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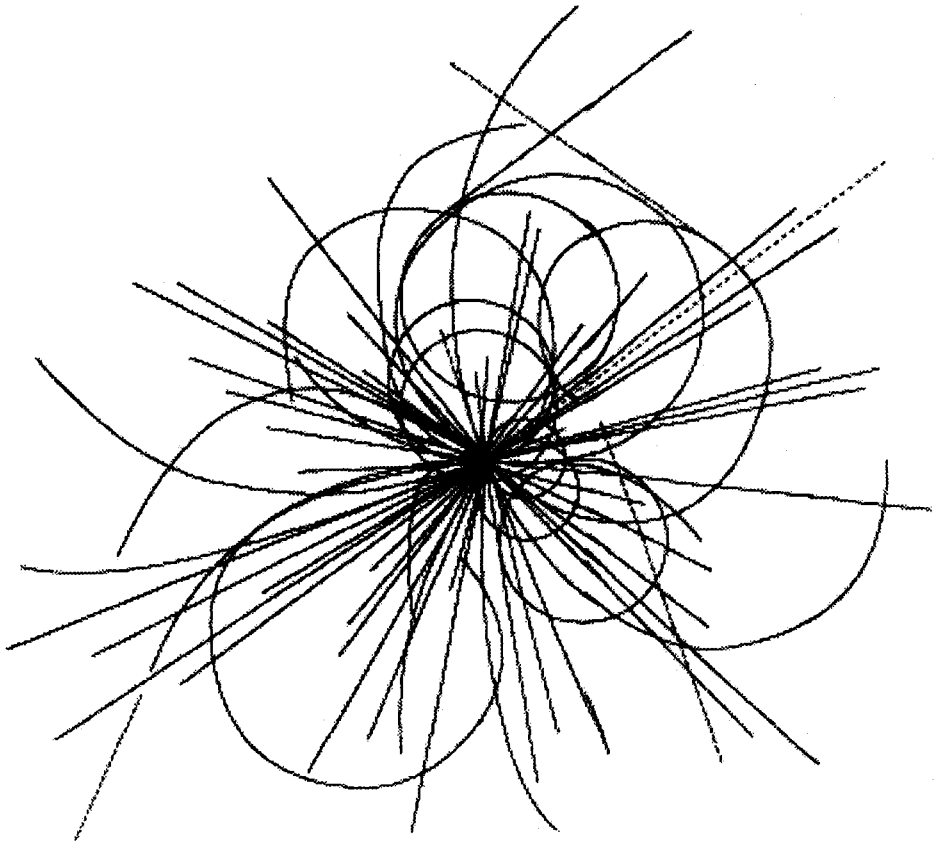

Superconducting Super Collider Laboratory

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# Slow Ground Motion and Operation of Large Colliders*

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# Slow Ground Motion and Operation of Large Colliders

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Slow ground motion with frequencies much less than characteristic frequencies of an accelerator usually has been considered as not seriously affecting machine operation because of assumed complete space and time coherence of the magnet displacements. In this paper, we analyze geophysical and numerous accelerator data on uncorrelated ground motion. We demonstrate that these data in many cases can be approximated by the "ATL law" that assumes diffusive wandering of the ground elements. An equation is derived that governs closed orbit distortion caused by random (though correlated) quadrupole displacements in a large circular accelerator. Estimates for the Superconducting Super Collider (SSC) are carried out and compared with the ability of the corrections system to maintain the orbit in the collider.

## 1 INTRODUCTION

In an ideal accelerator with well-aligned magnetic elements, the closed orbit passes through the centers of bending magnets and quadrupoles to provide optimal conditions for the operation of the machine. Any alignment errors cause a closed orbit distortion (COD), reduce the dynamic aperture of the ring, and, in an extreme situation, make it impossible to run the machine.[1] Typically, a correction system is used to counteract the magnetic errors that build up and accumulate during the operation of the machine. In large accelerators, such as the Superconducting Super Collider (SSC), Large Hadron Collider (LHC), and HERA, and which have many hundreds of magnetic elements, one of the most important sources of magnetic errors on a long time scale is the ground motion that displaces the magnets from their original position. Similar problems associated with the ground motion arise also for linear electron-positron colliders,[2, 3] B-factories, and a new generation of synchrotron radiation sources.

Numerous measurements of ground motion at accelerators (see, for example, Refs. 4–12) have shown that in the frequency range above 1 Hz "cultural noise" vibrations (produced by human activity) are dominant. The amplitude of this motion does not exceed 0.1 μm. Below 1 Hz the most visible vibrations are microseismic waves (with a period of approximately 7 s and a wavelength of several tens of kilometers) with the amplitudes 0.1–10 μm. In Ref. 13, COD in a periodic FODO lattice perturbed by a plane ground wave has been considered. It was found that from the point of view of COD the most dangerous wavelength is equal to the betatron wavelength, $\lambda \approx C/\nu$, where $C$ is the circumference of

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the ring and $\nu$ is the tune. Waves having the wavelength $\lambda \gg C/\nu$ perturb the beam motion in the ring only adiabatically, their response being proportional to a small factor of $(C/\nu\lambda)^2$.

In addition to microseismic noise, there exists an ambient low-frequency ground motion that is generated by local sources such as winds, temperature gradients, etc., and cannot be adequately treated as waves propagated in the ground. To study the effect of such motion one has to develop a theory that takes into account spatial correlation between displacements of two different points on the ground surface. In this paper we present our analysis of the problem.

In Section 2, a brief overview is given of the existing data on ground motion. Section 3 is devoted to the diffusive model of ground motion that is often referred to as the "ATL law." In Section 4, we formulate an equation that relates COD with the spatial correlation function of the ground displacements. The derivation of this equation is contained in Appendix B. In Appendix A, several methods are presented that allow us to numerically simulate random displacements governed by the ATL law.

## 2 ANALYSIS OF DATA FROM EXISTING ACCELERATORS

### *2.1 Ground Motion*

In recent years, measurements of ground motion in a wide range of frequencies have been performed at several accelerator sites around the world. Figure 1 shows the spectral density of the ground motion as a function of frequency $f$ for different places. A distinctive feature of these spectra is that all of them rapidly grow when $f$ goes to zero. A very rough interpolation of the spectra shown in Figure 1 in the region of low frequencies suggests a scaling law $S \propto f^{-3}$.

By definition, the integral of the spectral density over a frequency interval gives the averaged square of the displacement produced by the ground motion in this frequency range. If one uses interpolation $S \propto f^{-3}$ mentioned above, the resulting integral diverges when integrated from zero. This gives rise to an important question of how one can evaluate COD produced by the low-frequency ground motion. It is clear, however, that from the point of view of COD the divergence of the integral is not relevant to COD because, presumably, very low-frequency motion is well correlated on the ring diameter and displaces the accelerator ring as a rigid body. This kind of displacement cannot be detected as COD, because the orbit will be moving together with the magnets and diagnostic tools. Hence, for large machines, the most dangerous displacement will be

uncorrelated relative shifts of the quadrupoles that produce COD proportional to the number of the quads in the ring.

In addition to direct measurements of the ground motion, there are several observations of COD in existing accelerators. To compare these data for different machines, it has been proposed in Ref. 14 to convert them into displacements of the quadrupoles—assuming that the measured COD is solely due to independent displacements of the quads in the ring (and using the known lattice parameters of the ring). Of course, in this conversion one neglects the contribution to COD of the internal noise in the accelerator, so that the resulting data give an upper limit for the actual ground motion. The spectral densities of the displacements thus calculated for Tevatron, HERA (electron and proton rings), and KEK photon factory, along with a spectrum of COD in the Novosibirsk VEPP-3 electron storage ring, are shown in Figure 2. The dashed line in this figure represents the fit

$$S(f) = 0.03/f^2, \tag{1}$$

where $S$ is measured in $\mu\text{m}^2/\text{Hz}$ and $f$ in Hz. Point $a$ in Figure 2 represents the measurements performed at KEK photon factory[15] and corresponds to a COD of 100 μm in 2 h; point $b$ shows the results from HERA(e) with a COD of 200 μm in the time interval of 80 s.[4]

Some data at even lower frequencies (corresponding to a time period of a few days) have been reported from the KEK TRISTAN storage ring. Figure 3, taken from Ref. 16, shows COD in TRISTAN at 8.0 GeV. Full circles in the figure are the rms values of COD $\sigma_1$ as a function of time calculated with the use of the formula $\sigma_1^2 = N^{-1}\sum_{i=1}^{N} x_i^2$, where $x_i$ is the measured displacement of the CO relative to the "ideal" orbit at the location of the $i$-th beam position monitor (BPM) and $N = 392$ is the total number of BPMs. Open circles represent the value of $\sigma_2$, $\sigma_2^2 = N^{-1}\sum_{i=1}^{N}\left(x_i - x_{i0}\right)^2$, where $x_{i0}$ is the initial value of $x_i$ during an operation cycle between two successive corrections of the orbit. Note that the horizontal COD is smaller than the vertical one. It has been observed that when $\sigma_1$ reached values larger than 100–200 μm, the maximum beam current degraded significantly so that a correction of the orbit was needed toward the "ideal" one (sharp drops at points D, E, H, and some others in Figure 3). Using the data in Figure 3, we analyzed how CODs have been developing between the corrections. The results are summarized in Table 1, where the first column shows the time span in Figure 3, $T$ is the duration of the time interval between the corrections of the orbit, and $\sigma_2$ is the COD accumulated in this time. We have also

calculated the parameter $A = \sigma_2^2 T^{-1} C^{-1}$ ($C = 3000$ m is the TRISTAN circumference), the meaning of which is explained in Section 3 in connection with diffusive model of the ground motion.

