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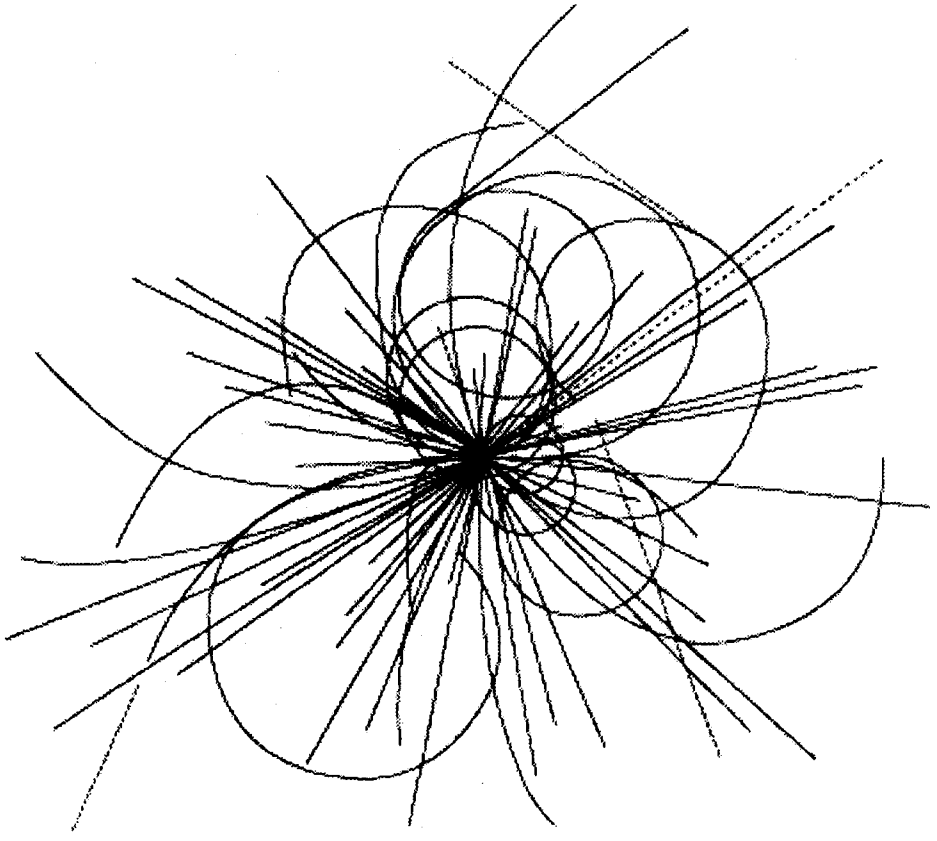

Superconducting Super Collider Laboratory

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Long-Term Tracking with Symplectic Implicit One-Turn Maps*

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Long-Term Tracking with Symplectic Implicit One-Turn Maps

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Abstract

Symplectic one-turn maps for long-term tracking of the Superconducting Super Collider (SSC) and its High Energy Booster (HEB) have been successfully used in dynamic aperture studies. We found that one-turn maps of about 5th order are sufficient for the SSC injection lattice, while for the HEB, slightly-higher-order (about 8th) one-turn maps are required. These results led to a tentative conclusion that the SSC is dominated by low-order resonances; that is, high-order (> 9) resonances do not play essential roles for the stability of the SSC.

Numerical particle tracking of circular accelerators, especially for a long term, has always consumed a great amount of computer time. In the past colleagues used scalar element-by-element accelerator tracking codes for such purposes. With such scalar tracking codes, only one particle can be tracked (advancement of phase-space coordinates) one element after another, then one turn after another, before another particle can be tracked. Therefore, most work was performed for short-term analyses, such as smear studies for predicting the linear aperture. However, after the SSC was proposed, it was realized that short-term studies alone were insufficient for deciding lattice issues, because the SSC is not as linear as the previous circular accelerators. Long-term stability and thus computational speed of particle tracking became critical issues for such a large nonlinear accelerator. The SSC contains more than 10,000 elements and needs to be operated for millions of turns. Therefore, long-term tracking studies with scalar element-by-element tracking codes are either not practical or virtually impossible. An improvement was then made to allow vectorized multi-particle tracking with the use of supercomputers [1]. Although the vectorized tracking speed was impressive (for a period of time) compared to scalar tracking, it still required more than 100 hours of Cray CPU time to obtain a survival plot up to 10^6 turns for the SSC. On the other hand, one-turn differential Lie algebraic maps had been attainable and used for order-by-order analysis [2]. It would be logical to consider using such one-turn maps for fast long-term tracking since a circular accelerator, whether large or small, would be represented by a single element, the one-turn map. However, whether a one-turn map can be used for long-term tracking had always been a controversial issue, although efforts to formulate one-turn-map tracking schemes had never been stopped (see reference [3] for a brief review).

