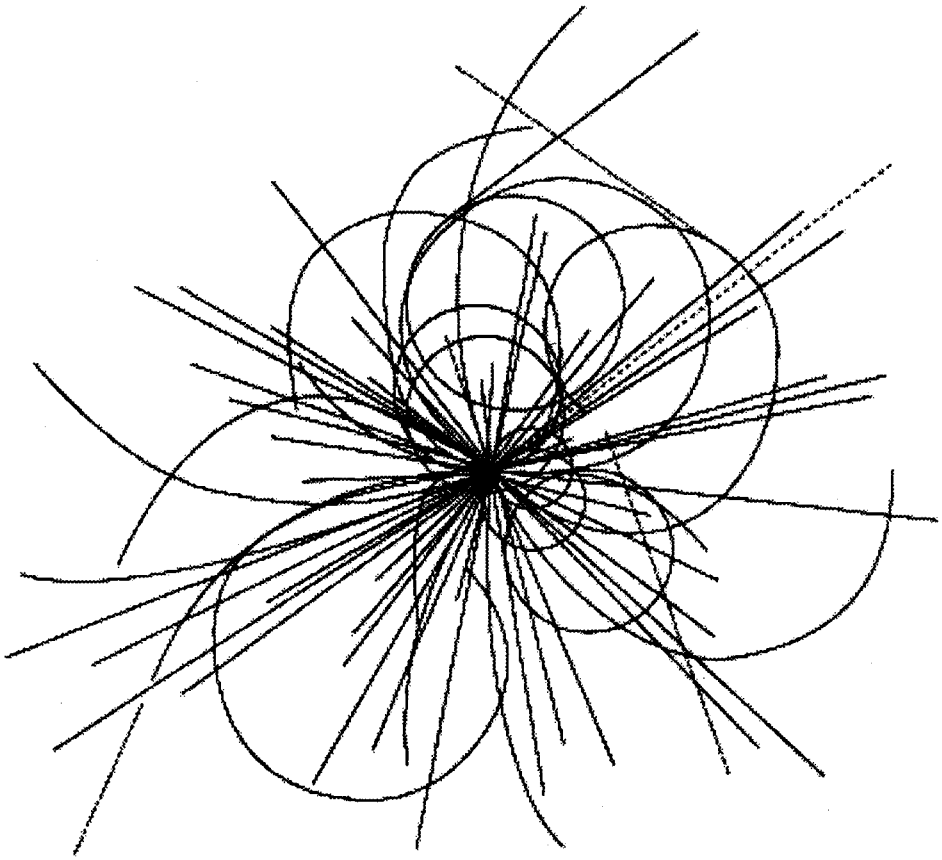
SSCL-Preprint-495

SSCL-Preprint-495
August 1993
Distribution Category: 414

G. Stupakov
V. Parkhomchuk
V. Shiltsev

Decoherence of a Gaussian Beam Due to Beam-Beam Interaction



Superconducting Super Collider Laboratory

SSCL-Preprint-495

Decoherence of a Gaussian Beam Due to Beam-Beam Interaction*

G. Stupakov, V. Parkhomchuk, and V. Shiltsev

Superconducting Super Collider Laboratory†
2550 Beckleymeade Ave.
Dallas, TX 75237

August 1993

---

*To be submitted to the "Particle Accelerators" Journal.
†Operated by the Universities Research Association, Inc., for the U.S. Department of Energy under Contract No. DE-AC35-89ER40486.

Decoherence of a Gaussian Beam Due to Beam-Beam Interaction

G. V. Stupakov, V. V. Parkhomchuk, and V. D. Shiltsev
Superconducting Super Collider Laboratory*
2550 Beckleymeade Ave.
Dallas, TX 75237

Using both analytical and computer simulation approaches, we compute the decoherence function that describes decay of the betatron oscillations of the centroid of an initially offset beam due to head-on beam-beam collisions. Based on this function, the decoherence time is estimated for the Superconducting Super Collider. We also discuss several definitions of the decoherence time arising in different beam dynamics problems.

