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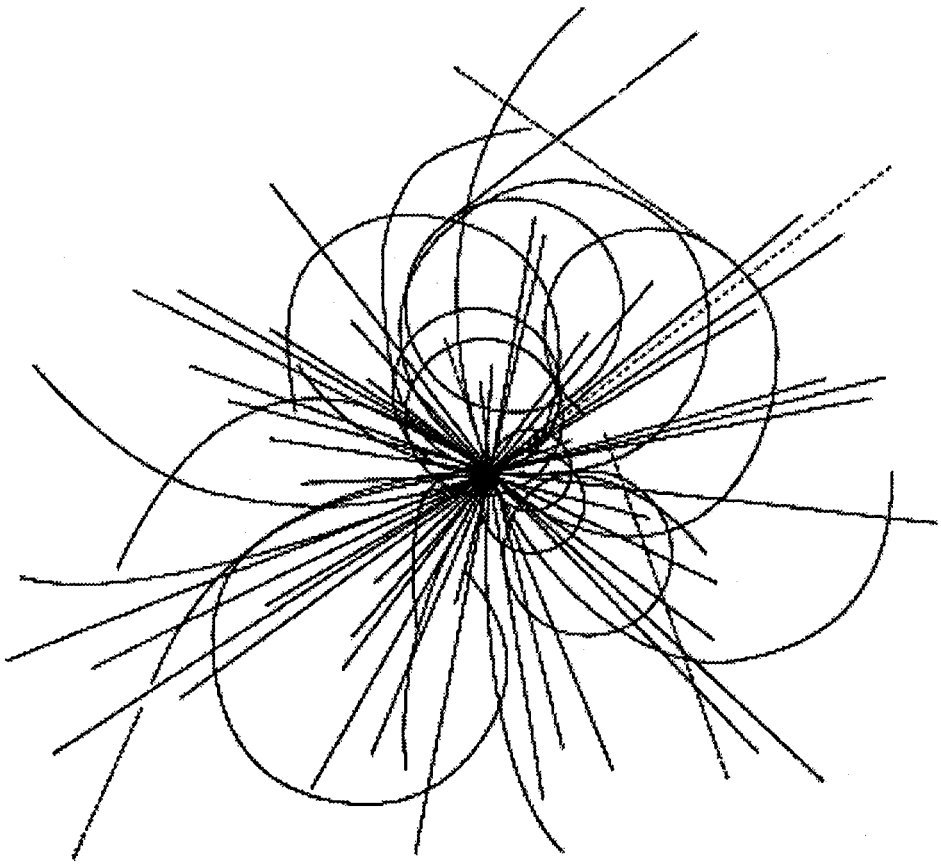

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CURRENT LOOP DECAY IN RUTHERFORD-TYPE CABLES

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INTRODUCTION

Recent measurements of superconducting particle accelerator magnets made of multi-strand Rutherford-type cable have shown that the magnetic field and its main harmonics oscillate along the magnet axis with a wavelength nearly equal to the cable transposition pitch length.[1,2] It was also observed that, at low transport current, the periodic magnetic field patterns can persist without any significant decay for more than 12 hours.[1]

The coincidence of the wavelength of the magnetic field oscillations with the cable transposition pitch suggests that slowly decaying current loops exist in the cable even at zero transport current.[3] These loops consist of currents flowing along the cable through one set of strands and returning through another set of strands. In this paper, we consider the process of current loop decay in a Rutherford-type cable.

MODELLING OF A RUTHERFORD-TYPE CABLE

The inner (outer) cables used in SSC 5-cm aperture dipole magnets consist of 30 (36) strands, twisted together, and shaped into a flat slightly keystoned cable. Due to the twisting, each strand goes successively from the inner edge of the cable to the outer edge, and back to the inner edge, over a distance, l_p, given by

$$l_p = \sqrt{4w^2 + p_c^2} \quad , \tag{1}$$

where w and p_c designate the width and the transposition pitch length of the cable.

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Over the distance l_p, the strand crosses over and has electrical contact with all the other cable strands. Estimate of the cross-over resistance between two cable strands, R_c, varies from 1 to 100 μΩ.[4] The electrical contact between a given strand and the rest of the cable can thus be characterized by a transverse conductance per unit length, G_t, given by

$$G_t = \frac{2(N-1)}{R_c\, l_p} \quad , \tag{2}$$

where N designates the number of cable strands. For $l_p \approx 100$ mm, $R_c \approx 5$ μΩ and $N = 30$, we have $G_t \approx 10^8\ \Omega^{-1}\mathrm{m}^{-1}$.

