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July 1993
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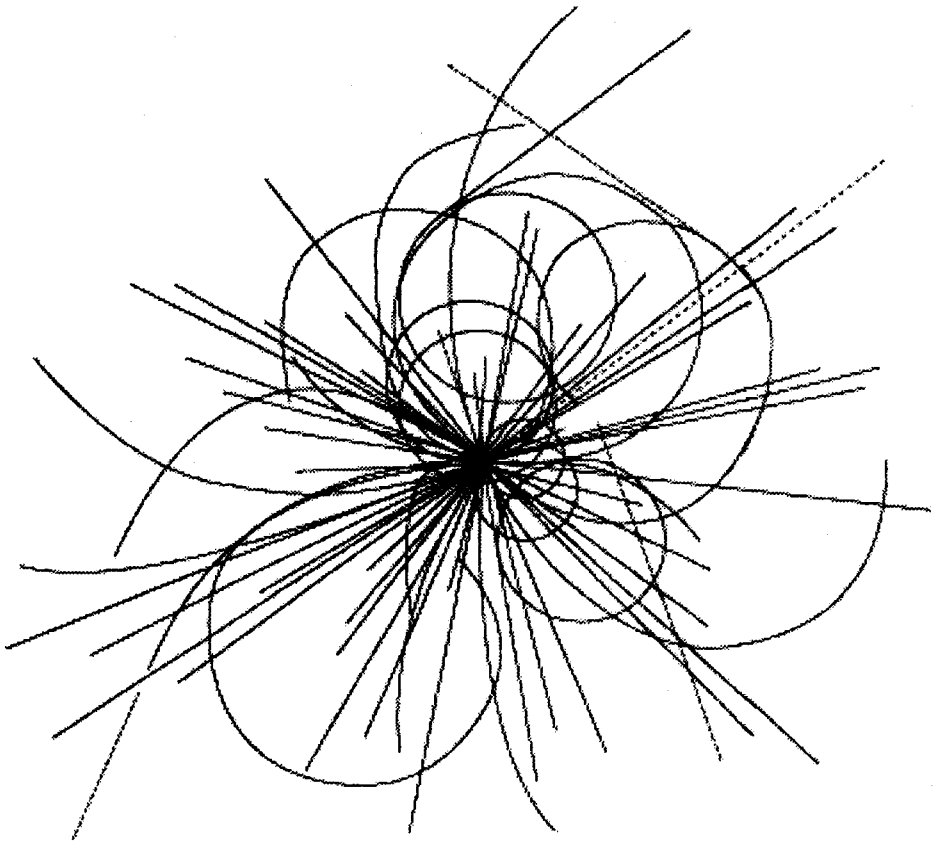

Superconducting Super Collider Laboratory

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Constant of Thermal Heat Conduction and Stabilization of the Bus Bar Conductor for Superconducting Accelerators*

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* To be presented at the International Cryogenics Engineering Conference, July 12–14, Albuquerque, N.M.
† Operated by the Universities Research Association, Inc., for the U.S. Department of Energy under Contract No. DE-AC35-89ER40486.

CONSTANT OF THERMAL HEAT CONDUCTION AND STABILIZATION OF THE BUS BAR CONDUCTOR FOR SUPERCONDUCTING ACCELERATORS

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ABSTRACT

Using the one-dimensional, time-independent conduction state, a constant of thermal heating conduction is given that brings about the known stabilization theorem and a closed expression for the bus bar to be cryogenically stable in superconducting accelerators.

