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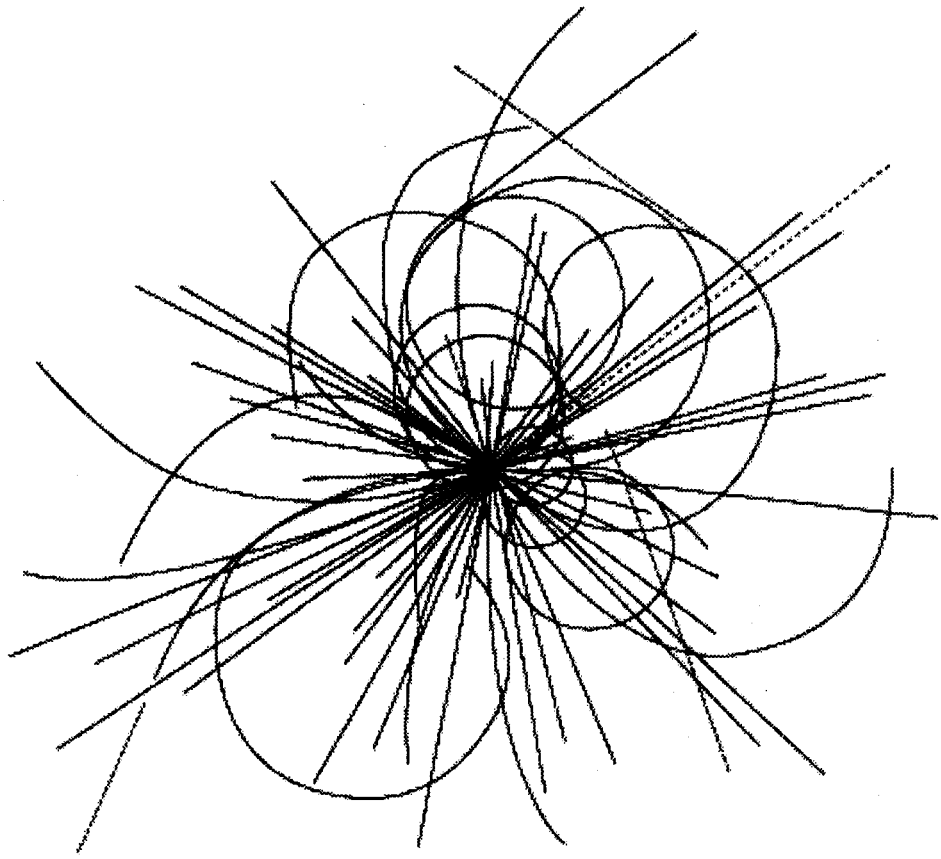

Superconducting Super Collider Laboratory

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Light-Quark, Heavy-Quark Systems: An Update*

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LIGHT-QUARK, HEAVY-QUARK SYSTEMS: AN UPDATE

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We review many of the recently developed applications of Heavy Quark Effective Theory techniques. After a brief update on Luke's theorem, we describe striking relations between heavy baryon form factors, and how to use them to estimate the accuracy of the extraction of $|V_{cb}|$. We discuss factorization and compare with experiment. An elementary presentation, with sample applications, of reparametrization invariance comes next. The final and most extensive chapter in this review deals with phenomenological lagrangians that incorporate heavy-quark spin-flavor as well as light quark chiral symmetries. We compile many interesting results and discuss the validity of the calculations.

1. INTRODUCTION

It seems hardly appropriate to devote any time to reviewing the fundamentals of Heavy Quark Effective Theory (HQET), both because this is a meeting of experts and because several good reviews of the subject are now available.[1, 2] Instead of wasting any space introducing conventions, I simply choose to use the notation of Ref. 2. Thus, I will be able to devote more energy towards a description of recent developments in this field.

I view this paper as updating and expanding on Ref. 2. There the HQET was presented and a few applications discussed at length. Other applications where briefly discussed. Much has changed since Ref. 2 was written, and it seems the time is ripe for an extension of that work. Because of time and space limitations this is not intended as an extensive overview of progress in the field since Ref. 2 was written. Rather, I shall pick and choose according to my taste, familiarity with the subjects, and what I perceived as relevant to the participants of the workshop.

2. AN UPDATE ON LUKE'S THEOREM

Presumably the best known consequence of heavy quark symmetries is that the form factors for semileptonic $B \to D$ and $B \to D^*$ decays are determined at the point of zero recoil (equal B and D velocities). Luke's theorem states that this normalization of the meson form factors has no $1/M_Q$ corrections.[3] It is not widely appreciated that Luke's original

proof did not exclude possible *short distance* corrections of order $(\alpha_s(m_c)/m_c)$. It turns out it is easy to extend Luke's proof to exclude corrections of this sort to any order in the strong coupling.[4]

Similarly, the normalization of form factors for $\Lambda_b \to \Lambda_c$ semileptonic decay is computable up to corrections of order $1/M_Q^2$.[4, 5]

3. HEAVY BARYON FORM FACTOR RELATIONS

3.1 Relations to First Order in $1/M_Q$

Six form factors encode the semileptonic decay amplitude $\Lambda_b \to \Lambda_c e\overline{\nu}$. The transition lends itself particularly well to HQET analysis because it is tightly constrained by the heavy quark spin symmetry.[6] Like their mesonic counterparts, the six form factors that parameterize this baryonic process are predicted at leading order in the $1/M_Q$ expansion in terms of a single Isgur-Wise function. In contrast with their mesonic counterparts, one can prove that this is still the case at order $1/M_Q$.[5] In other words, five relations among these six form factors remain after $O(1/m_c)$ and $O(1/m_b)$ corrections are included.

Remarkably, that such relations can be written is not precluded by short distance effects to any order in the strong coupling constant.[4] However the relations themselves get corrected order by order in perturbation theory. To see how this works, define the form factors through

$$\langle\Lambda_c(v',s')|V^\mu|\Lambda_b(v,s)\rangle = \overline{u}(v',s')[F_1(v{\cdot}v')\gamma^\mu + F_2(v{\cdot}v')v^\mu + F_3(v{\cdot}v')v'^\mu]u(v,s) \quad (1)$$

$$\langle\Lambda_c(v',s')|A^\mu|\Lambda_b(v,s)\rangle = \overline{u}(v',s')[G_1(v{\cdot}v')\gamma^\mu + G_2(v{\cdot}v')v^\mu + G_3(v{\cdot}v')v'^\mu]\gamma^5 u(v,s) \quad (2)$$

where v and s refer to the velocity and spin of the state Λ_b and of the Dirac spinor u. Then, the relations between form factors are[4]

$$\frac{F_1}{G_1} = 1 + \Big[\frac{\overline{\Lambda}}{2m_c} + \frac{\overline{\Lambda}}{2m_b}\Big]\frac{2}{(v{\cdot}v'+1)} + \frac{4}{3}\frac{\alpha_s(m_c)}{\pi}r + \frac{4}{3}\frac{\alpha_s(m_c)}{\pi}\frac{\overline{\Lambda}}{2m_c}\frac{2(1+r-v{\cdot}v'r)}{(v{\cdot}v'+1)} \quad (3)$$

$$\frac{F_2}{G_1} = \frac{G_2}{G_1} = -\frac{\overline{\Lambda}}{2m_c}\frac{2}{(v{\cdot}v'+1)} - \frac{4}{3}\frac{\alpha_s(m_c)}{\pi}r - \frac{4}{3}\frac{\alpha_s(m_c)}{\pi}\frac{\overline{\Lambda}}{2m_c}\frac{2(1+r-v{\cdot}v'r)}{(v{\cdot}v'+1)} \quad (4)$$

$$\frac{F_3}{G_1} = -\frac{G_3}{G_1} = -\frac{\overline{\Lambda}}{2m_b}\frac{2}{(v{\cdot}v'+1)} \quad (5)$$

where

$$r = \frac{\log(v{\cdot}v' + \sqrt{(v{\cdot}v')^2 - 1})}{\sqrt{(v{\cdot}v')^2 - 1}}\,. \quad (6)$$

and $\overline{\Lambda}$ is an undetermined constant with unit mass dimensions, expected to be of order of the hadronic scale, $\overline{\Lambda} \sim 500$ MeV. If in Eqs. 3 – 5 one sets $\alpha_s(m_c) = 0$ and $\overline{\Lambda} = 0$, one recovers the zeroth order results of Ref. 6, while the results of Ref. 5 are obtained by allowing $\overline{\Lambda} \neq 0$ but with $\alpha_s(m_c) = 0$. Clearly there are also corrections of order $\alpha_s(m_b)$ and of higher order in $1/M_Q$.

