STEWARD-1035

Fermilab Library
0 1160 0010026 7

17

PREPRINTS

of the

STEWARD OBSERVATORY

THE UNIVERSITY OF ARIZONA
TUCSON, ARIZONA 85721 U.S.A.

No. 1035

ON THE STABILITY OF NEUTRON-STAR WINDS

Fulvio Melia[1],
Department of Physics and Steward Observatory
University of Arizona
melia@nucleus.physics.arizona.edu

Mark Wardle, and Peter Heimberg
Department of Physics and Astronomy
Northwestern University

[1]Presidential Young Investigator and Alfred P. Sloan Fellow.

Submitted to the Editor of the Astrophysical Journal.

ABSTRACT

It is believed that radiatively driven mass outflows associated with type I cosmic X-ray bursts and fast X-ray transients are relevant to the precursors and apparent radius variations sometimes seen during the course of these events. Previous studies of neutron-star winds have assumed that the transition from subsonic to supersonic flow is smooth and describable by the steady conservation equations, but this assumption may be inappropriate if the wind is unstable. Earlier investigations into the question of whether such an instability arises were restricted to the case of a spherically symmetric isothermal flow (e.g., the solar wind), and concluded that (at least in this context) no growing perturbations can exist. In neutron-star systems, however, the winds are optically thick and driven predominantly by gradients in the radiation pressure. As such, it is not obvious that the previous analyses are relevant to these sources, given that optically thick winds have two distinct length scales, i.e., the length scale (corresponding to the radiation pressure scale height) over which the gas is accelerated, and the much smaller gas pressure scale height. Our goal in the present paper is to examine the stability of neutron-star winds in more detail than has previously been attempted, and to determine whether or not the introduction of temperature gradients can affect the stability of the flow. We find that the dispersion relation has three solutions, corresponding to outwardly and inwardly propagating sound waves, and a strongly attenuated entropy wave. Interestingly, the waves are always damped. We also show that when the transonic flow is adiabatic, the global perturbation analysis employed for isothermal winds carries over into this domain with similar conclusions, i.e., there is no indication of growing perturbations. We conclude that the stability of transonic flows is not affected by relaxing the isothermal constraint. Our results suggest that a consideration of earlier wind models as outer boundary conditions for calculations of the thermonuclear flash models are warranted, and that the use of X-ray burst sources as "standard candles" (with an Eddington-limited total luminosity) to determine the distance to the galactic center is valid.

Subject headings: instabilities—radiative transfer—stars: binaries—stars: neutron—stars: Winds—X-rays: Bursts

I. INTRODUCTION

Thermonuclear flashes in the surface layers of accreting neutron stars can release sufficient energy to produce mass ejection in the form of a wind. Observationally, the occurrence of such outflows during the peaks of many X-ray bursts is supported by at least three lines of evidence: (1) The peak luminosities of X-ray bursts are generally of the order of the Eddington limit (e.g., Joss and Rappaport 1984; Melia and Joss 1986 and references cited therein), suggesting that much higher luminosities are inhibited by the deposition of radiative energy into the kinetic and gravitational potential energy of a wind. (2) The apparent radius of the neutron star ($R_a \equiv [L/4\pi\sigma T_{col}^4]^{1/2}$, where L and T_{col} are the bolometric luminosity and color temperature, respectively) is seen to increase from ~ 10 km to a value several times larger during the peaks of the most luminous X-ray bursts from some sources. (3) Some "fast X-ray transients," which are very similar to type I bursts except that they last up to $\sim 10^3$ s, display distinct precursors. Lewin, Vacca, and Basinska (1984) and Tawara, Kii, and Hayakawa (1984) have suggested that the interval between the precursor and the main event corresponds to the emission of a strong wind by the neutron star. This phenomenon thus seems to represent a more extreme case of the apparent radius variations seen in some type I X-ray bursts.

Following initial discussions by Wallace, Woosley, and Weaver (1982) and Paczyński (1983), models for quasi-static winds from neutron stars have been developed by Ebisuzaki, Hanawa, and Sugimoto (1983), Kato (1983), Melia and Joss (1984), Quinn and Paczyński (1985), Paczyński and Prószyński (1986), Joss and Melia (1987), and Abramowicz, Ellis, and Lanza (1990) with the intent of formulating a set of outer boundary conditions for the hydrostatic interior of the neutron star. The assumption of quasi-static flow, permitting a time-independent treatment of the wind, was justified by the fact that all of the above phenomena are observed to occur on time scales, τ_{obs}, of seconds or longer. Thus, a quasi-static wind solution seemed to be self-consistent because $\tau_d < \tau_{flow} < \tau_{char} < \tau_{obs}$, where τ_d ($\approx 10^{-4}$ s) is the dynamical time scale at the surface of the neutron star, τ_{flow}

is the flow time required for a fluid element to travel from the base of the wind, where hydrostatic equilibrium is nearly maintained, to a level where the wind has essentially reached its terminal velocity, and $\tau_{char} \equiv M_w/\dot{M}_w$ is the time required for approximately time-independent conditions to be established in a wind with mass outflow rate $\dot{M}_w$ and total mass M_w above its base.

