SSCL-Preprint-534

SSCL-Preprint-534
November 1993
Distribution Category: 414

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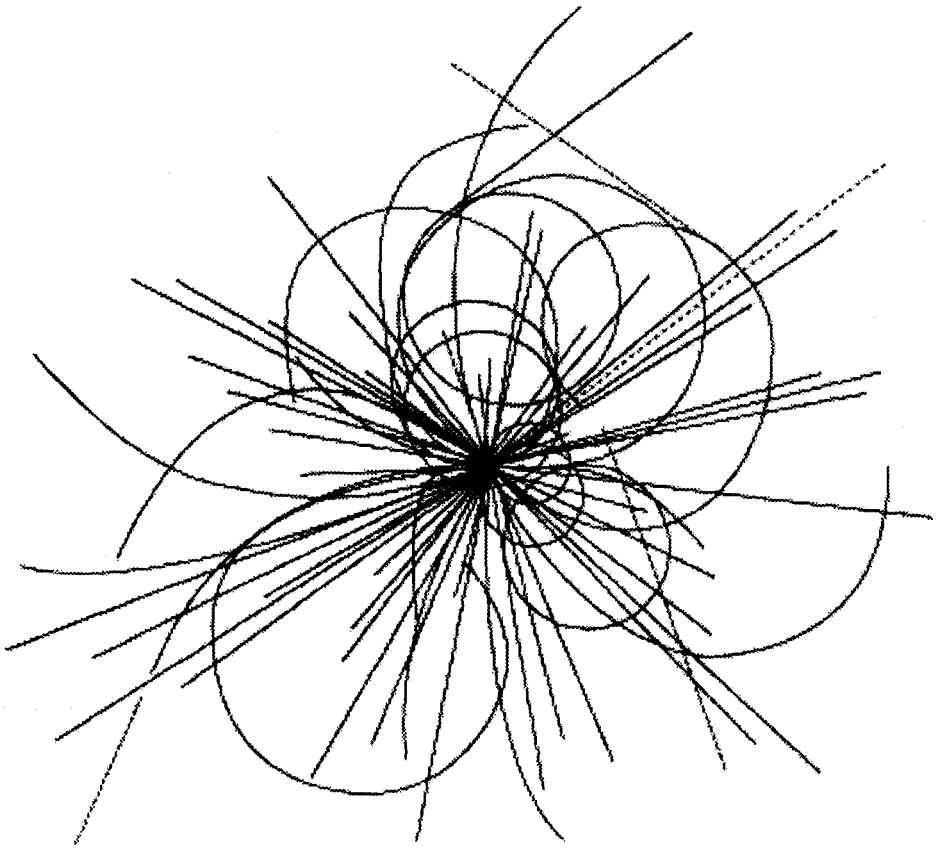

Superconducting Super Collider Laboratory

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The Magnitude of the Gluon Condensate and the Masses of c and b Quarks from the Families of $\mathcal{J}/\Psi$ and Υ Mesons*

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November 1993

* Submitted to *Physical Review Letters*.
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‡ Operated by the Universities Research Association, Inc., for the U.S. Department of Energy under Contract No. DE-AC35-89ER40486.

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The magnitude of the gluon condensate and the masses of c and b quarks from the families of $\mathcal{J}/\Psi$ and Υ mesons

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(July 21, 1993)

Abstract

It is shown that the model used by Shifman, Vainshtein, and Zakharov for the description of the experimental function $R_c(s)$ in the form of a sum of δ-functions and the plateau contradicts the Wilson operator expansion (O.E.) in the terms due to the gluon condensate. A QCD model with an infinite number of vector mesons does satisfy the requirements of the O.E. for masses and electronic width resonances close to the experimental values. The region

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of allowable values of the masses of c- and b-quarks and the gluon condensate compatible with the O.E. is obtained.

I. INTRODUCTION

The gluon condensate $\langle(\alpha_s/\pi)G^2\rangle$ is one of the fundamental characteristics of the QCD vacuum. In a basic paper [1], Shifman, Vainshtein, and Zakharov showed that the magnitude of the gluon condensate is nonzero and obtained for it the value

$$\langle(\alpha_s/\pi)G^2\rangle_{SVZ} = 0.012\ \mathrm{GeV}^4. \tag{1}$$

Since then, a large number of papers [2–8] have given various gluon condensate values that significantly differ from the value (1).

The analysis made for the families of $\mathcal{J}/\Psi$ and Υ mesons in Ref. [9] gives for the gluon condensate the value

$$0.05 \leq \langle(\alpha_s/\pi)G^2\rangle \leq 0.1\ \mathrm{GeV}^4, \tag{2}$$

rather different from value (1).

This paper is devoted to determining which of the values of the gluon condensate is correct. The plan of the article is as follows:

In Sec. II it will be shown the model used in Ref. [1] for the description of the experimental function $R_c(s)$ in the form of the sum of δ-functions and the plateau strongly contradicts the Wilson operator expansion (O.E.) in terms due to the gluon condensate.

In Sec. III we will develop a QCD model with an infinite number of vector mesons [9,10]. This model allows us to satisfy the requirements of the O.E. It will be shown that the basic equation (20) can be derived without the added assumption on $k \gg 1$.