Heavy quark symmetries give the value of the form factors at zero recoil. In the leading-log approximation

$$G_1(1) = \left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{a_I} \quad (7)$$

There are no corrections of order $1/M_Q$ to this relation.[5, 4] The counterpart of this prediction for mesons is used in the measurement of the mixing angle $|V_{cb}|$.

The form factor relations 3 – 5 provide a valuable means for assessing the uncertainty in future measurements of the mixing angle $|V_{cb}|$. It is reasonable to expect the prediction in Eq. 7 to hold to the same accuracy with which the form factors satisfy the predicted relations, at least for small or moderate $v{\cdot}v' - 1$.

3.2 *Relations To All Orders In* $1/m_c$

The relations above were obtained by expanding both in $1/m_c$ and $1/m_b$. Because the charm quark is only a few times heavier than typical hadronic scales, the corrections to the relations 3 – 5 may be large. Remarkably, Mannel and Roberts obtain four relations among the six form factors without assumptions on the size of m_c.[7] Expanding in $1/m_b$, *i.e.,* using the HQET for the b quark, the spin symmetry acting on the b quark alone is enough to limit to two the number of independent form factors in $\Lambda_b \to \Lambda_q$, where $q = u, c$:

$$\langle \Lambda_q(p', s')|\bar{q}\Gamma h_v^{(b)}|\Lambda_b(v, s)\rangle = \bar{u}(p', s')[f_1(v{\cdot}p') + \not{p} f_2(v{\cdot}p')]\Gamma u(v, s) \tag{8}$$

It is straightforward to write the six form factors in Eqs. 1 – 2 in terms of the two form factors in Eq. 8. Explicit relations between the form factors follow from eliminating $f_{1,2}$ from Eq. 8:

$$F_1 = G_1 - G_2 \tag{9}$$
$$F_2 = G_2 \tag{10}$$
$$F_3 = 0 \tag{11}$$
$$G_3 = 0 \tag{12}$$

These remarkably simple expressions receive corrections in order $1/m_b$ and $\alpha_s(m_b)/\pi$, but are valid for arbitrary m_q (provided $m_q < m_b$). Moreover, the perturbative corrections $\sim \alpha_s(m_b)/\pi$ are computable; the leading correction is obtained by replacing[8]

$$\Gamma \to \Gamma - \frac{\alpha_s(m_b)}{6\pi}\gamma_\mu \not{p}\Gamma \not{p}\gamma^\mu \tag{13}$$

in Eq. 8.

By taking the limit $m_b \to \infty$, one readily checks that Eqs. 3 – 5 are consistent with Eqs. 9 – 12.

4. FACTORIZATION

4.1 *Summary of Theory*

Consider purely hadronic B-meson decays into singly charmed final states. I have in mind the class of processes that includes $B \to D\pi$, $B \to D^*\pi$, $B \to D\rho$, etc. The interaction Hamiltonian density mediating these decays is

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [c_1 \bar{b}_L \gamma_\mu c_L \bar{u}_L \gamma^\mu d_L + c_2 \bar{b}_L \gamma_\mu T^a c_L \bar{u}_L \gamma^\mu T^a d_L] \ , \tag{14}$$

where $c_{1,2}$ are calculable short distance QCD corrections, T^a are color octet matrices, and q_L stands for a left handed quark. The second term in $\mathcal{H}$ arises from short distance QCD

effects. Factorization in a particular decay, say $B \to D\pi$ is the statement that the following equation is true:

$$\langle D\pi|\mathcal{H}|B\rangle = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* c_1 \langle D|\bar{b}_L \gamma_\mu c_L|B\rangle \langle \pi|\bar{u}_L \gamma^\mu d_L|0\rangle \tag{15}$$

If factorization holds, the rate for the hadronic decay (the left hand side in eq. (9)) is given in terms of a meson decay constant ($\langle \pi(q)|\bar{u}_L \gamma^\mu d_L|0\rangle = i f_\pi q^\mu$) and the form factors for $B \to D$ at a fixed momentum transfer (that is $\langle D|\bar{b}_L \gamma_\mu c_L|B\rangle$ at $q^2 = M_\pi^2$).

Whether a particular matrix element factorizes is a dynamical issue that involves non-perturbative strong interactions, and is therefore hard to settle from first principles. We do know, nevertheless, that factorization does not hold for a large class of two body decays. In the case of K decays, the $\Delta I = 1/2$ rule is a stark reminder that simple factorization does not hold. More recently, a wealth of evidence *against* factorization in D-meson decays (as in $D \to K\pi$) has been amassed.[9]

To my knowledge there are two known theoretical approaches to demonstrating factorization. It holds in leading order in the $1/N_c$ expansion, where N_c is the number of colors in QCD.[10] And it holds in the leading order in the $1/M_Q$ expansion.[11]

Now, these approaches are rather different. The large N_c limit is fairly democratic: effectively, it predicts factorization in any meson decay into two meson final states, regardless of which flavors are involved in the transition. It does not predict, as far as I can tell, factorization in baryon decays (because the number of non-spectator diagrams, each suppressed by $1/N_c$, scales like N_c).

The large M_Q limit is fairly restrictive as to which transitions may exhibit factorization. It must be a transition of the form M $\to$ M′X where M and M′ are heavy hadrons, with their masses in a fixed ratio, both scaling with the large parameter M_Q, and X is a hadronic state with small invariant mass, that is, it's mass does not grow with M_Q. To the extent that the b and c quarks can be considered heavy, this approach can be used for $B \to D\pi$, and even for baryons as in $\Lambda_b \to \Lambda_c \pi$. But in the case of D decays this approach says nothing, since the final state does not involve any heavy quarks.

I will have nothing to say about phenomenological approaches to factorization.[12] My interest here is on what can be obtained from first principles, even if only in some approximation. Clearly we have a better chance of learning about dynamics if we concentrate on results that follow directly from QCD than on phenomenological approaches. It is for this reason also that we have nothing to say about decays such as $B \to \psi K$ which may very well factorize, but we don't know of any first principles justification for that to be the case. (In fact, one expects factorization in the inclusive resonant rate $B \to \psi X_s$, where by resonant we mean that the ψ is directly produced. P-wave charmonium production in B-meson decays is known not to factorize.[13] Consequently nonresonant inclusive ψ production won't either).

4.2 Comparison With Experiment

The large N_c approach is far too democratic: experimentally it is found that factorization does not hold in decays of heavy mesons to light mesons, or in light-to-light decays. In this section I intend to investigate the predictions of the large mass limit as far as factorization is concerned.

We start by considering qualitative statements implied by the arguments of Ref. 11. Feynman diagrams that don't factorize on account of the light quark in the initial heavy meson ending up in the light hadron in the final state are suppressed by $1/M_Q$. Now, the

only diagrams that contribute to $\bar{B}^0 \to D^0\pi^0$ are of this kind (and therefore $\bar{B}^0 \to D^0\pi^0$ does not itself factorize). Hence if factorization is to hold to some accuracy ϵ, the rate for $\bar{B}^0 \to D^0\pi^0$ ought to be suppressed relative to the rate for $\bar{B}^0 \to D^+\pi^-$ or $B^- \to D^0\pi^-$ by roughly ϵ^2.

A quick glance at the particle data book shows that $\bar{B}^0$ decays into $D^+\pi^-$, $D^+\rho^-$, $D^+a_1(1260)^-$, $D^*(2010)^+\pi^-$, $D^*(2010)^+\rho^-$ and $D^*(2010)^+a_1(1260)^-$ have been observed and have branching fractions in the 0.3% to 1.8% range. Non of the corresponding decays into D^0 or $D^*(2010)^0$ plus a neutral light meson have been observed. An upper bound exists on the branching fraction for $\bar{B}^0 \to D^0\rho^0$ of 6×10^{-4}. This is all as expected from the factorization argument in the paragraph above.