1 INTRODUCTION

When a beam is kicked transversely, its centroid experiences decaying betatron oscillations. The decay is related to the tune spread in the beam, which—depending on a particular case—can be caused by nonlinearity of the lattice, chromaticity of the ring, or beam-beam interaction. Decoherence due to the first two sources of the tune spread has been previously studied by Meller et al.[1] However, for the Superconducting Super Collider (SSC) at its full energy of 20 TeV, the main source of the tune spread in the beam comes from the beam-beam interaction. For example, at 20 TeV an effective tune spread is about $7 \cdot 10^{-4}$ due to beam-beam effects,[2] $2 \cdot 10^{-5}$ due to residual chromaticity,[3] and about $5 \cdot 10^{-6}$ due to the lattice non-linearity.[3]

In this paper, we analyze the decoherence that is produced by head-on beam-beam collisions in the model of strong-weak beams and give estimates of the decoherence time.

The importance of the decoherence phenomenon is associated with the fact that in the course of the damping of betatron oscillations the transverse emittance of the beam increases. Characteristic time associated with the damping determines a scale on which coherent oscillations transform to an increment of the emittance. This time is one of the key parameters required for the design of the feedback system needed to suppress emittance growth of the beam.[2-5]

* Operated by the Universities Research Association, Inc., for the U.S. Department of Energy under Contract No. DE-AC35-89ER40486.

We use two different approaches to attack the problem. The first is based on an analytical consideration that reduces the problem of decoherence to calculation of a 3-dimensional integral. In addition, we performed a computer simulation in which an ensemble of particles was tracked in a linear lattice subject to beam-beam interaction. Both methods gave results that agree fairly well with one another.

2 ANALYTICAL CONSIDERATION

Let us consider two colliding proton round beams with a Gaussian distribution function characterized by the rms beam width σ. Due to beam-beam head-on collisions, a particle of a beam having the betatron amplitudes a_x and a_y in x and y directions, respectively, acquires a tune shift in x-direction, equal to[6]

$$\Delta\nu_x(\alpha_1,\alpha_2) = -\xi\int_0^1 du e^{-u(\alpha_1+\alpha_2)} I_0(\alpha_2 u)\left[I_0(\alpha_1 u) - I_1(\alpha_1 u)\right], \tag{1}$$

where $\alpha_1 = a_x^2/4\sigma^2$, $\alpha_2 = a_y^2/4\sigma^2$, $I_n(z)$ is the modified Bessel function of the n-th order, and ξ is the usual tune shift parameter, with $\xi = N_p r_p/4\pi\varepsilon$, where N_p is the number of particles in the bunch, r_p is the classical proton radius, and ε is the normalized emittance. Note that $\Delta\nu_x$ is negative for collisions of particles having the same charge sign. Because of the axial symmetry of the problem, all of our equations remain valid when $x \to y$ and $y \to x$.

In terms of the variables α_1 and α_2, the distribution function of the Gaussian beam has the form

$$\rho(\alpha_1,\alpha_2) = 4e^{-2(\alpha_1+\alpha_2)}, \tag{2}$$

where $\rho(\alpha_1,\alpha_2)$ is normalized so that

$$\int_0^\infty\int_0^\infty d\alpha_1 d\alpha_2 \rho(\alpha_1,\alpha_2) = 1. \tag{3}$$

Using Eqs. (1) and (2) one can find a distribution function over the tune shift, $f(\Delta\nu_x)$, such that $f(\Delta\nu_x)d\Delta\nu_x$ gives the probability for a particle to have the tune shift $\Delta\nu_x$ within the range $d\Delta\nu_x$. The function $f(\Delta\nu_x)$ is given by the following integral:

$$f(\Delta\nu_x) = \int_0^\infty\int_0^\infty d\alpha_1 d\alpha_2 \rho(\alpha_1,\alpha_2)\delta(\Delta\nu_x - \Delta\nu_x(\alpha_1,\alpha_2)). \tag{4}$$

