What is the effect of nucleus Coulomb field on particle correlations

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Abstract

A quantum approach to the particle correlations at small relative velocities in the presence of a strong Coulomb field is developed in the *adiabatic* approximation, allowing for a reliable determination of the space-time characteristics of particle production processes in heavy-ion reactions.

It is well-known (see, e.g., [1-3]) that the particle correlations at small relative velocities are sensitive to space-time characteristics of the production process due to the effects of quantum statistics (QS) and final state interactions (FSI). Usually, only the mutual interaction of two particles with a small relative velocity is taken into account. This is however questionable in the case of heavy-ion reactions when the particles are produced in a strong Coulomb field of residual nuclei. Sometimes the effect of the nucleus Coulomb field is taken into account by a classical shift of the particle momenta. Since this shift is typically several tens MeV/c, such a procedure strongly influences correlations of particles with different charge-to-mass ratio. Thus in ref. [4] it was argued that np correlations can be completely destroyed due to a strong Coulomb repulsion of the protons from the emitting nucleus. The "Coulomb shift" procedure however does not take into account that both the shift of particle momenta and the correlation effect develop simultaneously and cannot be separated in a simple way. At characteristic emission times τ of several hundreds fm/c or higher this complicated 3-body problem can be solved in the classical approximation. In fact, the classical trajectory calculations show that at $\tau > \frac{|a|}{\gamma v}$, where $|a|$, γ and v are Bohr radius, Lorentz factor and velocity of the particle pair, the effect of residual nucleus on particle correlations is of minor importance despite the one-particle spectra are substantially affected [5]. To extend the theoretical description to lower values of τ, we develop here a quantum approach in the *adiabatic* approximation, assuming the relative motion of the two particles much slower than their motion with respect to the Coulomb center.

Let us start with the usual assumption of sufficiently small density of the produced multiparticle system in the momentum space, such that the correlation of two particles with a small relative velocity is influenced by the effects of their mutual QS and FSI only. Define the correlation function $R(p_1, p_2)$ of the two particles as the ratio of their differential production cross section to the one which would be observed in the case of absence of the effects of QS and FSI. Introducing the normalized probability $W_S(x_1, p_1; x_2, p_2)$ of the emission of two noninteracting particles, with total spin S and 4-momenta p_1 and p_2, by the one-particle sources decaying at the space-time points $x_1 = \{t_1, \vec{r}_1\}$ and $x_2 = \{t_2, \vec{r}_2\}$, and assuming the momentum dependence of the emission probability inessential when varying the 4-momenta p_1 and p_2 by the amount characteristic for the correlation due

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to QS and FSI, we get the well-known result of Kopylov and Podgoretsky, modified by the substitution of the plane wave $e^{ip_1x_1+ip_2x_2}$ by the nonsymmetrized Bethe-Salpeter amplitudes in the continuous spectrum of the two-particle states $\psi^{S(+)}_{p_1p_2}(x_1,x_2)$ [3,6]:

$$\begin{aligned} R(p_1,p_2) &= \sum_S \int d^4x_1\, d^4x_2 W_S(x_1,p_1;x_2,p_2)|\psi^{S(+)}_{p_1p_2}(x_1,x_2)|^2 \\ &= \sum_S \rho_S(p_1,p_2)\langle|\psi^{S(+)}_{p_1p_2}(x)|^2\rangle_S. \end{aligned} \tag{1}$$

Here the mean $\langle f\rangle_S = \int d^4x_1\, d^4x_2 W_S f/\int d^4x_1\, d^4x_2 W_S$ is a function of p_1 and p_2, and

$$\rho_S(p_1,p_2) = \int d^4x_1\, d^4x_2 W_S(x_1,p_1;x_2,p_2), \quad \sum_S \rho_S(p_1,p_2) = 1, \tag{2}$$

describes the population of the total spin-S states, e.g., $\rho_S = (2S+1)/[(2s_1+1)(2s_2+1)]$ for unpolarized particles with spins s_1 and s_2. In the case of identical particles, the Bethe-Salpeter amplitude should be properly symmetrized:

$$\psi^{S(+)}_{p_1p_2}(x_1,x_2) \to [\psi^{S(+)}_{p_1p_2}(x_1,x_2) + (-1)^S\psi^{S(+)}_{p_2p_1}(x_1,x_2)]/\sqrt{2}. \tag{3}$$

