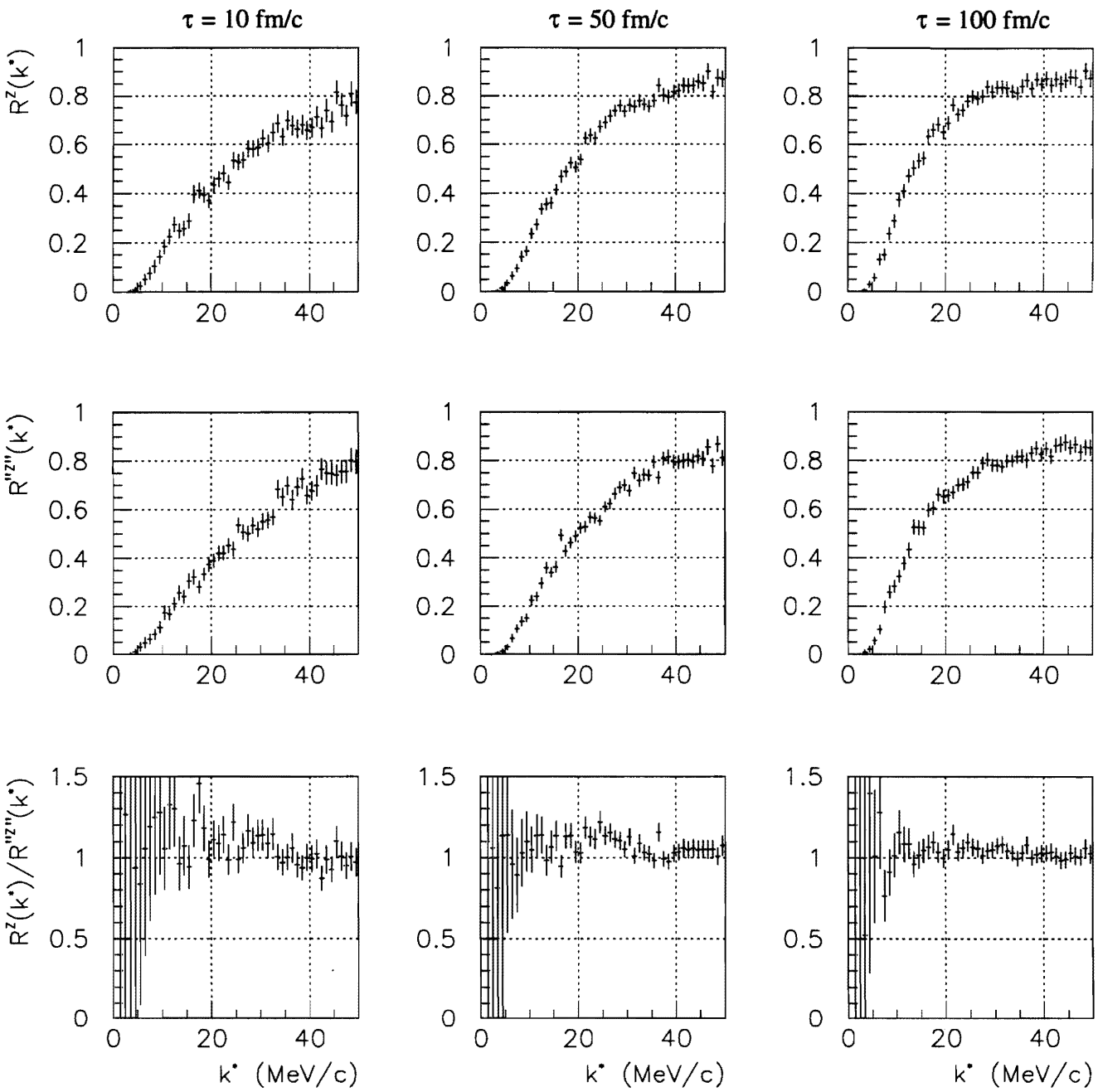


**Figure 2: The same as in Fig. 1 for the pd system.**

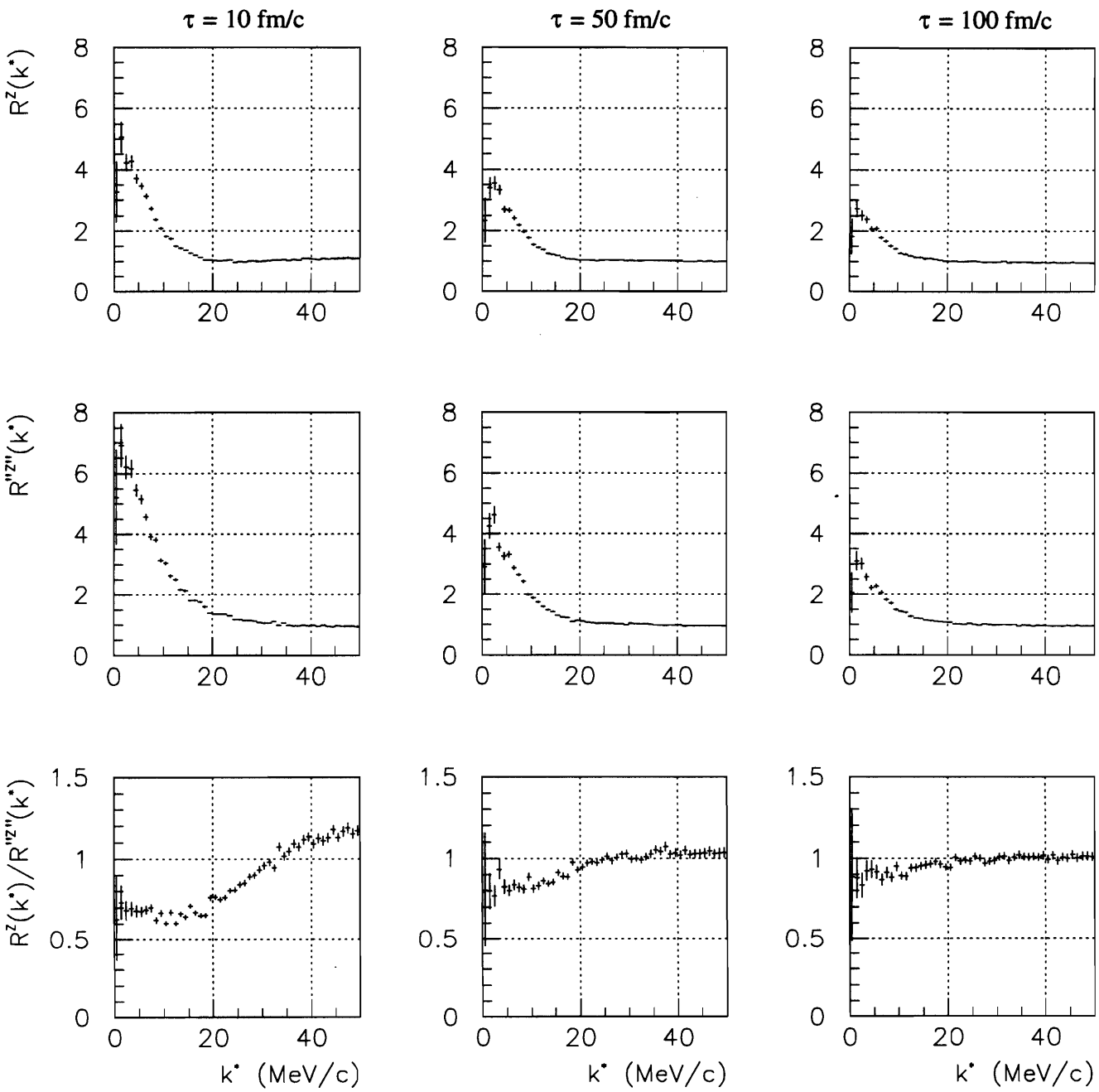


**Figure 3: The same as in Fig. 1 for the np system.**

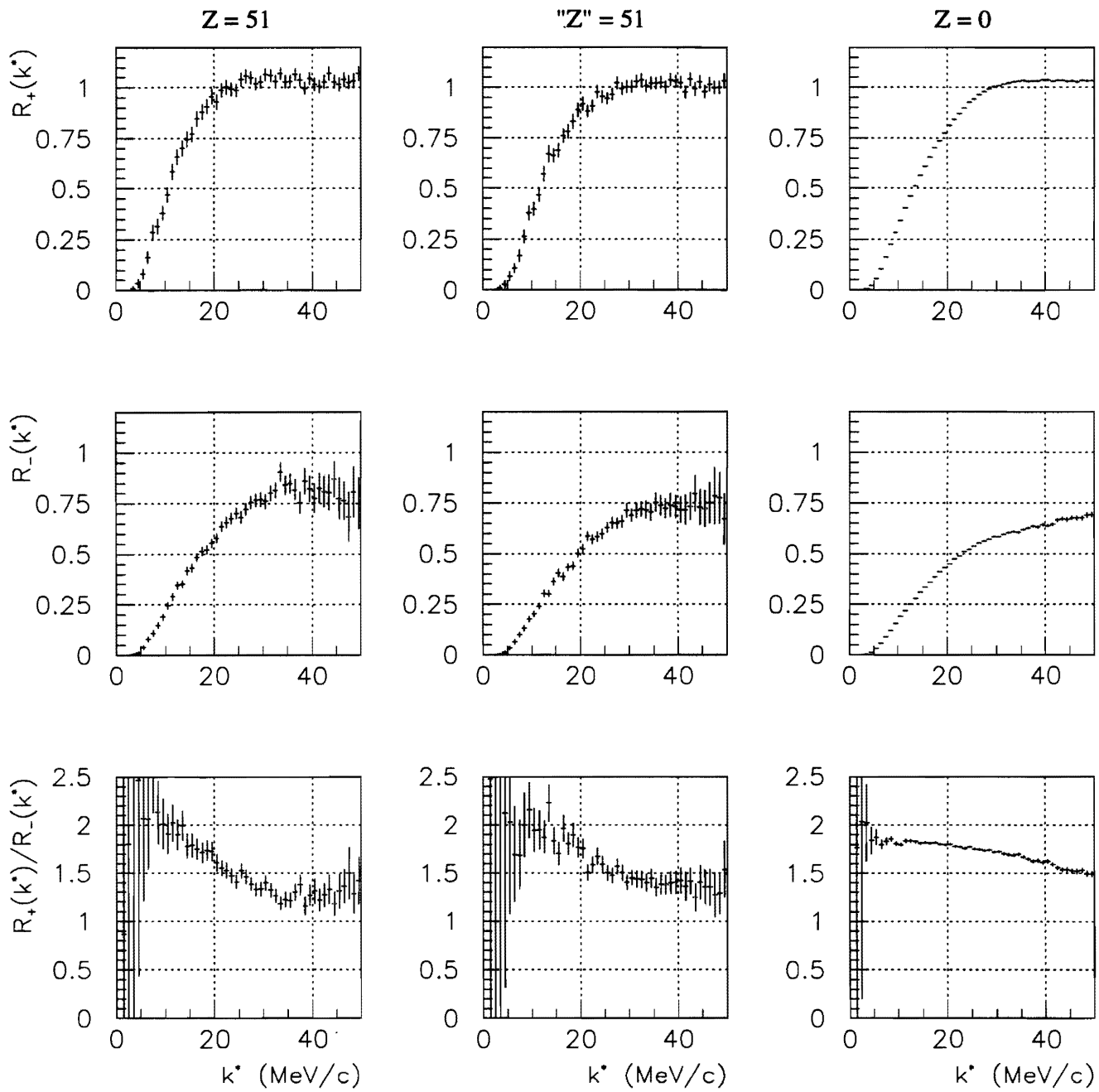


Figure 4: The pd correlation functions $R_+(\vec{v}\vec{k}^* \geq 0)$, $R_-(\vec{v}\vec{k}^* < 0)$ and their ratio calculated for the effective charge numbers of the emitting nucleus $Z = 51$, "$Z$" $= 51$ (only one-particle spectra are influenced by the nucleus charge) and $Z = 0$. The particles are assumed to be unpolarized and emitted isotropically according to Maxwellian law with the temperature 4 MeV. The distribution of space-time coordinates of the particle sources is approximated by Gaussian law with $\langle \frac{1}{3} r_i^2 \rangle^{1/2} \equiv r_0 = 3.5$ fm, $\langle t_i^2 - \langle t_i \rangle^2 \rangle^{1/2} \equiv \tau_0 = 50$ fm/c and $\langle t_p - t_d \rangle = -100$ fm/c.