Given $f(\Delta\nu_x)$, the average values $\langle\Delta\nu_x\rangle$ and $\langle(\Delta\nu_x)^2\rangle$ are reduced to the following integrals

$$\langle\Delta\nu_x\rangle = \int \Delta\nu_x f(\Delta\nu_x)d\Delta\nu_x = \int_0^\infty\int_0^\infty d\alpha_1 d\alpha_2 \rho(\alpha_1,\alpha_2)\Delta\nu_x(\alpha_1,\alpha_2), \tag{5}$$

$$\langle(\Delta\nu_x)^2\rangle = \int (\Delta\nu_x)^2 f(\Delta\nu_x)d\Delta\nu_x = \int_0^\infty\int_0^\infty d\alpha_1 d\alpha_2 \rho(\alpha_1,\alpha_2)\Delta\nu_x^2(\alpha_1,\alpha_2), \tag{6}$$

with the rms spread $\delta\nu_x$ given by

$$\delta\nu_x = \sqrt{\langle(\Delta\nu_x)^2\rangle - \langle\Delta\nu_x\rangle^2}. \tag{7}$$

3 DECOHERENCE FUNCTION

The most complete description of the decoherence process is provided by the so-called decoherence function that describes evolution of the beam centroid of a kicked beam. Let $\eta = x_c/\sqrt{\beta}$, where x_c is the centroid offset and β is the beta function. The decoherence function $K(N)$ is defined in Ref. 6 as

$$K(N) = \frac{\eta(N)}{\Delta\theta\sqrt{\beta_0}}, \tag{8}$$

where N is the number of turns, $\Delta\theta$ is the initial kick angle, and β_0 refers to the location of the kick. A mathematical expression for the decoherence function that corresponds to an infinitesimally small initial kick has been derived in Ref. 7. For our purposes it can be written as follows:

$$K(N) = \int_{-\infty}^{\infty} \sin\left(2\pi N(\nu_{x0} + \Delta\nu_x)\right) f(\Delta\nu_x) d\Delta\nu_x = \mathrm{Im}\, e^{2\pi i N \nu_{x0}} \int_{-\infty}^{\infty} e^{2\pi i N \Delta\nu_x} f(\Delta\nu_x) d\Delta\nu_x, \qquad (9)$$

where ν_{x0} is the unshifted tune of the beam. We restrict our attention here to considering only the modulus $\mathcal{K}(N)$ of the complex integral in Eq. (9); it has a meaning of the envelope (or amplitude) of the betatron oscillations produced by the initial kick,

$$\mathcal{K}(N) = \left| \int_{-\xi}^{0} \exp(2\pi i N \Delta\nu_x) f(\Delta\nu_x) d\Delta\nu_x \right|$$

$$= 4 \left| \int_0^{\infty} \int_0^{\infty} \exp\left(-2(\alpha_1 + \alpha_2) + 2\pi i N \Delta\nu_x(\alpha_1, \alpha_2)\right) d\alpha_1 d\alpha_2 \right|. \qquad (10)$$

In Eq. (10) we explicitly took into account that the beam-beam interaction produces a tune shift in the range $-\xi < \Delta\nu_x < 0$. It is interesting to note that the tune spread $\delta\nu_x$ (see Eq. (7)) can be related to the second derivative of $\mathcal{K}(N)$at $N = 0$,

$$\delta\nu_x = \frac{1}{2\pi} \sqrt{-\left.\frac{d^2\mathcal{K}}{dN^2}\right|_{N=0}}. \qquad (11)$$

4 RESULTS OF THE CALCULATIONS

We computed the decoherence function $\mathcal{K}(N)$ by two methods. In the first, we numerically integrated Eqs. (1) and (10), which together constitute a 3-dimensional integral. In the second, we performed a computer simulation in which an ensemble of up to $2 \cdot 10^5$ particles with an initial Gaussian distribution function has been tracked several thousand turns subject to a standard linear transfer map, followed by a kick due to the beam-beam interaction. The distribution function $f(\Delta\nu_x)$ of the ensemble has been computed, and with the use of the first part of Eq. (10), $\mathcal{K}(N)$ has been calculated by means of numerical integration. Both methods agree with one another within several percent.