The amplitude $\psi^{S(+)}_{p_1p_2}(x)$, depending only on the relative 4-coordinate $x \equiv \{t,\vec{r}\} = x_1 - x_2$, is obtained from the Bethe-Salpeter amplitude after separation of the two-particle c.m.s. motion: $\psi^{S(+)}_{p_1p_2}(x_1,x_2) = e^{iPX}\psi^{S(+)}_{p_1p_2}(x)$, where $X = [(p_1P)x_1 + (p_2P)x_2]/P^2$ is the c.m.s. 4-coordinate and $P \equiv 2p = p_1 + p_2$. At equal emission times $t^* = t_1^* - t_2^* = 0$ in the two-particle c.m.s. the amplitude $\psi^{S(+)}_{p_1p_2}(x)$ coincides with a stationary solution of the scattering problem $\psi^{S(+)}_{-\vec{k}^*}(\vec{r}^*)$, $\vec{k}^* = \vec{p}_1^* = -\vec{p}_2^*$, having at large r^* the asymptotics of superposition of the plane and diverging spherical waves. In fact, it can be shown [3] that the amplitude $\psi^{S(+)}_{p_1p_2}(x)$ can usually be substituted by this solution (*equal time* approximation).

We shall now consider particle correlations in the presence of a strong Coulomb field, e.g., in heavy-ion reactions. In this case the Bohr radius of the system *particle + residual nucleus* can be rather small (~ 0.6 fm for the system *proton + nucleus* with the charge number $Z = 51$) so that the Coulomb interaction in this system can be substantial even for relativistic particles. In such a situation, instead of the two-particle Bethe-Salpeter amplitude $\psi^{S(+)}_{p_1p_2}(x_1,x_2)$, the correlation function is determined by the amplitude $\psi^{S\{\alpha\}(+)}_{p_1p_2}(x_1,x_2)$ representing the solution of a complicated multibody problem, taking into account interaction between the two particles and also their interaction with the residual system described by the quantum numbers $\{\alpha\}$. For the correlation function, defined as the ratio of the two-particle production cross section to the one in the case of absence of the effects of QS and FSI between the two particles ($\psi \to \tilde{\psi}$), instead of Eq. (1), we have

$$R(p_1,p_2) = \frac{\sum_S \rho_S(p,p)\frac{1}{2}\langle|\psi^{S\{\alpha\}(+)}_{p_1p_2}(x_1,x_2) + (-1)^S\psi^{S\{\alpha\}(+)}_{p_2p_1}(x_1,x_2)|^2\rangle_S}{\sum_S \rho_S(p,p)\frac{1}{2}\langle|\tilde{\psi}^{S\{\alpha\}(+)}_{p_1p_2}(x_1,x_2)|^2 + |\tilde{\psi}^{S\{\alpha\}(+)}_{p_2p_1}(x_1,x_2)|^2\rangle_S} \tag{4}$$

for identical particles, and

$$R(p_1,p_2) = \frac{\sum_S \rho_S(p_1,p_2)\langle|\psi^{S\{\alpha\}(+)}_{p_1p_2}(x_1,x_2)|^2\rangle_S}{\sum_S \rho_S(p_1,p_2)\langle|\tilde{\psi}^{S\{\alpha\}(+)}_{p_1p_2}(x_1,x_2)|^2\rangle_S} \tag{5}$$

for nonidentical ones. Here $\langle f\rangle_S = \sum_{\{\alpha\}}\int d^4x_1\, d^4x_2 W_{S\{\alpha\}} f / \sum_{\{\alpha\}}\int d^4x_1\, d^4x_2 W_{S\{\alpha\}}$ and $\sum_{\{\alpha\}}$, taking into account the conservation laws, represents summation over the descrete quantum numbers and integration over the continuous ones. Again the smoothness condition is implied, i.e. the dependence of the emission probability of noninteracting particles $W_{S\{\alpha\}}(x_1, p_1; x_2, p_2)$ on the 4-momenta p_1 and p_2 is assumed inessential when varying them by the amount characteristic for QS and FSI.

