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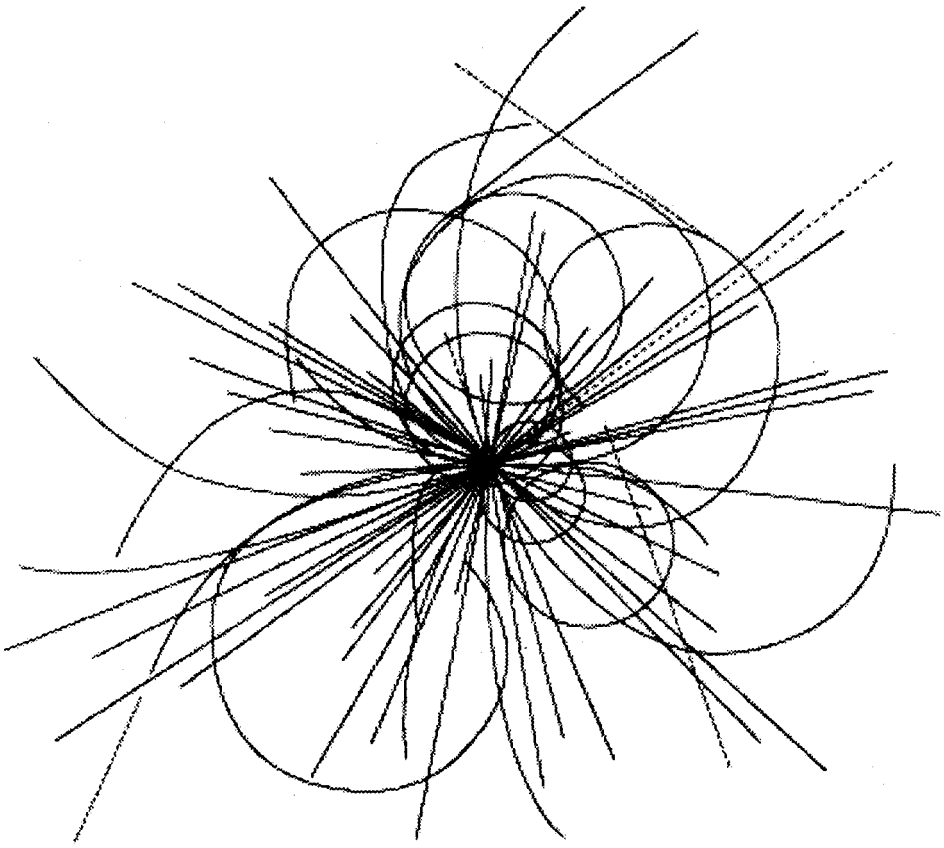
S. Meshkov

# Quark and Lepton Masses



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***To:*** All Preprint Recipients
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Please find enclosed a corrected version of page 6, section 5 Further Unifications. The corrections in the formulas found on this page are very important to the accuracy of the report. Please replace the incorrect page 6 with the new version of pages 5 and 6.

We are sorry for any inconvenience this may have caused and appreciate your help in this matter. If you have any questions, please feel free to give me a call.

Anita Lopez
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From the masses and $V_{CKM}$, we find that $M(-\frac{1}{3}) \equiv M$ can be written as:

$$M = \overline{m}_b \left( \frac{1}{3}\begin{pmatrix} 111 \\ 111 \\ 111 \end{pmatrix} + \frac{c_1}{6}\begin{bmatrix} 1 & 1 & -\frac{c_2}{c_1} \\ 1 & 1 & -\frac{c_2}{c_1} \\ -\frac{c_2}{c_1} & -\frac{c_2}{c_1} & -4\left(1-\frac{c_2}{c_1}\right) \end{bmatrix} + \frac{c_3}{\sqrt{3}}\begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix} \right) \quad (6)$$

The first matrix has the permutation symmetry $S(3)_L \times S(3)_R$; the second matrix, proportional to $c_1/6$, has the symmetry $S(2)_L \times S(2)_R$. The third matrix has the symmetry $S(1)_L \times S(1)_R$ and removes the last zero eigenvalue. For our case $c_1 = -5.5\, m_s / m_b$, and $c_2/c_1 = 0.77$.