INTRODUCTION

The superconducting bus bar cable makes the connection of every magnet in the ring of a superconducting accelerator.[1] This cable has very long extension and can suffer several perturbations during the operation of the machine. When a normal zone appears in this cable, the generated heat is transferred and removed by the helium flowing on the surface. This mechanism, the only means for the cable to recover its superconducting state, can be done by using a proper copper-to-superconducting (s.c.) ratio, λ, and having passages where the helium can flow on the surface. This is the cryostabilization method of s.c. cables,[2] which results from the Fourier conduction mechanism.[3] In this paper, it is shown that this criterion appears in a natural way from a "Constant of Thermal Heating Conduction" (CTHC), which is an extension of the concept of constant of motion in dynamical systems.[4] Using this CTHC, a closed expression is given for the copper-to-s.c. ratio as a function of the fraction of the surface wetted by the helium and the heat transfer function. Starting from the one-dimensional, time-independent heat equation, the CTHC is deduced using the same approach as in Reference 5. Finally, this constant is applied to the cryostabilization of a s.c. bus bar cable.

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CONSTANT OF THERMAL HEATING CONDUCTION

Once a quench appears in a s.c. cable, the normal zone moves longitudinally with a speed given by the magnitude of the longitudinal quench velocity. The transverse dimensions of the conductor are considered small enough so that the heat propagation in this direction can be neglected, restricting the problem to a one dimension problem. The normal zone becomes resistive, and its temperature θ, at the point z and at the time t, changes in accordance with the heat equation:

$$(\delta c_p)\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left(k(\theta)\frac{\partial\theta}{\partial z}\right) + \rho(\theta)j^2 - \frac{PH(\sigma)}{A}\ , \tag{1}$$

where (δc_p) represents the product of the density, δ, times the specific heat, c_p, averaged over all the components of the conductor; $k(\theta)$ is the thermal conductivity; ρj^2 is the Joule heating; P is the perimeter of the conductor in contact with the helium (liquid or gas), which has a cross section area A; $H(\sigma)$ is the heat transfer function, which depends on $\sigma = \theta - \theta_o$, where θ_o is the batch temperature (considered constant due to helium flow). A time-independent state of this system is the time-independent solution ($\partial\theta/\partial t = 0$, with $j = I/a_{cu}$ =constant) of Eq. (1), which is given by

$$\frac{d}{dz}\left(k(\theta)\frac{d\theta}{dz}\right) + \rho(\theta)j^2 - \frac{PH(\sigma)}{A} = 0\ . \tag{2}$$

This equation can be transformed to the dynamic system,

$$\frac{d\theta}{dz} = v/k(\theta) \tag{3a}$$

and

$$\frac{dv}{dz} = PH(\sigma)/A - \rho(\theta)j^2, \tag{3b}$$

by using the new variable $v = k(\theta)d\theta/dz$. A constant, K, associated to this system along the longitudinal direction, z, satisfies the following equation: $dK/dz = 0$, which brings about the partial differential equation

$$\frac{v}{k(\theta)}\frac{\partial K}{\partial\theta} + \left(\frac{PH(\sigma)}{A} - \rho(\theta)j^2\right)\frac{\partial K}{\partial v} = 0\ . \tag{4}$$

It can be solved by the characteristics method; the equations for the characteristics are given by

$$k(\theta)\ d\theta/v = dv/\left[PH(\sigma)/A - \rho(\theta)j^2\right] = dK/0\ . \tag{5}$$

From the first two terms of Eq. (5), a characteristic curve is obtained that can be used as the CTHC and is given by

$$K(\theta, v) = v^2/2 + \int^{\theta}\left[\rho(\zeta)j^2 - Ph(\zeta - \theta_o)/A\right]k(\zeta)\ d\zeta\ . \tag{6}$$

CRYOGENIC STABILIZATION AND CLOSED EXPRESSION FOR λ

Since K is a constant, for any two points of the conductor—for example, just at the boundary between the s.c. state and the normal zone ($z = 0$), $\theta(0) = \theta_g$, and $v(0) = v_o$, and the hottest point in the normal zone ($z = L$), $\theta(L) = \theta_{max}$, and $v(L) = v_L$—the following relation is established from Eq. (6):

$$\frac{1}{2}(v_L^2 - v_g^2) = \int_{\theta_g}^{\theta_{max}} \left[\frac{PH(\zeta - \theta_o)}{A} - \rho(\zeta) j^2 \right] k(\zeta)\, d\zeta \ . \tag{7}$$