TABLE 1: Development of COD between corrections.

| Time interval (h) | $\sigma_2$ (μm) | T (h) | $A = \sigma_2^2 T^{-1} C^{-1}$ (μm²/(s·m)) |
|---|---|---|---|
| 0–50 | 300 | 50 | $2 \cdot 10^{-4}$ |
| 66–96 | 140 | 30 | $0.6 \cdot 10^{-4}$ |
| 97–121 | 140 | 24 | $0.75 \cdot 10^{-4}$ |
| 123–135 | 220 | 12 | $3.5 \cdot 10^{-4}$ |
| 150–158 | 120 | 8 | $1.6 \cdot 10^{-4}$ |

Note that the values of $A$ are roughly constant, $A = (1.7 \pm 0.9) \cdot 10^{-4}$.

### 2.2 *Large Accelerators and Geophysical Phenomena*

There are several interesting observations of the influence of the earth's tides on large accelerators. Simultaneous measurements of the tide strain and the transverse beam motion in TRISTAN[17] showed that the vertical beam displacement oscillates with an amplitude of approximately 200 μm in a period of 1 day, which might result from the local deformations of the tunnel under the tide forces due to geological differences along the ring.

Due to the tidal forces, the circumference of the LEP electron-positron collider varies by approximately 1 mm over the distance of 27 km.[18] This minuscule effect produces detectable variations of the beam energy, which were measured using resonant depolarization technique to be $2 \cdot 10^{-5}$.

Other important factors that influence COD are local accelerator tunnel deformations caused by temperature variations and weather conditions. According to Refs. 9 and 17, the open air temperature variation (for example, from day to night) causes the TRISTAN tunnel deformations and results in a 0.1–0.2-mm COD in that machine. During seismic measurements at the UNK site (Protvino, Russia) an observable influence of the atmospheric pressure variation on the ground motion was detected.[5, 7] Recently this phenomenon was also discovered at KEK.[9] In Ref. 19, CODs in the arcs of the SSC during a strong storm were estimated to be in the 1-mm range. Note that the beam transverse size in the arcs of the SSC is about 100 μm.

## 3 DIFFUSIVE GROUND MOTION AND THE "ATL LAW"

The "ATL law" was originally proposed in Refs. 7 and 5 and developed in Ref. 20 to describe experimental data on relative displacement of two distant points on the ground surface. According to this law, the relative rms displacement $\delta x$ of two points located at a distance $L$ grows with time $T$:

$$\langle \delta x^2 \rangle = ATL, \tag{2}$$

where $A$ is a constant that roughly does not depend on the site. It was specially emphasized that such irregular diffusive motion occurs over a broad range of time intervals $T$ and distances $L$. The physical mechanisms responsible for the diffusive motion include groundwater-level variations, temperature and air-pressure fluctuations, microsesmic waves, earth tides, etc.

Originally it was found that Eq. (2) with the factor $A = (1.0 \pm 0.5) \cdot 10^{-4}$ μm²/(m·s) is in good agreement with the data obtained from relative displacement measurements by straight wire for $L$ varying from 1 m to 20 m and $T$ varying from tens of seconds up to tens of hours, and with optical measurements at the UNK site for $L$ up to 2 km and $T$ about 2 yr.[7] Also, analysis performed in Ref. 20 showed that Eq. (2) is confirmed by observation of SLAC Linear Collider SLC tunnel ($L$ = 3 km) displacements during 17 yr[12], and SLAC PEP tunnel displacements during 20 months over the circumference 2000 m.[6] Figure 4 shows the dependence of $\langle \delta x^2 \rangle / T$ versus the distance $L$. Different curves correspond to UNK, SLC, and PEP data mentioned above and to the ATL law with the coefficient $A = 10^{-4}$ μm²/(m·s). One can see that a general tendency $\delta x \propto \sqrt{T}$ is valid within a factor of 3.

Additional confirmations of the diffusive model can be found in COD measurements on existing machines. One of them has already been demonstrated in Table 1, where the factor $A$ has been evaluated using TRISTAN data. Also, a recent paper[21] reported about slow drifts of the closed orbit that lead to $1\sigma$ beam-beam separation at proton-electron collider HERA after 10 hr of operation, in good agreement with expectations due to the ATL law.

A different look at ATL dependence is provided by an analysis of the spectrum corresponding to Eq. (2). Using the relation $f \sim 1/T$, one can estimate the spectral density

$$S(f) \sim \langle \delta x^2 \rangle / f = AL/f^2 \propto \text{const}/f^2 . \tag{3}$$

This result is in agreement with Eq. (1), derived, as already pointed out, from COD data in different accelerators.

For computer simulations of the COD caused by diffusive ground motion, one has to be able to generate displacements that satisfy Eq. (2). In Appendix A, we describe several mathematical models that allow us to simulate this kind of random displacement.

Concluding of this section, we should note that the "ATL law" describes only irregular displacements of the surface of the ground caused by "natural" sources. In addition, there always exist "artificial" disturbances (especially for new tunnels), such as heavy construction work, excavation, settling of the ground, etc., that can superimpose on the diffusive ground motion.

## 4 CLOSED ORBIT DISTORTION IN FODO LATTICE WITH SPATIALLY CORRELATED DISPLACEMENTS OF THE QUADRUPOLES

To address the problem of COD caused by randomly displaced quadrupoles let us consider an accelerator ring having a FODO structure with *N/2* identical FODO cells. Each cell contains one focusing and one defocusing quadrupole located at a distance $l$ so that the circumference of the ring $C$ is $C = Nl$. Denote the focal length of each focusing quadrupole by $F$; correspondingly, the focal length of a defocusing quadrupole will be $-F$; the value of the beta function at the location of the quads will be denoted by $\beta_F$ and $\beta_D$, respectively. Let the betatron phase advance per cell be $\mu$ ; the betatron tune of the whole ring is $\nu = N\mu/4\pi$. The betatron phase advance $\phi_j$ for the $j$-th quadrupole relative to the 0-th quadrupole is equal to $\phi_j = j\mu/2$.

Assume that each quadrupole is randomly displaced in a transverse direction by the distance $\delta_i$. Using a well-known formula for the closed orbit distortion $\Delta x$, one can easily write the expression for the mean square of $\Delta x$ :

$$\langle \Delta x^2 \rangle = \frac{\beta}{4\sin^2 \pi\nu} \sum_{i,j} \frac{\sqrt{\beta_i \beta_j}}{F_i F_j} \langle \delta_i \delta_j \rangle \cos(\phi + \phi_i - \pi\nu) \cos(\phi + \phi_j - \pi\nu), \tag{4}$$

where $i$ and $j$ take values $0,1,\ldots N-1$, and $\beta$ is the value of the beta function at the observation point. The angular brackets in Eq. (4) stand for the averaging over the random values of $\delta_i$.

