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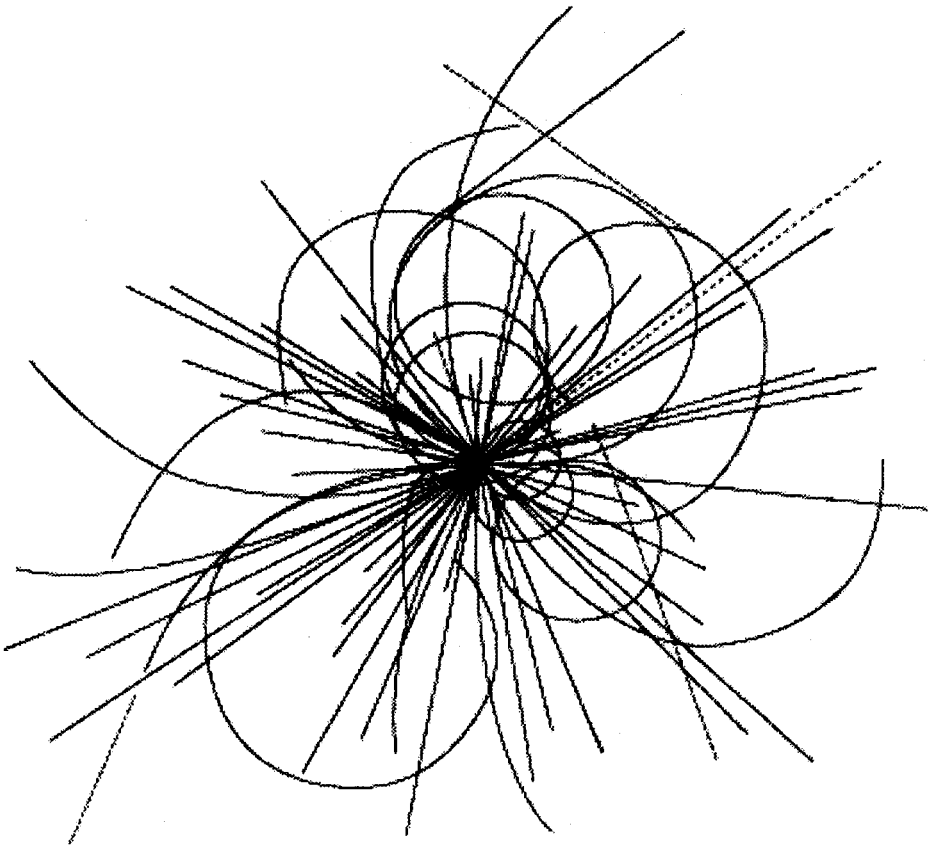

Superconducting Super Collider Laboratory

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STATISTICAL FACTORS TO QUALIFY THE SUPERCONDUCTING MAGNETS FOR THE SSC BASED ON WARM/COLD CORRELATIONS

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INTRODUCTION

All of the SSC production magnets will be measured at room temperature (warm), but only a fraction of these will be measured at liquid helium temperature (cold). The fractional information will then be analyzed to determine warm acceptance criteria for the field quality of the SSC magnets. Regarding predictors of the field quality based on partial information, there are several observations and studies based on the warm/cold correlation [Berk 88, Ferb 89, WaEt 92, PoEt 93]. A different facet of the acceptance test is production control, which interprets the warm/cold correlation to adjust the process parameters [Shap 92]. For these applications, we are evaluating statistical techniques relying on asymptotic estimators of the systematic errors and random errors, and their respective confidence intervals. The estimators are useful to qualify the population magnets based on a subset of sample magnets. We present the status of our work, including: i) a recapitulation of analytic formulas, ii) a justification based on HERA magnet experience, and iii) a practical interpretation of these estimators.