In Sec. IV we will perform numerical calculations for both meson families, where we will obtain the region of the allowable values of the gluon condensate and the masses of the c and b quarks consistent with the O.E.

II. WHY DOUBT THE VALIDITY OF THE GLUON CONDENSATE VALUE (1)?

The O.E. lies at the basis of the calculations of the gluon condensate value. The O.E. for the T-product of vector currents composed of c quarks has the form [1]:

$$i\int d^4x e^{iqx} T\left\{j^c_\mu(x), j^c_\nu(0)\right\} = (q_\mu q_\nu - q^2 g_{\mu\nu})\ (C_I I + C_G O_G + \cdots), \tag{3}$$

where $j^c_\mu(x) = \bar{c}\ (x)\gamma_\mu\ c(x)$, I is the unit operator and

$$C_I = \Pi^c_I(Q^2) = \frac{1}{12\pi^2 Q_c^2}\int\limits_{4m_c^2}^{\infty} \frac{R_c^{PT}(s)ds}{s+Q^2}, \tag{4}$$

where $Q^2 = -q^2$, Q_c is a c quark charge, and C_I is the coefficient function of the unit operator.

The function R_c^{PT} includes all corrections on α_s in perturbation theory (PT). Equation (4) has been written without the subtraction, because it uses derivatives of (4) with respect to Q^2 only. The remaining values are defined as: $O_G = G^a_{\mu\nu}(0)G^a_{\mu\nu}(0)$, where $G^a_{\mu\nu}$ is the operator of the gluon field.

$$C_G = \frac{\alpha_s}{\pi} P(Q^2),$$

$$P(Q^2) = \frac{1}{48Q^4}\left\{3(a+1)\left(1-\frac{1}{a}\right)^2 \frac{1}{2\sqrt{a}} \ln\frac{\sqrt{a}+1}{\sqrt{a}-1} - 3 + \frac{2}{a} - \frac{3}{a^2}\right\}, \tag{5}$$

$$a = 1 + \frac{4m_c^2}{Q^2}.$$

The O.E. is supposed to be valid for all positive Q^2. If we take the vacuum expectation value of (3) we obtain the theoretical definition of the polarization operator following from the O.E.:

$$\Pi^c_{Theor.}(Q^2) = \Pi^c_I(Q^2) + \langle(\alpha_s/\pi)G^2\rangle P(Q^2), Q^2 \geq 0. \tag{6}$$

The dispersion relation for $\Pi^c_{Exp.}(Q^2)$ permits expression of $\Pi^c_{Exp.}(Q^2)$ via the measuring value $R_c(s)$:

$$\Pi^c_{Exp.}(Q^2) = \frac{1}{12\pi^2 Q_c^2}\int\limits_{4m_c^2}^{\infty} \frac{R_c(s)ds}{s+Q^2}. \tag{7}$$

In the model of Ref. [1] it was

$$(\Pi^c_{Exp.}(Q^2))_{SVZ} = \frac{1}{12\pi^2 Q_c^2}\left\{\frac{9\pi}{\alpha^2}\sum_{i=0}^{5}\frac{\Gamma_i^{ee}M_i}{M_i^2+Q^2} + \int_{\overline{s}}^{\infty}\frac{R_c^{(1)}(s)ds}{s+Q^2}\right\}, \tag{8}$$

where Γ_i^{ee} is the electronic width of the i-resonance, M_i is the mass of this resonance, and $\alpha = 1/137$. The summation is over all 6 resonances of $\mathcal{J}/\Psi$ family. In Ref. [1] corrections of the first order in α_s are taken into account only because the function $R_c^{(1)}(s)$ is chosen in the form

$$R_c^{(1)}(s) = R_c^{(0)}(s)\left\{1 + \frac{\alpha_s}{3}\left[\frac{2\pi}{v} - \left(\frac{\pi}{2} - \frac{3}{4\pi}\right)(3+v)\right]\right\}, \tag{9}$$

where

$$R_c^{(0)}(s) = \frac{3}{2}Q_c^2 v(3-v^2) \tag{10}$$

$$Q_c = \frac{2}{3}, v = \left(1 - \frac{4m_c^2}{s}\right)^{1/2}, \alpha_s = 0.2.$$

For $s > \overline{s}$ $R_c^{(1)}$ is taken in the simplified form

$$R_c^{(1)}(s) = \frac{4}{3}\left(1 + \frac{\alpha_s}{\pi}\right)\Theta(s-\overline{s}). \tag{11}$$

The charmed quark mass $m_c = 1.26$ GeV, the value $\overline{s} = (4.2)^2$ GeV^2, and the gluon condensate (1) were found from a requirement of minimal discrepancy between the first ten moments of $(\Pi^c_{Exp.}(Q^2))_{SVZ}$ and $(\Pi^c_{Theor.}(Q^2))$. It can be shown that the formula (8) strongly contradicts the O.E. in the terms due to the gluon condensate. After subtracting from formulas (6) and (8) the contribution of the unit operator (4), we obtain:

$$\Delta\Pi^c_{Theor.}(Q^2) = \langle(\alpha_s/\pi)G^2\rangle P(Q^2) \tag{12}$$

$$(\Delta\Pi^c_{Exp.}(Q^2))_{SVZ} = \frac{1}{12\pi^2 Q_c^2}\left\{\frac{9\pi}{\alpha^2}\sum_{i=0}^{5}\frac{\Gamma_i^{ee}M_i}{M_i^2+Q^2} - \int_{4m_c^2}^{\overline{s}}\frac{R_c^{(1)}(s)ds}{s+Q^2}\right\}. \tag{13}$$

