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OPTIMIZATION OF WAVEFRONT SENSORS FOR THE HIGHEST ACCURACY AND SENSITIVITY

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ABSTRACT. The fundamental limit to accuracy and sensitivity of wavefront sensors for adaptive optics is set by photon noise. The mean square error of phase measurement is the sum of terms from spatial and temporal averaging of the measured quantities and the corresponding photon noise. We define two dimensionless quantities g and q that characterize the spatial and photon noise character of a wavefront sensor. The method of least squares is applied to determine the optimum scale of sampling, Dx, and integration time, Dt, for minimum error. We distinguish between two different domains, depending on the ratio Dx/r_0.

Once the limiting sensitivity and corresponding optimum sampling scales are known, the engineering goals for resolution and speed for the sensor detector and adaptive correction device follow. The acceptable detector noise, and the reference star magnitude or laser beacon power needed to reach a given wavefront accuracy are also determined.

Two wavefront sensors are analyzed, with the goals of optimized sensitivity for the distinct cases of stellar wavefronts and laser beacons. The first is a phase shifting stellar interferometer, designed to be used in a closed loop system operated in the domain $Dx < r_0$, at high Strehl ratio. For a bright star a residual wavefront error of 20 nm rms can be achieved, given good seeing and sampling scales of Dx = 40 mm and Dt = 400 μs. The second example is a Shack Hartmann sensor optimized for maximum sensitivity to laser beacons, in the domain $Dx > r_0$. It is designed for diffraction limited correction in the H and K infrared bands (rms wavefront error = 100 nm), and can make use of a commercially available continuous wave dye laser with a few watts power to create the sodium beacon. The optimum sampling scales in this case are 0.5 m and 1.2 msec. Fast CCDs with read noise < 3 e rms are required for both examples.

1. Measurement Errors

The image quality produced by adaptive optics depends on the accuracy and speed of both

the wavefront measurements and the corresponding correction applied through a deformable mirror. In this paper we will be concerned only with the fundamental measurement errors related to the character of the aberration produced by atmospheric turbulence, and photon noise in the wavefront being measured. Further system errors associated with the correction element can in principle be made as small as desired, and will not concern us here.

Adaptive correction is made on the basis of individual maps of the distorted wavefront. These cannot be obtained on an arbitrarily fine scale, or in an arbitrarily short time, when the photon flux to be measured is finite. In general a given measurement must have limited spatial resolution Dx, and be an average over some time interval Dt. Thus the fundamental errors of a wavefront sensor arise from the spatial and temporal averaging as well as from photon noise in the measurement. The properties of the atmospherically aberrated wavefront $\phi(x,t)$ are characterized statistically by phase structure functions:

$$<|\phi(x,t)-\phi(x+r,t)|^2> = 6.88\,(r/r_0)^{5/3}$$
$$<|\phi(x,t)-\phi(x,t+\Delta t)|^2> = (\Delta t/t_0)^{5/3} \qquad (1)$$

where r_0 is Fried's length measuring the scale of spatial aberration produced by atmospheric turbulence, and t_0 is the characteristic time scale of temporal evolution. From these functions the errors of wavefront averaging may be derived.

First, the error from the finite spatial resolution of the sensor pixels; no matter how accurately each pixel average is measured, a perfect description is lacking because the finer details of the instantaneous wavefront structure are missed. The general form of this error that follows from the above structure function is given by

$$\Delta_x = (gDx/r_0)^{5/3} \quad , \qquad (2)$$

where Δ_x is the mean square phase error in radians^2, averaged over the telescope pupil, Dx is the dimension of individually measured wavefront pixels or subapertures, g is a factor of order unity that relates the error in the full reconstructed wavefront to the errors of spatial averaging of the individual measurements. Its precise value depends on the character of the wavefront sensor. In section two below we will give a specific examples of this relationship derived for interferometric and Shack Hartmann sensors.