The cryostability criterion, $v_L^2 = v_o^2$, appears as a particular application of the constant (6). Assuming a cylindrically shaped bus bar s.c. cable, where a fraction, ξ, of its perimeter is in contact with helium, its perimeter is related to its cross section area, A, as $P = \xi 2\sqrt{\pi A}$. The Joule heating is due mainly to the copper matrix in the s.c. cable. If a_{cu} and a_{sc} are the total cross-section areas of copper and superconductor in the cable ($A = a_{cu} + a_{sc}$), and $\lambda = a_{cu}/a_{sc}$ is the copper-to-s.c. ratio, the cryostabilization criterion can be written as

$$\int_{\theta_g}^{\theta_{max}} \left[\frac{\rho(\zeta) I^2}{a_{sc}^2 \lambda^2} + \frac{2\xi\sqrt{\pi} H(\zeta - \theta_o)}{\sqrt{a_{sc}(1+\lambda)}} \right] k(\zeta)\, d\zeta \ . \tag{8}$$

Since the thermal conductivity is a positive defined function with respect to the temperature, it can be dropped from Eq. (13). Now, rearranging this expression with respect to λ, it follows that

$$\lambda^4 - q_*^2 \lambda^2 - q_*^2 = 0 \ , \tag{9}$$

where q_* is defined as

$$q_* = \left(\frac{I^2}{2 a_{sc}^{3/2} \sqrt{\pi}\, \xi} \right) \int_{\theta_g}^{\theta_{max}} \rho(\zeta)\, d\zeta \Big/ \int_{\sigma_g}^{\sigma_{max}} H(\sigma)\, d\sigma \ . \tag{10}$$

The polynomial (9) has one complex root (and its conjugate), one negative root, and one positive root that has physical meaning. This root is given by

$$\lambda_* = \frac{\sqrt{R_*^{2/3} - 4q_*^2/2}}{2R_*^{1/6}} \left[1 + \sqrt{1 + 2q_*^2 R_*^{1/2} \Big/ \left(R_*^{3/2} - 4q_*^2/3 \right)^{3/2}} \right] \ , \tag{11}$$

where R_* is defined as

$$R_* = \frac{q_*^4}{2} \left[1 + \sqrt{1 + \frac{256}{27 q_*^2}} \right] \ . \tag{12}$$

Eq. (11) represents a closed analytical expression for the cryogenic stabilization. Taking a constant resistivity of $\rho = 2 \times 10^{-10}\ \Omega$ m (true for $\theta_{max} < 20$ K, $RRR \approx 90$, $B = 0$), $a_{sc} = 5.1383$ mm^2, $I = 6.5$ kA, and $\theta_g = 4.5$ K, Figure 1 shows the copper to s.c. ratio for a bus bar cable as a function of the fraction of its perimeter wetted by helium. (Pool cooling is assumed with heat transfer values given in Reference 2.)