Quantitative, model independent,[14] tests of factorization are readily available. We will consider three kinds of such tests. The first two compare different two body decays which are related by a combined use of factorization and either isospin or heavy quark spin symmetries. In the third we compare some two body decays to corresponding semileptonic rates. The third is the most direct test, but is not available for as many processes. Also, it is interesting to see how well the other symmetries, and in particular heavy quark spin symmetry, work.

Using isospin symmetry on the factorized amplitudes, one obtains that the partial widths for the charged and the neutral meson decays into charmed two body decays should be equal. That is, one expects $\Gamma(\bar{B}^0 \to D^+\pi^-) \approx \Gamma(B^- \to D^0\pi^-)$ and similar relations for the other modes. These results are not predicted by isospin symmetry alone. The hamiltonian in Eq. 14 has $\Delta I = 0, 1$, while the B and D mesons are both $I = 1/2$ states, so the final $D\pi$ state is a combination of $I = 1/2$ and $I = 3/2$. There are three independent amplitudes, but they are not independent if factorization holds.

This can be tested assuming the total widths of the charged and neutral B-mesons are equal. It is seen that these relations hold to the present experimental accuracy. For example, the particle data book gives

$$\mathrm{Br}(B^- \to D^0\pi^-) = (3.8 \pm 1.1) \times 10^{-3} \tag{16}$$

while

$$\mathrm{Br}(\bar{B}^0 \to D^+\pi^-) = (3.2 \pm 0.7) \times 10^{-3} \tag{17}$$

and similar results for the other three modes mentioned above.

Since the factorized amplitude is given in terms of the semileptonic form factors, one can use heavy quark spin symmetry to relate the rates into D and D^* final states:

$$\Gamma(\bar{B} \to DX) = \Gamma(\bar{B} \to D^*X) \ . \tag{18}$$

This seems to work well, too. For example, from the particle data book

$$\mathrm{Br}(\bar{B}^0 \to D^*(2010)^+\pi^-) = (3.2 \pm 0.7) \times 10^{-3} \tag{19}$$

to be compared with $\mathrm{Br}(\bar{B}^0 \to D^+\pi^-)$ in Eq. 17 above. It is remarkable that both factorization and heavy quark spin symmetry can be tested simultaneously and that both seem to work rather well.

Table 1 shows CLEO II measured branching fractions.[15] The two columns are related by spin symmetry (if factorization holds). We group lines into pairs for the neutral and charged B decays. Thus the combined result of factorization, isospin symmetry, heavy

quark spin symmetry and the assumption of equal B^0 and B^+ lifetimes, is that all entries in each 2×2 block are equal. It can be seen that, within experimental errors this is the case. It is intriguing that the central values of all of the $\bar{B}^0$ decays are about 70% of the corresponding B^-. If this is a real effect it could be evidence against factorization. It could also be interpreted as evidence for different B^0 and B^+ lifetimes, $\tau(B^0)/\tau(B^+) \sim 0.7$. But this is hard to reconcile with direct results from the DELPHI[16] and ALEPH[17] experiments, which tend to favor $\tau(B^0)/\tau(B^+) > 1$.

Table 1. Some CLEO II Branching Fractions

Decay	Branching Fraction	Decay	Branching Fraction
$B^- \to D^0\pi^-$	$0.40 \pm 0.03 \pm 0.09$	$B^- \to D^*(2010)^0\pi^-$	$0.35 \pm 0.05 \pm 0.12$
$\bar{B}^0 \to D^+\pi^-$	$0.26 \pm 0.03 \pm 0.06$	$\bar{B}^0 \to D^*(2010)^+\pi^-$	$0.27 \pm 0.04 \pm 0.06$
$B^- \to D^0\rho^-$	$1.02 \pm 0.11 \pm 0.29$	$B^- \to D^*(2010)^0\rho^-$	$1.14 \pm 0.16 \pm 0.37$
$\bar{B}^0 \to D^+\rho^-$	$0.71 \pm 0.10 \pm 0.21$	$\bar{B}^0 \to D^*(2010)^+\rho^-$	$0.73 \pm 0.10 \pm 0.16$

If factorization holds, the degree of polarization in the decay $\bar{B}^0 \to D^*(2010)^+\rho^-$ can be predicted in terms of the degree of polarization in the semileptonic decay:[15]

$$\frac{\Gamma_L}{\Gamma}(\bar{B}^0 \to D^*(2010)^+\rho^-) = \frac{d\Gamma_L}{d\Gamma}(\bar{B}^0 \to D^*(2010)^+\ell\nu)|_{m^2_{\ell\nu}=m^2_\rho} \tag{20}$$

Here the differential rates on the right hand side are with respect to the invariant lepton pair mass, $m^2_{\ell\nu}$. The CLEO collaboration finds

$$\frac{\Gamma_L}{\Gamma}(\bar{B}^0 \to D^*(2010)^+\rho^-) = 0.90 \pm 0.07 \pm 0.05 \tag{21}$$

while the expected value from the semileptonic decay is 85% – 88%.

Finally, the most direct test of factorization is obtained by comparing directly both sides of Eq. 15, or equivalently by testing whether Bjorken's ratio

$$R_\pi \equiv \frac{\Gamma(\bar{B}^0 \to D^*(2010)^+\pi^-)}{d\Gamma(\bar{B}^0 \to D^*(2010)^+\ell\nu)/dm^2_{\ell\nu}|_{m^2_{\ell\nu}=M^2_\pi}} \tag{22}$$

agrees with the expectation from factorization:

$$R_\pi = 6\pi^2 f_\pi^2 c_1^2 \tag{23}$$

Similar expressions can be written with the pion replaced by some other final state. Experimentally, the ratios R_π and R_ρ for the neutral meson decay have been studied. The results of CLEO II measurements and the expectations from factorization are summarized in Table 2.[15]

Table 2. CLEO II Results on Bjorken's Ratios

	Experiment	Factorization
R_π	$1.3 \pm 0.2 \pm 0.3$	1.2 ± 0.2
R_ρ	$3.2 \pm 0.4 \pm 0.7$	3.3 ± 0.6

5. REPARAMETRIZATION INVARIANCE

There is an ambiguity in assigning a four-velocity, v, and residual momentum, k, to a particle in the HQET. Recall that only the momentum $p = Mv + k$ has physical significance. One may shift both the velocity and residual momentum to obtain the same physical momentum:

$$v \rightarrow v + q/M \tag{24}$$

$$k \rightarrow k - q \tag{25}$$

The only constraint on the vector q is that the new four-velocity be properly normalized:

$$(v + q/M)^2 = 1 \tag{26}$$

The effective field theory must be invariant under these reparametrizations.[18] The reparametrizations mix different orders in $1/M$. Hence, one can use reparametrization invariance to put constraints on the form of the $1/M$ corrections.[19]

As an example of an application consider the matrix element of the vector current between two pseudoscalar mesons. When using the HQET to order $1/M$ it is important to include in the description of the states both the velocity label v and the residual momentum k:

$$\langle v, k' | V_\mu | v, k \rangle = f_1 v_\mu + f_2(k_\mu + k'_\mu) + f_3(k_\mu - k'_\mu) \ . \tag{27}$$

Here V_μ stands for the heavy quark current including $1/M$ corrections. Now, in the "full theory", that is, the theory without any large mass expansion, there are only two independent form factors, usually denoted by f_+ and f_-. It shouldn't be necessary to introduce three form factors in the effective theory. This is implied by reparametrization invariance, which gives the relation

$$f_2 = \frac{1}{2M} f_1 \tag{28}$$

Of more practical importance is the use of reparametrization invariance to constrain the form of the heavy quark current in the effective theory. The heavy quark vector current has a $1/M$ expansion[2]

$$\sum_i C^{(i)}(v{\cdot}v')\mathcal{O}_i^{(0)} + \frac{1}{2M_Q}\sum_j D^{(j)}(v{\cdot}v')\mathcal{O}_j^{(1)} + \frac{1}{2M_{Q'}}\sum_j D'^{(j)}(v{\cdot}v')\mathcal{O}_j^{(1)} \tag{29}$$

where $\mathcal{O}_i^{(0)}$ and $\mathcal{O}_j^{(1)}$ stand for vector operators of dimension three and four respectively with $\overline{Q}'_{v'}Q_v$ quantum numbers, and their coefficients C, D and D' are perturbatively calculable. For example, at tree level the current is

$$\overline{Q}'_{v'}\gamma_\mu Q_v + \frac{1}{2M_Q}\overline{Q}'_{v'}\gamma_\mu i\not{D}Q_v - \frac{1}{2M_{Q'}}\overline{Q}'_{v'}i\overleftarrow{\not{D}}\gamma_\mu Q_v \tag{30}$$

where we have used the equations of motion, $v{\cdot}DQ_v = 0$. Now, the vector current in Eq. 29 will be reparametrization invariant if and only if it depends on the velocities v and v' in the combinations

$$v_\mu + k_\mu/M_Q \qquad \text{and} \qquad v'_\mu + k'_\mu/M_{Q'} \tag{31}$$

or in operator language

$$v_\mu + iD_\mu/M_Q \qquad \text{and} \qquad v'_\mu - i\overleftarrow{D}_\mu/M_{Q'} \tag{32}$$