Second, finite temporal resolution leads to errors, because when correction is applied the wavefront has changed from when it was measured. The time constant for temporal decorrelation is given by $t_0 = 0.314\ r_0/v_w$, where v_w, is a turbulence-weighted wind speed for all the atmospheric layers. The mean-square decorrelation error averaged over the telescope pupil, Δ_t depends on the degree of adaptive error averaged over the wavefront,

since the residual uncompensated fitting error is unaffected by temporal decorrelation. As a consequence the dependence of Δ_t on Dt, the measurement time interval, is quadratic for small Dt, turning to a 5/3-power at larger Dt. The turnover to the 5/3-law occurs at larger time intervals for low-order correction $(Dx \gg r_0)^2$, and for full correction (corresponding to $Dx \leq r_0$) the quadratic dependence holds only for $Dt/t_0 \ll r_0/D$. In this paper we will be concerned mainly with high-order correction, for which we are either in the transition between Dt^2 and $Dt^{5/3}$ dependence, or we are in the region where 5/3 has taken over. In either case, use of the 5/3 law represents an upper bound to the temporal error. Thus, we express the mean-square error as averaged over the pupil as

$$\Delta_t = \left(Dt/t_0\right)^{5/3} = \left(wDt/r_0\right)^{5/3} \quad . \tag{3}$$

Here, $w = r_0/t_0 = 3.2\ v_w$ is an effective speed relating spatial and temporal errors.

Now we consider the error arising because of photon noise in the light intensities measured by the wavefront sensor. For any given detector the propagation of the photon noise to the final wavefront mean square phase error Δ can be computed. The form of this error is the same for wavefront sensors based on local gradient or direct phase measurements. It changes depending on the measurement pixel size Dx. Thus we find

$$\begin{aligned} \Delta_p &= 1/FqDtDx^2 && \left(Dx < r_0\ ,\ case\ A\right) \\ \Delta_p &= 1/FqDtr_0^2 && \left(Dx > r_0\ ,\ case\ B\right) \end{aligned} \tag{4}$$

Here F is the flux of photons at the sensor per unit area of the telescope pupil and q is a parameter that characterizes photon sensitivity. It includes both the quantum efficiency of the detector and a factor of order unity that relates the photon noise in the local measurement to the corresponding phase error in the wavefront reconstructed over the telescope pupil. The denominators in these expressions are approximately equal to the number of photons detected in measurement time Dt from area Dx^2 or r_0^2 respectively. The two different cases A and B result from the transition from diffraction to seeing limit over the sampling aperture Dx. Thus accuracy of a Shack Hartmann sensor depends on the size of the individual images, which are inversely proportional to Dx when $Dx < r_0$, but is constant at the seeing limit for $Dx > r_0$. (In fact, a more careful analysis will show that the break may be closer to $Dx \approx 2 - 3\ r_0$ when an unresolved stellar wavefront is measured, because the instantaneous image is still diffraction limited). For an interferometer, the break may be interpreted as a reduction in fringe visibility when $Dx > r_0$.

A fourth error source will be considered later, namely the read noise of the imaging array in the wavefront sensor. It is not fundamental, and for the moment we will consider it to be negligible.

If the pixel size and integration time of a sensing cycle are fixed, the above equations can

be used directly to calculate noise performance. However, to find the ultimate accuracy limited only by photon noise, we shall suppose the wavefront sensor is configured with Dx and Dt adjusted so as to minimize the total wavefront error. In this limit, the accuracy of a sensor is characterized by just the two dimensionless parameters g and q. The atmospheric aberration is given by the spatial and temporal parameters r_0 and w, and the wavefront intensity by the photon flux F at the sensor. The optimum values of Dx and Dt and the corresponding mean square residual error may be determined by the least squares method as follows.