Formula (12) contains only the contribution of the operator connected with the gluon condensate, but formula (13) contains in addition higher dimensional operators. The contribution of operators of higher dimension is small at $Q^2 \geq 0$, so expressions (12) and (13) must be

close. The formulas (12) and (13) have different asymptotes if $m_c = 1.26\,\mathrm{GeV}$, $\overline{s} = 4.2\,\mathrm{GeV}^2$ (see Ref. [1]), since formula (13) contains terms in $1/Q^2$. The comparison between terms in $1/Q^4$ in formulae (12) and (13) gives

$$\langle(\alpha_s/\pi)G^2\rangle = \frac{1}{\pi^2 Q_c^2}\left\{\frac{9\pi}{\alpha^2}\sum_{i=0}^{5}\Gamma_i^{ee}M_i^3 - \int\limits_{4m_c^2}^{\overline{s}} R_c^{(1)}(s)s\,ds\right\}. \tag{14}$$

Substitution of m_c and $\overline{s}$ from Ref. [1] in formula (11) gives $\langle(\alpha_s/\pi)G^2\rangle = -3.85\ \mathrm{GeV}^4$. We shall attempt to improve the situation by changing the parameters in formula (13). Absence of $1/Q^2$ terms in asymptote (12) gives

$$\frac{9\pi}{\alpha^2}\sum_{i=0}^{5}\Gamma_i^{ee}M_i - \int\limits_{4m_c^2}^{\overline{s}} R_c^{(1)}(s)\,ds = 0\,. \tag{15}$$

After the integral in (15) has been taken, the parameters $m_c, \overline{s}$ can be expressed in terms of $\overline{v} = (1 - 4m_c^2/\overline{s})^{1/2}, M_i, \Gamma_i^{ee}$. The parameter $\overline{v}$, the masses M_i, and the widths Γ_i^{ee} were varied in order to make (12) and (13) maximally close. The masses M_i and the widths Γ_i^{ee} were varied within the limits of four experimental errors. It is the best result possible to achieve in the vicinity of (12) and (13) (its difference is less then 20%) at values $Q^2 > 4\ \mathrm{GeV}^2$ only. At $Q^2 = 0$ the right-hand sides of (12) and (13) differ by a factor of five. In our opinion it is too strong a violation of the O.E.. In the next section a model meeting O.E. requirements will be constructed.

III. THE QCD MODEL WITH AN INFINITE NUMBER OF VECTOR MESONS

In this section we intend to improve the QCD model with an infinite number of vector mesons suggested in Refs. [9,10]. We shall describe the families $\mathcal{J}/\Psi$ and Υ mesons in the same way. Let us consider the polarization operator $\Pi^a(Q^2)$ $a = c$ for the $\mathcal{J}/\Psi$ family, and $a = b$ for the Υ family.

The polarization operator $\Pi^a(Q^2)$ has the form [9]:

$$\Pi^a(Q^2) = \frac{1}{12\pi^2 Q_a^2}\frac{9\pi}{\alpha^2}\sum_{k=0}^{\infty}\frac{\Gamma_k^{ee}M_k}{s_k + Q^2} \tag{16}$$

in the approximation of an infinite number of narrow resonances, having masses M_k and electronic widths Γ_k^{ee}, $s_k = M_k^2$; Q_a is a quark charge.

If the widths and the masses of the vector mesons obey the conditions

$$M_k^2 - M_{k-1}^2 \ll M_k \Gamma_k \ll M_k^2 \tag{17}$$

for $k \geq 5$ (Γ_k is the total width of the k-th resonance), then after the sixth resonance the function $R(s)$ will be described by a smooth curve and all the formulas of the model under consideration will be applied as was shown in Ref. [10].

The polarization operator (16) can be rearranged in form with the separated unit operator. The remainder of the polarization operator can be associated with the gluon condensate and with the contribution of higher dimensional operators.

We transform the sum in (13) into an integral by means of the Euler-Maclaurin formula beginning from $k = k_0$, where k_0 is the middle resonance number from six discovered resonances ($k_0 = 2$ or $k_0 = 3$). For definiteness we have chosen $k_0 = 2$. Then we obtain:

$$\Pi^a(Q^2) = \frac{1}{12\pi^2 Q_a^2}\frac{9\pi}{\alpha^2}\left\{\int\limits_{4m_a^2}^{\infty}\frac{\Gamma_k^{ee}M_k}{s_k+Q^2}\frac{dk}{ds_k}ds_k - \int\limits_{4m_a^2}^{s_2}\frac{\Gamma_k^{ee}M_k}{s_k+Q^2}\frac{dk}{ds_k}ds_k + \right.$$
$$\frac{\Gamma_0^{ee}M_0}{s_0+Q^2} + \frac{\Gamma_1^{ee}M_1}{s_1+Q^2} + \frac{1}{2}\frac{\Gamma_2^{ee}M_2}{s_2+Q^2} - \frac{1}{12}\frac{d}{dk}\left(\frac{\Gamma_k^{ee}M_k}{s_k+Q^2}\right)|_{k=2} +$$
$$\left. \frac{1}{720}\frac{d^3}{dk^3}\left(\frac{\Gamma_k^{ee}M_k}{s_k+Q^2}\right)|_{k=2} - \cdots\right\}. \tag{18}$$

