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$\overline{B} \to \overline{K}e^+e^-$ in Chiral Perturbation Theory

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The combined use of chiral $SU(3)$ and heavy quark symmetries allows one to relate the hadronic form factors for the decay $\overline{B} \to \overline{K}e^+e^-$ to those for $\overline{B} \to \pi e^-\overline{\nu}$. We investigate departures from the symmetry limit which arise from chiral symmetry breaking. The analysis uses chiral perturbation theory and the heavy quark limit to compute the relevant hadronic matrix elements. We estimate the size of $SU(3)$ corrections by computing, at one loop order, the leading nonanalytic dependence on the light quark masses. The calculation is trustworthy only in the portion of the Dalitz plot in which the momentum of the kaon or pion is small. We find the corrections to be $\sim 40\%$.

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Flavor changing neutral transitions are suppressed in the standard model of electroweak interactions, because they do not occur at tree level, and at one loop because of the GIM mechanism. In $s \to d$ transitions GIM cancellations are very effective for diagrams involving virtual u and c quarks, while virtual t diagrams are doubly Cabibbo suppressed relative to the transitions mediated by u and c. By contrast, in $b \to s$ transitions it is those diagrams involving virtual u quarks which are doubly Cabibbo suppressed; then the GIM cancellation is rather ineffective, as it involves the very heavy t quark against the light (by comparison) c quark.

Hence processes involving $b \to s$ flavor change are interesting because, although rare, they are well within experimental reach. In fact, the first measurement of such a process was recently reported by the CLEO collaboration, who observe the process $B \to K^*\gamma$ with a branching fraction of $(4.5 \pm 1.9 \pm 0.9) \times 10^{-5}$ [1]. More importantly, however, processes involving $b \to s$ flavor change are interesting because, being rare, they are a quite sensitive probe of departures from standard expectations. There exist several studies of the effect of extensions to the standard model on the rates for this class of processes [2]. Predictions of exclusive event rates are uncertain, however, because they require the calculation of nonperturbative hadronic matrix elements. Inclusive rates, although they may be calculated more reliably, are considerably more difficult to measure.

In this paper, we will investigate the form factors which describe the rare decay $\overline{B} \to \overline{K}e^+e^-$. Isgur and Wise [3] have used heavy quark spin and flavor symmetries to relate the form factors for $\overline{B} \to \overline{K}e^+e^-$ to those for semileptonic D meson decay. Burdman and Donoghue [4] have instead related $\overline{B} \to \overline{K}e^+e^-$ to semileptonic $\overline{B}$ meson decay. This approach may seem reasonable, since it avoids the use of the heavy quark flavor symmetry, in particular the question of whether the heavy quark limit is a good approximation for charm. But the analysis invokes, as compensation, chiral $SU(3)$ symmetry. It is the purpose of this letter to investigate the validity of this latter approximation in this process.

We will compute violations to the $SU(3)$ symmetry limit, which arise from the light quark masses m_q, by means of a phenomenological lagrangian which displays simultaneously explicit chiral and heavy quark symmetries. This lagrangian is non-renormalizable, and in order to control the higher dimension terms, we consider only the portion of the Dalitz plot in which the momentum of the kaon or pion is small. We will compute one-loop expressions for the relevant form factors, retaining only terms, such as those of the form $m_q \ln m_q$, which depend nonanalytically on the symmetry breaking parameters. These

terms dominate the corrections in the theoretical limit of very small quark masses, and they cannot be reabsorbed into counterterms at higher order in the effective lagrangian.

With all these limitations, what is the interest in this computation? Although the validity of the symmetry relations between $\overline{B} \to \overline{K}\mathrm{e}^+\mathrm{e}^-$ and $\overline{B} \to \pi\mathrm{e}^-\overline{\nu}$ form factors will not be fully established, we will gain confidence in them if the nonanalytic corrections are small. Alternatively, large (order 100%) corrections would be an immediate indication of the breakdown of the relations. In this regard it is useful to keep in mind the case of the relation between kaon decay and the parameter B_K, which is invalidated by large corrections of precisely this sort [5].

The rare decay $\overline{B} \to \overline{K}\mathrm{e}^+\mathrm{e}^-$ occurs via the quark level transitions $b \to s\gamma$ and $b \to s\,\mathrm{e}^+\mathrm{e}^-$. These in turn are induced by loop processes at the weak scale, appearing at low energies as local nonrenormalizable operators with coefficients in which the leading logarithms have been resummed [6]. The three operators which will be relevant here are

$$\begin{aligned}
\mathcal{O}_7 &= \frac{e}{16\pi^2} m_b\, \bar{\mathrm{s}}_L \sigma^{\mu\nu} \mathrm{b}_R\, F_{\mu\nu}\,, \\
\mathcal{O}_8 &= \frac{e^2}{16\pi^2}\, \bar{\mathrm{s}}_L \gamma^\mu \mathrm{b}_L\, \bar{\mathrm{e}}\gamma_\mu \mathrm{e}\,, \\
\mathcal{O}_9 &= \frac{e^2}{16\pi^2}\, \bar{\mathrm{s}}_L \gamma^\mu \mathrm{b}_L\, \bar{\mathrm{e}}\gamma_\mu \gamma^5 \mathrm{e}\,,
\end{aligned} \tag{1}$$

assembled into an effective interaction Hamiltonian

$$\mathcal{H}_{\mathrm{eff}} = -\frac{4G_F}{\sqrt{2}}(s_3 + s_2 \mathrm{e}^{\mathrm{i}\delta})\left[c_7(\mu)\mathcal{O}_7 + c_8(\mu)\mathcal{O}_8 + c_9(\mu)\mathcal{O}_9\right]. \tag{2}$$

The total rate for the decay $\overline{B} \to \overline{K}\mathrm{e}^+\mathrm{e}^-$ is calculated from the matrix elements of these operators. The part of the computation which involves the leptons is perturbative and straightforward; however the same may not be said for the matrix elements of the flavor-changing quark operators between external hadron states. These typically must be parameterized in terms of a Lorentz-covariant decomposition,

$$\begin{aligned}
\langle \overline{K}(p_K)\,|\,\bar{\mathrm{s}}\gamma^\mu \mathrm{b}\,|\,\overline{B}(p_B)\rangle &= f_+\,(p_B + p_K)^\mu + f_-\,(p_B - p_K)^\mu\,, \\
\langle \overline{K}(p_K)\,|\,\bar{\mathrm{s}}\sigma^{\mu\nu} \mathrm{b}\,|\,\overline{B}(p_B)\rangle &= \mathrm{i}h\left[(p_B + p_K)^\mu (p_B - p_K)^\nu - (p_B + p_K)^\nu (p_B - p_K)^\mu\right],
\end{aligned} \tag{3}$$

in which the form factors f_+, f_- and h are scalar functions of the invariant momentum transfer $p_K \cdot p_B$. The differential partial decay width at fixed $\hat{s} = (p_{\mathrm{e}^+} + p_{\mathrm{e}^-})^2/m_B^2$ is then

given by

$$\frac{d\Gamma}{d\hat{s}}(\overline{B} \to \overline{K}e^+e^-) = \left|s_3 + s_2 e^{i\delta}\right|^2 \frac{G_F^2 m_B^5 \alpha^2}{3 \cdot 2^9 \pi^5}$$
$$\times \left[\left(1 - m_K^2/m_B^2\right)^2 - 2\hat{s}\left(1 + m_K^2/m_B^2\right) + \hat{s}^2\right]^{3/2} \tag{4}$$
$$\times \left[\left|c_8(m_b) f_+ + 2m_b c_7(m_b) h\right|^2 + \left|c_9(m_b) f_+\right|^2\right] .$$

The coefficients $c_7(m_b)$, $c_8(m_b)$ and $c_9(m_b)$ depend on short-distance physics and are discussed in detail in ref. [6].