Let us introduce the correlation function $K(n)$,

$$K(i-j)=\langle\delta_i\delta_j\rangle, \tag{5}$$

which, by definition, is an even periodic function of its argument,

$$K(m)=K(-m),\ \ K(m+N)=K(m). \tag{6}$$

Perform a discrete Fourier-transform of the correlation function:

$$K(n)=\frac{1}{N}\sum_{m=0}^{\frac{1}{2}N}k_m\cos\left(\frac{2\pi nm}{N}\right). \tag{7}$$

Due to the symmetry and periodicity of $K(n)$, only $N/2+1$ Fourier coefficients $k_m$ are needed to Fourier expand $K(n)$. The equation for the coefficients $k_m$ reads

$$k_m=2\sum_{i=0}^{N-1}K(i)\cos\left(\frac{2\pi im}{N}\right)\text{ for }0<m<N/2, k_0=\sum_{i=0}^{N-1}K(i),\ \ k_{N/2}=\sum_{i=0}^{N-1}(-1)^{-i}K(i). \tag{8}$$

A derivation presented in Appendix B, based on the assumption $N\gg 1$, gives the following result for $\langle\Delta x^2\rangle$ in terms of Fourier components of the correlation function:

$$\langle\Delta x^2\rangle=\frac{\beta N}{64F_0^2\sin^2(\pi\nu)}\left(k_{[\nu]}\left(\sqrt{\beta_F}-\sqrt{\beta_D}\right)^2+k_{\frac{1}{2}N-[\nu]}\left(\sqrt{\beta_F}+\sqrt{\beta_D}\right)^2\right), \tag{9}$$

where $[\nu]$ is the integer part of $\nu$. In the derivation, we kept only terms that scale $\propto N$ and neglected terms $\sim O(1)$. Eq. (9) implies that the dominant contribution to the COD comes from the correlation having the wavelength equal to the betatron wave length—a fact well-known from analysis of the role of magnetic field errors in an accelerator ring.[1]

In the limit when quadrupole displacements are fully uncorrelated, that is, $K(i-j)=\langle\delta^2\rangle$ for $i=j$ and $K(i-j)=0$ for $i\neq j$, from Eq. (8) it follows that $k_m=2\langle\delta^2\rangle$ and Eq. (9) reduces to

$$\langle\Delta x^2\rangle=\frac{\beta N\langle\delta^2\rangle}{16F_0^2\sin^2(\pi\nu)}(\beta_F+\beta_D). \tag{10}$$

This result can be easily obtained directly from Eq. (4) if one notes that on the right hand side nonvanishing terms have $i = j$ and substitutes the averaged value of $\cos^2(\phi_i - \pi\nu)$ by 1/2.

In the opposite limit of a fully correlated motion of the quadrupoles (say, in the vertical direction), the ring displaces rigidly. It is evident that the closed orbit moves together with the ring so that $\Delta x = \delta$. In this limit, COD can be considered as negligibly small because it does not depend on the number of the quadrupoles $N$. In our approach, the correlated motion corresponds to $K(i-j) = \langle \delta^2 \rangle$ for any $i$ and $j$. This gives $k_0 = N\langle \delta^2 \rangle$ and $k_m = 0$ for $m \neq 0$ which, according to Eq. (9), results in $\langle \Delta x^2 \rangle = 0$. The null result occurs because, as pointed out above, in the derivation of Eq. (9) we neglected terms that do not scale proportionally to $N$.

For a Gaussian correlation function, $K(n) = \text{const} \cdot e^{-n^2 l^2/2\Delta l^2}$, having the correlation length $\Delta l$, Fourier coefficients $k_m$ can be computed numerically. The result of the computation is shown in Figure 5 on which $\Delta x = \sqrt{\langle \Delta x^2 \rangle}$ is plotted as a function of $\Delta l/l$. Being normalized by the value of $\sqrt{\langle \Delta x^2 \rangle}$ at $\Delta l = 0$, this function rapidly falls with the increase of $\Delta l$.

Let us now apply Eq. (9) to the situation when quadrupole displacements are governed by the "ATL law." The "ATL law," mentioned in the previous section, states that:

$$\left\langle \left(\delta_i - \delta_j\right)^2 \right\rangle = ATl|i-j|, \tag{11}$$

which implies that

$$K(i-j) = D - B|i-j|, \tag{12}$$

where $B = ATl/2$ and $D$ is a constant related to a fully correlated motion of all of the quads. According to the above consideration, the constant part of $K(n)$ does not disturb the orbit so that $D$ can be neglected. Calculations of the Fourier spectrum of the correlation function (12) with $D = 0$ yield:

$$k_m = \frac{B\left(1-(-1)^m\right)}{N\sin^2(\pi m/N)}, \tag{13}$$

which gives the following equation for $\langle \Delta x^2 \rangle$:

$$\langle \Delta x^2 \rangle = \frac{\beta BN\left(\beta_F + \beta_D - 2\sqrt{\beta_F \beta_D}\cos(2\pi\nu/N)\right)}{4F^2 \sin^2(\pi\nu)\sin^2(2\pi\nu/N)}. \tag{14}$$

If one uses the following identity valid for a FODO lattice,

$$2\sqrt{\beta_F \beta_D} = (\beta_F + \beta_D)\cos(\mu/2),$$

then Eq. (14) takes a simpler form

$$\langle \Delta x^2 \rangle = \frac{\beta ATC(\beta_F + \beta_D)}{8F^2 \sin^2(\pi\nu)}. \tag{15}$$

Using Eq. (15) with $A = 10^{-4}$ μm$^2$ /(m·s), we have computed the displacement of the closed orbit at the SSC. The following SSC parameters were used: $\beta = 100$ m, $\mu = 90°$, $\beta_F = 305$ m, $\beta_D = 305$ m, $C = 87\ 120$ m, $F = 64$ m, $\nu = 122.265$. The rms displacement of the closed orbit $\sqrt{\langle \Delta x^2 \rangle}$ is presented in Table 2 for different time intervals *T*, along with the rms value of the relative displacement of two neighboring quadrupoles $\sqrt{\langle \delta x^2 \rangle} = ATl$.

TABLE 2: RMS displacement for different time intervals.

| $T$ | 1 s | 1 min | 1 h | 1 day | 1 month | 1 yr | 10 yr | 40 yr |
|---|---|---|---|---|---|---|---|---|
| $\sqrt{\langle \Delta x^2 \rangle}$ | 3.0 μm | 23 μm | 177 μm | 0.86 mm | 4.7 mm | 16.5 mm | 2.8 cm | 5.7 cm |
| $\sqrt{\langle \delta x^2 \rangle}$ | 0.1 μm | 0.74 μm | 5.7 μm | 28 μm | 153 mm | 0.54 mm | 1.7 mm | 3.4 mm |

Let us compare these results with the ability of the SSC correctors to displace the orbit in the ring. The maximum strength of a dipole corrector (there are one horizontal and one vertical corrector in each cell) is about 3 T·m,[22] which corresponds to the maximum angle correction $\Delta\theta_{max} = 45$ μrad at 20 TeV. According to Ref. 23, an allowable rms value of $\Delta\theta$ should be 4–5.5 times smaller than $\Delta\theta_{max}$, $\Delta\theta_{rms} = 8$–$11.5$ μrad. It corresponds to a "local" alignment tolerance (offset of a quadrupole from the line joining its nearest neighbors) of about $\delta x = \Delta\theta_{r.m.s.} F =$ 520–720 μm, which, according to Table 2, accumulates after only 1–2 yr. After this period of time one may expect that in several cells the maximum correction strength will not suffice to compensate for the accumulated misalignment. We have to emphasize here that "local" correction of the beam orbit does not return it to the original position, but instead makes it pass through the current centers of (displaced) quadrupoles. An attempt to keep the initial orbit in its original form would

result in its shift relative to the center of the vacuum pipe and a substantial decrease of the dynamic aperture after only 4 months of operation.

According to Ref. 23, the dominant physical constraint on the accuracy of the "global" survey at the SSC consists in the requirement that "the proton path length in the two halves of the ring (from the east campus area to the west campus area) must be equal to within 5 cm in order for collisions to take place at predicted locations." This allows us to estimate the time $T$ after which a mechanical realignment will be needed to compensate for the change of the perimeter $\Delta C = 5$ cm. Using $\Delta C \approx (ATC/2)^{1/2}$ one finds $T \approx 2$ yr. After this time it will be necessary to replace quads to their "ideal" initial positions.

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## APPENDIX A

To simulate the "ATL law" in computer codes for accelerator design, one needs algorithms that produce the required space and time dependencies of the ground displacements. Several methods were suggested for these purposes.

### A.1 SUMMATION OF RANDOM NUMBERS[24]

This method is applicable to one-dimensional problems when the displacing ground points are distributed along a straight line. Consider $N$ points located on a line so that the distance between two neighboring points is equal to $l$ (the full length of the line array is $lN$). Initially, at $t = 0$, the displacement $x$ of each point is equal to zero, $x_i^0 = 0$ for $i = 0, 1 \ldots N$. A recursive procedure to calculate the displacements at time $T = k\tau$ from given displacements at the previous moment $T = (k-1)\tau$ is the following:

$$x_i^k = x_i^{k-1} + \sqrt{A\tau l} \sum_{m=0}^{i} \Delta_m^k , \tag{A1}$$

where $A$ is the factor in the "ATL law" and $\Delta_m^k$ are independent random Gaussian variables having the properties

$$\langle \Delta_m^k \rangle = 0, \quad \langle \Delta_m^k \Delta_{m'}^{k'} \rangle = \delta_{mm'} \delta_{kk'},$$

where $\delta_{mm'}$ and $\delta_{kk'}$ are the Kroneker symbols. Taking the square of Eq. (A1) and averaging it, one easily finds that $\left\langle \left( x_i^k - x_{i+n}^k \right)^2 \right\rangle = Ak\tau nl = ATL$. If we want to model a circular machine, we have to impose a periodicity condition, $x_0^k = x_N^k$; in addition, one can subtract a constant component from $x_i^k$, putting $x_0^k = 0$. This will result in the following transformation:

$$x_i^k \to x_i^k - x_0^k - \left( x_N^k - x_0^k \right) \frac{i}{N}. \tag{A2}$$

The described algorithm is fast and gives time-dependent profiles of displacement that satisfy the "ATL law." Figure A.1 from Ref. 24 shows an example of its application for N = 100, $L = 3000\,\mathrm{m}$, and $T = 4$, 8, 12, and 16 yr. The maximum displacement is approximately 1.2 cm, which agrees well with many years of experimental observations at the SLAC 2-mile linac tunnel.