Consider, for example, the following leading term in Eq. 29

$$C^{(1)}(v\cdot v')\overline{Q}'_{v'}\gamma_\mu Q_v = \overline{Q}'_{v'}\left(\frac{1+\not{v}'}{2}\right)C^{(1)}(v\cdot v')\gamma_\mu\left(\frac{1+\not{v}}{2}\right)Q_v \tag{33}$$

It must appear in the following combination to be invariant under separate reparametrizations of v and v'

$$\begin{aligned}\overline{Q}'_{v'}&\left(\frac{1+\not{v}'-i\overleftarrow{\not{D}}/M_{Q'}}{2}\right)C^{(1)}((v'-i\overleftarrow{D}/M_{Q'})\cdot(v+iD/M_Q))\gamma_\mu\left(\frac{1+\not{v}+iD/M_Q}{2}\right)Q_v\\ &= C^{(1)}(v\cdot v')\left[\overline{Q}'_{v'}\gamma_\mu Q_v + \frac{1}{2M_Q}\overline{Q}'_{v'}\gamma_\mu i\not{D}Q_v - \frac{1}{2M_{Q'}}\overline{Q}'_{v'}i\overleftarrow{\not{D}}\gamma_\mu Q_v\right]\\ &\quad+\frac{dC^{(1)}}{dv\cdot v'}\left[\frac{1}{M_Q}\overline{Q}'_{v'}\gamma_\mu v'\cdot DQ_v - \frac{1}{M_{Q'}}\overline{Q}'_{v'}v\cdot\overleftarrow{D}\gamma_\mu Q_v\right]+\cdots\end{aligned} \tag{34}$$

In a similar manner the coefficients of other dimension four operators can be constrained by applying the same method to the other two dimension three operators, $\overline{Q}'_{v'}v_\mu Q_v$ and $\overline{Q}'_{v'}v'_\mu Q_v$.

The calculation leading to the $1/m_c$ corrections in $\Lambda_b \to \Lambda_c e\nu$ required the coefficients of the vector and axial currents to order $1/m_c$. It is easy to check that the coefficients used[4] to obtain the relations in Eqs. 3 – 5 satisfy the constraints from reparametrization invariance. The calculation there would have been simplified vastly had reparametrization invariance been used to obtain the result. (Alternatively, reparametrization invariance gives an independent test of the calculation).

6. CHIRAL SYMMETRY TOO

6.1 Generalities

Chiral symmetry and soft pion theorems have been used in particle physics for several decades now with great success. The most efficient way of extracting information from chiral symmetry is by writing a phenomenological lagrangian for pions that incorporates both the explicitly realized vector symmetry and the non-linearly realized spontaneously broken axial symmetry.[20] Theorems that simultaneously use heavy quark symmetries and chiral symmetries are most expediently written by means of a phenomenological lagrangian for pions and heavy mesons that incorporates these symmetries.[21, 22]

In the limit $m_b \to \infty$, the $\overline{B}$ and the $\overline{B}^*$ mesons are degenerate, and to implement the heavy quark symmetries it is convenient to assemble them into a "superfield" $H_a(v)$:

$$H_a(v) = \frac{1+\not{v}}{2}\left[\overline{B}^{*\mu}_a\gamma_\mu - \overline{B}_a\gamma^5\right]. \tag{35}$$

Here v^μ is the fixed four-velocity of the heavy meson, and a is a flavor $SU(3)$ index corresponding to the light antiquark. Because we have absorbed mass factors $\sqrt{2m_B}$ into the fields, they have dimension 3/2; to recover the correct relativistic normalization, we will multiply amplitudes by $\sqrt{2m_B}$ for each external $\overline{B}$ or $\overline{B}^*$ meson.

The chiral lagrangian contains both heavy meson superfields and pseudogoldstone bosons, coupled together in an $SU(3)_L \times SU(3)_R$ invariant way. The matrix of pseudogold-

stone bosons appears in the usual exponentiated form $\xi = \exp(i\mathcal{M}/f)$, where

$$\mathcal{M} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi_0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\frac{2}{3}}\,\eta \end{pmatrix}, \tag{36}$$

and f is the pion (or kaon) decay constant. The bosons couple to the heavy fields through the covariant derivative and axial vector field,

$$D^\mu_{ab} = \delta_{ab}\partial^\mu + V^\mu_{ab} = \delta_{ab}\partial^\mu + \tfrac{1}{2}\left(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger\right)_{ab}, \tag{37}$$

$$A^\mu_{ab} = \frac{i}{2}\left(\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger\right)_{ab} = -\frac{1}{f}\partial_\mu\mathcal{M}_{ab} + \mathcal{O}(\mathcal{M}^3). \tag{38}$$

Lower case roman indices correspond to flavor $SU(3)$. Under chiral $SU(3)_L \times SU(3)_R$, the pseudogoldstone bosons and heavy meson fields transform as $\xi \to L\xi U^\dagger = U\xi R^\dagger$, $A^\mu \to UA^\mu U^\dagger$, $H \to HU^\dagger$ and $(D^\mu H) \to (D^\mu H)U^\dagger$, where the matrix U_{ab} is a nonlinear function of the pseudogoldstone boson matrix $\mathcal{M}$.

The chiral lagrangian is an expansion in derivatives and pion fields, as well as in inverse powers of the heavy quark mass. The kinetic energy terms take the form

$$\mathcal{L}_{\rm kin} = \frac{1}{8}f^2\,\partial^\mu\Sigma_{ab}\,\partial_\mu\Sigma^\dagger_{ba} - \mathrm{Tr}\left[\overline{H}_a(v)iv\cdot D_{ba}H_b(v)\right], \tag{39}$$

where $\Sigma = \xi^2$. Here the trace is in the space of 4×4 Dirac matrices that define the "superfields" $H_a(v)$ in Eq. 35. The leading interaction term is of dimension four,

$$\mathcal{L}_{\rm int} = g\,\mathrm{Tr}\left[\overline{H}_a(v)H_b(v)\,\not{A}_{ba}\gamma^5\right], \tag{40}$$

where g is an unknown parameter, of order one in the constituent quark model. The analogous term in the charm system is responsible for the decay $D^* \to D\pi$. Expanding the term in the lagrangian in 40 to linear order in the Goldstone Boson fields, $\mathcal{M}$, we find the explicit forms for the $D^*D\mathcal{M}$ and $D^*D^*\mathcal{M}$ couplings

$$\left[\left(\frac{-2g}{f}\right)D^{*\nu}\partial_\mu\mathcal{M}D^\dagger + \mathrm{h.c.}\right] + \left(\frac{2gi}{f}\right)\epsilon_{\mu\nu\lambda\kappa}D^{*\mu}\partial^\nu\mathcal{M}D^{*\lambda}v^\kappa. \tag{41}$$

Using this one can compute the partial width

$$\Gamma(D^{*+}\to D^0\pi^+) = \frac{g^2}{6\pi f^2}|\vec{p}_\pi|^3 \tag{42}$$

$$\Gamma(D^{*+}\to D^+\pi^0) = \frac{g^2}{12\pi f^2}|\vec{p}_\pi|^3 \tag{43}$$

The ACCMOR collaboration has reported an upper limit of 131 KeV on the D^* width.[23] The branching fractions for $D^{*+} \to D^0\pi^+$ and $D^{*+} \to D^+\pi^0$ are $(68.1 \pm 1.0 \pm 1.3)\%$ and $(30.8 \pm 0.4 \pm 0.8)\%$, respectively, as measured by the CLEO collaboration.[24] Using $f = 130$ MeV, one obtains the limit $g^2 < 0.5$. Even if the D^* decay width is too small to measure, radiative D^* decays provide an indirect means for determining the coupling g, and provide a lower bound $g^2 \gtrsim 0.1$.[25]