The form factors f_+ and h which are needed for eq. (4) involve nonperturbative strong interactions and are in general incalculable. However the fact that the bottom quark is very massive compared to scales typical of QCD affords some simplifications,

$$\begin{aligned} &f_+ + f_- \sim m_b^{-1/2}, \qquad f_+ - f_- \sim m_b^{1/2}, \\ &h = \frac{f_+ - f_-}{2m_b}, \qquad s = (f_+ + f_-)m_b + (f_+ - f_-)p_K \cdot p_B/m_b . \end{aligned} \tag{5}$$

For completeness, we have included the scalar form factor s, which parameterizes the matrix element $\langle \overline{K}(p_K) \,|\, \bar{s}\, b \,|\, \overline{B}(p_B)\rangle$. Hence, in the simultaneous limits of chiral symmetry and $m_b \to \infty$, the form factors for the decay $\overline{B} \to \overline{K}e^+e^-$ are given simply in terms of the form factor f_+ which describes $\overline{B} \to \pi e^- \overline{\nu}$.

If we now restrict ourselves to that portion of the Dalitz plot in which the leptons are emitted back to back, and the kaon is very soft, we will be able to compute the hadronic matrix elements (3) in terms of two phenomenological parameters. These are the decay constant f_B of the $\overline{B}$ meson, and the axial coupling g of the pion to the $(\overline{B},\overline{B}^*)$ doublet. These constants appear as coefficients in a nonrenormalizable low-energy effective lagrangian in which both heavy quark and chiral $SU(3)$ symmetry are explicit. This is a framework within which the relations (5) arise naturally, and which also will allow us to compute the leading nonanalytic corrections which test the validity of $SU(3)$ symmetry in this process.

We begin with a brief synopsis of the formalism of heavy hadron chiral perturbation theory [7]. In the limit $m_b \to \infty$, the $\overline{B}$ and the $\overline{B}^*$ mesons are degenerate, and to implement the heavy quark symmetries it is convenient to assemble them into a "superfield" $H_a(v)$:

$$H_a(v) = \frac{1 + \not{v}}{2\sqrt{2}} \left[\overline{B}_a^{*\mu} \gamma_\mu - \overline{B}_a \gamma^5\right] . \tag{6}$$

Here v^μ is the fixed four-velocity of the heavy meson, and a is a flavor $SU(3)$ index corresponding to the light antiquark. Because we have absorbed mass factors $\sqrt{2m_B}$ into the fields, they have dimension 3/2; to recover the correct relativistic normalization, we will multiply amplitudes by $\sqrt{2m_B}$ for each external $\overline{B}$ or $\overline{B}^*$ meson.

The chiral lagrangian contains both heavy meson superfields and pseudogoldstone bosons, coupled together in an $SU(3)_L \times SU(3)_R$ invariant way. The matrix of pseudogoldstone bosons appears in the usual exponentiated form $\xi = \exp(\mathrm{i}\mathcal{M}/f)$, where

$$\mathcal{M} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi_0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\frac{2}{3}}\,\eta \end{pmatrix}, \tag{7}$$

and f is the pion (or kaon) decay constant. The bosons couple to the heavy fields through the covariant derivative and axial vector field,

$$\begin{aligned} D^\mu_{ab} &= \delta_{ab}\partial^\mu + V^\mu_{ab} = \delta_{ab}\partial^\mu + \tfrac{1}{2}\left(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger\right)_{ab}\,, \\ A^\mu_{ab} &= \frac{\mathrm{i}}{2}\left(\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger\right)_{ab} = -\frac{1}{f}\partial_\mu\mathcal{M}_{ab} + \mathcal{O}(\mathcal{M}^3)\,. \end{aligned} \tag{8}$$

Lower case roman indices correspond to flavor $SU(3)$. Under chiral $SU(3)_L \times SU(3)_R$, the pseudogoldstone bosons and heavy meson fields transform as $\xi \to L\xi U^\dagger = U\xi R^\dagger$, $A^\mu \to UA^\mu U^\dagger$, $H \to HU^\dagger$ and $(D^\mu H) \to (D^\mu H)U^\dagger$, where the matrix U_{ab} is a nonlinear function of the pseudogoldstone boson matrix $\mathcal{M}$.

The chiral lagrangian is an expansion in derivatives and pion fields, as well as in inverse powers of the heavy quark mass. The kinetic energy terms take the form

$$\mathcal{L}_{\rm kin} = \frac{1}{8}f^2\,\partial^\mu\Sigma_{ab}\,\partial_\mu\Sigma^\dagger_{ba} - \mathrm{Tr}\left[\overline{H}_a(v)\mathrm{i}v\cdot D_{ba}H_b(v)\right]\,, \tag{9}$$

where $\Sigma = \xi^2$. The leading interaction term is of dimension four,

$$g\,\mathrm{Tr}\left[\overline{H}_a(v)H_b(v)\,\not{A}_{ba}\gamma^5\right]\,, \tag{10}$$

where g is an unknown parameter, of order one in the constituent quark model. The analogue of this term in the charm system is responsible for the decay $D^* \to D\pi$, from which one may derive the limit $g^2 < 0.5$.