It seems reasonable to set the first term on the right-hand side of (18) equal to the first term on the right-hand side of (6):

$$\frac{1}{12\pi^2 Q_a^2}\frac{9\pi}{\alpha^2}\int\limits_{4m_a^2}^{\infty}\frac{\Gamma_k^{ee}M_k}{s_k+Q^2}\frac{dk}{ds_k}ds_k = \frac{1}{12\pi^2 Q_a^2}\int\limits_{4m_a^2}^{\infty}\frac{R_a^{PT}(s)ds}{s+Q^2}. \tag{19}$$

We consider the equality (19) as an ansatz, allowing separation of the large terms associated with the unit operator from small terms associated with the operator connected with the gluon condensate. Equation (19) has one and only one solution:

$$\Gamma_k^{ee} = \frac{2\alpha^2}{9\pi}R_a^{PT}(s_k)M_k^{(1)}. \tag{20}$$

In Eq. (20) and in the following formulas we use the notation

$$M_k^{(l)} \equiv \frac{d^l M_k}{dk^l}, \qquad s_k^{(l)} \equiv \frac{d^l s_k}{dk^l}. \tag{21}$$

It is significant that the derivation of formula (20) does not demand the additional assumption $k \gg 1$ as in Ref. [10]. It may be proven that the function $s_k \equiv s(k)$ given at $k = 0, 1, 2, \ldots$ can be continued by an analytical function of the complex variable k with the cut along the negative axes. We will not use the analytical properties of the function $s(k)$; it is suggested that the function $s(k)$ is continuous, is differentiable with respect to k, and has the necessary number of (not too great) derivatives with respect to k at $k = 2$. The derivatives $s_2^{(l)}$ will be considered as parameters.

In these notations the contribution of terms connected with the gluon condensate and higher dimensional operators is

$$\Delta\Pi^a(Q^2) = \frac{1}{12\pi^2 Q_a^2}\left\{\frac{R_a^{PT}(s_0)s_0^{(1)}}{s_0+Q^2} + \frac{R_a^{PT}(s_1)}{s_1+Q^2} - \int\limits_{4m_a^2}^{s_2}\frac{R_a^{PT}(s)ds}{s+Q^2} + \right.$$
$$\left.\frac{1}{2}W_{PT}(Q^2) - \frac{1}{12}W_{PT}^{(1)}(Q^2) + \frac{1}{720}W_{PT}^{(3)}(Q^2) - \cdots\right\}. \tag{22}$$

Here we have introduced the notations:

$$W_{PT}(Q^2) = \frac{R_a^{PT}(s_2)s_2^{(1)}}{s_2+Q^2}, W_{PT}^{(l)}(Q^2) = \frac{d^l}{dk^l}\left(\frac{R_a^{PT}(s_k)s_k^{(1)}}{s_k+Q^2}\right)|_{k=2}. \tag{23}$$

Because it is suggested that the contribution of higher dimensional operators is small, we obtain:

$$\Delta\Pi^a(Q^2) = \langle(\alpha_s/\pi)G^2\rangle P(Q^2). \tag{24}$$

The function $P(Q^2)$ is defined by formula (5) for $Q^2 > 0$. For $-4m_a^2 < Q^2 < 0$ the function $P(Q^2)$ is obtained by analytic continuation [9].

$$P(Q^2) = \frac{1}{48Q^4}\left\{\frac{3(a+1)(a-1)^2}{a^2}R - 3 + \frac{2}{a} - \frac{3}{a^2}\right\} \tag{25}$$

$$R = -\frac{1}{\sqrt{-a}}\arctan\frac{1}{\sqrt{-a}}$$

$$a = 1 + \frac{4m_c^2}{Q^2}.$$

The O.E. is suggested to be valid for the family of $\mathcal{J}/\Psi$ mesons for $Q^2 > 0$ and for the family of Υ mesons for $Q^2 > -4m_b^2 + 10$ GeV^2. The function $P(Q^2)$ for c and b quarks is shown in Fig. 1. The reasonableness in choosing the region of validity of the O.E. for the families of $\mathcal{J}/\Psi$ and Υ mesons can be seen from Fig. 1.