The function $\mathcal{K}(N)$ obtained as mentioned above is plotted in Fig. 1 as a function of the variable $2\pi N\xi$. On the same plot we also show a scale with number of turns N

corresponding to $\xi = 1.8 \cdot 10^{-3}$—the nominal value of ξ for the SSC with two interaction regions.

Fig. 2 shows the distribution function $f(\Delta\nu_x)$ found from the tracking simulations. This curve is not ideally smooth (as it has to be) simply because of the limited number of particles used in the simulations. The average values characterizing this function are

$$\sqrt{\left\langle (\Delta\nu_x)^2 \right\rangle} = 0.655\xi, \quad \langle \Delta\nu_x \rangle = -0.633\xi, \quad \delta\nu_x = 0.17\xi. \tag{12}$$

For comparison, we have also computed decoherence curves for large amplitudes of the initial kick. These curves are plotted in Fig. 3 (together with $\mathcal{K}(N)$ from Fig. 1) for initial kicks equal to σ and 4σ. They demonstrate the general trend of slower decoherence for stronger kicks.

5 DECOHERENCE TIME

Having found the decoherence curve we are now in a position to estimate the decoherence time for the decay of an initial kick. Notice that there is no unique, rigorous definition of the decoherence time. In this section, we present several definitions that arise in different beam dynamics problems.

One can simply define the decoherence time (number of turns) N_{decoh}, as the time needed for the amplitude to go down to half of its initial value, $\mathcal{K}(N_{decoh}) = 0.5$. From Fig. 1, one finds the following estimate for N_{decoh} so defined:

$$N_{decoh} \approx \frac{1}{\xi}. \tag{13}$$

For the SSC collider, where $\xi = 1.8 \cdot 10^{-3}$, this gives $N_{decoh} = 600$ turns.

For a feedback theory, a more relevant definition is associated with the behavior of $\mathcal{K}(N)$ in the region of small N. This is easy to justify if one takes into account that a strong feedback system damps the beam oscillations much more quickly before they decohere. This implies that only an initial phase of the decoherence process is involved in the residual emittance growth of the beam.[4] Because $\mathcal{K}(N)$ has a quadratic dependence in the limit $N \to 0$, an adequate definition of N_{decoh} in this case is

$$N_{decoh} = \left.\frac{1}{\sqrt{-\mathcal{K}''}}\right|_{N=0} = \frac{1}{2\pi\delta\nu_x}, \tag{14}$$

so that $\mathcal{K}(N) = 1 - \frac{1}{2}\left(N/N_{decoh}\right)^2$ for $N \ll N_{decoh}$. Our calculations give

$$N_{decoh} = \frac{0.95}{\xi}, \tag{15}$$

which results in $N_{decoh} = 530$ for the SSC.

Yet another N_{decoh} is related to the effect of external noise acting on the beam (without feedback). Assume that at each turn the beam experiences random, uncorrelated kicks of amplitude a_m. Its displacement after N turns is then given by

$$\Delta x_c(N) = \sum_{m=1}^{N} K(N-m) a_m . \tag{16}$$

Let us find the average square of the displacement:

$$\left\langle (\Delta x_c)^2 \right\rangle = \sum_{m=1}^{N} K^2(N-m) \left\langle a_m^2 \right\rangle = \left\langle a^2 \right\rangle \sum_{m=1}^{N} K^2(m), \tag{17}$$

where we have employed the condition of uncorrelated kicks $\left\langle a_m a_n \right\rangle = \left\langle a^2 \right\rangle \delta_{mn}$. Now we define the decoherence time N_{decoh} so that

$$\left\langle (\Delta x_c)^2 \right\rangle = \frac{1}{2} N_{decoh} \left\langle a^2 \right\rangle . \tag{18}$$

This gives us the relation,

$$N_{decoh} = 2\sum_{m=1}^{\infty} K^2(m) \approx \int_0^{\infty} \mathcal{K}^2(m) dm . \tag{19}$$

In Eq. (19) we have considered that $K(m)$ is a rapidly oscillating (with the betatron frequency) function, so that on the average its square is equal to half of its amplitude squared. Our calculations give

$$N_{decoh} = \frac{0.81}{\xi}, \tag{20}$$

with $N_{decoh} = 450$ for the SSC. Our computer tracking confirmed Eqs. (18) and (19) within an accuracy of the statistics of the calculations.