Transforming Eq. (6) into the heavy basis, the mass matrix may be written as

$$M_H = \overline{m}_b \begin{pmatrix} 0 & c_3 & 0 \\ c_3 & -\frac{1}{3}(c_1 - 2c_2) & \frac{1}{\sqrt{18}}(2c_1 - c_2) \\ 0 & \frac{1}{\sqrt{18}}(2c_1 - c_2) & 1 \end{pmatrix} \quad (7)$$

We may compare Eq. (7) with the Fritzsch matrix, $M_F$[13]:

$$M_F = \begin{bmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{bmatrix} \quad (8)$$

For $M_H$ to be put in the form of $M_F$, $c_2/c_1 = 1/2$.

Kaus and Meshkov[4c] found that in order to fit both the masses and $V_{CKM}$, a finite entry, D, is needed in the (2,2) matrix element of Equations (7) and (8).

It should be emphasized that this analysis has been carried out using the same texture for both the up and down matrices. Such a choice is arbitrary and as we shall discuss in the next section, many other choices are possible.

## 5. FURTHER UNIFICATIONS

Recently, a number of unification schemes based on various supersymmetric extensions (SU(5), SO(10) etc.) of the Standard Model have been explored. In each case, the authors have chosen mass matrices with different textures for up quarks than for down quarks and charged leptons. They start with a given choice of mass matrices at the unification point, run the couplings down to lower scales using the renormalization group, and predict various masses and CKM angles.

In order to show the commonality of some of the matrix choices, using the symmetry breaking scheme discussed in the previous section, Kaus and Meshkov have prepared a "Dictionary", displayed below, that provides a translation from the heavy basis to the democratic basis.

**DICTIONARY**

| HEAVY | | DEMOCRATIC | SYMMETRY | |
|---|---|---|---|---|
| $X\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}$ | $\longrightarrow$ | $\frac{X}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}$ | $S(3)_L \times S(3)_R$ | (9) |
| $Y\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}$ | $\longrightarrow$ | $\frac{Y}{\sqrt{3}}\begin{pmatrix}1&0&-1\\0&-1&1\\-1&1&0\end{pmatrix}$ | $S(1)_L \times S(1)_R$ | (10) |
| $Z\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}$ | $\longrightarrow$ | $\frac{\sqrt{2}Z}{3}\begin{pmatrix}1&1&-\frac{1}{2}\\1&1&-\frac{1}{2}\\-\frac{1}{2}&-\frac{1}{2}&1-3\end{pmatrix}$ | $S(2)_L \times S(2)_R$ | (11) |
| $W\begin{pmatrix}0&0&0\\0&1&0\\0&0&0\end{pmatrix}$ | $\longrightarrow$ | $\frac{W}{6}\begin{pmatrix}1&1&-2\\1&1&-2\\-2&-2&1+3\end{pmatrix}$ | $S(2)_L \times S(2)_R$ | (12) |

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# QUARK AND LEPTON MASSES

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## ABSTRACT

The BCS mechanism, augmented by an *ad hoc*, but reasonable symmetry breaking mechanism, describes both the quark and lepton masses, and is consistent with the constraints imposed by the Cabbibo, Kobayashi, Maskawa mixing matrix. This mechanism may be incorporated into the study of various extensions of the Standard Model such as SUSY SU(5). The symmetry breaking at the unification scale is related to that at experimentally accessible scales by running the coefficients of the terms in the mass matrices characterized by their symmetries.