Since charmed and beauty baryons are long lived, one can write down phenomenological lagrangians for their interactions with pions. These are as well justified and should be as good an approximation as the lagrangian for heavy mesons discussed above. The treatment is rather similar, and due to space limitations, we refer the interested reader to the literature.[26]

6.2 $B \to De\nu$ and $B \to D^*\pi e\nu$

As a first example of an application consider a soft pion theorem that relates the amplitudes for $B \to D^*e\nu$ and $B \to D^*\pi e\nu$.[22] The heavy quark current is represented in the phenomenological lagrangian approach by

$$J_\mu^{\bar{c}b} = \bar{h}_{v'}^{(c)}\gamma_\mu(1-\gamma_5)h_v^{(b)} \to \xi(v\cdot v')\mathrm{Tr}\overline{H}_a^{(c)}(v')\gamma_\mu(1-\gamma_5)H_a^{(b)}(v) + \cdots \tag{44}$$

where the ellipsis denote terms with derivatives, factors of light quark masses m_q, or factors of $1/M_Q$, and $\xi(v\cdot v')$ is the Isgur-Wise function. The leading term in Eq. 44 is independent of the pion field. Therefore, it is pole diagrams that dominate the amplitude for semileptonic $B \to D\pi$ and $B \to D^*\pi$ transitions; see Fig. 1. These pole diagrams are calculable in this approach, and are determined by the Isgur-Wise function and the coupling g.

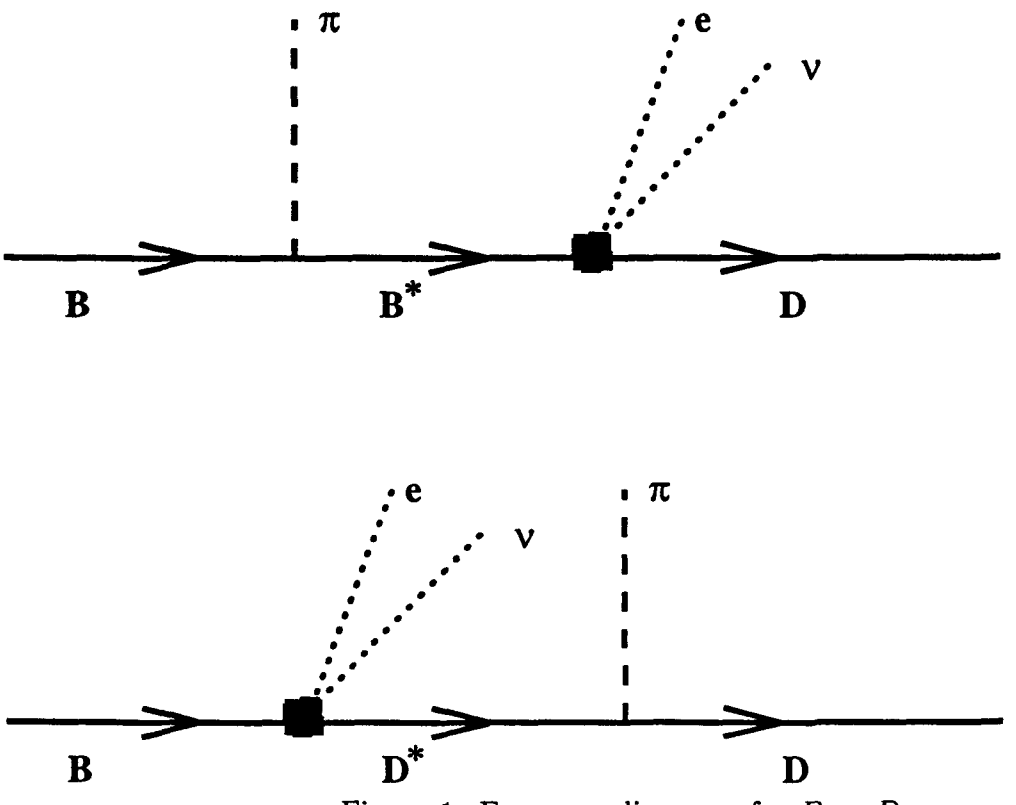

Figure 1. Feynman diagrams for $B \to De\nu$

A straightforward calculation gives

$$\begin{aligned}\langle D(v')\pi^a(q)|J_\mu^{\bar{c}b}|B(v)\rangle &= iu(B)^*\tfrac{1}{2}\tau^a u(D)\sqrt{M_B M_D}\frac{g}{f}\xi(v\cdot v') \\ &\times \left\{\frac{1}{v\cdot q}[i\epsilon_{\mu\nu\lambda\kappa}q^\nu v'^\lambda v^\kappa + q\cdot(v+v')v_\mu - (1+v\cdot v')q_\mu\right. \\ &\left. -\frac{1}{v'\cdot q}[i\epsilon_{\mu\nu\lambda\kappa}q^\nu v'^\lambda v^\kappa + q\cdot(v+v')v'_\mu - (1+v\cdot v')q_\mu\right\} \qquad (45)\end{aligned}$$

where $u(M)$ stands for the isospin wavefunction of meson M. A similar but lengthier expression is found for $B \to D^*\pi e\nu$.[22] If the coupling g is close to its upper limit, this process could be an important correction to the inclusive semileptonic rate. It may, perhaps, account for some of the anomalously large "D^{**}" contributions observed by CLEO.[27]

6.3 *Violations To Chiral Symmetry*

Phenomenological lagrangians are particularly well suited to explore deviations from symmetry predictions. In the context of heavy mesons, several quantities of considerable interest have been studied. Moreover, the self-consistency of the approach has been explored. It would be impossible to cover all of this in this talk. I will briefly comment on a few of those results, and invite you to consult the references for further details.

In order to study violations of chiral symmetry, one must introduce symmetry breaking terms into the phenomenological lagrangian. The light quark mass matrix $m_q = \mathrm{diag}(m_u, m_d, m_s)$ parametrizes the violations to flavor $SU(3)_V$. To linear order in m_q and lowest order in the derivative expansion, the correction to the phenomenological lagrangian is

$$\begin{aligned}\Delta\mathcal{L} &= \lambda_0\left[m_q\Sigma + m_q\Sigma^\dagger\right]^a{}_a \\ &+ \lambda_1 \mathrm{Tr}\bar{H}^{(Q)a}H_b^{(Q)}\left[\xi m_q\xi + \xi^\dagger m_q\xi^\dagger\right]^b{}_a \\ &+ \lambda_1' \mathrm{Tr}\bar{H}^{(Q)a}H_a^{(Q)}\left[m_q\Sigma + m_q\Sigma^\dagger\right]^b{}_b \qquad (46)\end{aligned}$$

The coefficients λ_0, λ_1 and λ_1' are determined by non-perturbative strong interaction effects, but may be determined phenomenologically. We postpone consideration of mass relations obtained from this lagrangian until we have introduced heavy quark spin symmetry breaking terms into the lagrangian too.

The decay constants for the D and D_s mesons, defined by

$$\langle 0|\bar{d}\gamma_\mu\gamma_5 c|D^+(p)\rangle = if_D p_\mu \qquad (47)$$

and

$$\langle 0|\bar{s}\gamma_\mu\gamma_5 c|D_s(p)\rangle = if_{D_s} p_\mu\ , \qquad (48)$$

determine the rate for the purely leptonic decays $D^+ \to \mu^+\nu_\mu$ and $D_s \to \mu^+\nu_\mu$. These are likely to be measured in the future.[28] In the chiral limit, where the up, down and strange quark masses go to zero, flavor $SU(3)_V$ is an exact symmetry and so $f_{D_S}/f_D = 1$. However $m_s \neq 0$, so this ratio will deviate from unity. Calculating this involves, at one loop, the Feynman diagrams in Fig. 2, where a dashed line stands for a light pseudoscalar propagator.