6 SUMMARY

In this paper, we have calculated the decoherence function for the head-on beam-beam collisions. This function gives the most complete description of the decoherence process, allowing, for example, calculation of the tune spread associated with the beam-beam interaction (see Eq. (11)). Via the Fourier transform, it is directly related to the distribution of the particles over the tune shift $\Delta\nu_x$. What is more important, this function gives us a natural characteristic time scale on which an initially driven betatron oscillation damps out. We have shown that, depending on the exact definition, the expression for N_{decoh} has a form, $N_{decoh} = \text{const}/\xi$, where ξ is the beam-beam tune shift and const ≈ 1. Our result generally agrees with the recent numerical study of the SSC beam decoherence.[8]

ACKNOWLEDGMENT

The authors are thankful to E. Tsyganov, whose critical remarks initiated this work.

REFERENCES

1. R.E. Meller et al., "Decoherence of Kicked Beams," SSC-N-360, 1987.

2. V.A. Lebedev, V.V. Parkhomchuk, V.D. Shiltsev, and A.N. Skrinsky, "Suppression of Emittance Growth Caused by Mechanical Vibrations of Magnetic Measurements in Presence of Beam-Beam Effects in the SSC," Preprint INP 91-120, Novosibirsk, 1991.

3. W. Chou and J.O. Peterson, "The SSC Transverse Feedback System," SSCL-623, 1993.

4. V.A. Lebedev, V.V. Parkhomchuk, V.D. Shiltsev, and G.V. Stupakov, "Emittance Growth due to Noise and Its Suppression with the Feedback System in Large Hadron Colliders," SSCL-Preprint-188, March 1993.

5. V.V. Parkhomchuk and V.D. Shiltsev, "Is Transverse Feedback Necessary for the SSC Emittance Preservation? (Vibration Noise Analysis and Feedback Parameters Optimizations)," SSCL-622, June 1993.

6. G. Lopez, "Head-On and Long Range Beam-Beam Tune Shifts Spread in the SSC," SSCL-442, May 1991.

7. A. Chao and G.V. Stupakov, "Collective Effects and Beam Decoherence," SSCL-621, April 1993.

8. E. Tsyganov et al., "Decoherence of the SSC Beam Oscillation due to Head-On Beam-Beam Collisions," SSCL-Preprint-481, 1993.

FIGURE CAPTIONS

FIGURE 1: Decoherence function $\mathcal{K}(N)$ versus $2\pi N\xi$.

FIGURE 2: Distribution function $f(\Delta v_x)$.

FIGURE 3: Decoherence curves for various initial offsets.