It can be shown that at $\langle r^*\rangle > |a|$, e.g., in the case of particles evaporated by a compound nucleus with the characteristic emission time $\tau > \frac{|a|}{\gamma v}$, and at k^* not too close to the classical boundary $k_b^* = (|a|\langle r^*\rangle/2)^{-1/2}$, the conditions of the applicability of the classical approach are fulfiled and this problem can be solved with the help of the classical trajectory calculations [5]. To get rid of the limitations of such calculations, we develop below the quantum approach and solve the problem of the two-particle correlations in the field of a heavy Coulomb center.

We consider the emitting nucleus N sufficiently heavy so that the recoil effects can be neglected and the c.m. of the system *particle + nucleus* can be identified with the rest frame of the nucleus, which we situate at the origin. Neglecting further the change of the nucleus electric charge during the process of particle emission, we approximate it by an effective charge Ze. Finally, neglecting the interaction of the particles 1 and 2 with other emitted particles, the quantum numbers $\{\alpha\}$ of the *residual system* are reduced to the effective charge number Z.

Let us start with the hypothetical case of particles which interact with the charge Ze but their mutual interaction is "switched off". In such a situation we can treat the systems (1, N) and (2, N) independently so that the interaction with the Coulomb center just leads to the substitution of the spatial parts of the plane waves $e^{ip_ix_i}$ by the usual Coulomb wave functions:

$$\begin{aligned} e^{-i\vec{p}_i\vec{r}_i} &\rightarrow e^{-i\vec{p}_i\vec{r}_i}\Phi_{\vec{p}_i}^{z_iZ}(\vec{r}_i), \\ \Phi_{\vec{p}_i}^{z_iZ}(\vec{r}_i) &= e^{i\delta_i}\sqrt{A_c(|\vec{p}_i|a_i)}F[\frac{-i}{|\vec{p}_i|a_i}, 1, i(\vec{p}_i\vec{r}_i + |\vec{p}_i|r_i)], \end{aligned} \tag{6}$$

where $|a_i| = 1/(\omega_i z_i Z e^2)$ is the Bohr radius of the system (i, N) generalized to the relativistic case by the substitution $m_i \rightarrow \omega_i$, $\delta_i = \arg\Gamma[1 + i/(|\vec{p}_i|a_i)]$ is the Coulomb s-wave shift, $A_c(|\vec{p}_i|a_i) = |\Phi_{\vec{p}_i}^{z_iZ}(0)|^2$ is the Coulomb penetration factor:

$$A_c(y) = \frac{2\pi}{y}/[\exp(\frac{2\pi}{y}) - 1] \tag{7}$$

and $F(\alpha, 1, z)$ is the confluent hypergeometrical function ($F \rightarrow 1$ at $r_i \ll |a_i|$). Thus

$$\tilde{\psi}_{p_1p_2}^{SZ(+)}(x_1, x_2) = e^{ip_1x_1+ip_2x_2}\Phi_{\vec{p}_1}^{z_1Z}(\vec{r}_1)\Phi_{\vec{p}_2}^{z_2Z}(\vec{r}_2) = e^{iPX}e^{-i\vec{k}^*\vec{r}^*}\Phi_{\vec{p}_1}^{z_1Z}(\vec{r}_1)\Phi_{\vec{p}_2}^{z_2Z}(\vec{r}_2). \tag{8}$$

Note that a small contribution of spin-dependent electromagnetic forces is neglected in Eq. (8) so that $\tilde{\psi}^{SZ} \equiv \tilde{\psi}^Z$ is independent of the total spin S of the particle pair. We can thus put $\sum_S \rho_S\langle|\tilde{\psi}^{SZ}|^2\rangle_S = \langle|\tilde{\psi}^Z|^2\rangle$ in the denominators of Eqs. (4) and (5).