Neglecting the up and down quark masses in comparison with the strange quark mass, this deviation has been calculated to be[29, 30]

$$f_{D_s}/f_D = 1 - \frac{5}{6}\left(1+3g^2\right)\frac{M_K^2}{16\pi^2 f^2}\ln\left(M_K^2/\mu^2\right) + \lambda(\mu)M_K^2 + \ldots \tag{49}$$

where the ellipsis denote terms with more powers of the strange quark mass (recall $M_K^2 \sim m_s$). The dependence of λ on the subtraction point μ cancels that of the logarithm. If μ is of order the chiral symmetry breaking scale then λ has no large logarithms and for very small m_s the explicit logarithm dominates the deviation of f_{D_S}/f_D from unity. In Eq. 49 the contribution from η loops has been written in terms of M_K using the Gell-Mann–Okubo formula $M_\eta^2 = 4M_K^2/3$, and the contribution from pion loops, proportional to $M_\pi^2 \ln M_\pi^2$, has been neglected. Numerically, using $\mu = 1$ GeV, the result is that

$$f_{D_s}/f_D = 1 + 0.064\ (1+3g^2), \tag{50}$$

or $f_{D_s}/f_D = 1.16$ for $g^2 = 0.5$.

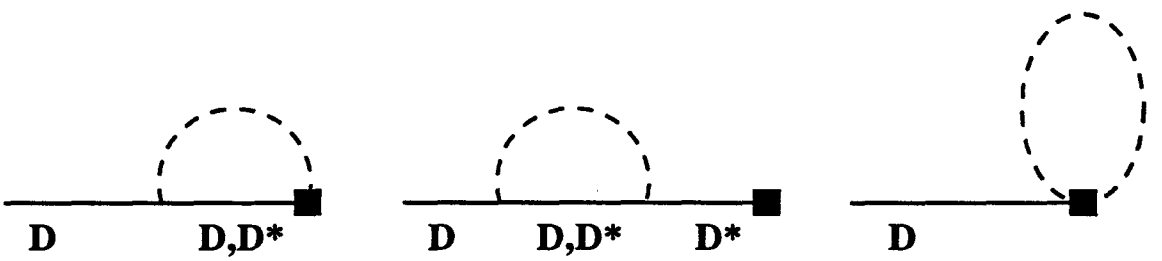

Figure 2. Feynman Diagrams in the calculation of f_{D_s}/f_D.

The same formula also holds for f_{B_s}/f_B. In fact, to leading order in $1/M_Q$ the ratio is independent of the the flavor of the heavy quark. Consequently,

$$\frac{f_{B_s}/f_B}{f_{D_s}/f_D} = 1 \tag{51}$$

to leading order in $1/M_Q$ and all orders in the light quark masses. Now, Eq. 51 also holds as a result of chiral symmetry, for any m_c and m_b. That is f_{B_s}/f_B and f_{D_s}/f_D are separately unity in the limit in which the light quark masses are equal. This means that deviations from unity in Eq. 51 must be small, $O(m_s) \times O(1/m_c - 1/m_b)$.[31] This ratio of ratios is observed to be very close to unity in a variety of calculations.[32] This may be very useful, since it suggests obtaining the ratio f_{B_s}/f_B of interest in the analysis of $B - \bar{B}$ mixing (see below) from the ratio f_{D_s}/f_D, measurable from leptonic D and D_s decays.

The hadronic matrix elements needed for the analysis of $B - \bar{B}$ mixing are

$$\langle \bar{B}(v)|\bar{b}\gamma^\mu(1-\gamma_5)d\ \bar{b}\gamma^\mu(1-\gamma_5)d|B(v)\rangle = \frac{8}{3}f_B^2 B_B\ , \tag{52}$$

$$\langle \bar{B}_s(v)|\bar{b}\gamma^\mu(1-\gamma_5)s\ \bar{b}\gamma^\mu(1-\gamma_5)s|B_s(v)\rangle = \frac{8}{3}f_{B_s}^2 B_{B_s}\ , \tag{53}$$

where the right hand side of these equations define the parameters B_{B_s} and B_B. In the $SU(3)_V$ symmetry limit $B_{B_s}/B_B = 1$. For non-zero strange quark mass, the ratio is no longer unity. The chiral correction is[29]

$$\frac{B_{B_s}}{B_B} = 1 - \frac{2}{3}\left(1 - 3g^2\right)\frac{M_K^2}{16\pi^2 f^2}\ln\left(M_K^2/\mu^2\right) . \tag{54}$$

Again, $M_\eta^2 = 4M_K^2/3$ has been used. Using $\mu = 1$ GeV, $f = f_K$, and $g^2 = 0.5$, the correction is $B_{B_s}/B_B \approx 0.95$.

Violations to chiral symmetry in $B \to D$ semileptonic decays have also been studied. One obtains that a different Isgur-Wise function must be used for each flavor of light spectator quark[30]

$$\frac{\xi_s(v{\cdot}v')}{\xi_{u,d}(v{\cdot}v')} = 1 + \frac{5}{3}g^2\Omega(v{\cdot}v')\frac{M_K^2}{16\pi^2 f^2}\ln\left(M_K^2/\mu^2\right) + \lambda'(\mu, v{\cdot}v')M_K^2 + \cdots \tag{55}$$

where

$$\Omega(x) = -1 + \frac{2+x}{2\sqrt{x^2-1}}\ \ln\left(\frac{x+1+\sqrt{x^2-1}}{x+1-\sqrt{x^2-1}}\right) + \frac{x}{4\sqrt{x^2-1}}\ \ln\left(\frac{x-\sqrt{x^2-1}}{x+\sqrt{x^2-1}}\right) \tag{56}$$

or, expanding about $x = 1$,

$$\Omega(x) = -\frac{1}{3}(x-1) + \frac{2}{15}(x-1)^2 - \frac{2}{35}(x-1)^3 + \cdots \tag{57}$$

Using $g^2 = 0.5$ and $\mu = 1$ GeV, and neglecting the counterterm one obtains

$$\frac{\xi_s(v{\cdot}v')}{\xi_{u,d}(v{\cdot}v')} = 1 - 0.21\,\Omega(v{\cdot}v') + \cdots \tag{58}$$

or a 5% correction at $v{\cdot}v' = 2$.

6.4 Violations to Heavy Quark Symmetry

In a similar spirit one can consider the corrections in chiral perturbation theory to predictions that follow from heavy quark spin and flavor symmetries. These are effects that enter at order $1/M_Q$, so the first step towards this end is to supplement the phenomenological lagrangian with such terms. In particular, the only $SU(3)_V$ preserving term of order $1/M_Q$ that violates spin symmetry in the lagrangian is[29]

$$\Delta\mathcal{L}_{\rm int} = \frac{\lambda_2}{M_Q}{\rm Tr}\bar{H}^{(Q)a}\sigma^{\mu\nu}H_a^{(Q)}\sigma_{\mu\nu} . \tag{59}$$

In addition there are contributions to the lagrangian in order $1/M_Q$ that violate flavor but not spin symmetries. These can be characterized as introducing M_Q dependence in the couplings g, λ_1 and λ_1' of Eqs. 40 and 46. At the same order as these corrections, there is a term that violates both spin and $SU(3)_V$ symmetries

$$\Delta\mathcal{L}_{\rm int} = \frac{\lambda_3}{M_Q}{\rm Tr}\left[\bar{H}^{(Q)a}\sigma^{\mu\nu}H_b^{(Q)}\sigma_{\mu\nu}\right]m_q{}^b{}_a \tag{60}$$