In 1990, with the use of Zmap, an 11th-order Taylor map of the SSC was tested for advancing the phase-space coordinates of particles turn-by-turn via direct evaluation of the truncated 11th-order Taylor map (not exactly symplectic due to truncation) [4]. The survival plot obtained was found to be roughly the same as that obtained previously with element-by-element tracking [1]. This was the first time that the one-turn map showed some promise for long-term tracking of a practical accelerator lattice, although there were still some concerns about the non-exact symplecticity [5]. The same one-turn map was also tested with 10th-order Taylor-map tracking, resulting in somewhat different survival plots. However, the 10th-order Taylor map—after it had been Lie-transformed (by Dragt-Finn factorization [6]) and re-expanded into an 11th- or 12th-order Taylor map to gain a higher degree of symplecticity—showed correct dynamic aperture up to 10^6 turns [7,8]. These results led us to conclude that a moderate-order (lower than 11th order), one-turn Taylor map is usually accurate enough, but its degree of symplecticity may not be enough for long-term tracking [8]. The wrong survival plots obtained with the direct Taylor-map tracking of 10th order are due not to inaccuracy of the map but to artificial diffusion of the particle orbits because of the lack of sufficient symplecticity. How to symplectify the Taylor map without imposing large spurious errors in the map becomes the key to success when using one-turn maps for long-term tracking.

To date, there have been several schemes developed for the purpose of symplectic one-turn-map tracking, most of which have not been tested for practical cases. There are Jolt factorization (Irwin factorization) [9], monomial factorization [10], integrable-polynomial factorization [11], fitted map [12], dynamical rescaling method [13], and the generating function method [14,15]. Based on the numerical procedure of [15] and with the use of Zlib [16], we have recently developed a program, called Zimaptrk, for performing symplectic implicit one-turn-map tracking of the SSC and the HEB. First, the one-turn map of the SSC or the HEB is separated into two maps, a symplectic linear transfer matrix followed by a nonlinear Taylor map to enhance numerical stability. Then, the nonlinear Taylor map

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is converted into an implicit type of mixed-variable, vector-power-series map without imposing a certain form of generating function. (Of course, one of the four types of generating functions exists implicitly.) Because this method does not impose a predetermined form (not type) of generating function, it provides the same degree of accuracy as the explicit Taylor map at the same order. This method has shown much success in our long-term trackings for the SSC and the HEB and has saved us enormous amounts of computer time [15,17].

Shown in Figure 1 is a sample case of using Zimaptrk for symplectic one-turn-map long-term tracking of the SSC injection lattices where survival plots are shown for an SSC injection lattice with 4-cm diameter dipoles. Trackings with the 4th-, 5th-, 6th-, and 7th-order maps show roughly the same dynamic aperture. Table 1 shows dynamic apertures at 500,000 turns for many cases of the HEB lattice. That the average dynamic aperture (8.3 mm) over 9 random seeds of Rev-1 lattice is significantly larger than that of the Rev-0 lattice helps lead to the decision of replacing the previously designed HEB lattice (Rev 0) with the newly improved HEB lattice (Rev 1). That low-order one-turn maps work for both the SSC and the HEB also lead to a tentative conclusion that the SSC is dominated by low-order resonances, that is, high-order (> 9) resonances do not play essential roles for the stability of the SSC.

In summary, since one is interested only in phase-space regions where one-turn Taylor maps converge, it is fine to use one-turn maps not only for order-by-order analysis but also for turn-by-turn tracking. It is especially economical to use one-turn maps for long-term tracking of large circular accelerators.

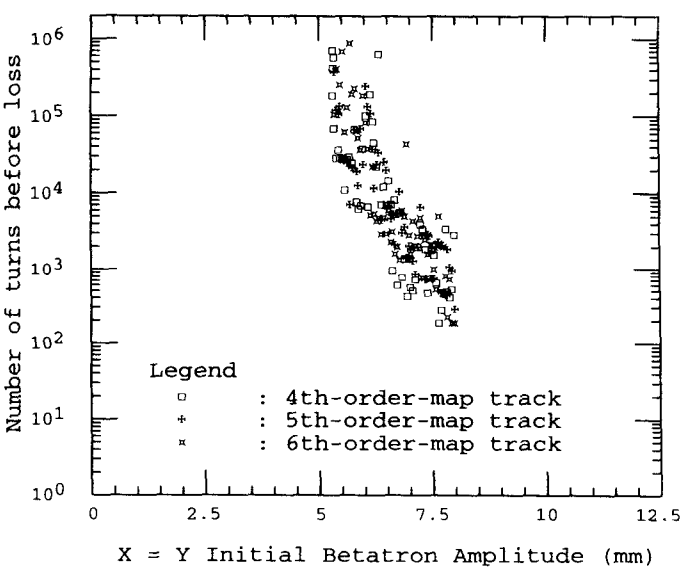

Figure 1. Survival plot.

Table 1
HEB Dynamic Aperture at 500,000 Turns

Seed Number	Rev 0 (mm)	Rev 1 (mm)
1	6.7	8.0
2	7.0	8.3
3	6.9	7.6
4	7.5	8.0
5	7.7	9.1
6	7.3	8.7
7	---	8.1
8	---	7.7
9	7.4	9.1
Average	(7.2)	8.3

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