The quark bilinears $J^\mu = \bar{\mathrm{s}}\gamma^\mu\mathrm{b}$ and $J^{\mu\nu} = \bar{\mathrm{s}}\sigma^{\mu\nu}\mathrm{b}$, whose hadronic matrix elements we must compute, may be matched onto operators in the chiral lagrangian written in terms of the meson fields. Heavy quark symmetry and the $SU(3)_L \times SU(3)_R$ transformation

properties of chiral currents dictate that this matching must to leading order take the universal form

$$\begin{aligned} \overline{\mathrm{q}}_{aL}\Gamma \mathrm{b} &\to c_L \,\mathrm{Tr}[\Gamma H_b(v)\xi^\dagger_{ba}]\,, \\ \overline{\mathrm{q}}_{aR}\Gamma \mathrm{b} &\to c_R \,\mathrm{Tr}[\Gamma H_b(v)\xi_{ba}]\,, \end{aligned} \tag{11}$$

for left- and right-handed light quark fields, where Γ is an arbitrary Dirac matrix. Then the two conditions

$$\begin{aligned} \langle 0 \,|\, \overline{\mathrm{q}}_a \gamma^\mu \gamma^5 \mathrm{b} \,|\, \overline{B}_a(p)\rangle &= \mathrm{i} f_B p^\mu\,, \\ \langle \pi(p') \,|\, \overline{\mathrm{q}}_a \gamma^\mu \gamma^5 \mathrm{b} \,|\, \overline{B}_a(p)\rangle &= 0\,, \end{aligned} \tag{12}$$

are sufficient to determine c_L and c_R,

$$c_L = c_R = \frac{\mathrm{i}}{2} f_B \sqrt{m_B}\,. \tag{13}$$

As we are working in the $SU(3)$ limit, the decay constant f_B is flavor symmetric. Note that the first of the conditions (12) is merely the definition of f_B, while the second reflects the invariance under parity of the strong interactions.

Decomposing the bilinears J^μ and $J^{\mu\nu}$ into chiral components, it is straightforward to perform the matching onto interactions in the effective lagrangian. We find the operators

$$\begin{aligned} \mathcal{O}^\mu &= \frac{\mathrm{i}}{4} f_B \sqrt{m_B}\, \left\{ \mathrm{Tr}\left[\gamma^\mu H_b(v)(\xi^\dagger + \xi)_{ba}\right] + \mathrm{Tr}\left[\gamma^5 \gamma^\mu H_b(v)(\xi^\dagger - \xi)_{ba}\right] \right\} \\ \mathcal{O}^{\mu\nu} &= \frac{\mathrm{i}}{4} f_B \sqrt{m_B}\, \left\{ \mathrm{Tr}\left[\sigma^{\mu\nu} H_b(v)(\xi^\dagger + \xi)_{ba}\right] + \mathrm{Tr}\left[\gamma^5 \sigma^{\mu\nu} H_b(v)(\xi^\dagger - \xi)_{ba}\right] \right\}\,. \end{aligned} \tag{14}$$

For the operators J^μ and $J^{\mu\nu}$, which carry strangeness, we take $a = 3$. Each of these relations is corrected at higher order in the chiral derivative expansion. Note that the first terms in (14) yield vertices with an even number of pseudogoldstone bosons, while the second terms yield those with an odd number.

We are now in a position to compute the hadronic matrix elements (3) in the effective theory.[1] The tree level Feynman diagrams are shown in fig. 1. There exist both pole graphs, fig. 1(a), in which a kaon is emitted via the interaction (10) and the virtual $\overline{B}_s^*$ meson is absorbed by one of the effective operators, and direct graphs, fig. 1(b), in which the effective operator both absorbs the $\overline{B}$ and emits the $\overline{K}$. The former are induced by the first terms in eqs. (14), while the latter are induced by the second terms.

[1] A recent preprint [8] has come to our attention, which also considers the process $\overline{B} \to \overline{K}\mathrm{e}^+\mathrm{e}^-$ in this theory. However the authors compute only the contributions of the operator $\mathcal{O}_7$, and they do not address the issue of $SU(3)$ violating corrections.

It is extremely straightforward to compute the desired amplitudes. For the vector and tensor currents, respectively, we find for the pole graphs

$$\begin{aligned}\mathcal{A}^{\mu}_{\text{pole}} &= -\frac{g f_B m_B}{f}\frac{1}{p_K\cdot v+\Delta}\left(p_K^{\mu}-p_K\cdot v v^{\mu}\right),\\ \mathcal{A}^{\mu\nu}_{\text{pole}} &= -\frac{g f_B m_B}{f}\frac{1}{p_K\cdot v+\Delta}\,\mathrm{i}\left(p_K^{\mu}v^{\nu}-p_K^{\nu}v^{\mu}\right),\end{aligned} \tag{15}$$

where $\Delta = m_{B_s^*} - m_B$, and $p_K\cdot v$ is the kaon energy in the $\overline{B}$ rest frame. For the point amplitudes, we find

$$\begin{aligned}\mathcal{A}^{\mu}_{\text{point}} &= -\frac{f_B m_B}{f}v^{\mu},\\ \mathcal{A}^{\mu\nu}_{\text{point}} &= 0.\end{aligned} \tag{16}$$

We may now solve for the form factors f_+, f_- and h, obtaining

$$\begin{aligned}f_{\pm} &= -\frac{f_B}{2f}\left[1\pm g\frac{m_B\mp p_K\cdot v}{\Delta+p_K\cdot v}\right],\\ h &= -\frac{f_B}{2f}\frac{g}{\Delta+p_K\cdot v}.\end{aligned} \tag{17}$$

Note that in the form factors $f_{\pm}$, the pole amplitudes dominate the direct ones by a factor $m_B/(p_K\cdot v+\Delta)$, so $f_{\pm}\to\mp g f_B m_B/2f(\Delta+p_K\cdot v)$ as $m_b\to\infty$. Substituting f_+ and h into eq. (4), we may now compute the partial decay rate. It is convenient to normalize to the semileptonic width $\Gamma(\overline{B}\to X_c e^-\overline{\nu})$, after which we obtain

$$\begin{aligned}\frac{1}{\Gamma(\overline{B}\to X_c e^-\overline{\nu})}\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{s}}(\overline{B}\to\overline{K}e^+e^-) =& \frac{\alpha^2}{8\pi^2}\frac{g^2 f_B^2}{4f^2}\frac{1}{(\hat{E}_K+\hat{\Delta})^2}\\ &\times\left[\left(1-\hat{m}_K^2\right)^2-2\hat{s}\left(1+\hat{m}_K^2\right)+\hat{s}^2\right]^{3/2}\\ &\times\left[\left|c_8(m_b)\hat{f}_+(\hat{s})+2c_7(m_b)\right|^2+\left|c_9(m_b)\hat{f}_+(\hat{s})\right|^2\right],\end{aligned} \tag{18}$$

where $\hat{m}_K^2 = m_K^2/m_B^2$, $\hat{\Delta}=\Delta/m_B$ and

$$\begin{aligned}\hat{E}_K &= E_K/m_B = (1+\hat{m}_K^2-\hat{s})/2,\\ \hat{f}_+ &= 1-\hat{E}_K+(\hat{E}_K+\hat{\Delta})/g.\end{aligned} \tag{19}$$

Our results so far assume an exact $SU(3)$ chiral symmetry among the light quarks. The virtue of this effective lagrangian formalism is that it allows us to make some estimate of the size of $SU(3)$ violating corrections. Of course, the leading corrections typically

involve new terms in the chiral lagrangian, whose coefficients must be fixed. Unfortunately, the current paucity of data on heavy meson interactions with pseudogoldstone bosons precludes any experimental determination of these coefficients. However, there are certain nonanalytic corrections, such as those of the form $m_\pi^2 \ln m_\pi^2$, which are independent of such new terms. These corrections are determined uniquely by loops in the flavor-conserving effective lagrangian, in which the $SU(3)$ violation enters indirectly via the pseudogoldstone boson masses. While such chiral logarithms are in fact dominant in the limit of very small light quark masses, for the physical pions and kaons this is unlikely to be the case. Still, we may hope that such loops at least indicate the magnitude of $SU(3)$ violation, even if they do not provide us with precise quantitative information. In particular, if the nonanalytic corrections are large ($\sim 100\%$), we will certainly know not to trust the results (17) and the extrapolation of matrix elements from $\overline{B} \to \pi$ to $\overline{B} \to \overline{K}$. However, if they are small we may gain some additional confidence that what we have done is sensible. In any case, this is the spirit in which we shall proceed.