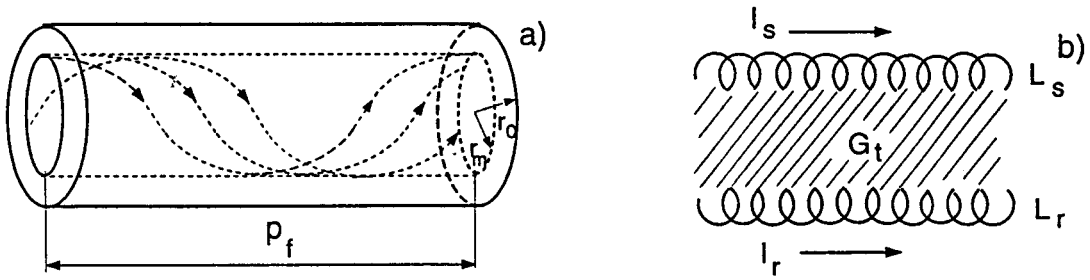

Figure 1. Rutherford-type cable: (a) strand geometry, (b) model describing the process of current redistribution between a given cable strand and the rest of the cable.

As shown in Fig. 1(a), the SSC strands consist of an inner core and an outer sheath of pure copper, surrounding an annular superconducting multifilamentary composite. The filaments of the composite are twisted, with a twist pitch, p_f, different from p_c. While in the superconducting state, the current is confined within the helicoidal filaments. Such a current distribution produces a longitudinal field, H_{sl}, in the strand inner core, and a circular field, H_{sc}, in the strand outer sheath.[5] Calculating the magnetic energy associated with H_{sl} and H_{sc} allows one to estimate their contributions, L_{sl} and L_{sc}, to the strand self-inductance. These contributions can readily be expressed per strand unit length as

$$L_{sl} = \mu_0 \frac{\pi\, r_m^2}{p_f^2} \quad , \tag{3a}$$

$$L_{sc} = \frac{\mu_0}{2\pi} \ln\left(\frac{r_0}{r_m}\right) \quad , \tag{3b}$$

where μ_0 is the magnetic permeability of vacuum, r_m is the average radius of the strand multifilamentary area and r_0 is the strand outer radius. For $r_0 \approx 0.4$ mm, $r_m \approx 0.3$ mm, and $p_f \approx 15$ mm, we have $L_{sl} \approx 2\ 10^{-9}$ H/m and $L_{sc} \approx 6\ 10^{-8}$ H/m.

In this paper, we shall assume that the process of current redistribution between a given strand of a Rutherford-type cable and the remaining $(N-1)$ strands of the cable can be described by the model given in Fig. 1(b). Here, G_t is the transverse conductance calculated above, and L_s and L_r are two effective inductances per unit length defined as

$$L_s = L_{sl} + L_{sc} \quad , \tag{4a}$$

$$L_r = \frac{L_s}{N-1} \quad . \tag{4b}$$

CURRENT REDISTRIBUTION AT LOW TRANSPORT CURRENT

Applying Faraday's law to the circuit of Fig. 1(b) we get

$$L_s \frac{dI_s}{dt} + E_s - L_r \frac{dI_r}{dt} - E_r = \frac{1}{G_t} \frac{d^2 I_s}{dx^2} \quad , \tag{5}$$

where I_s (resp., I_r) and E_s (resp., E_r) are the current and the electric field in the given strand (resp., the remaining $(N-1)$ strands). Let us now assume that I_s and I_r can be written

$$I_s = I^* + \delta I \quad , \qquad (6a) \qquad\qquad I_r = (N-1)\, I^* - \delta I \quad , \qquad (6b)$$

where I^* is constant and uniform and $\delta I << I^*$. If I^* is much less than the strand critical current, I_c, we have: $E_s = E_r = 0$, and Eq. (5) becomes

$$(L_s + L_r)\frac{d(\delta I)}{dt} = \frac{1}{G_t}\frac{d^2(\delta I)}{dx^2} \quad . \qquad (7)$$

With the boundary conditions $\delta I(x=0,t) = \delta I(x=l,t) = 0$, where l is the strand length, the solution of Eq. (7) can be written as a sum of time-decaying waves

$$\delta I(x,t) = \sum_{k=1,2,\ldots} I_{k0} \sin\left(\frac{2\pi x}{l_k}\right)\exp\left(-\frac{t}{\tau_k}\right) \quad , \qquad (8)$$

where I_{k0} is the initial wave amplitude, and l_k and τ_k are given by

$$l_k = \frac{2l}{k} \quad , \qquad (9a) \qquad\qquad \tau_k = (L_s + L_r)\, G_t \left(\frac{l_k}{2\pi}\right)^2 \quad . \qquad (9b)$$

It follows from Eqs. (9) that the larger the wavelength, the longer the time constant. This can be understood if one considers that in our model, the inductances are directly proportional to the wavelength, while the transverse resistance is inversely proportional to l_k. The largest possible wave corresponds to $k = 1$. With the values of G_t, L_s and L_r calculated above, and $l \approx 10^3$ m, we have $\tau_1 \approx 7\ 10^5$ s.