## 1. INTRODUCTION

Physicists have long sought their Holy Grail: a Unified Model of all interactions. In the 19th century, the then separate fields of light, electricity and magnetism were unified into electromagnetism, elegantly formulated in Maxwell's equations. Contemporaneously, the field of chemistry, was combined with a growing knowledge of the charge structure and masses of various atoms and the synthesis was summarized, empirically, in the Periodic table. Not all attempts at unification have been so successful. For example, even Einstein failed in his attempt to unify gravity and electromagnetism.

In the early 70's (plus or minus a few years), Glashow, Weinberg, and Salam [1] successfully unified the weak and electromagnetic interactions using the $SU(2)_W \times U(1)_Y$ symmetry. This led to the successful predictions (not postdictions) that $M_W = 80$ GeV and $M_Z = 91$ GeV, where the W boson carries the weak force and the Z boson is the heavy photon. By 1972, Quantum Chromodynamics (QCD) was suggested as the theory of strong interactions. The strong force was due to the exchange of massless gluons. This theory was

formulated by Bardeen, Fritzsch, Gell-Mann, Leutwyler, and Nambu [2]. In the late seventies and early eighties, there arose the hope that one might construct a Unified Theory of the strong, weak, and electromagnetic interactions , as for, example, in the SU(5) model of Georgi and Glashow[3]. This SU(5) model lead to the prediction that the proton should decay, with a lifetime of approximately $10^{29}$ years. In 1984, the decay was looked for in the mode $p \rightarrow \pi^0 e^+$ in water Cerenkov counters at both the Cleveland Salt Mine(IMB) and Kamioka, Japan. The bad news, at least for the SU(5) proponents, is that the proton lives!, with a lifetime $\tau_p > 10^{32}$ years. The good news is that these detectors were great for detecting neutrinos from SN1987A.

## 2. STANDARD MODEL

Clearly, SU(5) doesn't work; but what does? The obvious choice is: $SU(3)_C \times SU(2)_W \times U(1)_Y$. This leads to the Standard Model, a gauge theory for which the forces are mediated by the exchange of $J^P = 1^-$ bosons between fermions. This Standard Model "explains" all of the currently known phenomena in Elementary Particle physics.

As we all know, in this world we don't get something for nothing! The price that we pay for the success of the Standard Model is that there are lots of parameters, $\equiv 21$. Of these, the majority are the 12 quark and lepton masses, falling into three generations. There are six quarks and six leptons. In addition, there are 5 gauge bosons all with $J^P = 1^-$. Of these, the photon, $W^\pm$ , and Z are directly observable; only the gluon is not. In addition there are the coupling constants $\alpha$, $\alpha_s$ and $\sin^2\theta_W$. A symmetry breaking mechanism is needed, because all bosons and fermions are taken to be massless originally, and we need to make most of them massive. This implies the possible existence of massive Higgs bosons, with as yet undetermined masses.

Information about the parameters themselves must come from *outside* the standard model. What is our current knowledge of these parameters, in particular, the masses? The boson masses are quite well understood; the Higgs mass (masses) is totally open. The situation for the fermion masses is difficult. We have no known calculation of *any* fermion mass from first principles. Our *best hope* is to get relations among masses of the various subsets of fermions listed below:

| | | | |
|---|---|---|---|
| Quarks: | $q = +2/3$ | $3(u,c,t)$ | (t mass unknown, but $> 110$GeV) |
| | $q = -1/3$ | $3(d,s,b)$ | |
| Leptons: | $q = -1$ | $3(e^-,\mu^-,\tau^-)$ | |
| | $q = 0$ | $3(\bar{\nu}_e,\bar{\nu}_\mu,\bar{\nu}_\tau)$ | |

Note that only the lepton masses have been measured directly.

## 3. BCS MECHANISM

In this paper, I present the latest results that Peter Kaus and I have obtained in our continuing program for describing the fermion masses[4a,b,c]. Our work was originally motivated by the qualitative similarity of the quark and charged lepton spectra, with their large gaps (Fig. 1), to nuclear spectra and BCS superconductivity. Analogous work along these lines has also been carried out by Nambu[5], Fritzsch[6], Koide[7], and in a different vein, by Bardeen, Hill, and Lindner[8], and Miransky, Tanabashi, and Yamawaki[9].