On the right-hand side of (24) corrections in α_s are not taken into account; consequently, on the left-hand side, too, these corrections can be disregarded, i.e., we can make the replacement $R_a^{PT}(s) \to R_a^0(s) = (3/2)Q_a^2 v(3-v^2)$.[1] As a result we write the left-hand side of (22) in the form

$$\Delta\Pi^a(Q^2) = \frac{1}{8\pi^2}\left\{W_0 + W_1 - 2\frac{4m_a^2}{Q^2}F + \frac{1}{2}W_2 - \frac{1}{12}W_2^{(1)} + \frac{1}{720}W_2^{(3)} - \right.$$
$$\left. -\frac{1}{30240}W_2^{(5)} + \frac{1}{129600}W_2^{(7)} - \frac{1}{47900160}W_2^{(9)} + \cdots\right\}. \qquad (26)$$

In (26) we use the notations

$$W_k = \frac{u_k}{s_k + Q^2}, u_k = v_k(3 - v_k^2)s_k^{(1)}, k = 0, 1, 2 \ldots$$

$$F = \frac{Q^2}{8m_a^2}\int_{4m_a^2}^{s_2} \frac{v(3-v^2)ds}{s+Q^2} = \frac{1}{a-1}\left[\ln\frac{1+v_2}{1-v_2} - \frac{(3-a)\sqrt{a}}{2}\ln\frac{\sqrt{a}+v_2}{\sqrt{a}-v_2}\right] - v_2 \qquad (27)$$

$$a = 1 + \frac{4m_a^2}{Q^2}.$$

For $-4m_a^2 < Q^2 < 0$ the following formula holds for the function F:

$$F = \frac{1}{a-1}\left[\ln\frac{1+v_2}{1-v_2} - (3-a)\sqrt{-a}\,\mathrm{arctg}\,\frac{v_2}{\sqrt{-a}}\right]. \qquad (28)$$

---

[1]It is supposed that the masses of the resonances considered do not depend on α_s. As an illustration we refer to Ref. [11], where it is shown that the mass difference between a proton and a neutron may not be electromagnetic but has its origin in the difference of the masses of u- and d-quarks.

The superscripts on W_2 are explained by (21). For $Q^2 \to \infty$ the right-hand side of (24) takes the form

$$\langle(\alpha_s/\pi)G^2\rangle P(Q^2) \to -\frac{1}{12}\langle(\alpha_s/\pi)G^2\rangle\frac{1}{Q^4}\,. \tag{29}$$

Consequently, the terms $1/Q^2$ on the left-hand side of (21) should cancel, and we obtain the equation:

$$\begin{aligned} C_2 = u_0 + u_1 - 2s_2v_2^3 + \frac{1}{2}u_2 - \frac{1}{12}u_2^{(1)} + \frac{1}{720}u_2^{(3)} - \frac{1}{30240}u_2^{(5)} + \\ + \frac{1}{1209600}u_2^{(7)} - \frac{1}{47900160}u_2^{(9)} + \cdots = 0\,. \end{aligned} \tag{30}$$

Equating the terms $1/Q^4$ as $Q^2 \to \infty$ on both sides of (24), we obtain:

$$\begin{aligned} \langle(\alpha_s/\pi)G^2\rangle_\infty = \frac{3}{2\pi^2}\Big\{u_0s_0 + u_1s_1 - 2(4m_a^2)^2F_4 + \frac{1}{2}u_2s_2 - \frac{1}{12}(u_2s_2)^{(1)} + \\ + \frac{1}{720}(u_2s_2)^{(3)} - \frac{1}{30240}(u_2s_2)^{(5)} + \frac{1}{1209600}(u_2s_2)^{(7)} - \\ - \frac{1}{47900160}(u_2s_2)^{(9)} + \cdots\Big\} \end{aligned} \tag{31}$$

$$F_4 = \frac{1}{2(4m_a^2)^2}\int\limits_{4m_c^2}^{s_2} sv(3-v^2)ds = \frac{v_2(3-v_2^2)}{4(1-v_2^2)^2} - \frac{3}{8}\ln\frac{1+v_2}{1-v_2}\,, \tag{32}$$

where $\langle(\alpha_s/\pi)G^2\rangle_\infty$ is the value of the gluon condensate from (31). In Eqs. (22), (25), and (26) the terms not written out have been omitted because of the smallness of the coefficients in these terms.

The unobserved parameters $s_2^{(k)}(k = 2, ...10)$ may be connected with the measured quantities $s_i, s_i^{(1)}(i = 0, 1, 3, 4, 5)$ by Taylor expansion:

$$s_i = \sum_{k=0}^{10}\frac{(i-2)^k}{k!}s_2^{(k)},\ i = 0, 1, 2, 3, 4, 5 \tag{33}$$

$$s_i^{(1)} = \sum_{k=0}^{9}\frac{(i-2)^k}{k!}s_2^{(k+1)},\ i = 0, 1, 2, 3, 4, 5\,. \tag{34}$$

The Taylor expansions are cut off at $k = 10$ so that Eqs. (33) and (34) contain the same parameters $s_2^{(2)}, s_2^{(3)}, \ldots, s_2^{(10)}$ as Eqs. (26), (30), and (31).