The structure of these solutions is qualitatively similar to that of models for the solar wind, as first discussed by Parker (1960). At the bottom of the outflow, the fluid velocities are very subsonic, and hydrostatic equilibrium is an excellent approximation. On the other hand, the terminal velocity of the fluid in the outer zone is highly supersonic. Between these two extremes, the solutions pass through a singular point, corresponding to the level r_c in the neutron-star atmosphere where the fluid velocity v is equal to the isothermal sound speed $c_s \equiv (R_g T/\mu)^{1/2}$ (in terms of the kinetic temperature T and the molecular weight μ of the particulate gas; Melia 1987). For simplicity, the transition from subsonic to supersonic flow has always been assumed smooth and describable by the "usual" steady conservation equations (see § II below). However, this assumption is inappropriate if the outwardly expanding flow is unstable against the amplification of acoustic waves in the region where $v \sim c_s$.

The question of whether or not such an instability arises has already received some attention, particularly in the context of the solar wind. For example, Parker (1966), Carovillano and King (1966), and Jockers (1968) studied the perturbation problem of a spherically symmetric isothermal flow and concluded that (at least under these conditions) no growing perturbations can exist (see also Cannon and Thomas 1977). The situation with neutron stars is distinctly different, however, in that the winds in these systems are optically thick and radiation dominated. As we shall see, this added feature introduces a new length scale into the problem that renders the earlier analyses inappropriate for our discussion. Our goal in the present paper is to examine optically thick winds in more detail than has previously been attempted and, in so doing, to determine if growing fluctuations can affect the overall structure of the expanding atmosphere.

II. BASIC EQUATIONS

Our analysis will be based on the formalism described by Joss and Melia (1987). This assumes a weakly magnetized, nonrotating neutron star with nominal parameters: mass $M = 1.41\ M_\odot$ and (preflash) radius $R = 10$ km. In addition, the wind is assumed to be spherically symmetric and its interaction with the surrounding accretion disk is neglected (cf. Melia and Joss 1985). The chemical abundances (by mass) are taken to be $(X, Y, Z) = (0.69, 0.30, 0.01)$.

The relevant fluid equations are the continuity equation, the equations of momentum and energy conservation, and the equation of heat transport, in Eulerian form, under the constraints of spherically symmetric flow. In our present analysis, we will allow for time-dependence, so that our stability criteria include the outward spatial amplification of any given disturbance. For the equation of continuity, we have

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial (r^2 \rho v)}{\partial r} = 0\,, \tag{1}$$

where $\rho(r,t)$ and $v(r,t)$ are the fluid density and velocity, respectively, as functions of radius r and time t. The equation of momentum conservation takes the form

$$\rho\frac{\partial v}{\partial t} + \rho v\frac{\partial v}{\partial r} = -\frac{\partial P}{\partial r} - \frac{GM\rho}{r^2}\,, \tag{2}$$

where $P(r,t)$ is the total pressure (i.e., the sum of gas pressure P_g and radiation pressure P_r).

Neglecting convective heat transport (see Joss and Melia 1987), the equation of conservation of energy can be written as

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial r} + P\left[\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) + v\frac{\partial}{\partial r}\left(\frac{1}{\rho}\right)\right] = -\frac{1}{r^2\rho}\frac{\partial (r^2 F_R)}{\partial r}\,, \tag{3}$$

where $u(r,t) = u_g + u_r$ is the total energy per unit mass (u_g and u_r being the internal energy per unit mass of the particulate gas and radiation, respectively). For simplicity, we will restrict our attention to wind solutions whose critical point lies at sufficient optical depth in the wind that LTE is a good approximation in the transonic region. As such, the

gas and radiation temperatures are equal, and the thermodynamic properties of the fluid are essentially those of an ideal gas plus blackbody radiation:

$$P_g = \frac{R_g}{\mu}\rho T, \qquad u_g = \frac{3}{2}\frac{R_g}{\mu}T, \qquad P_r = \frac{a}{3}T^4, \qquad u_r = \frac{a}{\rho}T^4 , \tag{4}$$

where R_g is the gas constant, and $T = T(r,t)$. In terms of the parameter $\beta \equiv P_g/P$, we may also write

$$P = \frac{R_g \rho T}{\mu\beta}, \qquad u = \frac{R_g T}{\mu}\left(\frac{3}{\beta} - \frac{3}{2}\right) . \tag{5}$$