CURRENT REDISTRIBUTION AT LARGE TRANSPORT CURRENT

Let us know consider the case when I^* is close to the strand critical current, I_c. In this case, we need to take into account the influence of the longitudinal electrical field. For a superconducting multifilamentary strand, it is well known that

$$E = E_0 \exp\left(\frac{I - I_c}{I_0}\right) \quad , \qquad (10)$$

where E_0 and I_0 are two constant parameters. (For SSC inner strands, $E_0 \approx 10^{-5}$ V/m, $I_0 \approx I_c/30$, and I_c(4.3 K,5 T) $\approx$ 500 A.) If $\delta I << I_0$, it can be shown that

$$E_s = R^* \left(I_0 + \delta I\right) \quad , \qquad (11a) \qquad\qquad E_r = R^* \left(I_0 - \frac{\delta I}{N-1}\right) \quad , \qquad (11b)$$

where R^* is a resistance per unit length given by

$$R^* = \frac{E_0}{I_0}\exp\left(\frac{I^* - I_c}{I_0}\right) \quad . \qquad (12)$$

Introducing the above expressions of E_s and E_r into Eq. (5) yields

$$(L_s + L_r)\frac{d(\delta I)}{dt} = \frac{1}{G_t}\frac{d^2(\delta I)}{dx^2} - \frac{N}{N-1} R^* \delta I \quad . \qquad (13)$$

Once again, the solution of Eq. (13) can be written as a sum of time-decaying waves

$$\delta I(x,t) = \sum_{k=1,2,\ldots} I_{k0} \sin\left(\frac{2\pi x}{l_k}\right) \exp\left(-\frac{t}{\tau_k}\right) \exp\left(-\frac{t}{\tau^*}\right) \quad , \tag{14}$$

where

$$\tau^* = \frac{(N-1)(L_s + L_r)}{N R^*} \quad . \tag{15}$$

If $I^* > 3I_c/4$, then $R^* > 10^{-10}$ Ω/m. With the values of L_s and L_r calculated above, we have $\tau^* << \tau_1$. For long waves and at large transport current, the time constant of current redistribution is thus τ^*, which does not depend on the wavelength. Fig. 2 presents a plot of τ^* as a function of the cable transport current, $I_t = N I^*$. For this computation, we assumed $I_c = I_c(4.35\ \mathrm{K}, B)$, where $B \approx 10^{-3} I_t$ corresponds to the field on the pole turn of the inner coil of SSC 5-cm-aperture dipole magnet. It appears that, for $I_t > 6500$ A, the characteristic time of the current redistribution process decreases sharply.

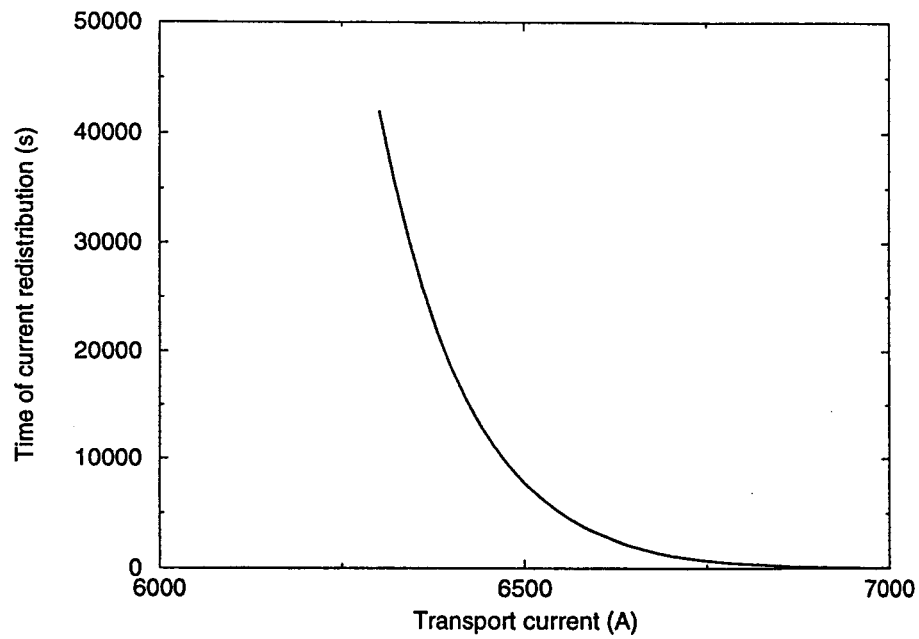

Figure 2. Dependence of τ^* on cable transport current for the pole turn of SSC dipole magnet.

SUMMARY

We have shown that the time constant of current redistribution between one strand of a Rutherford-type cable and the rest of the cable depends on the transport current. At low transport current, the process is dominated by the transverse conductance, resulting in a long time constant, while at large transport current, the series resistance forces the current to redistribute much faster.

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