For case A $Dx < r_0$, we have

$$\Delta^A = \left(gDx/r_0\right)^{5/3} + \left(wDt/r_0\right)^{5/3} + 1/\left(FqDtDx^2\right) \quad . \tag{5}$$

Setting $\partial\Delta^A/\partial Dx = 0$ and $\partial\Delta^A/\partial Dt = 0$ to find the minimum value, we obtain

$$Dx^A_{opt} = 2^{12/35}\left(5/3\ Fq/w\right)^{-3/14}\ g^{-4/7}\ r_0^{5/14} \quad , \tag{6}$$

and

$$Dt^A_{opt} = 2^{-9/35}\left(5/3\ Fq\right)^{-3/14}\ g^{3/7}\ w^{-11/14}\ r_0^{5/14} \quad , \tag{7}$$

and hence

$$\Delta^A_{\min} = 7/6\ 2^{11/7}\left(5/3\ Fqr_0^3/wg^2\right)^{-5/14} \quad . \tag{8}$$

Equation 8 is appropriate for the limiting performance of adaptive optics systems that aim at very accurate wavefront correction, with small scale sampling of the wavefront and high Strehl.

For case B, $x > r_0$, which is appropriate for large scale sampling of a laser beacon, we have

$$\Delta^B = \left(gDx/r_0\right)^{5/3} + \left(wDt/r_0\right)^{5/3} + 1/\left(FqDtr_0^2\right) \quad . \tag{9}$$

Setting $\partial\Delta^B/\partial Dt = 0$, we obtain for the optimum value of Dt

$$Dt^B_{opt} = \left(5/3\ Fq\right)^{-3/8}\ w^{-5/8}\ r_0^{-1/8} \quad , \tag{10}$$

when

$$\Delta^B_{t\ \min} = \left(gDx/r_0\right)^{5/3} + 8/3\ \left(5/3\ Fqr_0^3/w\right)^{-5/8} \quad . \tag{11}$$

The error from spatial undersampling is independently set. A reasonable goal would be to keep it at no more than the second term in equation (11), in which case we find

$$Dx^{B}_{opt} = (8/3)^{3/5} \left(5/3 Fq/w\right)^{-3/8} g^{-1} r_0^{-1/8} \quad , \tag{12}$$

and for the minimum total error

$$\Delta^{B}_{\min} = 16/3 \left(5/3 \; Fqr_0^3/w\right)^{-5/8} \quad . \tag{13}$$

A number of interesting results about the limiting performance of adaptive optics may be deduced from these expressions. Consider first the dependence of residual error on atmospheric conditions and photon flux. It is sometimes argued that adaptive optics will be able to correct for seeing, reducing the need to build on sites with the best seeing. However, we see that the mean square residual error after correction does depend on r_0, as approximately r_0^{-1} for small subapertures, and as r_0^{-2} for larger, and thus better seeing will result in better correction. Put another way, for the same measurement accuracy, the photon flux required for given precision of correction in both cases depends on r_0^{-3}.

The gain in wavefront accuracy with increased flux is weak when high accuracy measurements are to be made, (case A). Thus from the -5/14 power in equation 8 we see that a ten fold increase in flux allows only a 2.3 fold improvement in mean square error, or 1.5 in the rms error. The dependence on wind speed and detector quantum efficiency is similarly weak. Laser beacon sensing for the correction of infrared images falls in the domain of case B. The mean square error depends on flux and hence laser output as the -5/8 power. There is no dependence of the residual error on telescope aperture seen in the above equations. This is correct for errors in the wavefront being sensed. However, when a laser beacon wavefront is sensed with a large telescope, it will be different from a coincident stellar wavefront because of focus anisoplanitism.

The possibility of making sensing aberrations at one wavelength to correct the wavefront at another was proposed by Woolf.[3] Neglecting chromatic effects in the aberration, the mean square residual error Δ_o at the observation wavelength λ_o is given by

$$\Delta_0 = \Delta_w(\lambda_w/\lambda_0)^2 \quad , \tag{14}$$

where Δ_w is the error given by equations 8 and 13 above at the sensor wavelength, λ_w.

In these equations r_0 is the value appropriate to the measurement wavelength, and it can be related to r_0 at the standard wavelength of 0.5 μm by $r_0(\lambda_w) = r_0(0.5)(\lambda_w/0.5)^{6/5}$. Suppose we are using a continuum stellar source to measure the wavefront aberration, and are free to choose the optimum wavelength for the minimum absolute wavefront error.