Since we expect the largest corrections to come from the large K and η masses, it is appropriate to simplify the calculation by making two approximations. First, we shall set $m_{\pi^\pm} = m_{\pi^0} \equiv m_\pi$ and $m_{K^\pm} = m_{K^0,\overline{K}^0} \equiv m_K$. Second, we shall set all mass splittings between the various flavor and spin states of the B mesons to zero when they appear in loops. (Note that we do *not* ignore the splitting Δ when it appears in a pole, as in eq. (15).) In order to focus on $SU(3)$ violation, we will compute separately the corrections to the matrix elements for $B^- \to \pi^-$ and $B^- \to K^-$. For each nonvanishing graph, we will present the nonanalytic dependence on the pion masses and on the momentum of the external pion or kaon, giving the answer as a fractional correction to the tree level result. At the end we will assemble the various pieces and provide a numerical estimate of the size of these leading nonanalytic contributions to the violation of chiral $SU(3)$ symmetry.

It will be convenient to express the results in terms of a few general Feynman integrals. After applying dimensional regularization to the ultraviolet divergences, there will be nonanalytic dependence not only on the pion masses and the external momenta, but on the renormalization scale μ as well. Since it is precisely this behavior in which we are interested, we will drop any additional constants which may appear.

The first two integrals have no Lorentz dependence. They are

$$\begin{aligned} \mathrm{i}\int \frac{\mathrm{d}^{4-\epsilon}p}{(2\pi)^{4-\epsilon}} \frac{1}{p^2-m^2} &= \frac{1}{16\pi^2} I_1(m) + \dots , \\ \mathrm{i}\int \frac{\mathrm{d}^{4-\epsilon}p}{(2\pi)^{4-\epsilon}} \frac{1}{(p^2-m^2)(p\cdot v-\Delta)} &= \frac{1}{16\pi^2}\frac{1}{\Delta} I_2(m,\Delta) + \dots , \end{aligned} \tag{20}$$

where

$$
\begin{aligned}
I_1(m) &= m^2 \ln(m^2/\mu^2)\,, \\
I_2(m,\Delta) &= -2\Delta^2 \ln(m^2/\mu^2) - 4\Delta^2 F(m/\Delta)\,.
\end{aligned}
\tag{21}
$$

The function $F(x)$ will appear frequently. It is most convenient to write it in a form where the smooth transition between the regimes $x < 1$ and $x > 1$ is apparent:

$$
F(x) = \begin{cases} \sqrt{1-x^2}\,\tanh^{-1}\sqrt{1-x^2}\,, & x \le 1 \\ -\sqrt{x^2-1}\,\tan^{-1}\sqrt{x^2-1}\,, & x \ge 1 \end{cases}
\tag{22}
$$

The third integral is a two-index symmetric tensor:

$$
\begin{aligned}
J^{\mu\nu}(m,\Delta) &= \mathrm{i}\int \frac{\mathrm{d}^{4-\epsilon}p}{(2\pi)^{4-\epsilon}} \frac{p^\mu p^\nu}{(p^2-m^2)(p\cdot v-\Delta)} \\
&= \frac{1}{16\pi^2}\Delta\left[J_1(m,\Delta)g^{\mu\nu} + J_2(m,\Delta)v^\mu v^\nu\right] + \ldots\,,
\end{aligned}
\tag{23}
$$

where

$$
\begin{aligned}
J_1(m,\Delta) &= (-m^2 + \frac{2}{3}\Delta^2)\ln(m^2/\mu^2) + \frac{4}{3}(\Delta^2 - m^2)F(m/\Delta)\,, \\
J_2(m,\Delta) &= (2m^2 - \frac{8}{3}\Delta^2)\ln(m^2/\mu^2) - \frac{4}{3}(4\Delta^2 - m^2)F(m/\Delta)\,.
\end{aligned}
\tag{24}
$$

Finally, we have an integral which can be derived from $J^{\mu\nu}$,

$$
\begin{aligned}
K^{\mu\nu}(m,\Delta_1,\Delta_2) &= \mathrm{i}\int \frac{\mathrm{d}^{4-\epsilon}p}{(2\pi)^{4-\epsilon}} \frac{p^\mu p^\nu}{(p^2-m^2)(p\cdot v-\Delta_1)(p\cdot v-\Delta_2)} \\
&= \frac{1}{16\pi^2}\left[K_1(m,\Delta_1,\Delta_2)g^{\mu\nu} + K_2(m,\Delta_1,\Delta_2)v^\mu v^\nu\right] + \ldots \\
&= \frac{1}{\Delta_1-\Delta_2}\left[J^{\mu\nu}(m,\Delta_1) - J^{\mu\nu}(m,\Delta_2)\right]\,.
\end{aligned}
\tag{25}
$$

We will need only the limit $K(m,\Delta) = K_1(m,\Delta,0)$, which takes the simple form

$$
K(m,\Delta) = J_1(m,\Delta) - \frac{2\pi}{3}\frac{m^3}{\Delta}\,,
\tag{26}
$$

and we note that $K(m,0) = -I_1(m)$.