Moreover, the appropriate equation of heat transport is the radiative diffusion equation:

$$F_R = -\frac{ac}{3\kappa\rho}\frac{dT^4}{dr} , \tag{6}$$

where κ is the opacity coefficient.

III. THE TRANSONIC REGIME

When quasi-static conditions are assumed, the problem can be reduced to the solution of two simultaneous, first-order, ordinary, differential equations in two unknowns (together with the constitutive relations: the equation of state, the internal energies of the gas and radiation, and the radiative opacities). Taking v and T to be the dependent variables and r to be the independent variable, we can combine equations (1), (2) and (4) (with $\partial/\partial t \to 0$) to obtain one of the two requisite relations (Melia 1987):

$$\frac{dv}{dr} = \frac{N}{D} , \tag{7}$$

where

$$N \equiv \frac{2R_g}{\mu}\frac{T}{r} - \frac{GM}{r^2} - \left(\frac{R_g}{\mu} + \frac{4a}{3}\frac{T^3}{\rho}\right)\frac{dT}{dr} , \tag{8}$$

and

$$D \equiv v - \frac{R_g}{\mu}\frac{T}{v} . \tag{9}$$

The other relation can easily be obtained from equations (1), (2), (3), and (6).

The critical point corresponds to the level in the flow where $D = 0$ (i.e., where $v = c_s$, as indicated above). In order for the solution to be regular at this point, it is necessary that N also vanish there, which will then allow a smooth transition from subcritical to supercritical flow.

IV. SHORT-WAVELENGTH FLUCTUATIONS IN AN ISOTHERMAL FLOW (APPLICATION TO THE SOLAR WIND)

In his discussion of spherically symmetric isothermal perturbations of the solar wind, Jockers (1968) treated the problem with mixed initial and boundary-value conditions, in which the perturbations of velocity and density are specified everywhere at the initial time, and in addition the velocity perturbation at the base of the corona is known for all subsequent times. The solutions to the singular eigenvalue problem for the amplitudes of the velocity and pressure perturbations can be represented as a linear superposition of normal modes and are temporally stable and bounded at the critical point.

As a prelude to our study of a radiation dominated flow, we will in this section reconsider the perturbation of a spherically symmetric, isothermal wind in the short wavelength limit (i.e., $\lambda \ll r$, or $1/kr \ll 1$, where $k \equiv 2\pi/\lambda$), for which a local analysis is adequate. Although an analysis in this wavelength regime may not be sufficient to establish the global stability of an isothermal wind whose length scale is set by the gas pressure scale height $l_g \equiv P_g/(\partial P_g/\partial r)$ ($\sim r$), it will nonetheless be instructive to compare this result with that of the optically thick flow in which $l_g \ll l_r \sim r$, where $l_r \equiv P_r/(\partial P_r/\partial r)$ is the corresponding radiation pressure scale height and $\lambda \sim l_g$.

When the flow is isothermal, we may dispense with the energy equation (3), and simplify the momentum conservation equation as follows:

$$\rho\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r}\right) = -c_s^2\frac{\partial \rho}{\partial r} - \frac{GM\rho}{r^2} . \tag{12}$$

The stationary solutions to equations (1) and (12) are well known, and may be written in the form

$$4\pi r^2 \rho v = \dot{M} , \tag{13}$$

and

$$\left(\frac{v}{c_s}\right)^2 - \ln\left(\frac{v}{c_s}\right)^2 = 4\ln\left(\frac{r}{r_c}\right) + \frac{4r_c}{r} - K , \tag{14}$$

where $\dot{M}$ is the total mass flux (which is constant for a stationary flow) and $K = -3$ for a transonic solution (Parker 1966). To determine the manifestation of any time-dependent disturbance or instability in the transonic regime, we now linearize the conservation equations (1) and (12), and consider perturbations about the time-independent zeroth order quantities ρ_0 and v_0 (corresponding to the solutions described in equations [13] and [14]) by writing