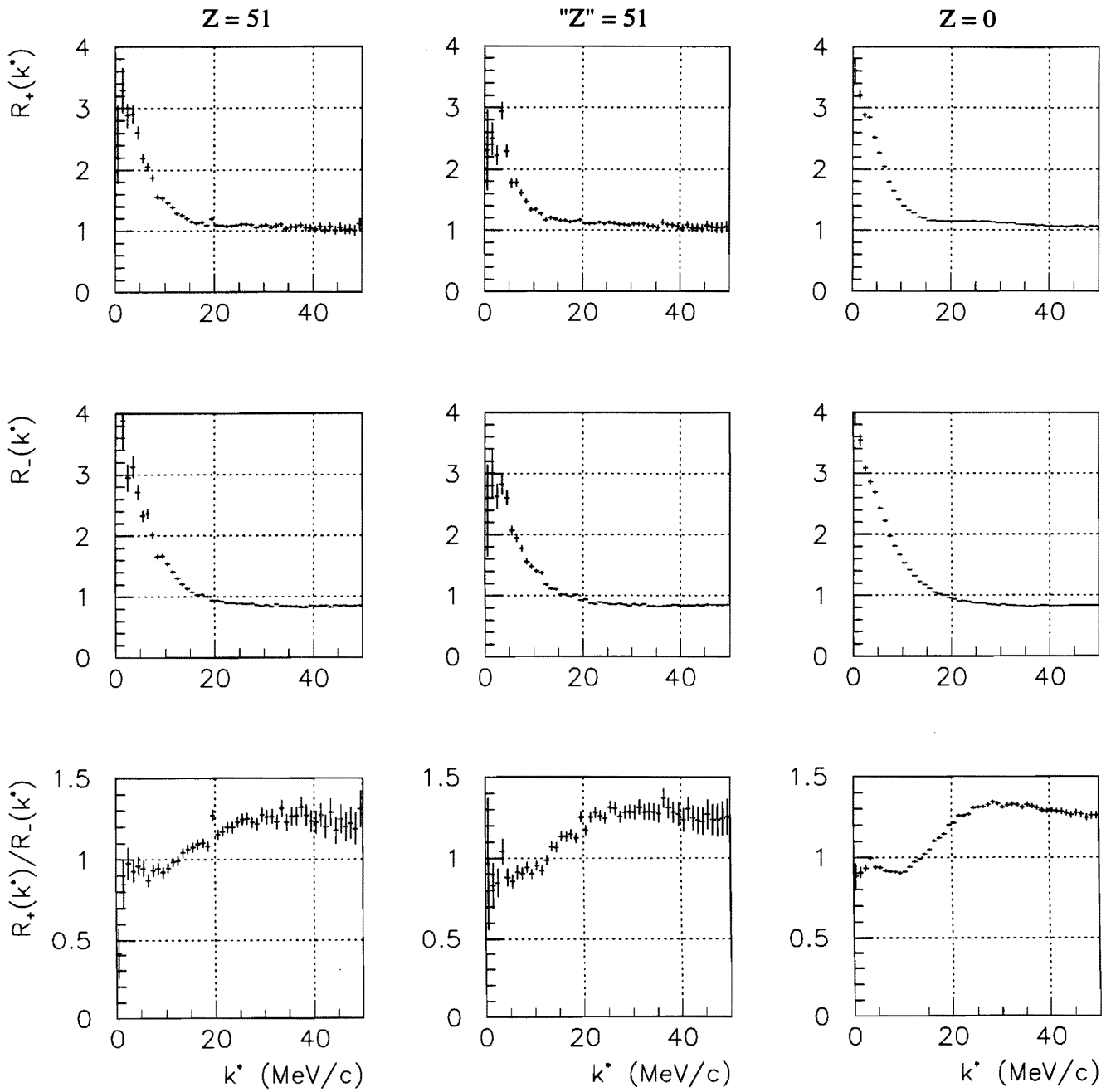


**Figure 5: The same as in Fig. 4 for the np system. In particular, $\langle t_{\mathrm{n}} - t_{\mathrm{p}} \rangle = -100$ fm/c.**