Spin symmetry violation is responsible for "hyperfine" splittings in spin multiplets. To leading order these mass splittings are computed in terms of the spin symmetry violating coupling of Eq. 59

$$\Delta_B \equiv M_{B^*} - M_B = -\frac{8\lambda_2}{m_b} \tag{61}$$

That the mass splittings scale like $1/M_Q$ seems to be well verified in nature:

$$\frac{M_{D^*} - M_D}{M_{B^*} - M_B} \approx \frac{M_B}{M_D} \tag{62}$$

Table 3. Measured Mass Splittings

$X - Y$	$M_X - M_Y$ (MeV)
$D_s - D^+$	99.5 ± 0.6[33]
$D^+ - D^0$	$4.80 \pm 0.10 \pm 0.06$[34]
$D^{*+} - D^{*0}$	$3.32 \pm 0.08 \pm 0.05$[34]
$D^{*0} - D^0$	$142.12 \pm 0.05 \pm 0.05$[34]
$D^{*+} - D^+$	$140.64 \pm 0.08 \pm 0.06$[34]
$D_s^* - D_s$	141.5 ± 1.9[33]
$B_s - B$	82.5 ± 2.5[34] or 121 ± 9[35]
$B^0 - B^+$	0.01 ± 0.08[33]
$B^* - B$	$46.2 \pm 0.3 \pm 0.8$[36] or 45.4 ± 1.0[35]
$B_s^* - B_s$	47.0 ± 2.6[35]
$(D^{*0} - D^0)$ $-(D^{*+} - D^+)$	$1.48 \pm 0.09 \pm 0.05$[34]

Armed with the machinery of chiral lagrangians that include both spin and chiral symmetry violating terms, one can compare hyperfine splitting for different flavored mesons. There is a wealth of experimental information to draw from; see Table 3. Breaking of flavor $SU(3)_V$ and heavy quark flavor symmetries by electromagnetic effects is not negligible. It is readily incorporated into the lagrangian in terms of the charge matrices $Q_Q = \mathrm{diag}(2/3, -1/3)$ and $Q_q = \mathrm{diag}(2/3, -1/3, -1/3)$,[37] which must come in bilinearly. For example, terms involving Q_q^2 correspond to replacing $m_q \to Q_q$ in Eqs. 46 and 60. The electromagnetic effects of the light quarks can be neglected if one considers only mesons with d and s light quarks. The electromagnetic shifts in the hyperfine splittings Δ_{X_q} and Δ_{X_q} ($X = D, B$, $q = d, s$) differ on account of different b and c charges, but they cancel in the difference of splittings

$$\Delta_{X_s} - \Delta_{X_d} = (M_{X_s^*} - M_{X_s}) - (M_{X_d^*} - M_{X_d}) \tag{63}$$

The only term in the phenomenological lagrangian that enters this difference is Eq. 60. This

immediately leads to

$$(M_{B_s^*}-M_{B_s})-(M_{B_d^*}-M_{B_d}) = (m_c/m_b)\left(\frac{\bar{\alpha}_s(m_c)}{\bar{\alpha}_s(m_b)}\right)^{9/25}\left[(M_{D_s^*}-M_{D_s})-(M_{D_d^*}-M_{D_d})\right] \quad (64)$$

We have included here the short distance QCD effect that is usually neglected.[38]

The accuracy with which Eq. 64 holds is to be much better than the separate relations for each hyperfine splitting in Eq. 61. Recall that $SU(3)_V$ breaking by light quark masses and electromagnetic interactions have been accounted for in leading order. Moreover, the result is trivially generalized by replacing the quark mass matrix in Eqs. 46 and 60, by an arbitrary function of the light quark mass matrix. It is seen from Table 3 that this relation works well. The left side is 1.2 ± 2.7 MeV while the right side is 3.0 ± 6.3 MeV.

Since both sides of Eq. 64 are consistent with zero and both are proportional to the interaction term in Eq. 60, it must be that the coupling λ_3 is very small.[37] From the difference of hyperfine splittings in the charm sector

$$-\frac{8\lambda_3}{m_c}(m_s - m_d) = 0.9 \pm 1.9 \text{ MeV} \quad (65)$$

while

$$M_{D_s} - M_{D_d} = 4\lambda_1(m_s - m_d) - \frac{12\lambda_3}{m_c}(m_s - m_d) = 99.5 \pm 0.6 \text{ MeV} \quad (66)$$

leading to $|\lambda_3/\lambda_1|$ less than ~ 20 MeV. This is smaller than expected by about an order of magnitude. With such a small coefficient it is clear that the next-to-leading terms and the loop corrections may play an important role. In particular they may invalidate the simple $1/M_Q$ scaling of Eq. 64.[39] There is no obvious breakdown of chiral perturbation theory, even though the leading coupling (λ_3) is anomalously small.[40]

At one loop, the expressions for the mass shifts involve large $O(m_s \ln m_s)$ and $O(m_s^{3/2})$ (non-analytic) terms.[30, 40] The coupling λ_3 is not anomalously small at one loop. Instead, the smallness of the difference of hyperfine splittings in Eq. 64 is the result of a precise cancellation between one loop and tree level graphs. Explicitly,[40]

$$\left(M_{X_s} - M_{X_s^*}\right) - \left(M_{X_d} - M_{X_d^*}\right) = \frac{5}{3}g^2\left(\frac{8\lambda_2}{M_Q}\right)\frac{M_K^2}{16\pi^2 f^2}\ln\left(M_K^2/\mu^2\right) - \frac{8\lambda_3}{M_Q}m_s \quad (67)$$

With $g^2 = 0.5$ and $\mu = 1$ GeV, the chiral log is 30 MeV, so the λ_3 counterterm must cancel this to a precision of better than 10%.

The $1/M_Q$ corrections to the masses M_X and M_{X^*} drop out of the combination $M_X + 3M_{X^*}$. The combination $(M_{X_s} + 3M_{X_s^*}) - (M_{X_d} + 3M_{X_d^*})$ is a measure of $SU(3)_V$ breaking by a non-vanishing m_s (or $m_s - m_d$ if the d quark mass is not neglected). It can be computed in the phenomenological lagrangian. To one loop[40]

$$\frac{1}{4}\left(M_{X_s} + 3M_{X_s^*}\right) - \frac{1}{4}\left(M_{X_d} + 3M_{X_d^*}\right) = 4\lambda_1 m_s - g^2\left(1 + \frac{8}{3\sqrt{3}}\frac{1}{2}\right)\frac{M_K^3}{16\pi f^2}$$
$$-4\lambda_1 m_s\left(\frac{25}{18} + \frac{9}{2}g^2\right)\frac{M_K^2}{16\pi^2 f^2}\ln\left(M_K^2/\mu^2\right) \quad (68)$$

The pseudoscalar splittings $(M_{D_s} - M_{D_d})$ and $(M_{B_s} - M_{B_d})$ have been measured; see Table 3. Also, $\frac{1}{4}(M_{X_s} + 3M_{X_s^*}) - \frac{1}{4}(M_{X_d} + 3M_{X_d^*}) = \frac{3}{4}[(M_{X_s^*} - M_{X_s}) - (M_{X_d^*} - M_{X_d})] + (M_{X_s} - M_{X_d})$,

and the term in square brackets is less than a few MeV, as we saw above. The combination $(M_{X_s}+3M_{X_s^*})-(M_{X_d}+3M_{X_d^*})$ in Eq. 68 is first order in m_s but has no corrections at order $1/M_Q$. Thus, one expects a similar numerical result for B and D systems. Experimentally, $(M_{B_s}-M_{B_d})/(M_{D_s}-M_{D_d})$ is consistent with unity; see Table 3. The formula in Eq. 68 has a significant contribution from the M_K^3 term which is independent of the splitting parameter λ_1. The M_K^3 term gives a negative contribution to the splitting of ~ -250 MeV for $g^2=0.5$. The chiral logarithmic correction effectively corrects the tree level value of the parameter λ_1; for $\mu=1$ GeV and $g^2=0.5$, the term $4\lambda_1 m_s$ gets a correction ≈ 0.9 times its tree level value. Thus, the one-loop value of $4\lambda_1 m_s$ can be significantly greater than the value determined at tree-level of approximately 100 MeV.