$$\rho(r,t) = \rho_0(r) + \rho_1(r,t)\,, \tag{15}$$

and

$$v(r,t) = v_0(r) + v_1(r,t)\,. \tag{16}$$

This yields

$$\frac{1}{v_0}\frac{\partial}{\partial t}\left(\frac{\rho_1}{\rho_0}\right) + \frac{\partial}{\partial r}\left(\frac{v_1}{v_0}\right) + \frac{\partial}{\partial r}\left(\frac{\rho_1}{\rho_0}\right) = 0\,, \tag{17}$$

and

$$\frac{\partial v_1}{\partial t} + \frac{\partial}{\partial r}(v_0\, v_1) + c_s^2\frac{\partial}{\partial r}\left(\frac{\rho_1}{\rho_0}\right) = 0\,. \tag{18}$$

We now seek wave-like solutions with the usual normal mode expansion of ρ_1 and v_1, viz.

$$\rho_1 = \rho_{10}\,\frac{\exp(ikr - i\omega t)}{r}\,, \tag{19}$$

and

$$v_1 = v_{10}\,\frac{\exp(ikr - i\omega t)}{r}\,, \tag{20}$$

where ρ_{10} and v_{10} are constants, and we consider both propagation and amplification of the disturbance by admitting real and imaginary parts of ω. With this assumed form of ρ_1 and v_1, equations (17) and (18) reduce to the dispersion relation

$$\xi^2 + \xi\left[\frac{-2\,v_0}{c_s} + \frac{i}{k}\,\frac{2}{c_s}\,\frac{\partial v_0}{\partial r}\right] + \left[\frac{v_0^2}{c_s^2}\left(1 + \frac{1}{k^2r^2}\right)\right.$$
$$\left. -\frac{1}{k^2c_s^2}\left(\frac{\partial v_0}{\partial r}\right)^2 - \left(1 + \frac{1}{k^2r^2}\right) - \frac{2}{k^2rv_0}\,\frac{\partial v_0}{\partial r} - \frac{1}{k^2v_0^2}\left(\frac{\partial v_0}{\partial r}\right)^2 - \frac{i2v_0}{kc_s^2}\,\frac{\partial v_0}{\partial r}\right] = 0\,, \tag{21}$$

where $\xi \equiv \omega/k\,c_s$. Thus, to order $(1/kr)$ (with $1/kr \ll 1$), we recover the two solutions

$$\xi_1 = \frac{v_0}{c_s} + 1 - \frac{i}{kc_s}\,\frac{\partial v_0}{\partial r}\,, \tag{22}$$

and

$$\xi_2 = \frac{v_0}{c_s} - 1 - \frac{i}{kc_s}\,\frac{\partial v_0}{\partial r}\,, \tag{23}$$

which clearly correspond to the outwardly and inwardly propagating waves, respectively. Since $\partial v_0/\partial r > 0$ (see equation [14] above), it is evident that all wave fluctuations in a spherically symmetric, isothermal flow (with $kr \gg 1$) are damped, in agreement with the more general analysis of earlier workers (e.g., Jockers 1968).

V. SHORT-WAVELENGTH FLUCTUATIONS IN A RADIATION DOMINATED FLOW (APPLICATION TO NEUTRON-STAR WINDS)

The essential difference between the isothermal flow discussed above and one in which the dominant driving mechanism is due to gradients in the radiation pressure is that in the latter situation, the length scale (corresponding to the radiation pressure scale height $l_r \equiv P_r/(\partial P_r/\partial r)$) over which the gas is accelerated can be significantly larger than the gas pressure scale height $l_g \equiv P_g/(\partial P_g/\partial r)$. It is not difficult to show from equation (4) that

$$\frac{l_g}{l_r} = \frac{4\tau}{\tau + 2 + \eta}\,, \tag{24}$$

where

$$\tau \equiv -\frac{\partial \ln T}{\partial \ln r}\,, \tag{25}$$

and

$$\eta \equiv \frac{\partial \ln v}{\partial \ln r}\,. \tag{26}$$

It is clear from equation (2) that a steady transonic flow has $\eta \sim 4\tau/\Pi$, where $\Pi \equiv P_g/P_r$, and so $l_g \ll l_r$ when $\Pi \ll 1$, i.e., when the wind is radiation dominated. On the other hand, the "acceleration length scale" $l_{acc} \equiv v/(\partial v/\partial r)$ is given approximately by the expression $l_{acc} \approx P_g/(\partial P_r/\partial r)$ near the critical point r_c, where $v \approx c_s$. That is,

$l_{acc} \approx l_g\,\Pi\,(\eta+2+\tau)/4\tau$, which in our case reduces to $l_{acc} \approx l_g$ since $\eta \gg \tau$. As such, the dominant fluctuation modes associated with the plasma are expected to have wavelengths $\lambda \sim l_{acc} \approx l_g \ll l_r \sim r$. The isothermal analysis discussed above suggests that growing perturbations on the larger length scale (i.e., $l_r \sim r$) are unlikely (see also §VI below).