The electronic widths of the resonances Γ_k^{ee} are given by (20). For $R_a^{PT}(s)$ we shall use the formula

$$R_a^{PT}(s_i) = R_c^{(0)}(s_i)\mathcal{D}_i(v_i)\,, \tag{35}$$

where $R_a^{(0)}(s)$ is given by (10) and for function $D_k(v_k)$ the following expression holds:

$$\mathcal{D}_i(v_i) = \frac{4\pi\alpha_s/3v_i}{1 - exp(-4\pi\alpha_s/3v_i)} - \frac{1}{3}\left(\frac{\pi}{2} - \frac{3}{4\pi}\right)(3 + v_i)\alpha_s\,. \tag{36}$$

In (35) we have taken into account terms of the first order in α_s and Coulomb terms of all orders in α_s/v_k. The Coulomb poles are taken into account in the same way as in Ref. [9].

Using Eqs. (16), (25) , (28), and (31), the unobserved parameters $s_2^{(k)}(k = 2, ...10)$ as well as the electronic widths Γ_0^{ee} and Γ_1^{ee} are expressed by the masses $M_i(i = 0, ...5)$ and the electronic widths $\Gamma_i^{ee}(i = 2, ...5)$.

Note that the order of the gluon condensate magnitude is $\langle(\alpha_s/\pi)G^2\rangle \sim s_0 s_0^{(1)}$ from dimensionality (31). It is $\langle(\alpha_s/\pi)G^2\rangle \sim 30$ GeV^4 for the $\mathcal{J}/\Psi$ family and $\langle(\alpha_s/\pi)G^2\rangle \sim 1000$ GeV^4 for the Υ family. As obtained in this work, the value $\langle(\alpha_s/\pi)G^2\rangle \approx 0.05 \div 0.1$ GeV^4 from the analysis of the $\mathcal{J}/\Psi$ and Υ families is due to great cancellations in Eqs. (26) and (31). Analogous cancellations occur in the model in Ref. [1]. These cancellations make calculations very difficult. There is a very great sensitivity in all results of the values of resonance masses M_i and electronic widths Γ_i^{ee}. A negligibly small variation of the magnitudes M_i and Γ_i^{ee} gives disagreement in (24).

IV. NUMERIC CALCULATIONS AND RESULTS

Let us point out the necessity of fine adjustment at the masses and the electronic widths of the mesons considered. The right-hand and left-hand sides of (24) differ several times at the experimental values of masses $(M_i)_{Exp.}(i = 0, 1, ...5)$ and electronic widths $(\Gamma_i^{ee})_{Exp.}(i = 2, ...5)$. That is why the masses and the electronic widths of resonances were varied in addition to varying of quarks masses and gluon condensate. The masses $M_i(i = 0, 1, ...5)$

and the widths $\Gamma_i^{ee}(i = 2, ...5)$ varied near their experimental values. The fulfillment of (30) was required. The minimizing function was written in the form

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{P_a(Q_i^2)}{\langle(\alpha_s/\pi)G^2\rangle P(Q_i^2)} - 1 \right)^2 /0.2^2 + \sum_{i=0}^{5} \frac{(M_i^2 - M_{iExp.}^2)^2}{(\Delta M_{iExp.}^2)^2} + \sum_{i=0}^{5} \frac{(\Gamma_i^{ee} - \Gamma_{iExp.}^{ee})^2}{(\Delta\Gamma_{iExp.}^{ee})^2}, \tag{37}$$

where

$$P_a(Q^2) = \Delta\Pi^a(Q^2) - \frac{1}{8\pi^2} C_2 \frac{1}{Q^2 - s_2}. \tag{38}$$

$\Delta\Pi^a(Q^2)$ is defined by (26) and C_2 is defined by (30). ΔM_i^2 , $\Delta\Gamma_i^{ee}$ are experimental errors of M_i^2 , Γ_i^{ee}. N is the number of points used to check the right-hand and left- hand sides of (24). The final number chosen was $N = 120$, and the region of the comparison chosen was $0 < Q^2 < 150$ GeV^2 for the $\mathcal{J}/\Psi$ family and $-4m_b^2 + 10$ GeV$^2 < Q^2 < 500$ GeV^2 for the Υ family. The right-hand and left-hand sides of (24) coincided with an accuracy of 1% at large Q^2. The minimum of the function (37) is reached at the masses M_i and the electronic widths Γ_i^{ee} of resonances given in Table I, with the mass of the c-quark $m_c =$ 1.302 GeV $\langle(\alpha_s/\pi)G^2\rangle = 0.0526$ GeV^4 (from the $\mathcal{J}/\Psi$ family) and the mass of the b-quark $m_b = 4.547$ GeV $\langle(\alpha_s/\pi)G^2\rangle = 0.0615$ GeV^4 (from the Υ family). At these values right-hand and left-hand sides of (24) coincide with an accuracy better than 3% for the $\mathcal{J}/\Psi$ family and with accuracy better than 9% for the Υ family. Thereafter, the masses of the quarks and the magnitude of the gluon condensate were varied over the region where the right-hand and left-hand sides of (24) coincided with an accuracy better than 20%; this region is called the permissible region. The permissible region for the c-quark is presented in Fig. 2 and for the b-quark in Fig. 3. The best magnitude of the gluon condensate follows from Figs. 2 and 3:

$$0.04 \leq \langle(\alpha_s/\pi)G^2\rangle \leq 0.105 \text{ GeV}^4. \tag{39}$$

The magnitude of the gluon condensate obtained from (31), $\langle(\alpha_s/\pi)G^2\rangle_\infty$, practically coincides with $\langle(\alpha_s/\pi)G^2\rangle$. The masses and the electronic widths of the resonances were varied in the permissible region from $(M_i)_{min}$, $(\Gamma_i^{ee})_{min}$ to $(M_i)_{max}$, $(\Gamma_i^{ee})_{max}$ (Table I).