## A.2 FRACTAL MODELING OF GROUND MOTION[20]

This technique is based on the "fractal model" proposed in Ref. 20. The model considers the ground as a set of separated blocks with different characteristic sizes $R$. The physical mechanism that makes blocks move is the release of its deformation energy. The model assumes that each block randomly displaces, with the rms value of the displacement being proportional to $R$. The rms time interval between the successive displacements is proportional to $R^\gamma$, where $\gamma$ is a parameter. The model assumes also that the number of blocks $N(R)$ in a unit volume of ground having the size $R$ depends on $R$ as $N(R) \propto R^{-D}$. The parameters $D$ and $\gamma$ must satisfy the condition $\gamma = 2 - D/2$.

In computer simulations described in Ref. 20, the total number of blocks was about 4000. Each block was considered as a two-dimensional square; the sizes of blocks had been chosen to be $R = 1, 2, 4, 8, 16, 32, 64$ (in arbitrary units), and the parameters of the model were $D = 1.9$, $\gamma = 1.05$. The centers of the blocks were randomly distributed in the half-space. To model the surface of the ground, 64 points were evenly distributed along the boundary of the half-space, and displacement of each point was determined as the sum of the displacements of blocks located under it. At each time step the blocks having the smallest dimension $R = 1$ were randomly displaced with displacement rms value equal to 1;

blocks having $R=2$ were displaced after $2^{\gamma}$ time steps randomly with rms value of the displacement equal to 2, etc. An example of the resulting profile is presented in Figure A.2.

## A.3 FOURIER TRANSFORMATION OF RANDOM FUNCTIONS [25]

In Ref. 25 it has been suggested to generate an $n$-dimensional hypersurface satisfying the "ATL law" as a sum of random waves:

$$x(\mathbf{r},t)=\int d^n\mathbf{k}|\mathbf{k}|^{-(n+1)/2}F(\mathbf{k},t)\exp(-i\mathbf{kr}), \tag{A3}$$

where $F(\mathbf{k},t)$ is a random function satisfying the following conditions:

$$\langle F(\mathbf{k},t)F(\mathbf{k}',t)\rangle = At\delta^n(\mathbf{k}+\mathbf{k}'),\quad F^*(\mathbf{k},t)=F(-\mathbf{k},t). \tag{A4}$$

It is easy to verify that the surface generated by $x(\mathbf{r},t)$ satisfies the "ATL law" $\left\langle (x(\mathbf{r},t)-x(\mathbf{r}+L\mathbf{a},t))^2\right\rangle \propto AtL$, where $\mathbf{a}$ is a unit vector. A two-dimensional surface generated using this method is presented in Figure A.3.

## APPENDIX B

We start from Eq. (4):

$$\langle \Delta x^2\rangle = \frac{\beta}{4\sin^2\pi\nu}\sum_{i,j}\frac{\sqrt{\beta_i\beta_j}}{F_iF_j}\langle\delta_i\delta_j\rangle\cos\left(\phi+\frac{i\mu}{2}-\pi\nu\right)\cos\left(\phi+\frac{j\mu}{2}-\pi\nu\right). \tag{B1}$$

First, separate Eq. (B1) into three sums:

$$\langle \Delta x^2\rangle = \langle \Delta x^2\rangle_{FF}+\langle \Delta x^2\rangle_{DD}+\langle \Delta x^2\rangle_{FD}, \tag{B2}$$

$$\langle \Delta x^2\rangle_{FF} = \frac{\beta\beta_F}{4F^2\sin^2\pi\nu}\sum_{i,j-\text{even}}\langle\delta_i\delta_j\rangle\cos\left(\phi+\frac{i\mu}{2}-\pi\nu\right)\cos\left(\phi+\frac{j\mu}{2}-\pi\nu\right), \tag{B3}$$

$$\langle \Delta x^2\rangle_{DD} = \frac{\beta\beta_D}{4F^2\sin^2\pi\nu}\sum_{i,j-\text{odd}}\langle\delta_i\delta_j\rangle\cos\left(\phi+\frac{i\mu}{2}-\pi\nu\right)\cos\left(\phi+\frac{j\mu}{2}-\pi\nu\right), \tag{B4}$$

$$\langle \Delta x^2 \rangle_{FD} = -\frac{\beta\sqrt{\beta_F \beta_D}}{2F^2 \sin^2 \pi\nu} \sum_{\substack{i-\text{even},\\ j-\text{odd}}} \langle \delta_i \delta_j \rangle \cos\left(\phi + \frac{i\mu}{2} - \pi\nu\right) \cos\left(\phi + \frac{j\mu}{2} - \pi\nu\right). \tag{B5}$$

The first sum, $\langle \Delta x^2 \rangle_{FF}$, accounts for the focusing quadrupoles ($i$ and $j$ take even values); the second one, $\langle \Delta x^2 \rangle_{DD}$, gives the contribution of the defocusing quadrupoles ($i$ and $j$ take odd values); and the third sum, $\langle \Delta x^2 \rangle_{FD}$, results from the cross terms in which $i$ takes even and $j$ takes odd values.

Let us consider $\langle \Delta x^2 \rangle_{FF}$. Introducing the correlation function according to Eq. (5), $K(i-j) = \langle \delta_i \delta_j \rangle$, we have

$$\langle \Delta x^2 \rangle_{FF} = \frac{\beta\beta_F}{4F^2 \sin^2 \pi\nu} I_{FF}, \tag{B6}$$

where

$$\begin{aligned} I_{FF} &= \sum_{i,j-\text{even}} K(i-j) \cos\left(\phi + \frac{i\mu}{2} - \pi\nu\right) \cos\left(\phi + \frac{j\mu}{2} - \pi\nu\right) \\ &= \frac{1}{2} \sum_{i,j-\text{even}} K(i-j) \left[\cos\left(\frac{\mu}{2}(i-j)\right) + \cos\left(2\phi - 2\pi\nu \frac{\mu}{2}(i+j)\right)\right]. \end{aligned} \tag{B7}$$

For a given difference $(i-j)$, the sum $(i+j)$ takes $(N-|i-j|)/2$ values from the interval $[|i-j|, 2(N-2)-|i-j|]$. Performing the summation over $(i+j)$, first, we find that the first term in Eq. (B7) has to be multiplied by $(N-|i-j|)/2$ because it does not depend on $(i+j)$. The second term oscillates as a function of $(i+j)$, taking positive and negative values so that in the limit $N \gg 1$ its contribution compared with the first term is $\sim O(N^{-1})$. We neglect this term in what follows:

$$I_{FF} \approx \frac{1}{4} \sum_{i-j} (N-|i-j|) K(i-j) \cos\left(\frac{\mu}{2}(i-j)\right), \tag{B8}$$

where $(i-j)$ takes even values from $-(N-2)$ to $(N-2)$. Using Fourier representation for $K$, Eq. (7), one finds

$$I_{FF} \approx \frac{1}{4N} \sum_{m=0}^{N/2} k_m \sum_n (N-|n|) \cos\left(\frac{\mu}{2} n\right) \cos\left(\frac{2\pi nm}{N}\right), \tag{B9}$$

where we have denoted $n = i - j$. With some straightforward algebra, the sum over $n$ in Eq. (B9) can be evaluated as

$$\sum_n (N - |n|)\cos\left(\frac{\mu n}{2}\right)\cos\left(\frac{2\pi nm}{N}\right) = \\ = \frac{\sin^2(\pi\nu)}{4}\left[\sin^{-2}\left(\frac{2\pi(m-\nu)}{N}\right) + \sin^{-2}\left(\frac{2\pi(m+\nu)}{N}\right)\right] . \tag{B10}$$