If we now return to the set of equations (1), (2), and (3), and consider perturbations in the temperature T as well as ρ and $\mathbf{v}$, so that in addition to equations (15) and (16) we have

$$T(r,t) = T_0(r) + T_1(r,t)\,, \tag{27}$$

we then obtain the modified momentum conservation equation (cf. eq. [18]):

$$\rho_0 \frac{\partial v_1}{\partial t} + \rho_0 v_0 \frac{\partial v_1}{\partial r} + \rho_0 \frac{\partial v_0}{\partial r} v_1 + \left(v_0 \frac{\partial v_0}{\partial r} + \frac{\partial}{\partial r}\frac{\partial P_0}{\partial \rho_0} + \frac{GM}{r^2}\right)\rho_1$$

$$+ \left(\frac{\partial P_0}{\partial \rho_0}\right)\frac{\partial \rho_1}{\partial r} + \left(\frac{\partial P_0}{\partial T_0}\right)\frac{\partial T_1}{\partial r} + \left(\frac{\partial}{\partial r}\frac{\partial P_0}{\partial T_0}\right) T_1 = 0\,, \tag{28}$$

where P_0 is given by eq. (5) in terms of the steady state quantities, and

$$P_1 \equiv \frac{\partial P_0}{\partial T_0} T_1 + \frac{\partial P_0}{\partial \rho_0}\rho_1\,. \tag{29}$$

In a similar fashion, the conservation of energy equation becomes

$$\left[\frac{\partial u_0}{\partial T_0}\right]\frac{\partial T_1}{\partial t} + \left[\frac{\partial u_0}{\partial \rho_0} - \frac{P_0}{\rho_0^2}\right]\frac{\partial \rho_1}{\partial t} + \left[v_0 \frac{\partial u_0}{\partial T_0} - \frac{c}{\kappa\rho_0}\,\mathrm{Div}\left(\frac{1}{\rho_0}\frac{\partial P_{r0}}{\partial T_0}\right)\right.$$

$$\left. - \frac{c}{\kappa\rho_0^2}\left(\frac{\partial}{\partial r}\frac{\partial P_{r0}}{\partial T_0}\right)\right]\frac{\partial T_1}{\partial r} + \left[v_0 \frac{\partial u_0}{\partial \rho_0} - \frac{v_0 P_0}{\rho_0^2} + \frac{c}{\kappa\rho_0^3}\frac{\partial P_{r0}}{\partial r}\right]\frac{\partial \rho_1}{\partial r}$$

$$+ \left[v_0\left(\frac{\partial}{\partial r}\frac{\partial u_0}{\partial T_0}\right) - \frac{v_0}{\rho_0^2}\frac{\partial \rho_0}{\partial r}\left(\frac{\partial P_0}{\partial T_0}\right) - \frac{c}{\kappa\rho_0}\,\mathrm{Div}\left(\frac{1}{\rho_0}\frac{\partial}{\partial r}\frac{\partial P_{r0}}{\partial T_0}\right)\right] T_1$$

$$+ \left[v_0\left(\frac{\partial}{\partial r}\frac{\partial u_0}{\partial \rho_0}\right) - \frac{v_0}{\rho_0^2}\frac{\partial \rho_0}{\partial r}\left(\frac{\partial P_0}{\partial \rho_0}\right) - \frac{1}{\rho_0^2}\,\mathrm{Div}F_{R0} + \frac{c}{\kappa\rho_0}\,\mathrm{Div}\left(\frac{1}{\rho_0^2}\frac{\partial P_{r0}}{\partial r}\right)\right]\rho_1$$

$$+ \left[-\frac{c}{\kappa}\frac{1}{\rho_0^2}\frac{\partial P_{r0}}{\partial T_0}\right]\frac{\partial^2 T_1}{\partial r^2} + \left[\frac{\partial u_0}{\partial r} - \frac{P_0}{\rho_0^2}\frac{\partial \rho_0}{\partial r}\right] v_1 = 0\,, \tag{30}$$

where $\mathrm{Div}A \equiv (1/r^2)\,\partial(r^2 A)/\partial r$ for any variable A, and F_{R0} is given by eq. (6) in terms of the steady state quantities.