The difference between the right-hand and left-hand sides of (24) is minimal at masses M_i^{Best} and electronic widths Γ_i^{Best}, less than 3% for the $\mathcal{J}/\Psi$ family for all positive Q^2, and less than 9% for the Υ family for all Q^2 from interval $-4m_b^2 + 10\ \mathrm{GeV}^2 < Q^2 < \infty$. $(M_i)_{min}$, $(\Gamma_i^{ee})_{min}$, $(M_i)_{max}$, $(\Gamma_i^{ee})_{max}$ are minimal (maximal) value of masses and electronic widths of mesons such that the right-hand and left-hand sides of (24) differ by less than 20% if Q^2 is in the interval where the O.E. is valid.

ACKNOWLEDGMENTS

The authors are grateful to Frank Paige for his review of the questions concerned, and we appreciate his comments. We also thank the staff of the SSC Computer Center. We appreciate the efforts of Linda Fowler and thank her for her contribution in preparation of this article for publication. The Superconducting Super Collider Laboratory is operated by the Universities Research Association, Inc., for the U.S. Department of Energy under contract No. DE-AC35-89ER40486.

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TABLES

TABLE I. The domain of variations of masses and the electronic widths of resonances, in GeV.

	$\mathcal{J}/\Psi$ family					
	0	1	2	3	4	5
$M^{Best}_{i,Theor.}$	3.09688	3.68605	3.77464	4.00219	4.08354	4.40874
$M^{max}_{i,Theor.}$	3.09790	3.68705	3.78337	4.02603	4.14069	4.42646
$M^{min}_{i,Theor.}$	3.09576	3.68539	3.76689	4.00013	4.08176	4.39045
$M_{i,Exp.}$	3.09693	3.686	3.7699	4.04	4.159	4.415
$Exp.err.$	0.00009	0.0001	0.0025	0.01	0.02	0.006

	0	1	2	3	4	5
$\Gamma^{Best}_{i,Theor.}$	5.2896	1.9379	0.31942	1.6366	0.67065	0.51277
$\Gamma^{max}_{i,Theor.}$	6.1576	2.0227	0.3999	1.8489	1.4910	0.8740
$\Gamma^{min}_{i,Theor.}$	5.0393	1.6147	0.2593	1.4665	0.1142	0.2006
$\Gamma_{i,Exp.}$	5.36	2.14	0.26	0.75	0.77	0.47
$Exp.err.$	0.29	0.21	0.04	0.15	0.23	0.1

	Υ family					
	0	1	2	3	4	5
$M^{Best}_{i,Theor.}$	9.46032	10.0220	10.3553	10.5841	10.8506	11.0102
$M^{max}_{i,Theor.}$	9.46074	10.0239	10.3570	10.5870	10.8623	11.0349
$M^{min}_{i,Theor.}$	9.45986	10.0220	10.3543	10.5736	10.8494	11.0031
$M_{i,Exp.}$	9.46032	10.0233	10.3553	10.58	10.865	11.019
$Exp.err.$	0.00022	0.00031	0.0005	0.0035	0.008	0.008

	0	1	2	3	4	5
$\Gamma^{Best}_{i,Theor.}$	1.0217	0.64051	0.34043	0.35361	0.37848	0.10799
$\Gamma^{max}_{i,Theor.}$	1.1876	0.6452	0.4105	0.3536	0.4500	0.1899
$\Gamma^{min}_{i,Theor.}$	1.0198	0.5913	0.3404	0.3174	0.3617	0.0725
$\Gamma_{i,Exp.}$	1.34	0.56	0.44	0.24	0.31	0.13
$Exp.err.$	0.04	0.09	0.04	0.05	0.07	0.03

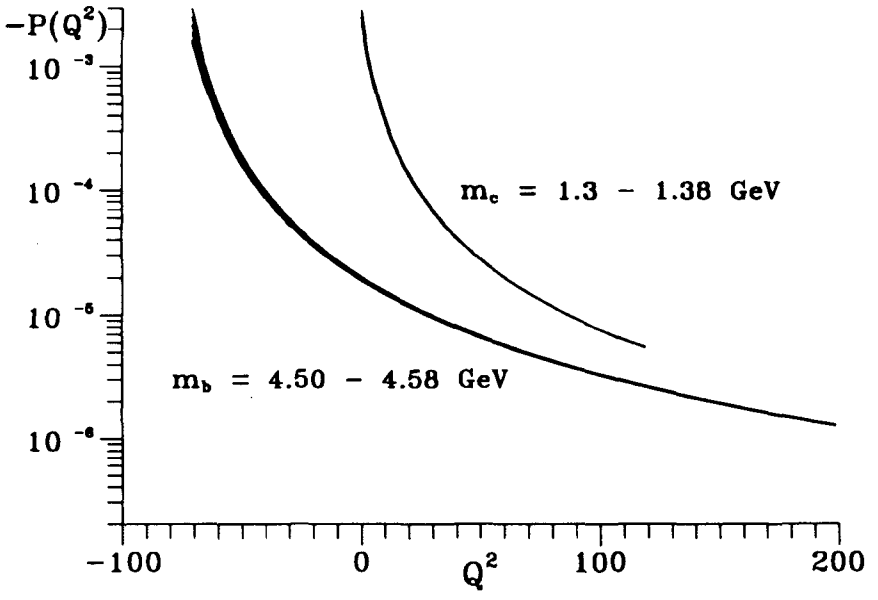

FIG. 1. The function $P(Q^2)$ for c- and b-quarks.