To carry out the summation over $m$, we note that, in the limit $N \gg 1$, Eq. (B10) represents a function of $m$ that has two narrow peaks around $m = [\nu]$ and $m = N/2 - [\nu]$, where $[\nu]$ denotes an integer part of $\nu$. Assuming that $k_m$ is a smooth function of $m$ in the vicinity of the peaks, we can approximate

$$\frac{k_m}{\sin^2(2\pi(m-n)/N)} \approx \frac{N^2 k_{[\nu]}}{4\pi^2(j-\Delta\nu)^2}, \tag{B11}$$

where $j = m - [\nu]$ and $\Delta\nu = \nu - [\nu]$ is the fractional part of $\nu$. Also,

$$\frac{k_m}{\sin^2(2\pi(m+n)/N)} \approx \frac{N^2 k_{N/2-[\nu]}}{4\pi^2(j-\Delta\nu)^2}, \tag{B12}$$

where now $j = m + [\nu] - N/2$. Extending summation over $j$ from $-\infty$ to $+\infty$ one finds

$$I_{FF} = \frac{N\sin^2(\pi\nu)}{16\pi^2}\left(k_{[\nu]} + k_{N/2-[\nu]}\right)\sum_{-\infty}^{\infty}\frac{1}{(j-\Delta\nu)^2} = \frac{N}{16}\left(k_{[\nu]} + k_{N/2-[\nu]}\right), \tag{B13}$$

where we have used the identity

$$\sum_{-\infty}^{\infty}\frac{1}{(j-\Delta\nu)^2} = \frac{\pi^2}{\sin^2(\pi\Delta\nu)}.$$

Returning to Eq. (B6) we have

$$\langle\Delta x^2\rangle_{FF} = \frac{N\beta\beta_F}{64F^2\sin^2(\pi\nu)}\left(k_{[\nu]} + k_{N/2-[\nu]}\right). \tag{B14}$$

In a similar way one can find that

$$\langle \Delta x^2 \rangle_{FF} = \langle \Delta x^2 \rangle_{DD}, \tag{B15}$$

$$\langle \Delta x^2 \rangle_{FD} = -\frac{N\beta\sqrt{\beta_F \beta_D}}{32F^2 \sin^2(\pi\nu)} \left( k_{[\nu]} - k_{N/2-[\nu]} \right), \tag{B16}$$

which for the total $\langle \Delta x^2 \rangle$ gives Eq. (9).

REFERENCES


1. E.D. Courant and H.S. Snyder, *Ann. Phys.*, vol. **3**, pp. 1–48 (1958).
2. T.O. Raubenheimer, "The Preservation of Low-Emittance Flat Beams," *Proc. of IEEE 1993 Particle Accelerator Conference,* 17–20 May 1993, Washington, D.C.
3. V.D. Shiltsev, "Emittance Preservation Issues in Linear Collider VLEPP," INP Preprint 91-86, Novosibirsk, 1991 (in Russian).
4. W. Decking, K. Floetman, J. Rossbach, "Measurement of Slow Closed Orbit Motion in Correlation with Ground Motion," *Proc. of II European Part. Acc. Conf.,* Nice, 12–16 March 1990, pp. 1449–1451, and DESY-M-90-02, Feb. 1990.
5. B.A. Baklakov, P.K. Lebedev, V.V. Parkhomchuk, A.A. Sery, V.D. Shiltsev, A.I. Sleptsov, "Investigation of Seismic Vibrations and Relative Displacement of Linear Collider VLEPP Elements," *Proc. of 1991 IEEE Part. Accel. Conf.,* San Francisco, USA, pp. 3273–3275, May 1991.
6. G.E. Fischer, in *Summary and Presentation of the Workshop on Vibrational Control and Dynamic Alignment Issues at the SSC*, ed., H.J. Weaver, SSCL-SR-1185, Feb. 1992.
7. B.A. Baklakov, P.K. Lebedev, V.V. Parkhomchuk, A.A. Sery, V.D. Shiltsev, A.I. Sleptsov, "Exploration of Correlation and Power Characteristics of Earth Surface Motion in the UNK Site," Preprint INP 91-15, Novosibirsk, 1991 (in Russian).
8. V.A. Lebedev, P.K. Lebedev, V.V. Parkhomchuk, V.D. Shiltsev, "Transverse Vibrations of Electron Beam and Ground Motion Measurements at VEPP-3 Storage Ring," INP Preprint 92-39, Novosibirsk, 1992, and *Proc. of 1992 Int. Part. Acc. Conf.,* Dubna, Russia, October 1992.
9. R. Sugahara et al., "Measurements of Seismic Motion and Displacement of the Floor at the TRISTAN Ring, and the Alignment Issues," KEK-Preprint-92-200, Feb. 1993.
10. V.E. Balakin et al., "Measurements of Seismic Vibrations in the CERN TT2A Tunnel for Linear Collider Studies," CLIC-Note-191, 1993.
11. V.V. Parkhomchuk, V.D. Shiltsev, H.J. Weaver, "Measurements of the Ground Motion Vibrations at the SSC," SSCL-Preprint-323, May 1993.
12. G.E. Fischer and P. Morton, "Ground Motion Tolerances for the SSC," SSC-55, 1986.
13. J. Rossbach, "Closed Orbit Distortions of Periodic FODO Lattices due to Plane Ground Waves," *Particle Accelerators,* vol. **23**, pp. 121–132 (1988)
14. V.V. Parkhomchuk and V.D. Shiltsev, "Is Transverse Feedback Necessary for the SSC Emittance Preservation?," SSCL-622, April 1993.
15. Y. Kobayashi et al., "A Test of 3-GeV Operation at the Photon Factory Storage Ring," *Proc. of III European Part. Acc. Conf.,* Berlin, 24–28 March 1992, pp. 3–7.

16. H. Koiso, S. Kamada, and N. Yamamoto, "Observation of Changes of the Closed Orbit in the TRISTAN Main Ring," *Particle Accelerators,* 1990, vol. **27**, pp. 71–76.
17. S. Takeda, N. Nakanishi, K. Kudo, and A. Akiyama, "Vertical Displacement of the Base of TRISTAN," KEK-Preprint-92-67, July 1992, and *Proc. of XV Int. Conf. on High Energy Accel.,* Hamburg, Germany, July 1992.
18. *CERN Courier,* vol. **33**, No.1, Jan/Feb 1993, p. 4.
19. V.A. Lebedev, V.V. Parkhomchuk, V.D. Shiltsev, A.N. Skrinsky, "Suppression of Emittance Growth Caused by Mechanical Vibrations of Magnetic Elements in Presence of Beam-Beam Effects in the SSC," Preprint INP 91-120, Novosibirsk, 1991.
20. V.V. Parkhomchuk and V.D.Shiltsev, "Fractal Model of the Ground Motion," Preprint INP 92-31, Novosibirsk, 1992 (in Russian).
21. R. Brinkmann and F. Willeke, "First Experience with Colliding Electron-Proton Beams in HERA," *Proc. of IEEE 1993 Particle Accelerator Conference,* 17–20 May 1993, Washington, D.C.
22. Element Specification, Collider Accelerator Arc Sections, SSCL, Number E10-000027, August 1992.
23. *Site-Specific Conceptual Design of the Superconducting Super Collider,* eds., J.R. Sanford and D.M. Matthews, SSCL-SR-1056, July 1990, p. 277.
24. V.V. Parkhomchuk, in *Summary and Presentation of the Workshop on Vibrational Control and Dynamic Alignment Issues at the SSC,* ed., H.J. Weaver, SSCL-SR-1185, Feb. 1992.
25. N. Yamamoto, in *Proc. of Int. Workshop on Emittance Preservation in Linear Colliders,* KEK, Tsukuba, Japan, April 1993 (to be published).
26. H.K. Given, "Variations in Broadband Seismic Noise at IRIS/IDA Stations in the USSR with the Implementation for Event Detection," *Bulletin of the Seismological Society of America,* vol. **80**, No.6, p. 2072, Dec. 1990.

**FIGURE CAPTIONS**

FIGURE 1: Spectra of ground motion at the SSC site and other accelerators. SSC, Ref. 11; VEPP-3, Ref. 8; UNK, Ref. 7; IDA, Ref. 26; Fischer, Ref. 12.

FIGURE 2: Spectra of quadrupole vibrations calculated from beam data (from Ref. 14).

FIGURE 3: Variations of the closed orbit with time at TRISTAN (from Ref. 16).