The mass spectra of the charge +2/3 and -1/3 quarks, and the charged leptons are quite similar. For each charge, there is a high state (t, b, $\tau$), with a large gap between it and two low-lying levels, which are basically degenerate and close to zero. We invoke a BCS theory to describe the spectra, where a BCS theory or mechanism refers to Cooper pair formation through attractive forces between some constituent fermions. This analogy has been discussed in detail in earlier work and will not be discussed here.

## 4. SYMMETRY BREAKING

Successful symmetry breaking schemes that yield the correct light quark masses as well as the correct CKM matrix have been constructed by Kaus and Meshkov[4c], Fritzsch and Plankl[10], Koide[7], Tanimoto[11], and Matumoto[12]. They have assumed that the up and down matrices have the same texture. It is useful to shuttle between two rank one bases:

a) Democratic(coherent, ur-quark) basis;

$$M_O = 1/3\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{1}$$

b) Heavy basis;

$$M_H = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2}$$

where $M_O$ and $M_H$ are related by

$$M_O = U_O^{-1} M_H U_O, \tag{3}$$

and

$$U_O = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix} \tag{4}$$

If all six quark masses vanish, the Lagrangian of the standard model is invariant under the chiral symmetry $U(3)_L \times U(3)_R$, where U(3) is the symmetry group connecting the three generations. The indices L (R) refer to left-handed (right-handed) quark fields. The $U(3)_L \times U(3)_R$ symmetry is exact only if all quark masses vanish or are equal; in this limit all weak mixing angles vanish. The real world is far different from this limit, with $m_t >> m_c >> m_u$, $m_b >> m_s >> m_d$, $m_\tau >> m_\mu >> m_e$. To obtain one heavy state and two massless ones, we break $U(3)_L X U(3)_R$ to $S(3)_L x S(3)_R$; S(3) is the group of permutations of three elements.

$S(3)_L \times S(3)_R$ is the symmetry of the democratic matrix $M_O$. At this stage, only the third family is massive and the other two are massless. As remarked earlier, we can also represent this state of affairs by $M_H$, but the symmetry-breaking chain is made manifest using the democratic matrix. To break the degeneracy of the two massless states we break the $S(3)_L$ x $S(3)_R$ to $S(2)_L \times S(2)_R$. This is accomplished by adding to $M_O$ a matrix of the form

$$M_1 = \begin{bmatrix} 1 & 1 & a \\ 1 & 1 & a \\ a & a & b \end{bmatrix} \tag{5}$$

By so doing, we violate the $S(3)_L \times S(3)_R$ symmetry, but maintain an $S(2)_L \times S(2)_R$ symmetry. Now only the first family remains massless. To render it massive, the symmetry $S(2)_L \times S(2)_R$ must be broken; i.e., an additional matrix $M_2$ must be added which has nonequal values for the matrix elements $M_{11}$, $M_{12}$, $M_{21}$, and $M_{22}$.

In our earlier work we found the corrections to the $M_O$ matrix that yield the mass matrix $M_D$, which when diagonalized yields the observed quark masses. We do this by demanding that our transformation matrices $U\left(-\frac{1}{3}\right)$ are consistent with the requirements imposed by the current numerical values of the elements of the Cabbibo, Kobayashi, Maskawa mixing matrix $V_{CKM}$.