With these integrals in hand, we now turn to the set of Feynman graphs which we must compute. The diagrams fall into three classes: those which correct the pole amplitudes $\mathcal{A}_{\rm pole}$, those which correct the point amplitudes $\mathcal{A}_{\rm point}$, and those which correct both. In the last class is the wavefunction renormalization of the B^- meson, depicted in fig. 2. This graph is universal, independent of the external pion momentum or flavor. The result may

be obtained from ref. [9]. For both $\mathcal{A}_{\rm pole}$ and $\mathcal{A}_{\rm point}$, we find a fractional correction to the tree amplitude of

$$\frac{g^2}{16\pi^2 f^2}\left[-\frac{9}{4}I_1(m_\pi)-\frac{3}{2}I_1(m_K)-\frac{1}{4}I_1(m_\eta)\right]. \tag{27}$$

There are two nonzero graphs which correct the point amplitude $\mathcal{A}_{\rm point}$, depicted in fig. 3. Although we have seen that the form factors of interest are actually dominated by the pole amplitude, we will include these diagrams for completeness. The diagram in fig. 3(a) yields a fractional correction to the matrix element for $B^- \to \pi^-$ of

$$\frac{1}{16\pi^2 f^2}\left[-\frac{5}{12}I_1(m_\pi)-\frac{1}{3}I_1(m_K)-\frac{1}{12}I_1(m_\eta)\right], \tag{28}$$

while for $B^- \to K^-$ the result is

$$\frac{1}{16\pi^2 f^2}\left[-\frac{1}{4}I_1(m_\pi)-\frac{1}{2}I_1(m_K)-\frac{1}{12}I_1(m_\eta)\right]. \tag{29}$$

The graph in fig. 3(b) requires a two-pion interaction which arises from the V^μ_{ab} part of the heavy meson kinetic energy term (9). It also depends on $E_\pi = p_\pi \cdot v$, the energy of the external pion (or kaon) in the rest frame of the B^-. For $B^- \to \pi^-$, we find

$$\frac{1}{16\pi^2 f^2}\left[I_1(m_\pi)+\frac{1}{2}I_1(m_K)+2I_2(m_\pi,E_\pi)+I_2(m_K,E_\pi)\right]. \tag{30}$$

For $B^- \to K^-$ we obtain

$$\frac{1}{16\pi^2 f^2}\left[I_1(m_K)+\frac{1}{2}I_1(m_\eta)+2I_2(m_K,E_K)+I_2(m_\eta,E_K)\right]. \tag{31}$$

The diagrams in fig. 3(c) and fig. 3(d) vanish.

There are four nonzero graphs which correct the pole amplitude $\mathcal{A}_{\rm pole}$, depicted in fig. 4. The diagram in fig. 4(a) is simple, since it is independent of the external pion momentum. For $B^- \to \pi^-$ we find the fractional correction

$$\frac{1}{16\pi^2 f^2}\left[-\frac{3}{4}I_1(m_\pi)-\frac{1}{2}I_1(m_K)-\frac{1}{12}I_1(m_\eta)\right], \tag{32}$$

while for $B^- \to K^-$ we obtain

$$\frac{1}{16\pi^2 f^2}\left[-I_1(m_K)-\frac{1}{3}I_1(m_\eta)\right]. \tag{33}$$

The graph in fig. 4(b) is equally straightforward. The correction to $B^- \to \pi^-$ is given by

$$\frac{1}{16\pi^2 f^2}\left[-\frac{2}{3}I_1(m_\pi) - \frac{1}{3}I_1(m_K)\right] , \tag{34}$$

while for $B^- \to K^-$ it is

$$\frac{1}{16\pi^2 f^2}\left[-\frac{1}{4}I_1(m_\pi) - \frac{1}{2}I_1(m_K) - \frac{1}{4}I_1(m_\eta)\right] . \tag{35}$$

The diagrams in fig. 4(c) and (d) actually consist of two graphs. Since the interaction term (10) contains a $\overline{B}^*$-$\overline{B}^*$-π coupling as well as $\overline{B}^*$-$\overline{B}$-π, the heavy meson line can take either the form $\overline{B}$—$\overline{B}^*$—$\overline{B}$—$\overline{B}^*$ or the form $\overline{B}$—$\overline{B}^*$—$\overline{B}^*$—$\overline{B}^*$. In fig. 4(c) the second possibility gives twice the former. We find a somewhat more complicated dependence on the external momentum $p_\pi \cdot v$, which is expressed in terms of the integral $J^{\mu\nu}$. However, we can resum this contribution into the denominator of the $\overline{B}^*$ propagator, at which point it is consistent with our approximations to subtract the term which renormalizes the meson mass. This procedure introduces the limit $K(m,\Delta)$ of the general integral $K^{\mu\nu}$. For $B^- \to \pi^-$ we then find the correction

$$\frac{g^2}{16\pi^2 f^2}\left[\frac{9}{2}K(m_\pi, E_\pi) + 3K(m_K, E_\pi) + \frac{1}{2}K(m_\eta, E_\pi)\right] , \tag{36}$$

and for $B^- \to K^-$ we obtain

$$\frac{g^2}{16\pi^2 f^2}\left[6K(m_K, E_K) + 2K(m_\eta, E_K)\right] . \tag{37}$$

In fig. 4(d), the second possibility gives minus twice the first. The momentum dependence enters through the limit $K(m,\Delta)$ of the general integral $K^{\mu\nu}$. The fractional correction to $B^- \to \pi^-$ may then be written

$$\frac{g^2}{16\pi^2 f^2}\left[-\frac{1}{2}K(m_\pi, E_\pi) + \frac{1}{6}K(m_\eta, E_\pi)\right] , \tag{38}$$

while for $B^- \to K^-$ we obtain

$$\frac{g^2}{16\pi^2 f^2}\left[-\frac{1}{3}K(m_\eta, E_K)\right] . \tag{39}$$

The diagrams in fig. 4(e)–(g) vanish identically.

Finally, for both $\mathcal{A}_{\rm pole}$ and $\mathcal{A}_{\rm point}$ we must include the wavefunction renormalization of the external pseudogoldstone boson, as shown in fig. 5. The pion self-interaction is induced by the kinetic energy term (9). For $B^- \to \pi^-$ we find the fractional correction

$$\frac{1}{16\pi^2 f^2}\left[-\frac{2}{3}I_1(m_\pi) - \frac{1}{3}I_1(m_K)\right] , \tag{40}$$

while for $B^- \to K^-$ we obtain

$$\frac{1}{16\pi^2 f^2}\left[-\frac{1}{4}I_1(m_\pi) - \frac{1}{2}I_1(m_K) - \frac{1}{4}I_1(m_\eta)\right] . \tag{41}$$

We now assemble these various amplitudes into an estimate of the size of $SU(3)$ corrections in this process. We begin with the pole amplitudes, because they dominate the observable form factors in the limit $m_b \to \infty$. Although one could simply add together the diagrams in fig. 2, fig. 4 and fig. 5, it is more reasonable to absorb some of the corrections into a renormalization of the heavy meson decay constant f_B. Since in $\mathcal{A}_{\rm pole}$ the pion or kaon is emitted before the flavor-changing operator $\mathcal{O}^\mu$ or $\mathcal{O}^{\mu\nu}$ acts, it is either f_{B_d} (for $B^- \to \pi^-$) or f_{B_s} (for $B^- \to K^-$) which is relevant to the amplitude. In fact, this would be precisely the combined effect of fig. 4(a) and half of fig. 4(c), if the momentum of the external pion or kaon were set to zero. The relation between the bare parameter f_B and the renormalized decay constants, computed in the same chiral logarithmic approximation, is given by [9]

$$\begin{aligned} f_B &= f_{B_d}\left\{1 + \frac{1}{16\pi^2 f^2}\left(\frac{1}{2} + \frac{3}{2}g^2\right)\left[\frac{3}{2}I_1(m_\pi) + I_1(m_K) + \frac{1}{6}I_1(m_\eta)\right]\right\} , \\ f_B &= f_{B_s}\left\{1 + \frac{1}{16\pi^2 f^2}\left(\frac{1}{2} + \frac{3}{2}g^2\right)\left[2I_1(m_K) + \frac{2}{3}I_1(m_\eta)\right]\right\} . \end{aligned} \tag{42}$$