As in the case of the isothermal wind, we now seek wave-like solutions with the normal mode expansion of ρ_1 and v_1 given by equations (19) and (20), and the additional mode expansion for the temperature:

$$T_1 = T_{10}\,\frac{\exp(ikr - i\omega t)}{r}\,. \tag{31}$$

After a long algebraic calculation, it is possible to reduce the three equations (17), (28), and (30) to the cubic dispersion relation

$$\xi^3 + \xi^2\left[(\gamma-3)\mathcal{M} + i\frac{1}{3}\alpha\,kr\,\mathcal{M}\right] + \xi\left[-\frac{1}{9}\epsilon\mathcal{M}^2 - i\frac{2}{3}\alpha\,kr\,\mathcal{M}^2\right]$$

$$+ \left[\frac{1}{9}\epsilon\mathcal{M}^3 + i(\mathcal{M}^2-1)\frac{1}{3}\alpha\,kr\,\mathcal{M}\right] = 0\,, \tag{32}$$

which in keeping with our assumed small wavelength, has terms only to highest order in $1/kr$ ($\ll 1$). Here

$$c_{s0} \equiv \left(\frac{R_g T_0}{\mu}\right)^{1/2}\,, \tag{33}$$

$$\mathcal{M} \equiv \frac{v_0}{c_{s0}}\,, \tag{34}$$

$$\alpha \equiv \frac{c}{\kappa\rho_0 v_0 r}\,, \tag{35}$$

$$\epsilon \equiv \frac{4u_{r0}}{v_0^2}\,, \tag{36}$$

and

$$\gamma \equiv \frac{2}{3}\alpha + \tau + \frac{1}{3}\alpha\eta + \frac{1}{3}\alpha\tau\,. \tag{37}$$

This equation has the following three solutions:

$$\xi_1 = \mathcal{M} + 1 - i\frac{2}{\mathcal{M}\alpha\,kr\,\Pi_0}\,, \tag{38}$$

$$\xi_2 = \mathcal{M} - 1 - i\frac{2}{\mathcal{M}\alpha\,kr\,\Pi_0}\,, \tag{39}$$

and

$$\xi_3 = \mathcal{M}(1-\gamma) - i\frac{1}{3}\alpha\, kr\, \mathcal{M}\,. \tag{40}$$

As before (see eqns [22] and [23]), the first two solutions correspond to the outwardly and inwardly propagating waves, respectively. In this case, a rapidly decaying additional mode ξ_3—the entropy wave—appears because the temperature is also allowed to vary. Since both ξ_1 and ξ_2 always occupy the bottom half of the complex plane, this interesting result (together with equations [22] and [23]) shows that all short wavelength fluctuations are damped, regardless of whether or not the flow is isothermal.

VI. A GLOBAL ANALYSIS OF ADIABATIC WINDS

We can carry this discussion on the stability of optically-thick winds one step further in the case where the transonic flow is approximately adiabatic. In neutron-star winds, for example, the radiative flux

$$F_A \equiv \frac{u_r \dot{M}}{4\pi r^2} \tag{41}$$

advected across the sonic point is sometimes greater than the corresponding flux F_R diffusing through the plasma (Joss and Melia 1987). Under these conditions, any deviations from an adiabatic flow should be sufficiently small to allow the use of analytic solutions (analogous to [14]) of the adiabatic flow equations in a global stability analysis (see also Aikawa 1979 for the case $\gamma = 1$ and Bondi 1952).

Putting

$$P = P_0 \left(\frac{\rho}{\rho_0}\right)^{\gamma}\,, \tag{42}$$

and

$$c_a \equiv \left(\frac{\gamma P}{\rho}\right)^{1/2}\,, \tag{43}$$

the momentum equation (2) reduces to

$$(c_a^2 - v^2)\frac{d\ln v}{d\ln r} = \frac{GM}{r} - 2c_a^2\,, \tag{44}$$

or in terms of the velocity

$$v_a = \left(\frac{GM}{2r_a}\right)^{1/2} \tag{45}$$

at the adiabatic sonic point r_a (where $c_a = v_a$),

$$\left[\left(\frac{c_a}{v_a}\right)^2 - \left(\frac{v}{v_a}\right)^2\right]\frac{d\ln(v/v_a)}{d\ln(r/r_a)} = 2\left[\frac{r_a}{r} - \left(\frac{c_a}{v_a}\right)^2\right]\,. \tag{46}$$