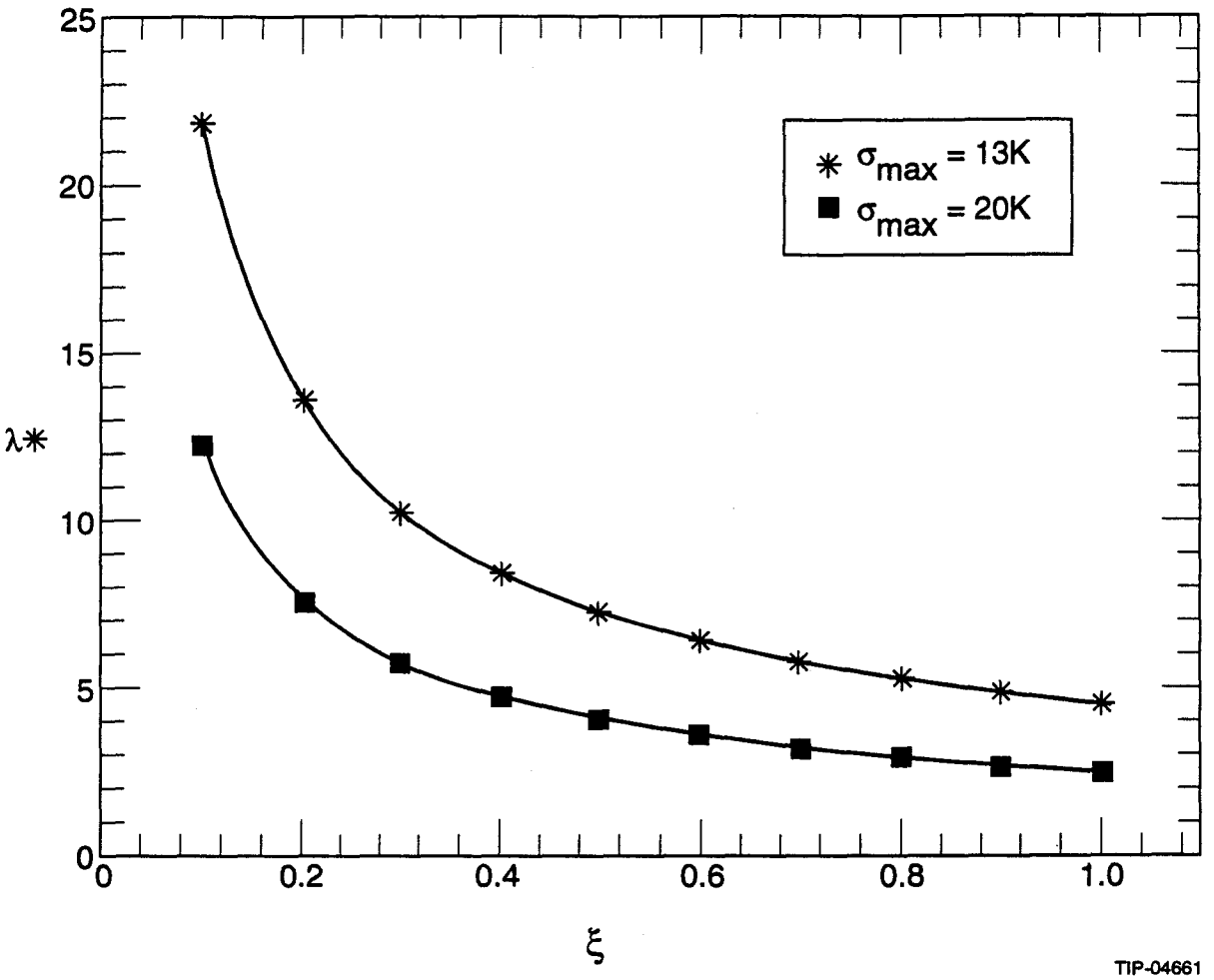

Figure 1. Copper-to-superconducting ratio λ_* for a bus bar cable as a function of the fraction of its perimeter ξ wetted by helium.

CONCLUSIONS AND COMMENTS

A constant of thermal heating equation was found from the one-dimensional, time-independent heat equation. The known cryogenic stability criterion appears as a consequence of this constant, and a closed expression for the copper-to-s.c. ratio was found for a cylindrical bus bar. The approach can be easily extended to other geometries. λ_* is an increasing function with respect to q_*: if $q_*^1 < q_*^2$, then $\lambda_*(q_*^1) < \lambda_*(q_*^2)$, which indicates the change needed in the bus bar cu:sc ratio when the parameters θ_o, θ_{max}, $H(\sigma)$, ξ, I, a_{sc}, and ρ change.

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