Similarly, it is appropriate to renormalize the pseudogoldstone boson decay constant f to f_π or f_K, for which we have [10]

$$\begin{aligned} f &= f_\pi\left\{1 - \frac{1}{16\pi^2 f^2}\left[2I_1(m_\pi) + I_1(m_K)\right]\right\} , \\ f &= f_K\left\{1 - \frac{1}{16\pi^2 f^2}\left[\frac{3}{4}I_1(m_\pi) + \frac{3}{2}I_1(m_K) + \frac{3}{4}I_1(m_\eta)\right]\right\} . \end{aligned} \tag{43}$$

Note that in the amplitudes $\mathcal{A}_{\rm pole}$ and $\mathcal{A}_{\rm point}$, f appears in the denominator.

In estimating the diagrams, we take the masses $m_\pi = 140$ MeV, $m_K = 490$ MeV, and $m_\eta = 550$ MeV. Since the largest corrections are come from the K and η masses, we take

the pseudogoldstone boson decay constant f to be $f_K \approx 165$ MeV. To be conservative, we take the renormalization scale $\mu = 1$ GeV, since this choice magnifies the effect of the chiral logarithms. For the same reason we choose the coupling g to be as large as possible; since from the width for $D^* \to D\pi$ we have $g^2 \leq 0.5$, we take $g^2 = 0.5$ in our estimates.[2] Finally, when they appear we take the external pseudogoldstone boson energies to be equal to their masses, $E_\pi = m_\pi$. This is consistent with the soft pion limit in which we are working, and simplifies our estimates.

Assembling the corrections as we have described, and replacing $f_B \to f_{B_d}$, $f \to f_\pi$ in $B^- \to \pi^-$ and $f_B \to f_{B_s}$, $f \to f_K$ in $B^- \to K^-$, we obtain a residual correction to the dominant pole amplitudes $\mathcal{A}_{\rm pole}$, for $B^- \to \pi^-$ of -13% and for $B^- \to K^-$ of -51%. Hence, in this approximation where we keep only the nonanalytic dependence on the masses, we find $SU(3)$ violation at the level of $\sim 40\%$. For the point amplitudes $\mathcal{A}_{\rm point}$, we must include the diagrams in fig. 2, fig. 3 and fig. 5, plus the decay constant redefinitions (42) and (43). We then find a correction of 1% to the amplitude for $B^- \to \pi^-$, while the correction to $B^- \to K^-$ is 13%.

Finally, we may use our results to estimate the $SU(3)$ corrections to the coupling constant g which multiplies the interaction term (10). This is given by the graphs in fig. 4(b) and (d), plus the wavefunction renormalization on the external meson (fig. 2(a)) and pseudogoldstone boson (fig. 5(a)) lines. The only new piece is the B_s wavefunction renormalization; like that for the B^-, it may be obtained from ref. [9], and is given by

$$\frac{g^2}{16\pi^2 f^2}\left[-3I_1(m_K) - I_1(m_\eta)\right] . \tag{44}$$

The tree level amplitude due to the interaction (10) is proportional to g/f; at one loop, for an external pion this will become g_π/f_π, and for an external kaon g_K/f_K. Hence we must also include the correction (43) in computing g_π and g_K. Assembling the results, we find $g_\pi \approx 1.14\,g$ and $g_K \approx 1.21\,g$. This effect is in part $SU(3)$ symmetric; $SU(3)$ violation appears at the level of only $\sim 7\%$. As we have noted, however, the $SU(3)$ conserving correction has an impact on the extraction of the parameter g from the decay $D^* \to D\pi$.

[2] In fact, we would be justified in using the amplitudes in fig. 2 and fig. 4(b)–(d) to correct the prediction for $D^* \to D\pi$ and extract experimentally a "renormalized" g. Doing this would tighten the experimental upper limit on g by approximately 15%; instead of the limit $g^2 < 0.5$, we would have $g^2 < 0.4$. However, to be conservative, we do not include this additional restriction here.

The violation of $SU(3)$ symmetry at the 40% level which we have found is substantial, but not necessarily so much so that we would consider the entire computation to be untrustworthy. Indeed, we do not find nonanalytic corrections at the level of 100%, such as plague other processes. Of the 40% correction, half of it comes from resolving the flavor ambiguities in the decay constants via the replacements (42) and (43). Of course, we should stress that by itself the computation of the nonanalytic corrections proves nothing, since the analytic corrections due to higher order terms in the phenomenological lagrangian could still be large and spoil the desired relations. Rather, we view our calculation as helping to build confidence that using $SU(3)$ symmetry to compute the form factors for $\overline{B} \to \overline{K}e^+e^-$ may indeed be a sensible treatment of the nonperturbative matrix elements.

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Figure Captions

Fig. 1. Tree level amplitudes for $\overline{B} \to \overline{K}$. The solid line represents the heavy meson, the dashed lines pseudogoldstone bosons. The solid square indicates the insertion of the flavor-changing operator $\mathcal{O}^{\mu}$ or $\mathcal{O}^{\mu\nu}$. (a) the pole amplitude $\mathcal{A}_{\rm pole}$; (b) the point amplitude $\mathcal{A}_{\rm point}$.

Fig. 2. Diagrams contributing to the wavefunction renormalization of the external B^-. (a) correction to $\mathcal{A}_{\rm pole}$; (b) correction to $\mathcal{A}_{\rm point}$.

Fig. 3. Diagrams which correct the point amplitude $\mathcal{A}_{\rm point}$.

Fig. 4. Diagrams which correct the pole amplitude $\mathcal{A}_{\rm pole}$.

Fig. 5. Diagrams contributing to the wavefunction renormalization of the external pseudogoldstone boson. (a) correction to $\mathcal{A}_{\rm pole}$; (b) correction to $\mathcal{A}_{\rm point}$.