From the masses and $V_{CKM}$, we find that $M(-\frac{1}{3}) \equiv M$ can be written as:

$$M = \overline{m}_b \left( \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{c_1}{6}\begin{bmatrix} 1 & 1 & -\frac{c_2}{c_1} \\ 1 & 1 & -\frac{c_2}{c_1} \\ -\frac{c_2}{c_1} & -\frac{c_2}{c_1} & -4\left(1-\frac{c_2}{c_1}\right) \end{bmatrix} + \frac{c_3}{\sqrt{3}}\begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix} \right) \quad (6)$$

The first matrix has the permutation symmetry $S(3)_L \times S(3)_R$; the second matrix, proportional to $c_1/6$, has the symmetry $S(2)_L \times S(2)_R$. The third matrix has the symmetry $S(1)_L \times S(1)_R$ and removes the last zero eigenvalue. For our case $c_1 = -5.5\, m_s / m_b$, and $c_2/c_1 = 0.77$.

Transforming Eq. (6) into the heavy basis, the mass matrix may be written as

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We may compare Eq. (7) with the Fritzsch matrix, $M_F$[13]:

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For $M_H$ to be put in the form of $M_F$, $c_2/c_1 = 1/2$.

Kaus and Meshkov[4c] found that in order to fit both the masses and $V_{CKM}$, a finite entry, D, is needed in the (2,2) matrix element of Equations (7) and (8).

It should be emphasized that this analysis has been carried out using the same texture for both the up and down matrices. Such a choice is arbitrary and as we shall discuss in the next section, many other choices are possible.

## 5. FURTHER UNIFICATIONS

Recently, a number of unification schemes based on various supersymmetric extensions (SU(5), SO(10) etc.) of the Standard Model have been explored. In each case, the authors have chosen mass matrices with different textures for up quarks than for down quarks and charged leptons. They start with a given choice of mass matrices at the unification point, run the couplings down to lower scales using the renormalization group, and predict various masses and CKM angles.

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DICTIONARY

| HEAVY | | DEMOCRATIC | SYMMETRY | |
|---|---|---|---|---|
| $X\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}$ | $\xrightarrow{\cdot}$ | $\frac{X}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}$ | $S(3)_L \times S(3)_R$ | (9) |
| $Y\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}$ | $\longrightarrow$ | $\frac{Y}{\sqrt{3}}\begin{pmatrix}1&0&-1\\1&-1&1\\-1&1&0\end{pmatrix}$ | $S(1)_L \times S(1)_R$ | (10) |
| $Z\begin{pmatrix}0&0&0\\0&0&1\\0&0&0\end{pmatrix}$ | $\longrightarrow$ | $\frac{\sqrt{2}Z}{3}\begin{pmatrix}1&1&-\frac{1}{2}\\1&1&-\frac{1}{2}\\-\frac{1}{2}&-\frac{1}{2}&1-3\end{pmatrix}$ | $S(2)_L \times S(2)_R$ | (11) |
| $W\begin{pmatrix}0&0&0\\0&1&0\\0&0&0\end{pmatrix}$ | $\longrightarrow$ | $\frac{W}{6}\begin{pmatrix}1&1&-2\\1&1&-2\\-2&-2&1+3\end{pmatrix}$ | $S(2)_L \times S(2)_R$ | (12) |

Some further simplifications of Equations (11) and (12) in the democratic basis are given below.

$$\begin{pmatrix} 1 & 1 & -\frac{1}{2} \\ 1 & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & -2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 6 \end{pmatrix} \tag{13}$$

$$\begin{pmatrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ -2 & -2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - 2\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & -\frac{3}{2} \end{pmatrix} \tag{14}$$

From the relations given above, it is clear that "nice" texture in the heavy basis translates into "nice" $S(3) \rightarrow S(2)$ breaking in the democratic basis. We see that texture translates into Permutation Symmetry.

## 6. GEORGI- JARLSKOG MATRICES

The Georgi- Jarlskog[14] SU(5) matrices have been used by Ramond and his coworkers[15] and by Dimopoulos, Hall, and Raby[16] in their recent works on SUSY SU(5). The Georgi- Jarlskog SU(5) matrices for the q = -1/3 quark sector and the q = -1 charged lepton sector have a different texture than the original Fritzsch matrix which they used for the q = +2/3 quark sector.