Let us now adiabatically "switch on" the interaction between particles 1 and 2, i.e. consider relative motion of the two particles at characteristic distances much slower compared with their motion with respect to the Coulomb center. It is clear that in such a case the plane wave $e^{-i\vec{k}^*\vec{r}^*}$ in Eq. (8) will be substituted by the Bethe-Salpeter amplitude $\psi_{p_1p_2}^{S(+)}(x)$ describing the relative motion of isolated interacting particles and that the

Coulomb functions $\Phi^{z_i Z}_{\vec{p}_i}(\vec{r}_i)$ will be only slightly modified. Neglecting this modification, we arrive at our main result:

$$\psi^{SZ(+)}_{p_1 p_2}(x_1, x_2) = e^{iPX} \psi^{S(+)}_{p_1 p_2}(x) \Phi^{z_1 Z}_{\vec{p}_1}(\vec{r}_1) \Phi^{z_2 Z}_{\vec{p}_2}(\vec{r}_2). \tag{9}$$

We will call it *adiabatic* approximation using a close analogy with the quantum theory of the motion of electrons and nuclei in a molecule. In the adiabatic approximation the fast motion of electrons in a molecule is separated from the slow motion of nuclei, leading to the corresponding factorization of the total wave function. In our problem, the fast motion is the motion of the two considered particles with respect to the Coulomb center, and the slow one is their relative motion.

Analyzing the distortions of the Coulomb potentials $V_i(r_i)$ between the particles and the Coulomb center due to the interaction between the two particles, we have found [7] the following sufficient applicability conditions of the *adiabatic* approximation in the region of small $k^* <\sim 1/r^*$ we are interested in:

$$1/|a| \ll |\vec{p}_i|, \qquad 1/[r^* + r^{*2}/|f^S(k^*)|] \ll |\vec{p}_i|. \tag{10}$$

These conditions simply reflect the fact that the Coulomb potentials $V_i(r_i)$ are distorted to the extent determined by the ratios of the momenta $1/|a|$ and $1/[r^* + r^{*2}/|f^S(k^*)|]$, characteristic for Coulomb and strong interaction between the two particles (at $k^* <\sim 1/r^*$), to the total particle momenta $|\vec{p}_i|$. As practically $|\vec{p}_i|$ is larger than the classical boundary value $p_{bi} = (|a_i|\langle r_i\rangle/2)^{-1/2}$, we can substitute it by this value to get still more sufficient conditions. In the case of two protons emitted by a typical nucleus with $Z = 51$ and $\langle r_i\rangle = 6$ fm ($p_{bi} = 154$ MeV/c, $a = 58$ fm, $f^0(0) = 7.8$ fm), these conditions are not very restrictive: $41 \gg 1$, $r^* \gg 1.1$ fm. In the case of heavier particles or fragments the first condition becomes stronger while the second one is weakened. E.g., for two deuterons ($p_{bi} = 218$ MeV/c): $32 \gg 1$, $r^* \gg 0.8$ fm.

For quantitative estimates we assume that, in the case of absence of FSI, the compound nucleus isotropically emits unpolarized particles with the energies distributed according to Maxwellian law with the temperature $T = 4$ MeV. The distribution of the 4-coordinates of the particle sources is approximated by Gaussian law

$$W_S(x_1, p_1; x_2, p_2) \propto \rho_S(p_1, p_2) \exp(-\frac{\vec{r}_1{}^2}{2r_0^2} - \frac{t_1^2}{2\tau_0^2}) \exp(-\frac{\vec{r}_2{}^2}{2r_0^2} - \frac{t_2^2}{2\tau_0^2}), \tag{11}$$