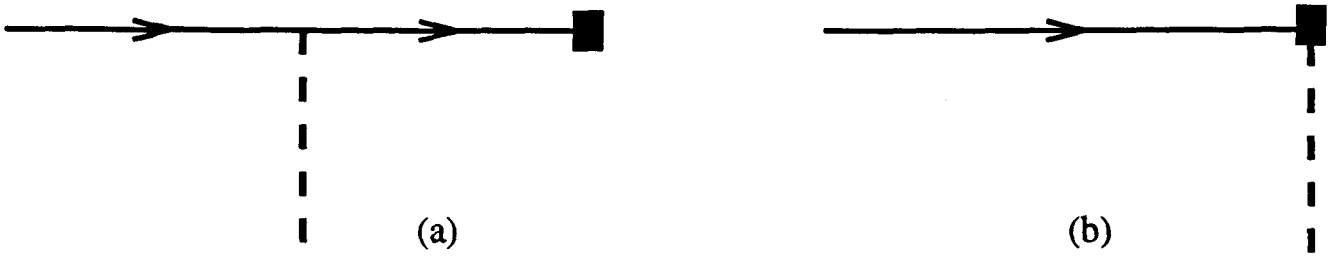

Fig 1

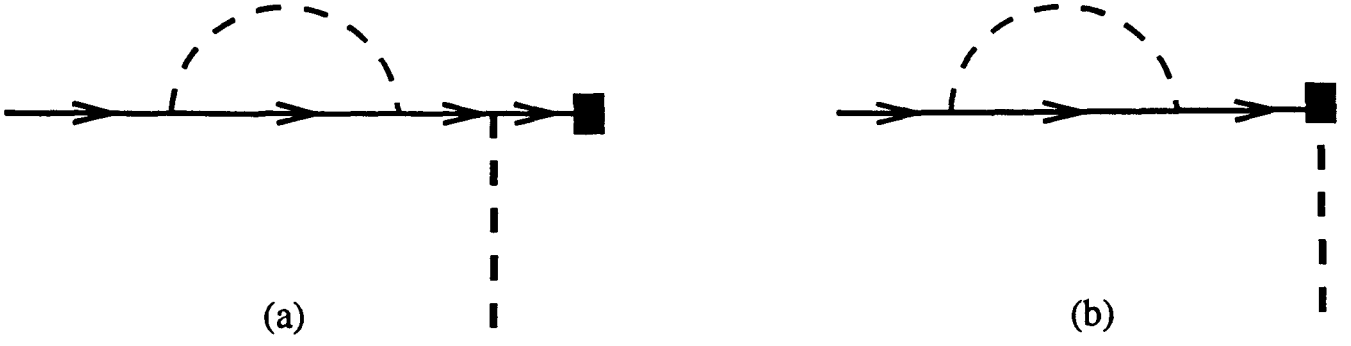

Fig 2

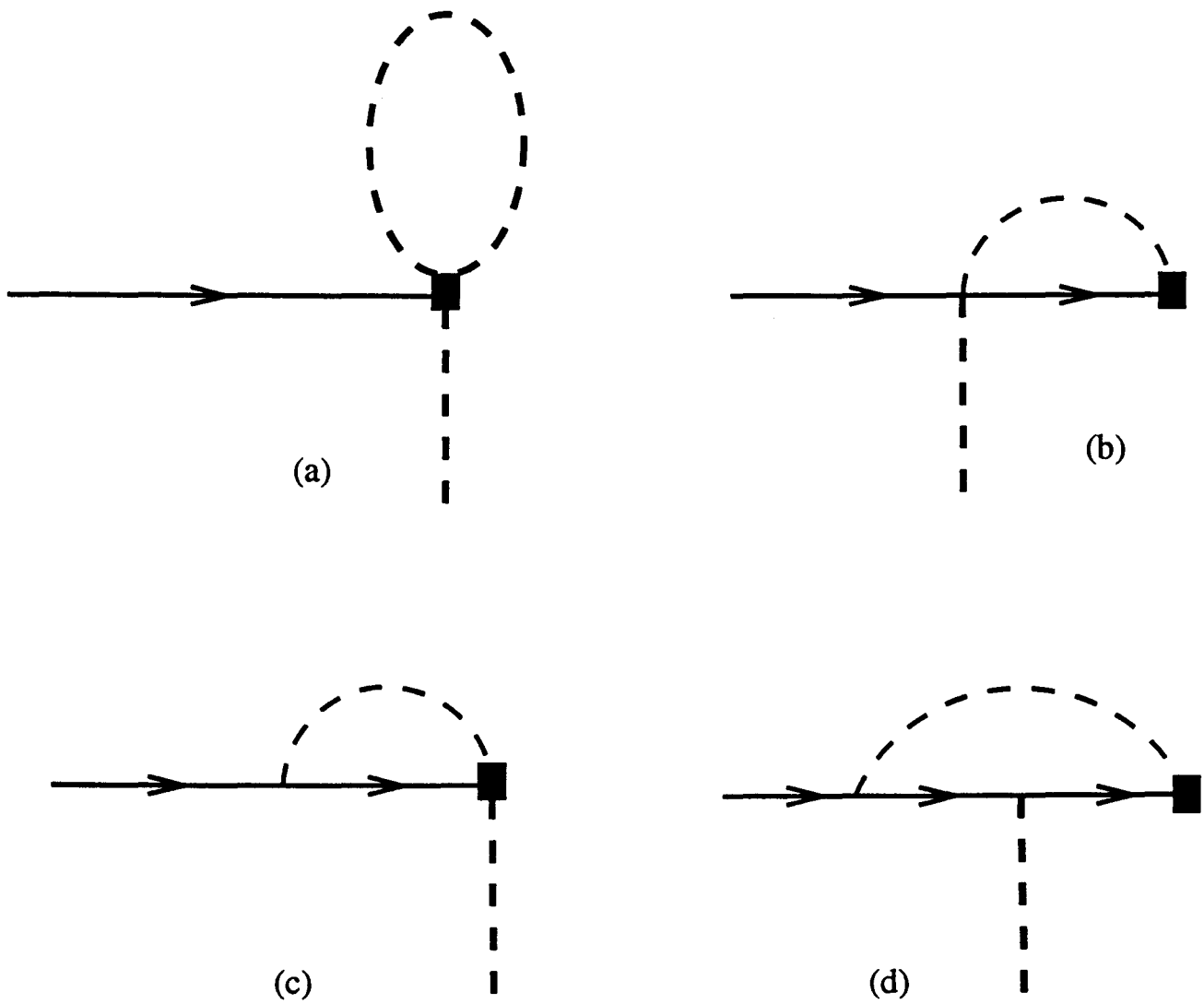