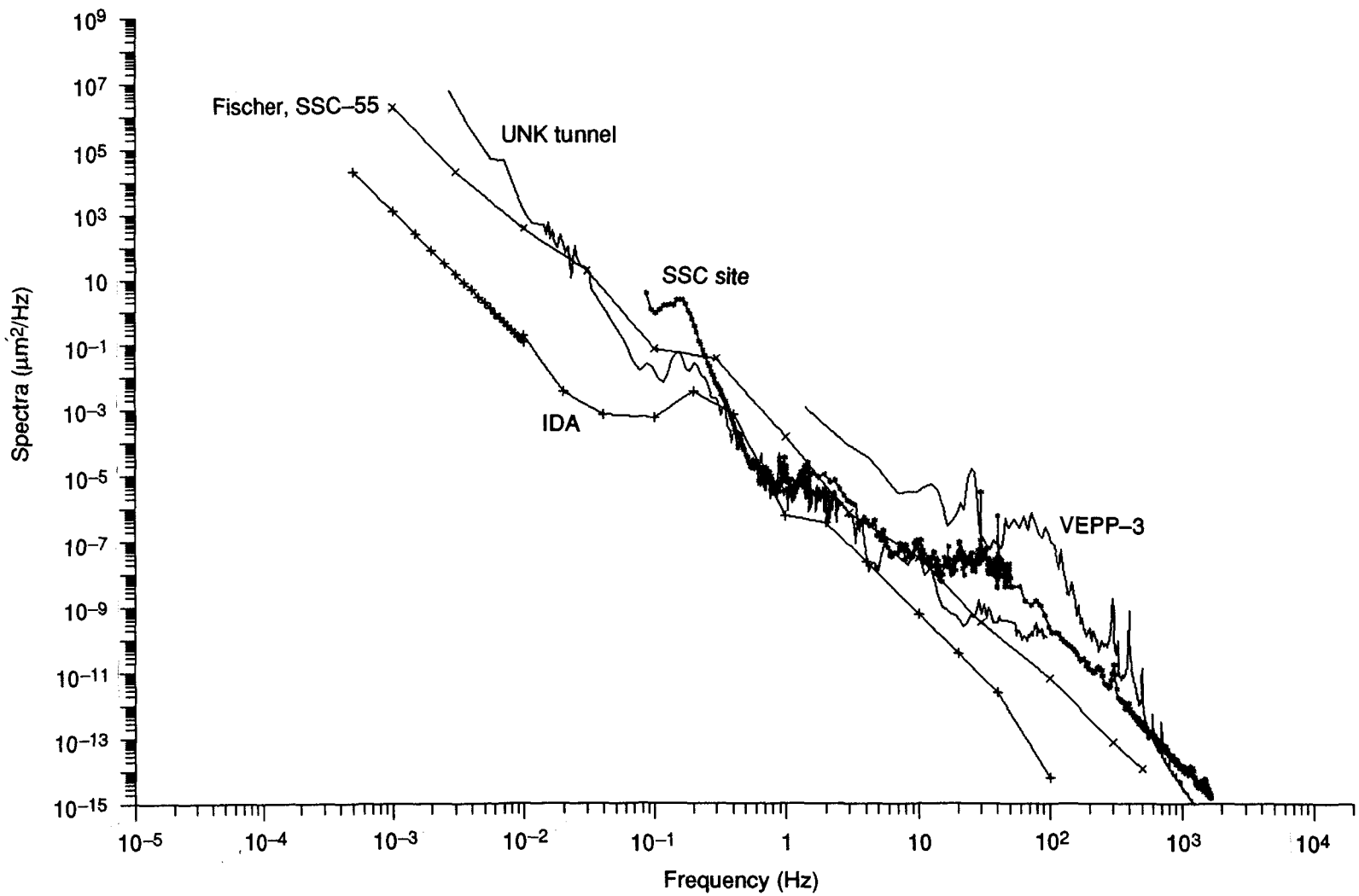
FIGURE 4: Dependence of $\langle \delta x^2 \rangle / T$ versus $L$.

FIGURE 5: Closed orbit distortion as a function of the correlation length $\Delta l$.

FIGURE A.1: SLAC tunnel displacements calculated according to ATL law (from Ref. 24).

FIGURE A.2: Ground surface displacements calculated using the fractal model (from Ref. 20).

FIGURE A.3: Two-dimensional ground surface calculated by the method of Ref. 25.


Fischer, SSC–55
UNK tunnel
SSC site
IDA
VEPP–3
Spectra (μm²/Hz)
Frequency (Hz)
10⁹
10⁷
10⁵
10³
10
10⁻¹
10⁻³
10⁻⁵
10⁻⁷
10⁻⁹
10⁻¹¹
10⁻¹³
10⁻¹⁵
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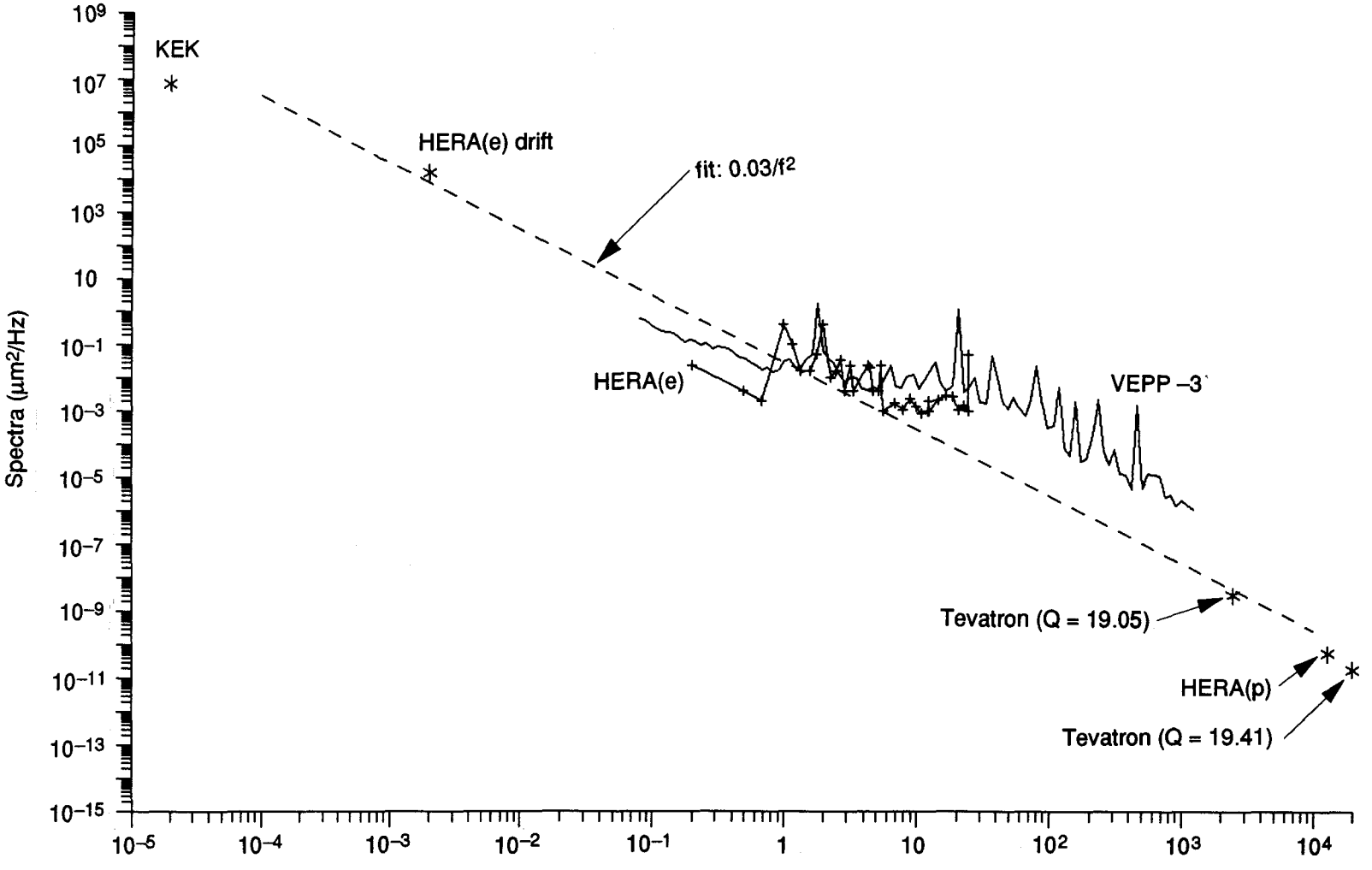
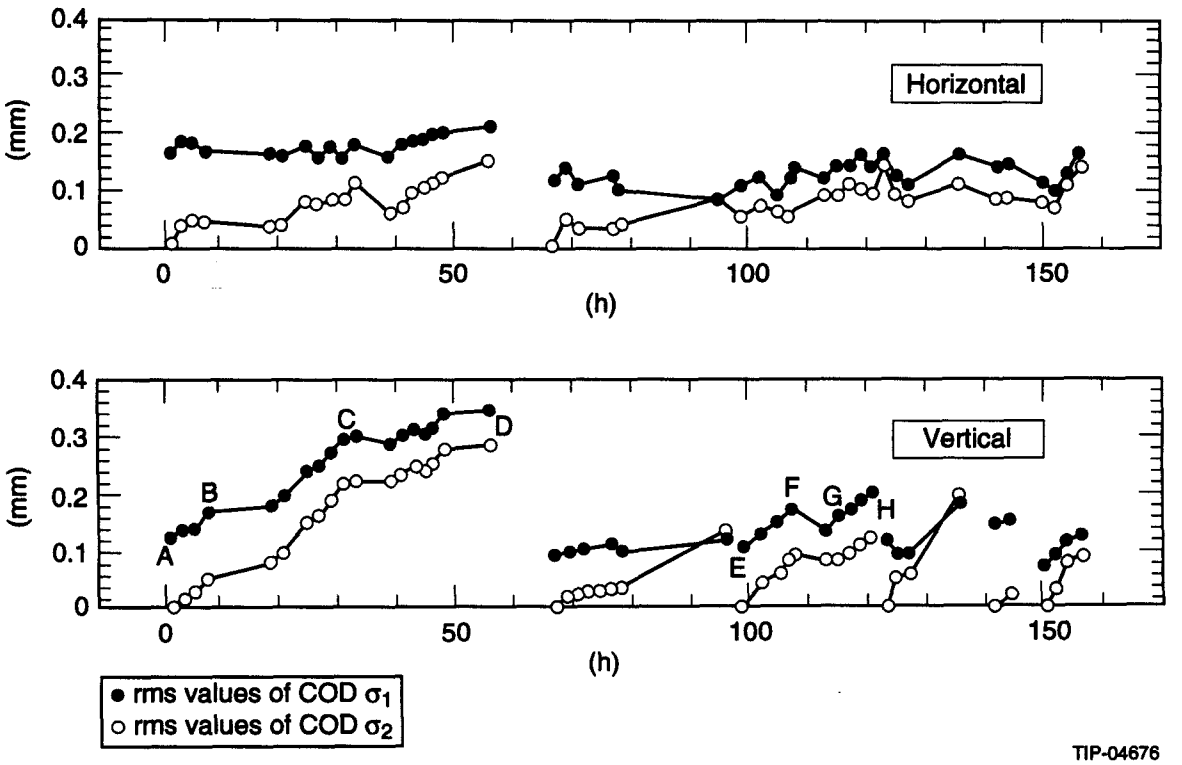
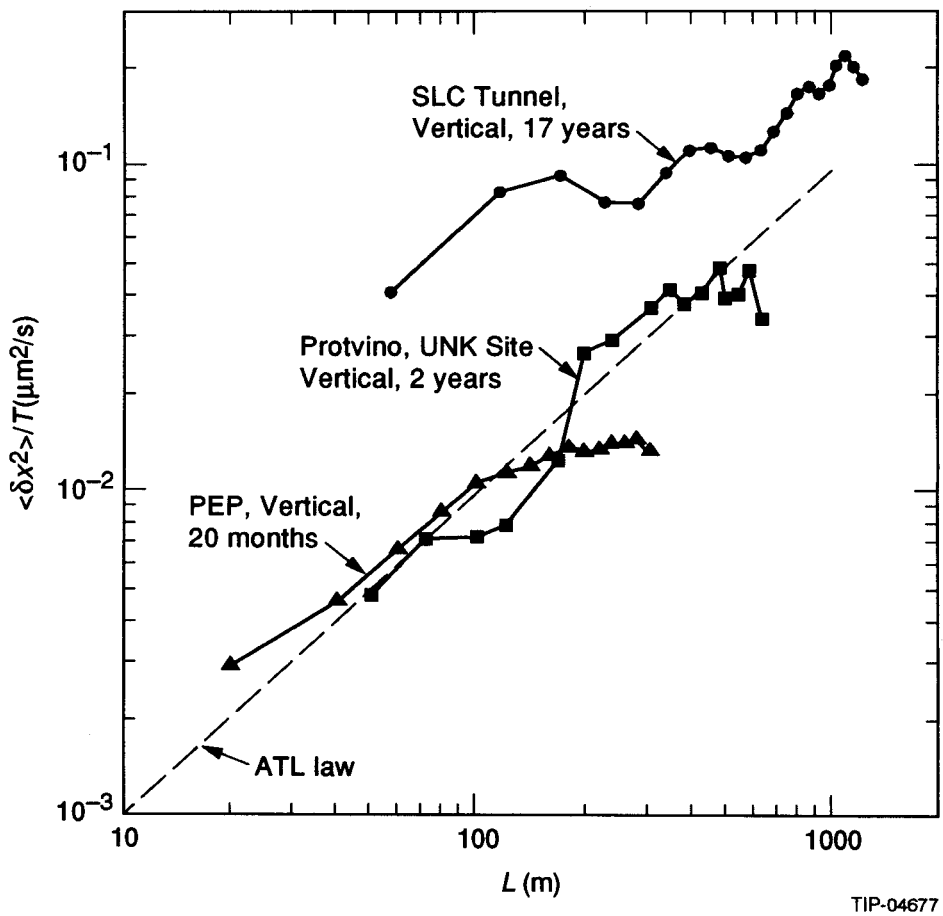
10⁻⁴
10⁻³
10⁻²
10⁻¹
1
10
10²
10³
10⁴

