Fermilab

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Masayoshi Wake

Direct Calculation of Magnetic Field from The Measured Strand Positions

The behavior of the magnetic field components in SSC magnets have not been completely understood. The comparison between the strand positions in the cut up of the magnet and the measured magnetic field is expected to give a good study on the field behavior. This note describes the calculation method of harmonic components from the measured strand positions.

Field Expression

It is convenient to use complex coordinate system to handle 2-dimensional magnetic field, in which the position (x, y) is represented by:

$$z = x + iy \tag{1}$$

The magnetic field vector in this coordinate system is also expressed as

$$H = H_x + iH_y \tag{2}$$

The definition of harmonic components in our familiar notation

$$B_y + iB_x = B_0 \sum_{n=0}^{\infty} (b_n + ia_n)(x + iy)^n \tag{3}$$

is represented as

$$iH^* = \sum_{n=0}^{\infty} C_n (\frac{z}{\rho})^n \tag{4}$$

where,

$$iC_n^* = \frac{B_0}{\mu_0}(a_n + ib_n) \tag{5}$$

and ρ is the normalization radius. We use $1cm$ for it *i.e.* $\rho = 0.01$ in MKSA unit.

*Distribution: R.Bossert, J.Carson, S.Delchamps, T.Jaffery, W.Koska, M.Lamm, G.Pewitt, J.Strait, J.Jayakumar, J.Seuntjens

Single Filament Field

The magnetic field for current I at the origin gives

$$H_\theta = \frac{I}{2\pi r}, \qquad H_r = 0 \tag{6}$$

therefore

$$H_x = \frac{-Iy}{2\pi(x^2+y^2)} \quad , \quad H_y = \frac{Ix}{2\pi(x^2+y^2)}. \tag{7}$$

Then the complex notation becomes,

$$iH^*(z) = \frac{I(x-iy)}{2\pi(x^2+y^2)} = \frac{I}{2\pi z} \tag{8}$$

By shifting origin, the field at z_0 when the current is at z is

$$iH^*(z_0) = \frac{I}{2\pi(z_0 - z)} \tag{9}$$

When a current is placed in a circular hole of iron with infinite permeability, the field in the iron free space is equivalent to the superposition of the current produced field and mirror image produced field. Since the mirror image of the current is created at the position z' where $z^*z' = R^2$. The image field is

$$\frac{I}{2\pi[z - (\frac{R^2}{z_0^*})]} \tag{10}$$

then the total field becomes

$$iH^*(z_0) = \frac{I}{2\pi}[\frac{1}{z_0 - z} + \frac{1}{z_0 - (\frac{R^2}{z^*})}] \tag{11}$$

Multipole Expansion

The Taylor expansion of this equation gives,

$$iH^*(z_0) = \frac{I}{2\pi}\sum_{n=0}^{\infty}[z^{-(n+1)} + (\frac{z^*}{R^2})^{(n+1)}]z_0^n \tag{12}$$

With the scaling at normalization radius,

$$iH^*(z_0) = \frac{I}{2\pi\rho}\sum_{n=0}^{\infty}[(\frac{z}{\rho})^{-(n+1)} + (\frac{z^*}{\rho R^2})^{(n+1)}](\frac{z_0}{\rho})^n \tag{13}$$

Comparison of this equation with equation(4), one can find

$$C_n = \frac{I}{2\pi\rho}[(\frac{z}{\rho})^{-(n+1)} + (\frac{z^*}{\rho R^2})^{(n+1)}] \tag{14}$$

When there are many strands (6024 strands in SSC dipole magnets), harmonic components are the sum of the contributions from strand current. The total harmonic component is given by

$$C_n = \sum_k \frac{I_k}{2\pi\rho}[(\frac{z_k}{\rho})^{-(n+1)} + (\frac{z_k^*}{\rho R^2})^{(n+1)}] \tag{15}$$

where I_k and z_k are the current and position of the kth strand. In our normal notation using b_n and a_n,

$$b_n = \frac{1}{B_0}\sum_k \frac{\mu_0 I_k}{2\pi\rho} Re[(\frac{z_k}{\rho})^{-(n+1)} + (\frac{z_k^*}{\rho R^2})^{(n+1)}] \tag{16}$$

$$a_n = \frac{1}{B_0}\sum_k \frac{\mu_0 I_k}{2\pi\rho} Im[(\frac{z_k}{\rho})^{-(n+1)} + (\frac{z_k^*}{\rho R^2})^{(n+1)}] \tag{17}$$

Computer Program

Since we use $[T]$ for field, $[cm]$ for length, $[kA]$ for current and $1cm$ for the normalization radius, equation (15) is rewritten as:

$$(b_n + ia_n)B_0 = 0.02\sum_k I_k[z_k^{-(n+1)} + (\frac{z_k^*}{R^2})^{(n+1)}] \tag{18}$$