Then we find from equation 8 that in case A the absolute error depends on $(\lambda_w^2/ F(\lambda_w))^{-5/14}$. Thus the wavelength should be chosen such that $\lambda_w^2/F(\lambda_w)$ is minimized, favoring the use of shorter wavelengths. For case B, the corresponding result is that $1/(F\ (\lambda_w)^5\ \lambda_w^2)$ should be minimized, favoring the use of longer wavelengths.

The flux needed to reach given phase error or Strehl ratio at different observed wavelengths λ_o is also of interest. For case B, appropriate for laser correction of the infrared, the flux or laser power must be proportional to $\lambda_o^{-3.2}$. It is for this reason that laser correction is much easier at longer wavelengths. For case A the dependence is even steeper, as $\lambda_o^{-5.2}$.

Finally, we use the above analysis to obtain expressions for the number of photons detected by each pixel of the imaging detector of the optimized wavefront sensor, and hence the limit of acceptable detector noise. The total number of photons N detected to sense either phase over one pixel on an interferometer, or to measure an individual star centroid in a Shack Hartmann sensor is given by

$$N = F\eta Dx^2 Dt \quad , \tag{15}$$

where η is the effective quantum efficiency (transmission plus detector quantum efficiency). Substituting the optimum values of Dx and Dt derived above, we find

$$\begin{aligned} N &= 14/5\ \eta/q\ 1/\Delta^A_{\min} \qquad (case\ A) \quad , \\ N &= 2^{14/5}/5\ \eta/q\ 1/g^2\ (\Delta^B_{\min})^{-1/5} \qquad (case\ B) \quad . \end{aligned} \tag{16}$$

While specific values of N require knowledge of η/q and g for individual sensor implementations, we can make some general observations. A striking feature is that the optimum number of photon detections in an optimized system is very small, and independent of detector quantum efficiency; η/q and g are both independent of detector efficiency. Since 4 - 8 pixels reads are involved in the measurement of phase or star position, the number of detected photons to be recorded by individual pixels is around N/4 ~ N/8. N itself is already a small number, as η/q and g are close to unity. For case B the dependence of N on wavefront error is extremely weak, and in any well optimized system N will always be small. For case A, N depends inversely on $\Delta^A_{\min}$, and will become larger when accurate measurements with very high Strehl are undertaken.

From the above analysis a critical evaluation can be made of alternative forms of wavefront sensor for different applications. First, on the assumption that adequately low noise array detectors will be available, we can characterize the performance of any wavefront sensing detector by the parameters q and g. Then for a given strength of atmospheric aberration and photon flux, the optimal spatial and temporal sampling scales

Dx and Dt can be derived. The system component requirements follow, including speed, resolution and noise. In the next section we give examples of this type of analysis for two wavefront sensors.

2. Two Specific Wavefront Sensors and Their Errors

A sensor optimized for case A, small scale sampling and high accuracy, is described by Angel [4] and a prototype has been operated at the telescope by D'nardo Colucci [5]. In brief, the sensor is based on phase shifting interferometry, as used for accurate metrology in the optics shop. The aberrated wavefront enters a Mach Zehnder interferometer ahead of the focus. The aberration in one beam is largely removed by passing the star image through a spatial filter consisting of a diaphragm the size of the diffraction limited image core. After the beams are recombined, interference with the aberrated beam results in white light fringes showing the aberration at images of the pupil formed at both interferometer outputs. A piezo driven mirror is used to introduce stepped phase changes between the two beams, with the fringe patterns recorded for each phase setting. Phase maps reconstructed from the fringe images are used in a closed loop adaptive system to maintain high Strehl ratio, obtained initially by a bootstrap method. The reference beam is then strong and the arrangement gives high photon efficiency with low systematic errors. From the analysis given by Angel, we find the appropriate parameters for this device, incorporating a CCD of high quantum efficiency are $g=0.4$, $\eta=0.4$ and $q=0.2$.