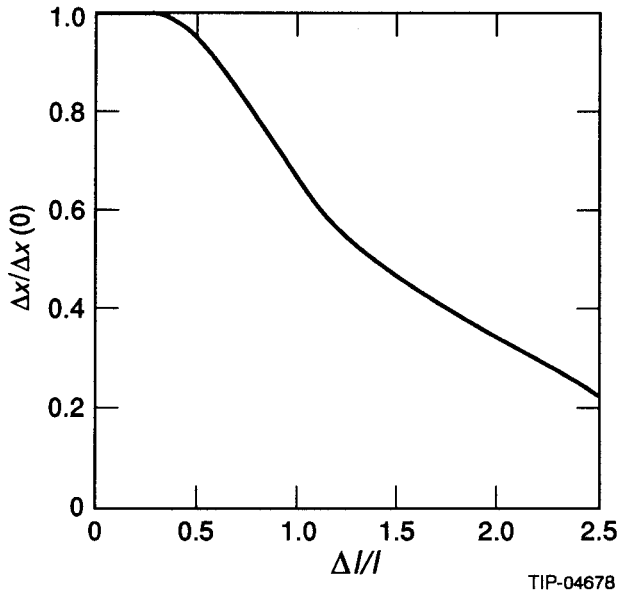

Spectra (μm²/Hz)
KEK
HERA(e) drift
fit: 0.03/f²
HERA(e)
VEPP –3
Tevatron (Q = 19.05)
HERA(p)
Tevatron (Q = 19.41)


0.4
0.3
0.2
0.1
0
(mm)
Horizontal
0
50
100
150
(h)
0.4
0.3
0.2
0.1
0
(mm)
Vertical
A
B
C
D
E
F
G
H
0
50
100
150
(h)
● rms values of COD σ1
○ rms values of COD σ2

TIP-04676


SLC Tunnel,
Vertical, 17 years
Protvino, UNK Site
Vertical, 2 years
PEP, Vertical,
20 months
ATL law
$10^{-1}$
$10^{-2}$
$10^{-3}$
10
100
1000
$L$ (m)
$<\delta x^2>/T(\mu m^2/s)$