NOTATIONS

The requirements for the dipole field quality of an ensemble of magnets building the collider rings are given by i) the systematic multipole errors (SE) and ii) the random multipole errors (RE). The systematic errors are confined mainly to control the tune shift component of the linear aperture criterion, while the random errors are to fit the smear component of the linear aperture inside the physical aperture. To estimate the two errors of a full set of magnets to build a ring, we are using measurements of a subset of magnets (measured respectively at a magnet, at a z-scan position, at a repetition, either warm or cold), using a harmonics-measurement instrumentation named "Mole." Let's introduce some letters and indexes for the description of the quantities:

- χ: a nominal multipole coefficient, i.e., $\sim \{ a_n, b_n ; n = 1, .. \infty \}$
- x : a measurement of χ
- Subscript w (or c, or s) : refers to warm (or cold, or subset) measurements
- M (or K) : total number of magnets measured warm (or cold), $K \leq M$
- P: total number of positions within a magnet
- L: total number of repetitions at a given position within a magnet
- Superscript m (or p, or l) : magnet (or position, or repetition) index, $1 \leq m \leq M$, $1 \leq p \leq P$, $1 \leq l \leq L$. According to the above definitions, the l-th warm (cold) measurement, at the

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p-th position of the m-th magnet is written: $x_w^{l,p,m}$ ($x_c^{l,p,m}$). Regarding the usage of the indexes, it is noteworthy that i) a missing superscript is after processing on the corresponding index and ii) a missing subscript implies non-specification of the condition [KiE 93]. The SE is defined as $\mathbf{E}[\chi_c^m]$ an ensemble average of the magnet mean (χ_c^m), and the $(RE)^2$ as $\mathbf{E}[\{\chi_c^m - \mathbf{E}[\chi_c^m]\}^2]$.

ESTIMATION PROBLEMS

The first question was raised when we had successfully manufactured several full size R&D dipole magnets; do these magnets promise that consistent reproduction of the assembly processes will meet the requirements? Or, what kind of additional information do we need to promise that magnets will meet these requirements? The same question could be asked while the production is proceeding (especially when the consistency assumptions are not valid). We are following the framework of statistics and estimation theory to conclude the answers analytically. Specifically, we set three distinct questions:

- Q1: Based on K magnets measured warm (or, cold), how can we estimate the mean (or, variance) of M warm (or, cold) measurements to be produced later?
- Q2: Based on K magnets measured warm and cold, and M measured warm, how can we estimate the mean (variance) of a cold measurement to be produced later?
- Q3: Based on K magnets measured warm and cold, and M warm how can we estimate the mean (variance) of M cold measurements to be produced later?

In short, the information to predict a cold measurement consists of three features coming out of: i) inter-relation of the subset cold measurements, ii) inter-relation of the ensemble warm measurements, and iii) co-relation of the cold and the warm measurements. A simple case assumes that the warm (or cold) measurements quantities are statistically independent and they are linearly co-related, then the relations are well described by the statistical averages. For the case, we can derive i) asymptotic estimators of the systematic errors and random errors, and their respective confidence intervals. In reality, considering that the manufacturing procedures keep the time-history, e.g., aging of the tools and continuity of the processing, we recommend incorporating the dependencies among the warm (or cold) measurements. For the second case, we are considering auto-regressive-moving-average (ARMA) estimators of the cold measurements, and their respective confidence intervals, which are more tightly bound and useful for production control because they utilize the on-going history of the production [BoJe75].

ASYMPTOTIC ESTIMATORS

The aforementioned questions on the estimation problems can be answered by applying the results to the prediction model selection and several estimators corresponding to each parameter, respectively, derived in [KiEt93]. We use a model including the ratio (λ) of measurement noise variances, and the linear prediction coefficients μ and β.

I. Based on a sample of K magnets measured cold; estimate the mean value of the measurements for the ensemble of M magnets.

i) calculate the mean $x_{c,K}$ and variance $\sigma_{c,K}$, ii) then, an α-confident estimate of the cold value χ_c is confined: $|\chi_c - x_{c,K}| < Q(\alpha/2)\ \sigma_{c,K}\{\frac{M-K}{MK}\}^{1/2}$

where $Q(\alpha/2)$ is the Gaussian percentile, and the probability that the nominal value χ_c satisfies the inequality is $(1-\alpha)$. A reasonable number of $Q(\alpha/2)$ is 4, for $\alpha < 10^{-4}$.