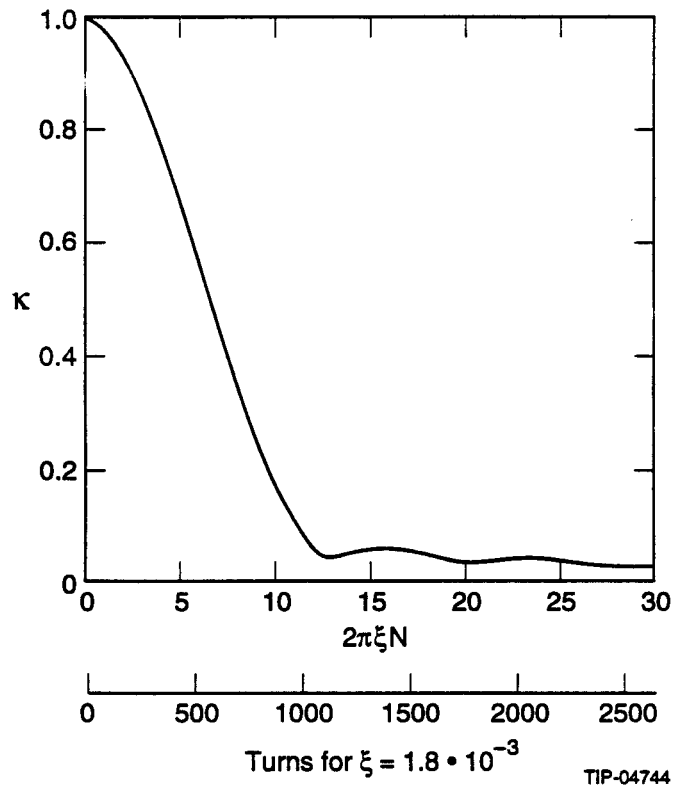

1.0
0.8
0.6
κ
0.4
0.2
0
0
5
10
15
20
25
30
2πξN
0
500
1000
1500
2000
2500
Turns for ξ = 1.8 • 10^-3
TIP-04744

Figure 1

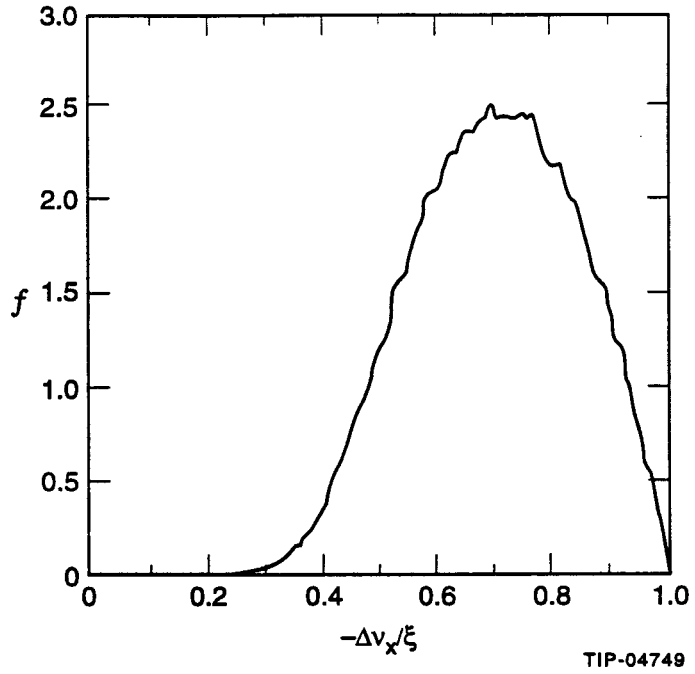


Figure 2

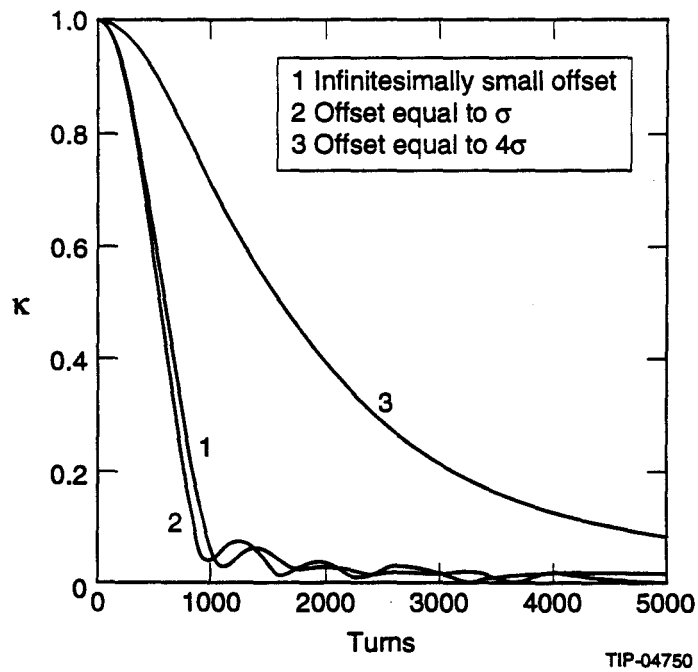

1.0
0.8
0.6
0.4
0.2
0
κ
0
1000
2000
3000
4000
5000
Turns
1 Infinitesimally small offset
2 Offset equal to σ
3 Offset equal to 4σ
1
2
3
TIP-04750

Figure 3