TIP-04677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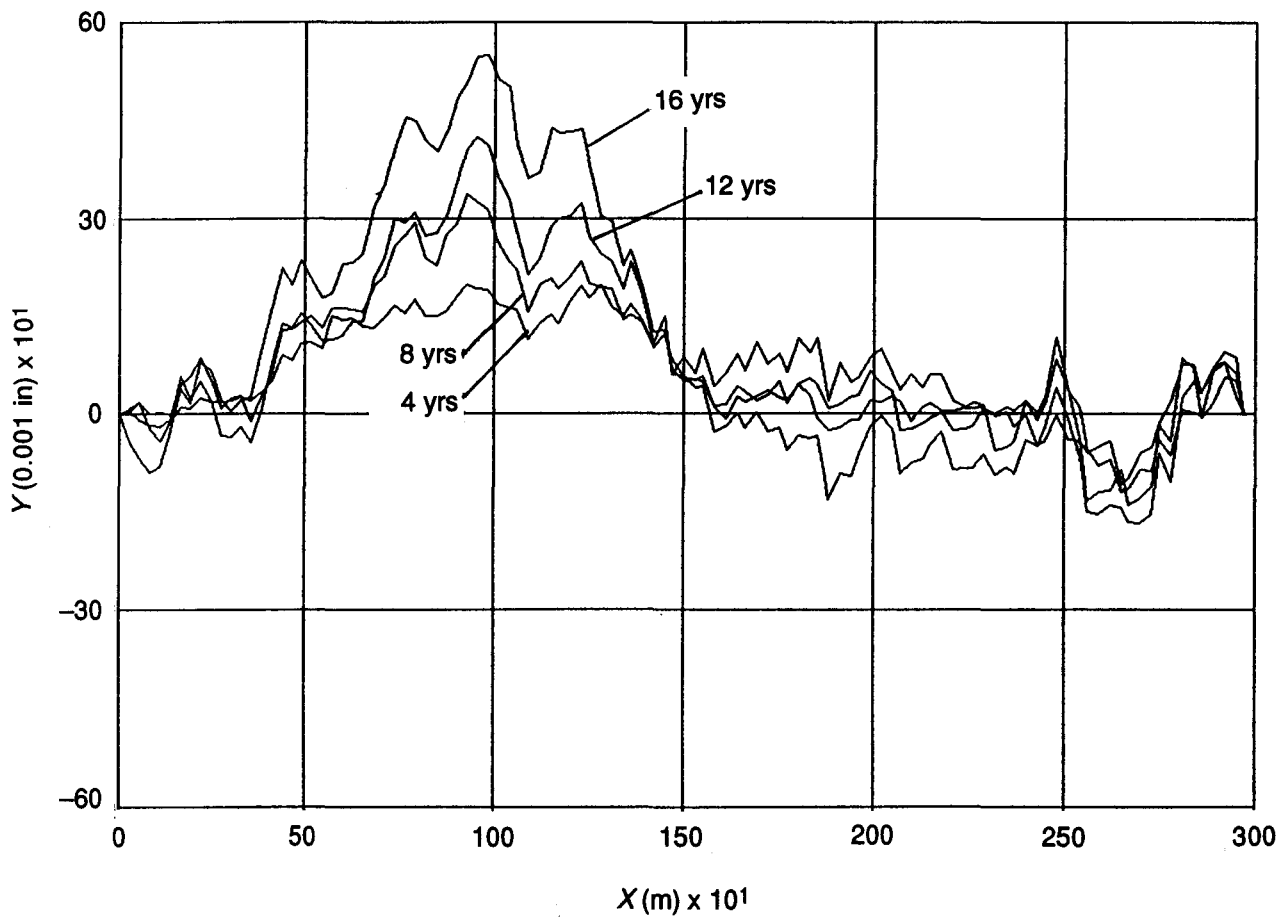
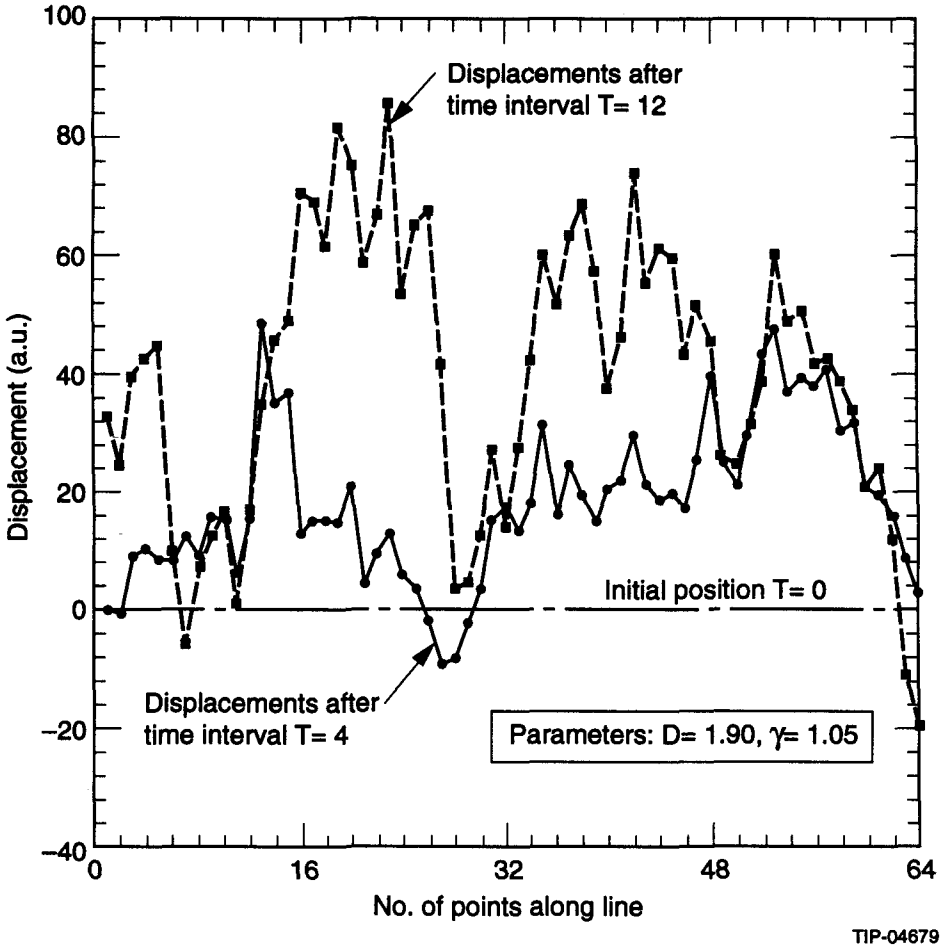
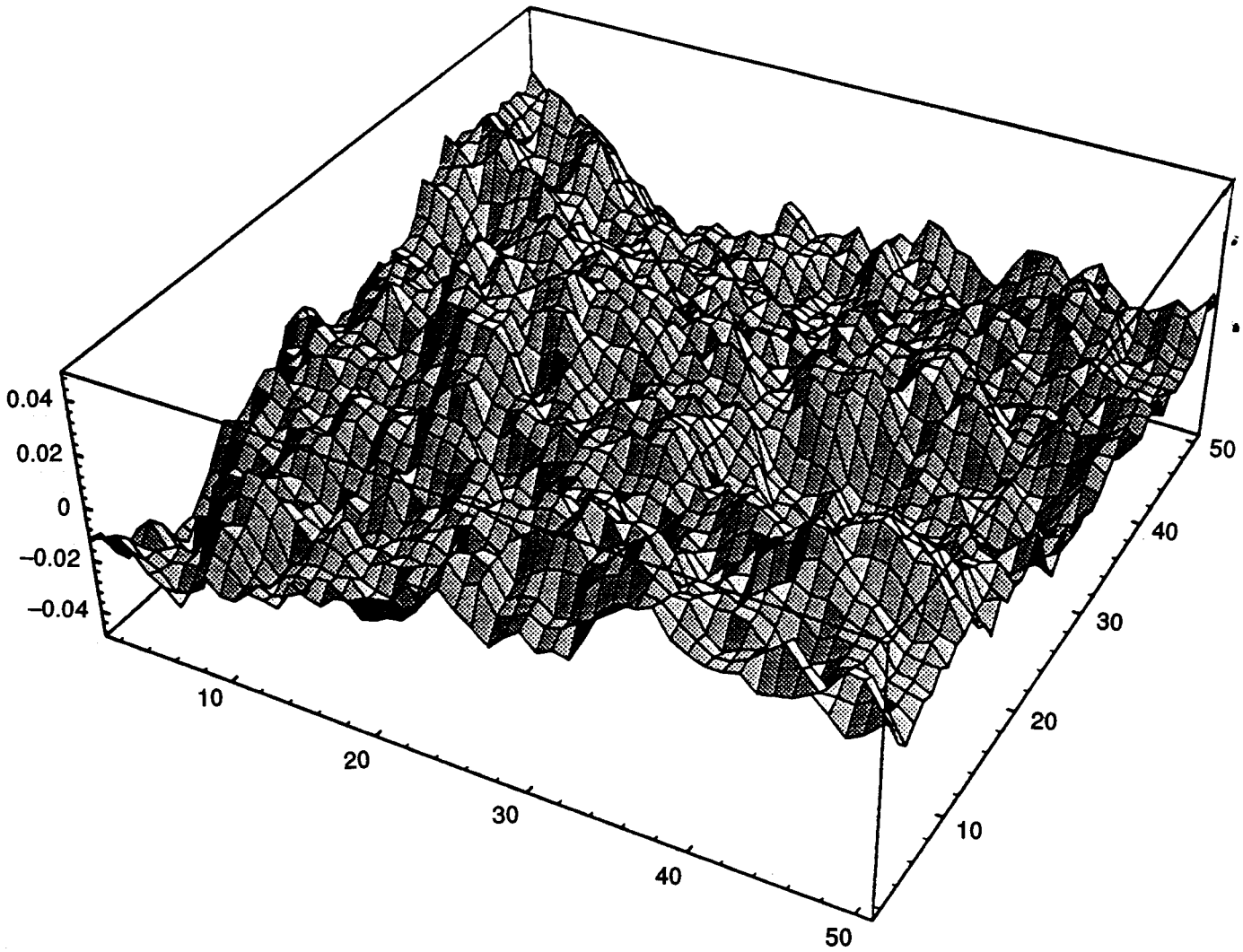
0.8
0.6
0.4
0.2
0
Δx/Δx (0)
0
0.5
1.0
1.5
2.0
2.5
Δl/l
TIP-04678


60
30
0
−30
−60
0
50
100
150
200
250
300
16 yrs
12 yrs
8 yrs
4 yrs
Y (0.001 in) x 10¹
X (m) x 10¹


Displacements after
time interval T= 12
Initial position T= 0
Displacements after
time interval T= 4
Parameters: D= 1.90, γ= 1.05
Displacement (a.u.)
No. of points along line
100
80
60
40
20
0
−20
−40
0
16
32
48
64

TIP-04679


0.04
0.02
0
−0.02
−0.04
10
20
30
40
50
10
20
30
40
50