This sensor is of special interest, since we argue that the g and q values represent the limiting performance possible with any sensor. Thus when viewing a bright star in good seeing, the projected optimized wavefront errors represent as perfect correction as is possible with astronomical adaptive optics. For the case of a wavefront of a star with I magnitude = 2, the photon flux $F = 10^9$ photons/m^2/sec. In good seeing conditions, r_0 in the I band is 0.3 m and we take $v_w = 10$ m/sec. Then from equation 8 the optimized mean square error is 0.02 radians^2, or 20 nm rms wavefront error. Thus adaptive optics can achieve image quality with Strehl near 98% in the near infrared. The optimal values of Dx and Dt corresponding to this limit from equations 6 and 7 are 40 mm and 400 microseconds respectively. From equation 16, case A, we find for the above values of η/q and g that N=270. This number of detected photons is divided between 8 pixel reads for a phase measurement. Thus the average signal per read is 34 e, with an rms error = 6 e. Detector read noise < 3 electrons rms is needed to avoid increasing the photon noise.

As a second case, we consider the sensor optimized for case B, $Dx > r_0$, described by Sandler et al.[1] This is a Shack Hartmann sensor, designed to measure the wavefront from a sodium beacon. The mean square wavefront error at the laser wavelength λ_w to photon noise is given [1] as

$$\Delta_p = 9.7\omega^2/N\ (1+4n^2/N) \tag{17}$$

where ω is the angular width of the laser beacon in units of the subaperture diffraction limit, λ_w/Dx. The numerical factor was evaluated for a specific reconstruction geometry corresponding to large subaperture. N is the number of photons detected per subaperture in time Dt, given by equation 15. We shall suppose that an optimally transmitted laser beacon has width λ_w/r_0, and thus $\omega = Dx/r_0$. Combining these results, and neglecting for the moment detector noise, we obtain

$$\Delta_p = 9.7\ /\ F\eta Dtr_o^2 \quad . \tag{18}$$

Comparing equation 18 with equation 4 (case B) we see that $\eta/q = 9.7$. We adopt η = 0.5 as an achievable goal for overall transmission, including losses in the wavefront sensor and detector quantum efficiency. It then follows that for this sensor q=0.05. The second parameter g that characterizes the spatial averaging error is for a Shack Hartmann sensor given approximately by g=0.3. The sensor parameters q and g for this system, 0.05 and 0.3, may be compared with those of the phase shifting interferometer, 0.2 and 0.4.

With the aid of equation 13 above, we can now calculate the laser power P needed to achieve given accuracy. Suppose for example that correction to 100 nm rms is required to give diffraction limited imaging in the near infrared at 1.6 and 2.2 microns wavelength. The photon flux returned from a sodium beacon is given by F = 300,000 P.[1] We find from the above that the laser power required is 4 watts, assuming wavefront aberration parameters $r_0(0.59\mu m) = 0.2$ m and $v_w = 20$ m/sec. The corresponding optimum sampling values are $Dt_{opt} = 1.2$ msec and $Dx_{opt} = 0.5$ m. We note that the laser power requirement is critically dependent on the laser beacon having angular size limited by seeing. Explicit in the determination of q above is the assumption of an instantaneous angular extent λ/r_0, in this example 0.6 arcsec. If the laser beacon image size were increased, for example to 2 arcsec in diameter, the laser power requirement would be increased as the square, in this case to 44 watts. In order to obtain a beam of the smallest possible diameter, the laser and its projection optics should be diffraction limited, and the expanded beam should have a beam diameter of $\sim 2r_0$ [6]. In practice, it maybe advantageous for a large telescope to use Dx and Dt larger than the optimum found here, if focus anisoplanitism becomes the dominant error and is not corrected through the use of multiple laser beacons.

The number of photons detected per measurement from equation 16, case B, is N=150. It is proposed that the motion of each subaperture image be measured with a quad cell from 4 pixels on a CCD, as described by Wittman et al. [7] The average number of photon events recorded per pixel read in this case is thus 38, with an error of 6e rms. Thus again the detector noise n of ≤ 3 e rms is needed to avoid significant noise increase. The

technical challenge in this case is easier, because the detector frame rate is now 800 Hz, instead of 10 kHz for the phase shifting interferometer, and the number of pixels is far less, only a few thousand.

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