Fig 3

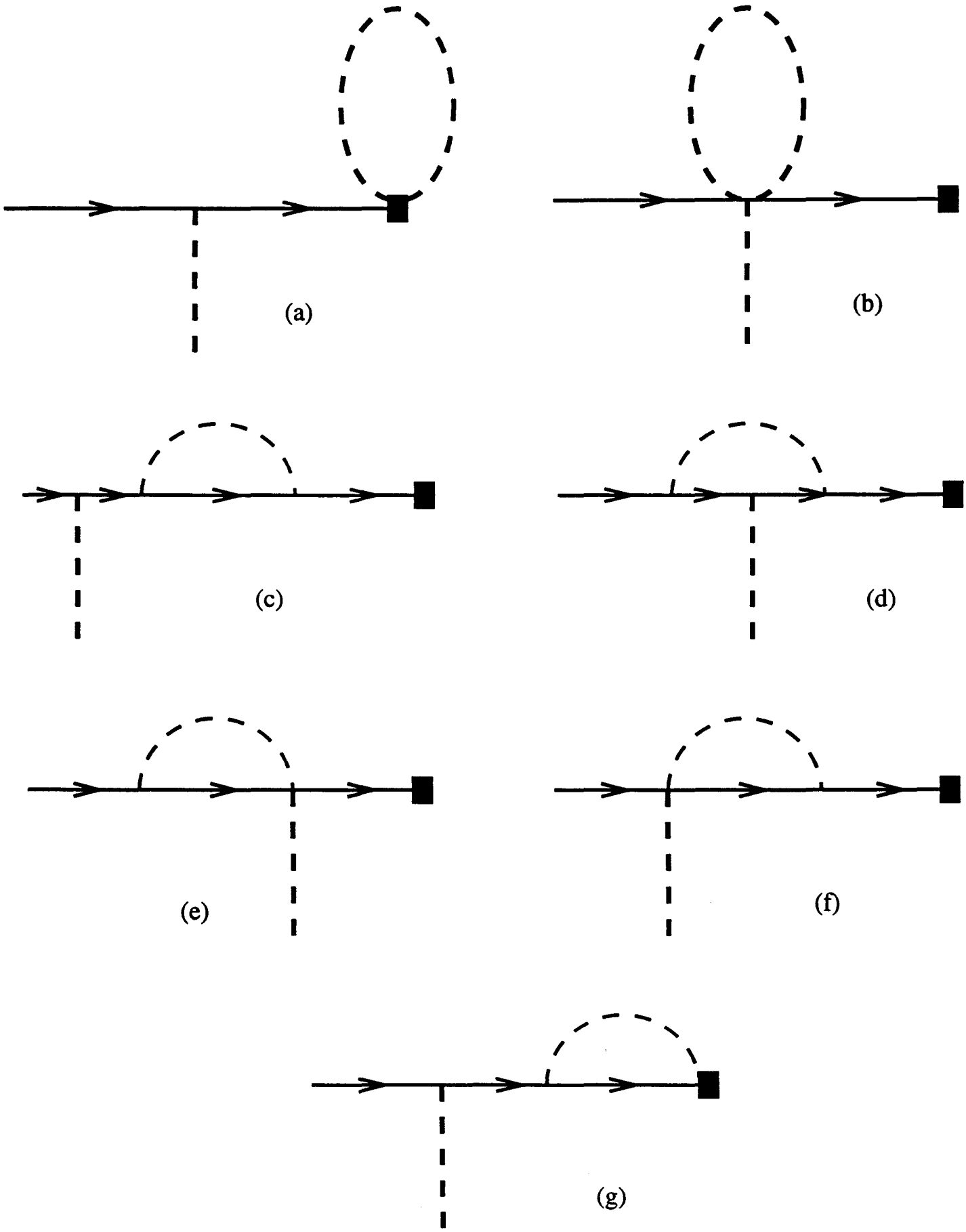

Fig 4

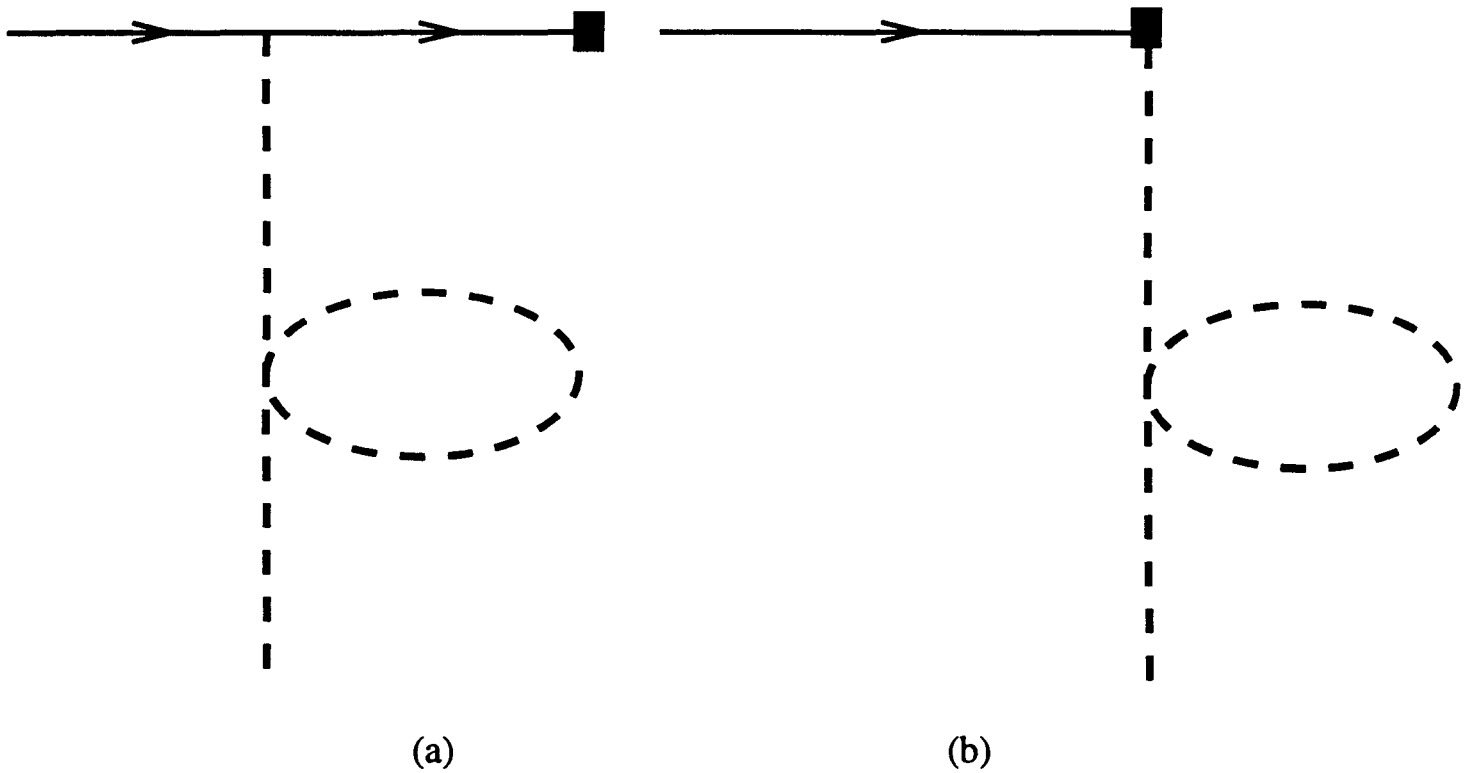

Fig 5