II. Based on M warm and K cold measurements, estimate the cold multipoles of a given magnet.

i) calculate x_W and σ_W, ii) calculate $x_{c,K}$, $\sigma_{c,K}$, iii) based on the ratio λ, calculate the μ and β, iv) then, for the LSE type, an α-confident estimate of the cold value χ_c^m is

$$|\chi_c^m - \mu x_w^m - \beta| < Q(\alpha/2) \left\{ \frac{V_S[x_c^m - \mu x_w^m - \beta]}{K} + \frac{(x_w^m - x_{w,K})^2 V_S[x_c^m - \mu x_w^m - \beta]}{\Sigma_{m=1}^{K} (x_w^m - x_{w,K})^2} \right\}^{1/2}$$

III. Based on M warm and K cold measurements, estimate the mean of M cold measurements.

i) calculate x_W and σ_W, ii) calculate $x_{(c-w),K}$, iii) calculate $\sigma_{(c-w),K}$, iv) based on λ, calculate μ, v) then, an α-confident estimate of the cold mean χ_c is

$$|\chi_c - \mu (x_W + x_{(c-w),K}) - (1-\mu) x_{c,K}| < Q(\alpha/2) \left\{ \frac{\sigma_W^2}{M} + \frac{\sigma_{(c-w),K}^2}{K} \right\}^{1/2}$$

Implications: Simple factors to accept 10,000 magnets based on K cold and M ($K \le M \le 10{,}000$) warm measurements are: $\{ \sigma_W^2 + \sigma_{(c-w),K}^2 \}^{1/2} < RE$, and

$$|\mu (x_W + x_{(c-w),K}) + (1-\mu) x_{c,K}| + 4 \left\{ \frac{\sigma_W^2}{M} + \frac{\sigma_{(c-w),K}^2}{K} \right\}^{1/2} < SE$$

where the inter-relation information is in x_W, $x_{c,K}$ and σ_W, while the correlation information is in $\sigma_{(c-w),K}$ and μ.

Examples: We have tested the above rules for 5 Fermilab-General Dynamics Magnets. To compare the relative significance of each multipole, normalized numbers of the estimated RE are plotted in Figure 1. The sliced bars represent the estimated RE normalized by the required RE, and the plain bars the contribution from the correlation uncertainty.

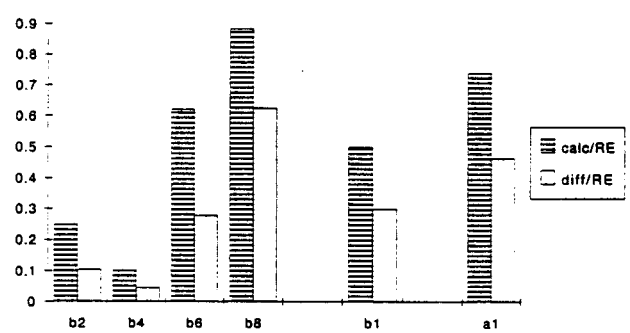

Figure 1. The ratio of the estimated and the required random errors, based on DCA311-315 (sliced bars), and the plain bars for the contribution of the correlation uncertainty.

Regarding SE computations, we do not have enough samples to draw any conclusion, even after the allowed multipoles are assumed to be significantly reduced by a modest redesign [WaEt 92]. To check further the validity of the proposed formulas, we have run simulations on the normal sextupole coefficient b_2 measurements of 220 HERA magnets produced by ANSALDO. Among the 220 Ansaldo magnets, 40 collared-coil assemblies were taken apart and re-assembled after the first set of warm magnetic measurements in order to correct some of their multipole coefficients by shimming the coils. These corrections constitute a deviation in the assembly process, and these 40 magnets are excluded from the study. Also, in order to retain information about equipment wear or personnel training, the magnet index, m, follows the manufacturing sequence. Figure 2 shows the cold and warm b_2 data for the ANSALDO magnets. Figure 2.a shows b_2 values