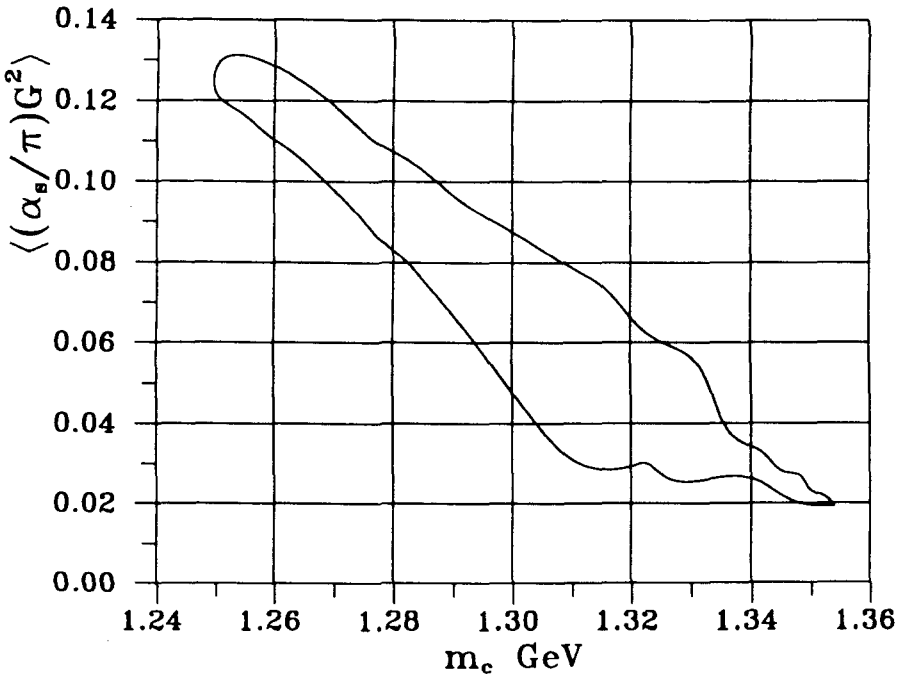

FIG. 2. The domain of allowable magnitudes of gluon condensate and c-quark mass. The difference is less than 20% within the curve for all $Q^2 > 0$.

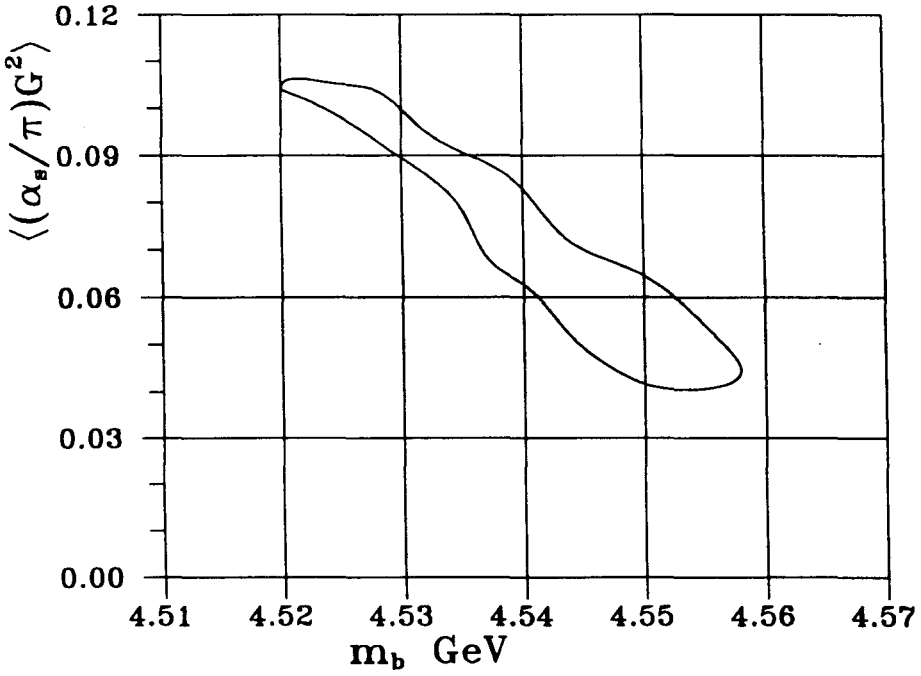

FIG. 3. The domain of allowable magnitudes of gluon condensate and b-quark mass. The difference is less than 20% within the curve for all Q^2 from interval $-4m_b^2 + 10\ \mathrm{GeV}^2 < Q^2 < \infty$.