with the parameter $r_0 = 3.5$ fm roughly corresponding to the mass number of the emitting nucleus $A = 120$. Requiring the same dispersion of the difference $t = t_1 - t_2$ of the emission times for the Gaussian law and the exponential decay law, the parameter τ_0 can be identified with the emitter lifetime τ. For evaporation processes τ is typically several hundreds fm/c (leading to $\langle r^*\rangle \simeq v\tau$ of several tens fm). For the effective charge number of the residual nucleus we put $Z = 51$, which leads to about twice as large mean kinetic energy of the emitted protons or deuterons as compared with that of neutrons. The mean velocity of pp, pd or np pairs at small values of k^* is $\langle v\rangle \sim 0.15$. The above parameters roughly describe particle emission in the reaction $^{40}Ar +^{108} Ag$ at 44 MeV/nucleon [5].

Instead of the 6-dimensional correlation function $R(p_1, p_2)$ we calculate the 1-dimensional one $R^Z(k^*)$ corresponding to Eq. (4) or (5) with the nominator and denominator integrated over 1-particle spectra $d^3\sigma_i(\vec{p}_i)/d^3\vec{p}_i$. To separate pure effect of the nucleus Coulomb field on particle correlations, we compare the correlation function $R^Z(k^*)$ with

the one $R^{"Z"}(k^*)$ taking into account the effect of the nucleus Coulomb field on one-particle spectra but not on particle correlations. For instance, for nonidentical particles

$$R^{"Z"}(k^*) = \frac{\sum_S \int d^3\sigma_1(\vec{p}_1)\, d^3\sigma_2(\vec{p}_2)\delta(k^* - |\vec{p}_1^*|)\rho_S(p_1,p_2)\langle|\psi^{S(+)}_{p_1p_2}(x_1,x_2)|^2\rangle_S\langle|\Phi^{z_1Z}_{\vec{p}_1}(\vec{r}_1{}')\Phi^{z_2Z}_{\vec{p}_2}(\vec{r}_2{}')|^2\rangle}{\int d^3\sigma_1(\vec{p}_1)\, d^3\sigma_2(\vec{p}_2)\delta(k^* - |\vec{p}_1^*|)\langle|\Phi^{z_1Z}_{\vec{p}_1}(\vec{r}_1{}')\Phi^{z_2Z}_{\vec{p}_2}(\vec{r}_2{}')|^2\rangle}. \quad (12)$$

Our results confirm the classical trajectory calculations showing negligible effect of the residual nucleus on particle correlations at large emitter lifetimes $\tau > \frac{|a|}{\gamma v}$. In Fig. 1 we present the results of quantum calculations for np system in the region of smaller lifetimes where the classical approach is not valid. Our calculations show an increasing influence of the residual nucleus with decreasing τ leading to a suppression of the correlation. Though this suppression appeares much less important than expected from the simple "Coulomb shift" procedure [4]. For example, at $\tau = 100$ (10) fm/c the np correlation function is suppressed at small relative momenta by ~ 10 (35)% only. A noticeable suppression at $\tau < 50$ fm/c is obtained also for pd system ($\sim 15\%$) and even for pp system ($\sim 10\%$).

In conclusion we summarize our results. In the *adiabatic* approximation we have developed quantum approach to the correlations of particles produced with a small relative velocity in the field of a heavy Coulomb center. We have demonstrated that, contrary to some expectations, substantial correlations of particles with different charge-to-mass ratio survive even in a strong Coulomb field of residual nuclei in heavy-ion reactions. In these processes, the quantum *adiabatic* approach to the particle correlations at small relative velocities can be used for a quantitative study of the space-time picture of particle production.