This is simple enough to calculate from the measured strand positions. To keep the accuracy of calculation, it is recommended to use repeated multiplications instead of using logarithmic conversion. The specific part of the program is coded as follows. The accuracy of the calculation can be tested by the error in the asymmetric terms. It produced 10^{-20} unit of asymmetric terms. Therefore this calculation should be good enough to the accuracy of the position measurement. The assumption of the circular bore of the iron may be the limitation.

```
subroutine cal(nd,np)
complex*16 dn,fn,cn,zn,zz
common /geome/ Rf,z(ns),c(ns)
common /field/ dn(ns,np), fn(ns,np), cn(ns,np)
do k=1,nd                       !  nd strands in total
facc=0.02*c(k)                  !  current c(k) in each strand
zz=dconjg(z(k))/Rf/Rf           !  Rf is the iron bore radius
dn(k,1)=2.0D-02*c(k)/z(k)       !  z(k) is the strand position
fn(k,1)=2.0D-02*zz              !
cn(k,1)=dn(k,1)+fn(k,1)         !
do j=2,np                       !  calculate up to np th pole
dn(k,j)=dn(k,j-1)/z(k)          !  current part (tesla)/(z/unit)**n
fn(k,j)=fn(k,j-1)*zz            !  iron part (tesla)/(z/unit)**n
cn(k,j)=dn(k,j)+fn(k,j)         !  total (tesla)/(z/unit)**n
enddo                           !
enddo                           !
return                          !
end                             !
```

Test Run

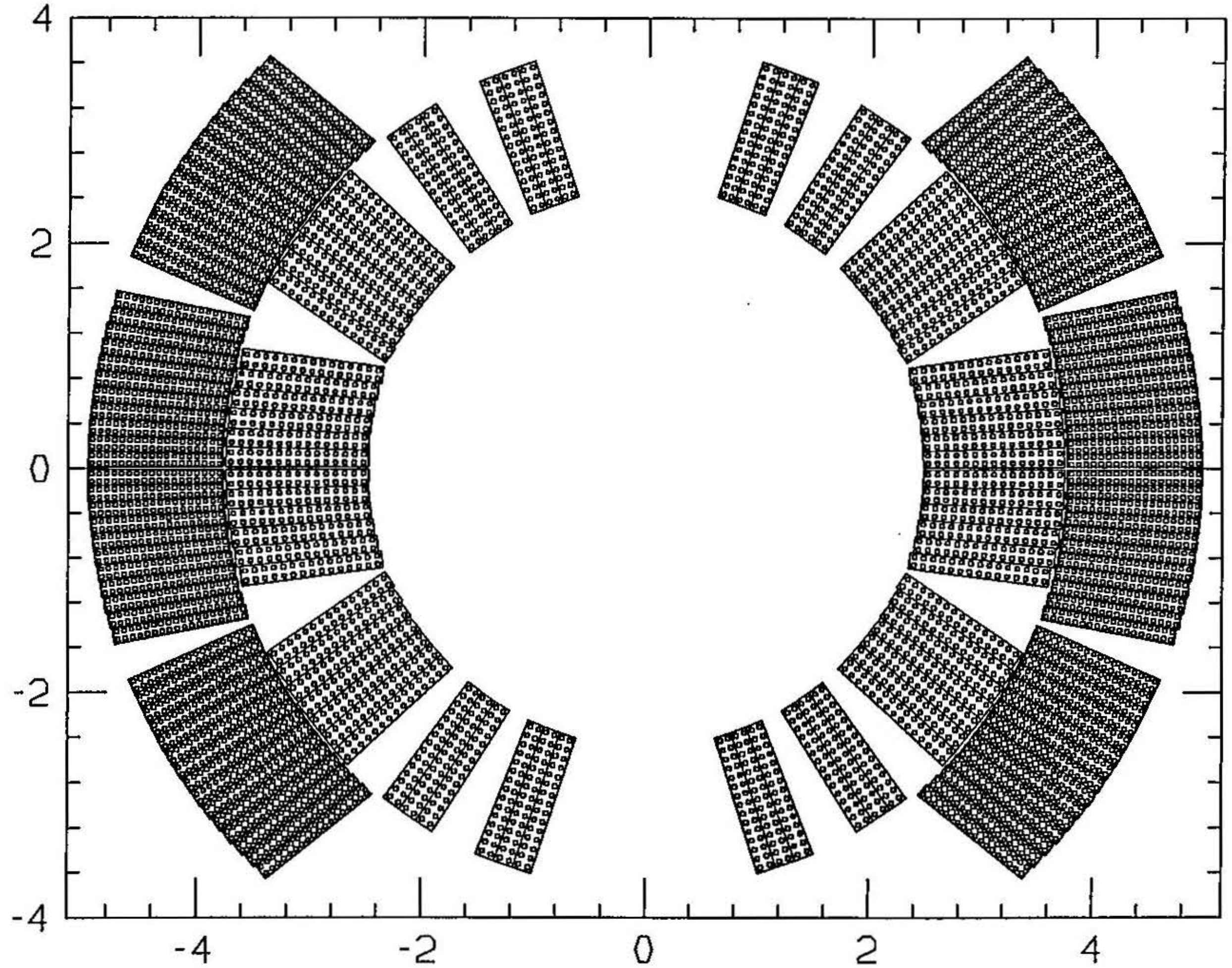
Simulating the measurement results, a set of strand position data was created using equal distance in the cable cross section. Figure 1 is the tested geometry. The calculated result from this geometry was very close to other design calculations as listed in the following table. Difference between integral formula analytic calculation [1] may be because of the current density distribution.

	Tf	b2	b4	b6	b8
Akbar result	1.045	+0.23	0.06	-0.02	+0.04
Yellow book	1.045	-0.18	-0.04	0.00	+0.05
M.W.(R=6.7792)	1.047	-0.28	0.06	-0.03	+0.04
(R=6.7488)	1.049	-0.19	0.06	-0.03	+0.04

Application

Starting DSA326, several magnets are planed to be cut after cold test including field measurement. Position measurements are now under taken at SSCL. The comparison between the position of the conductor and the field will be made with this calculation.

[1] Akbar Mokhtarani, Fermilab TS-SSC 92-028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