It is not difficult to show that the stationary solutions to this equation are

$$\frac{1}{2}\left(\frac{v}{v_a}\right)^2 + \frac{1}{\gamma-1}\left(\frac{c_a}{v_a}\right)^2 - \frac{2r_a}{r} = K_a \qquad (\gamma \neq 1)\,, \tag{47}$$

and

$$\frac{1}{2}\left(\frac{v}{v_a}\right)^2 - \ln\left(\frac{v}{v_a}\right) - 2\left[\ln\left(\frac{r}{r_a}\right) + \frac{r_a}{r}\right] = -\frac{3}{2} \qquad (\gamma = 1)\,, \tag{48}$$

where K_a is a constant determined by the physical conditions at $r = r_a$. That is,

$$K_a = \frac{5-3\gamma}{2(\gamma-1)}\,. \tag{49}$$

The global perturbation analysis may now be treated as a mixed initial and boundary-value problem along the lines set forth by Jockers (1968) in his treatment of an isothermal wind. Following this earlier work, we now linearize equations (1) and (2) for an adiabatic flow (equation [24]), getting

$$(c_a^2 - v_0^2)\frac{dz}{dr} + \left(\beta + 2\frac{dv_0}{dr}\right)v_1 + \left[\frac{dc_a^2}{dr} - \beta v_0\right]z = 0\,, \tag{50}$$

and

$$(c_a^2 - v_0^2)\frac{d}{dr}\left(\frac{v_1}{v_0}\right) - \left(\beta + 2\frac{dv_0}{dr}\right)v_1 + \left[\frac{\beta c_a^2}{v_0} - \frac{dc_a^2}{dr}\right]z = 0\,, \tag{51}$$

where $z \equiv \rho_1/\rho_0$, and $\partial z_1/\partial t = \beta z$, $\partial v_1/\partial t = \beta v_1$. If we now define the new variables

$$\zeta \equiv z\exp\left\{\int_{r_0}^{r}\left(\frac{dc_a^2}{dr} - \beta v_0\right)(c_a^2 - v_0^2)^{-1}\,dr\right\} \tag{52}$$

and

$$\eta \equiv -\frac{1}{\beta}\frac{v_1}{v_0}(c_a^2 - v_0^2)\exp\left\{\int_{r_0}^{r}\left(\frac{dc_a^2}{dr} - \beta v_0\right)(c_a^2 - v_0^2)^{-1}\,dr\right\}\,, \tag{53}$$

where r_0 is the base radius of the wind, then these two equations reduce to

$$\frac{d\zeta}{dr} - \frac{v_0\beta(\beta + 2dv_0/dr)\eta}{c_a^2 - v_0^2} = 0 \tag{54}$$

and

$$\frac{d\eta}{dr}\omega_0 - \zeta = 0 \,, \tag{55}$$

where

$$\omega_0 \equiv \left[\frac{c_a^2}{v_0} - \frac{1}{\beta}\frac{dc_a^2}{dr}\right]^{-1} \,. \tag{56}$$

As in the isothermal case, the proof of stability consists in using equations (54) and (55) to show that if $Re(\beta) \geq 0$, this eigenvalue β is real, and then to use this fact to demonstrate that there are no eigenvalues with $Re(\beta) > 0$.

Constructing the integral

$$\int_{r_0}^{r_s}\left(\frac{d\zeta}{dr}\eta^* - \frac{d\zeta^*}{dr}\eta\right)dr = \frac{(\beta - \beta^*)}{|\beta|^2}\int_{r_0}^{r_s}|\zeta|^2\frac{dc_a^2}{dr}\,dr \,, \tag{57}$$

we can evaluate the l.h.s. explicitly to give

$$(\beta - \beta^*)\left[\int_{r_0}^{r_s}\frac{2v_0|\eta|^2(dv_0/dr + Re(\beta))}{(c_a^2 - v_0^2)^2}\,dr - \frac{1}{|\beta|^2}\int_{r_0}^{r_s}|\zeta|^2\frac{dc_a^2}{dr}\,dr\right] = 0 \,. \tag{58}$$

Both terms are positive definite as long as $dc_a^2/dr < 0$, so that under this condition, $\beta = \beta^*$ as required. The second part of the proof (that $Re(\beta) < 0$) follows exactly the procedure of the isothermal case since $\omega_0 > 0$ when β is real and positive. As such, an adiabatic, spherically symmetric wind is stable against spherically symmetric initial and boundary perturbations, where the boundary perturbations are applied at the inner boundary of the outflow.