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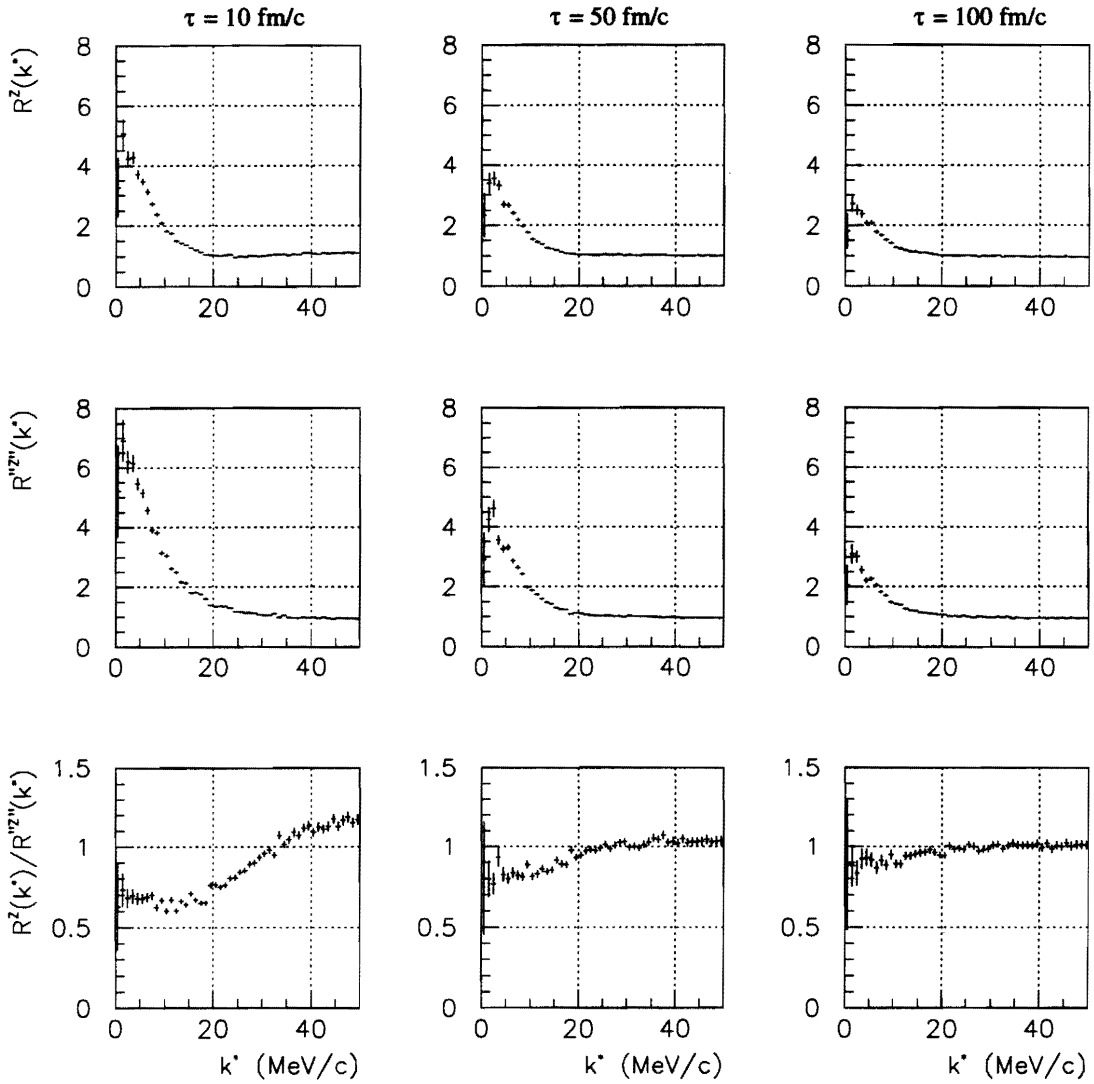

Figure 1: The np correlation functions R^Z and $R^{"Z"}$ (only one-particle spectra are influenced by the nucleus charge) and their ratio calculated for the effective charge number of the emitting nucleus $Z = 51$. The particles are assumed to be unpolarized and emitted isotropically according to Maxwellian law with the temperature 4 MeV. The distribution of space-time coordinates of the particle sources is approximated by Gaussian law in Eq. (11) with $r_0 = 3.5$ fm, $\tau_0 \equiv \tau = 10,\ 50$ and 100 fm/c.