Chiral perturbation theory can be used to predict the leading corrections to the form factors for semileptonic $B\to D$ or D^* decays which are generated at low momentum, below the chiral symmetry breaking scale. Of particular interest are corrections to the predicted normalization of form factors at zero recoil, $v\cdot v'=1$. According to Luke's theorem (see section 2), long distance corrections enter first at order $1/M_Q^2$. Deviations from the predicted normalization of form factors that arise from terms of order $1/M_Q^2$ in either the lagrangian or the current are dictated by non-perturbative physics. But there are computable corrections that arise from the terms of order $1/M_Q$ in the lagrangian. These must enter at one-loop, since Luke's theorem prevents them at tree level, and result from the spin and flavor symmetry breaking in the hyperfine splittings Δ_D and Δ_B. Retaining only the dependence on the larger Δ_D, the correction to the matrix elements at zero recoil are[41]

$$\langle D(v)|J_\mu^{\bar{c}b}|B(v)\rangle = 2v_\mu\left(1-\frac{3g^2}{2}\left(\frac{\Delta_D}{4\pi f}\right)^2\left[F(\Delta_D/M_\pi)+\ln(\mu^2/M_\pi^2)\right]+C(\mu)/m_c^2\right) \tag{69}$$

$$\langle D^*(v,\epsilon)|J_\mu^{\bar{c}b}|B(v)\rangle = 2\epsilon_\mu^*\left(1-\frac{g^2}{2}\left(\frac{\Delta_D}{4\pi f}\right)^2\left[F(-\Delta_D/M_\pi)+\ln(\mu^2/M_\pi^2)\right]+C'(\mu)/m_c^2\right) \tag{70}$$

where C and C' stand for tree level counter-terms and

$$F(x)\equiv\int_0^\infty dz\frac{z^4}{(z^2+1)^{3/2}}\left(\frac{1}{[(z^2+1)^{1/2}+x]^2}-\frac{1}{z^2+1}\right) \tag{71}$$

As before, no large logarithms will appear in the functions C and C' if one takes $\mu\approx 4\pi f\sim 1$ GeV. With this choice, formally, their contributions are dwarfed by the term that is enhanced by a logarithm of the pion mass. Numerically, with $g^2=0.5$ the logarithmically enhanced term is -2.1% and -0.7% for D and D^*, respectively.

The function F accounts for effects of order $(1/m_c)^{2+n}$, $n=1,2,\ldots$ It is enhanced by powers of $1/M_\pi$ over terms that have been neglected. Consequently it is expected to be a good estimate of higher order $1/m_c$ corrections. With $\Delta_D/M_\pi\approx 1$, one needs $F(1)=14/3-2\pi$ and $F(-1)=14/3+2\pi$ for a numerical estimate; with μ and g^2 as above, this term is 0.9% and -2.0% for D and D^*, respectively.

6.5 Trouble on the Horizon?

I would like to point out a peculiar aspect of this result. The function $F(x)$ can be expanded in x starting at order x, as expected.[41] But it can also be expanded in $1/x$, and

the leading term is a logarithmic singularity $\sim -2\ln x$. Physically this limit corresponds to $M_\pi \to 0$ (rather than the absurd alternative $\Delta_D \to \infty$), and the logarithmic singularity is canceled by the $\ln(\mu^2/M_\pi^2)$ in Eqs. 69 and 70. Thus, the expansions in powers of x and $1/x$ correspond, in terms of physical limits, to expansions in powers of $1/m_c$ and M_π, respectively. These are alternative, but not equivalent, expansions. This troubles me some. It seems to indicate that the order of the limits $1/m_c \to 0$ and $M_\pi \to 0$ matters. But the phenomenological lagrangian for pions and heavy mesons implicitly assumes that one can systematically expand about the origin in $1/m_c$ – M_π space.

Frequently the non-analytic corrections to relations that follow from the symmetries are uncomfortably large. A case of much interest is the relation between the form factors $f_\pm$ and h for $B \to K$ transitions, relevant to the short distance process $b \to se^+e^-$,

$$\langle \overline{K}(p_K)\,|\,\overline{s}\gamma^\mu b\,|\,\overline{B}(p_B)\rangle = f_+\,(p_B+p_K)^\mu + f_-\,(p_B-p_K)^\mu\,, \tag{72}$$

$$\langle \overline{K}(p_K)\,|\,\overline{s}\sigma^{\mu\nu} b\,|\,\overline{B}(p_B)\rangle = ih\,[(p_B+p_K)^\mu(p_B-p_K)^\nu - (p_B+p_K)^\nu(p_B-p_K)^\mu]\,, \tag{73}$$

and the form factors for $B \to \pi e\nu$,

$$\langle \overline{\pi}(p_\pi)\,|\,\overline{u}\gamma^\mu b\,|\,\overline{B}(p_B)\rangle = \hat{f}_+\,(p_B+p_\pi)^\mu + \hat{f}_-\,(p_B-p_\pi)^\mu\,. \tag{74}$$

In the combined large mass and chiral limits only one of these form factors is independent:

$$m_b h = f_+ = -f_- = \hat{f}_+ = -\hat{f}_- \tag{75}$$

In this limit, the ratio of rates for $B \to Ke^+e^-$ and $B \to \pi e\nu$ is simply given, in the standard model of electroweak interactions, by $|V_{ts}/V_{ub}|^2$, times a perturbatively computable function of the top quark mass. If the relation 75 held to good accuracy one could thus measure a ratio of fundamental standard model parameters.*

The non-analytic, one-loop corrections to the relations in Eq. 75 have been computed.[42] The results are too lengthy to display here. Numerically, the violation to $SU(3)_V$ symmetry is found to be at the 40% level.†

The phenomenological lagrangian that we have been considering extensively neglects the effects of states with heavy-light quantum numbers other than the pseudoscalar – vector-meson multiplet. The splitting between multiplets is of the order of 400 MeV and is hardly negligible when one considers $SU(3)_V$ relations involving both π and K mesons. For example, consider the effect of the scalar – pseudovector-meson multiplet. One can incorporate its effects into the phenomenological lagrangian. To this end, assemble its components into a "superfield", akin to that in Eq. 35 for the pseudoscalar – vector multiplet:[43]

$$S_a(v) = \frac{1+\not{v}}{2}\left[B'^{\mu}_{1a}\gamma_\mu\gamma^5 - B^*_{0a}\right]\,. \tag{76}$$

The phenomenological lagrangian has to be supplemented with a kinetic energy and mass for S,

$$\mathrm{Tr}\left[\overline{S}_a(v)(iv\cdot D_{ba} - \Delta\delta_{ba})S_b(v)\right]\,, \tag{77}$$

*Another application of this relation was discussed by I. Dunietz in this workshop. Assuming factorization in $B \to \psi X$, ratios of CKM elements can be extracted from these two body hadronic decays. For more details, consult the talk by Dunietz, these proceedings.

†The large violation of $SU(3)_V$ symmetry affects as well the results of Dunietz (see previous footnote).

where Δ is the mass splitting for the excited S from the ground state H, and with coupling terms

$$g' \operatorname{Tr}\left[\overline{S}_a(v) S_b(v) \not{A}_{ba} \gamma^5\right] + (h \operatorname{Tr}\left[\overline{H}_a(v) S_b(v) \not{A}_{ba} \gamma^5\right] + \text{h.c.}) . \tag{78}$$

In terms of these one can now compute additional corrections to quantities such as f_{D_s}/f_D in Eq. 49. Numerically the corrections are not small,[44] $f_{D_s}/f_D = 1 + 0.13h^2$ for $M_{D_0^*} =$ 2300 MeV (or $f_{D_s}/f_D = 1 + 0.08h^2$ for $M_{D_0^*} = 2400$ MeV), assuming the strange mesons to be 100 MeV heavier. Similarly, corrections to the Isgur-Wise function can be computed, and are not negligible.[44]

7. CONCLUSIONS

Applications of heavy quark symmetries and of heavy quark effective theory methods abound. Many specific predictions have been made and can be tested. If the predictions work well we may feel confident in using these methods for a more lofty goal, that of interpreting experiments, be it for the measurement of fundamental parameters (as in $|V_{cb}|$) or in probing new physics at very short distances (as in $B \to K\ell^+\ell^-$).

Theorists are starting to understand the precision and limitations of the method. The warning flags of the previous section are a sign of the maturity research in this field has attained.

This is not to say the work is done. Many open questions remain. A salient issue is that of computation of form factors for semileptonic $b \to u$ decays. Even the inclusive rate cannot be computed at large electron energies,[‡] where it is measured with an aim at determining $|V_{ub}|$. Some remaining issues require improved input from experiment. For example, a better measurement of the entries in Table 1 and of the lifetimes of B^+ and B^0 would settle the issue of factorization discussed above.

Regardless of the nature of the machine that conducts the next generation beauty and charm experiments, Heavy Quark Effective Theory methods will play a salient role in the interpretation of the results.

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[‡]Space limitations keep me from discussing recent work on this issue. The interested reader may consult the literature.[45]

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