as a function of the sequence. It appears that b_2 varies widely (the mean and standard deviation is about 2 units both for cold and warm measurements). Figure 2.b shows b_2 cold as a function of b_2 warm; note that the Pearson correlation coefficient is 0.961, $\mu = 0.761$ and $\beta = 0.199$ based on the Least-Squared estimator (LSE), or $\mu = 0.824$ and $\beta = 0.069$ based on the Maximum-likelihood estimator (MLE). Let's look at the case III, as an acceptance tool.

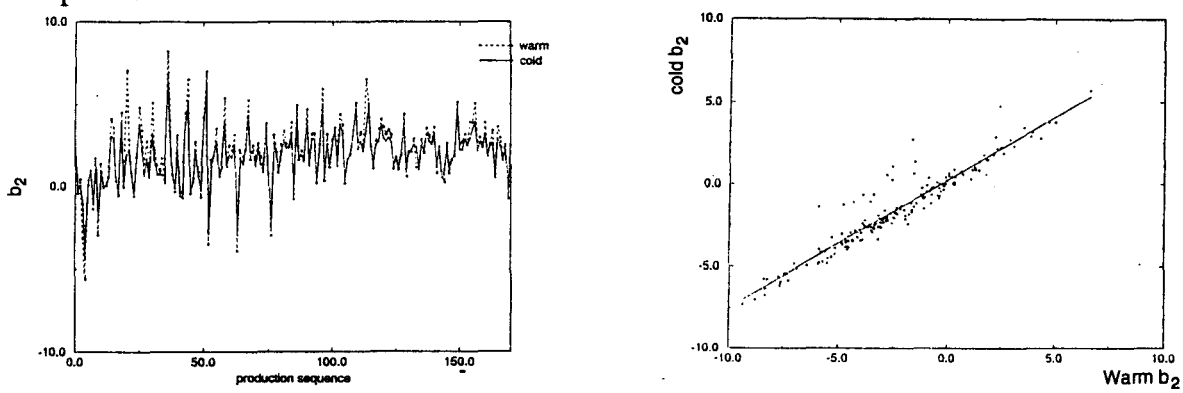

Figure 2. Cold and warm measurement data of b_2 for the HERA/ANSALDO magnets; a) b_2 as a function of product sequence; b) b_2 cold versus b_2 warm (an offset of 12.897 units)

IV. Based on M warm and 20 cold measurements, estimate the mean of M cold measurements.

We have 20 magnets measured both warm and cold, and all the magnets produced subsequently were only measured warm. Based on this information, we can estimate the mean of the cold measurements of M magnets. Figure 3 shows the cold estimate of the ensemble mean of M magnets. Two different estimators are shown: the circle for the simplified (asymptotic) and the X for the proposed. It appears that the proposed estimator is closer to the asterisk which is the mean valued calculated from the cold measurements. It is noticeable here that the confidence width of about 1 unit is not significant where the HERA magnets' reference radius is 2.5 cm and the maximum deviation of b_2 error is given by 10 units.

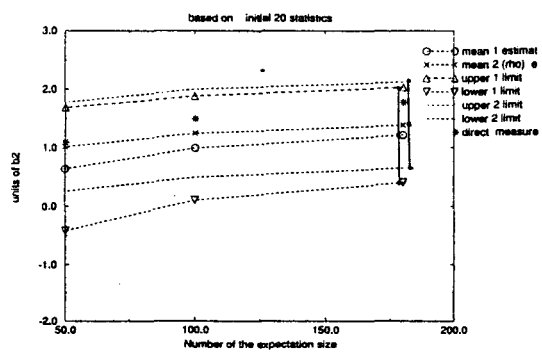

Figure 3. b_2 cold estimate of the ensemble mean of M magnet. The circle for the simple estimator and the X for the proposed estimator.

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