VII. CONCLUSIONS

We have reconsidered the question of stability in a spherically symmetric, transonic flow, though this time with an application to optically-thick, radiation dominated neutron-star winds. In these systems, the mass outflux results from gradients in the radiation pressure over a distance l_r that is very large compared to the gas pressure scale height l_g.

Our analysis has shown that the introduction of a temperature gradient does not affect the stability of the outflow against transonic, wave-like fluctuations with $\lambda \sim l_g \ll l_r \sim r$, since both the outwardly and inwardly propagating acoustic waves are always damped. We have also demonstrated that when the transonic flow is adiabatic (a good approximation for these radiation-dominated expanding coronae), the wind is globally stable against initial and boundary perturbations.

We therefore conclude that inasmuch as the earlier wind calculations were used to determine the outer boundary conditions for the nuclear burning fuel at the base of the wind, those constraints are valid. In addition, our calculations would seem to indicate that the use of X-ray burst sources as "standard candles" (with $L_{max} \approx L_{ed}$) to determine the distance to the galactic center is probably warranted, since the wind can in fact regulate L by reducing its value below L_{ed} while the particle mechanical energy and number flux correspondingly increase, as indicated by the earlier studies of quasi-static, neutron-star winds.

This research was supported in part by NSF grant PHY 88-57218, NASA grant NAGW-2380, and the Alfred P. Sloan Foundation. Most of the numerical calculations were carried out at the National Center for Supercomputing Applications in Urbana-Champaign.

REFERENCES

Abramowicz, M.A., Ellis, G.F.R., and Lanza, A. 1990, *Ap. J.*, in press.

Aikawa, T. 1979, *Astrop. Sp. Science*, **66**, 277.

Bondi, H. 1952, *M.N.R.A.S.*, **112**, 195.

Cannon, C.J., and Thomas, R.N. 1977, *Ap. J.*, **211**, 910.

Carovillano, R.L., and King, J.H. 1966, *Ap. J.*, **145**, 426.

Ebisuzaki, T., Hanawa, T., and Sugimoto, D. 1983, *Pub. Astr. Soc. Japan*, **35**, 17.

Jockers, K. 1968, *Solar Physics*, **3**, 603.

Joss, P.C., and Melia, F. 1987, *Ap. J.*, **312**, 700.

Joss, P.C., and Rappaport, S.A. 1984, *Ann. Rev. Astr. Ap.*, **22**, 537.

Kato, M. 1983, *Pub. Astr. Soc. Japan*, **35**, 33.

Landau, L.D., and Lifshitz, E.M. 1959, *Fluid Mechanics*, (London: Pergamon Press), p. 256.

Lewin, W.H.G., Vacca, W.D., and Basinska, E. M. 1984, *Ap. J. (Letters)*, **277**, L17.

Melia, F. 1987, *J. Comp. Physics*, **74**, 488.

Melia, F., and Joss, P.C. 1984, in *AIP Conf. Proc., No. 115, High-Energy Transients in Astrophysics*, ed. S.E. Woosely (New York: AIP), p. 330.

Melia, F., and Joss, P.C. 1985, *Ap. J.*, **295**, 98.

Melia, F., and Joss, P.C. 1986, in *IAU Conference Proceedings No. 89, "Radiation Hydrodynamics In Stars and Compact Objects"*, Eds. D. Mihalas and K.-H. Winkler, Springer Verlag, New York, p. 283.

Paczyński, B. 1983, *Ap. J.*, **267**, 315.

Paczyński, B., and Prószyński, M. 1986, *Ap. J.*, **302**, 519.

Parker, E.N. 1960, *Ap. J.*, **132**, 175.

Parker, E.N. 1966, *Ap. J.*, **143**, 32.

Quinn, T., and Paczyński, B. 1985, *Ap. J.*, **289**, 634.

Tawara, Y., Kii, T., and Hayakawa, S. 1984, in *AIP Conf. Proc., No. 115, High-Energy Transients in Astrophysics*, ed. S.E. Woosley (New York: AIP), p. 257.

Wallace, R.K., Woosley, S.E., and Weaver, T.A. 1982, *Ap. J.*, **258**, 696.

AUTHORS' ADDRESS:

Peter Heimberg and Mark Wardle: Department of Physics and Astronomy,
Northwestern University, Evanston, IL 60208

Fulvio Melia: Department of Physics and Steward Observatory,
University of Arizona, Tucson, AZ 85721
melia@nucleus.physics.arizona.edu