These matrices, all originally written in the heavy quark basis, are listed below, as is the Kaus-Meshkov modification of the Fritzsch matrix. They were originally motivated by the desire to obtain:

a) $m_b = m_\tau$. This comes from the **5** in SU(5). (15)

b) $m_s = m_\mu / 3$. To get this term requires the addition of the **45** representation of SU(5). (16)

c) $m_d = 3m_e$. (17)

$$M_D = \begin{pmatrix} 0 & E & 0 \\ E & F & 0 \\ 0 & 0 & G \end{pmatrix} \tag{18}$$

$$M_l = \begin{pmatrix} 0 & E & 0 \\ E & -3F & 0 \\ 0 & 0 & G \end{pmatrix} \tag{19}$$

$$M_{U_F} = \begin{pmatrix} O & A & O \\ A & O & B \\ O & B & C \end{pmatrix} \tag{20}$$

$$M_{U_{KM}} = \begin{pmatrix} O & A & O \\ A & D & B \\ O & B & C \end{pmatrix} \tag{21}$$

Using our dictionary, we translate the Georgi- Jarlskog mass matrices from the heavy basis to the democratic basis, and list the mass matrices in both bases on the following page.

**HEAVY BASIS** **DEMOCRATIC BASIS**

$$M_D = \begin{pmatrix} O & E & O \\ E & F & O \\ O & O & G \end{pmatrix} = \frac{G}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{F}{6}\begin{pmatrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ -2 & -2 & 1+3 \end{pmatrix} + \frac{E}{\sqrt{3}}\begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix} \tag{22}$$

$$M_L = \begin{pmatrix} O & E & O \\ E & -3F & O \\ O & O & G \end{pmatrix} = \frac{G}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} - \frac{F}{2}\begin{pmatrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ -2 & -2 & 1+3 \end{pmatrix} + \frac{E}{\sqrt{3}}\begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix} \tag{23}$$

$$M_{U_F} = \begin{pmatrix} O & A & O \\ A & O & B \\ O & B & C \end{pmatrix} = \frac{C}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{\sqrt{2}B}{3}\begin{pmatrix} 1 & 1 & -\frac{1}{2} \\ 1 & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 1-3 \end{pmatrix} + \frac{A}{\sqrt{3}}\begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix} \tag{24}$$

$$M_{U_{KM}} = \begin{pmatrix} O & A & O \\ A & D & B \\ O & B & C \end{pmatrix} = \frac{C}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{D}{6}\begin{pmatrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ -2 & -2 & 1+3 \end{pmatrix} + \frac{\sqrt{2}B}{3}\begin{pmatrix} 1 & 1 & -\frac{1}{2} \\ 1 & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 1-3 \end{pmatrix} + \frac{A}{\sqrt{3}}\begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix} \tag{25}$$

$S(3)_L \times S(3)_R$ $S(2)_L \times S(2)_R$ $S(2)_L \times S(2)_R$ $S(1)_L \times S(1)_R$

The linear combinations are written in terms of the functions E,F, G and A, B, C, D, whose running must be investigated in order to relate symmetry breaking at the unification scale to symmetry breaking at experimentally accessible scales. Analogous expansions may be made for the Giudice ansatz[17]:

$$M_{U_G} = \begin{pmatrix} 0 & 0 & b \\ 0 & b & 0 \\ b & 0 & a \end{pmatrix} \tag{26}$$

In principle, we need not restrict ourselves to a particular form such as the Georgi- Jarlskog, Fritzsch, Kaus-Meshkov or Giudice matrices. One can, in fact, do a general analysis for the running of the coefficients of each matrix

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{27}$$

and of

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \tag{28}$$

Such a calculation is currently underway, in collaboration with Pierre Ramond.

## 7. CONCLUSIONS

The BCS mechanism, augmented by an *ad hoc*, but reasonable symmetry breaking mechanism, describes both the quark and lepton masses, and is consistent with the constraints imposed by the Cabbibo, Kobayashi, Maskawa mixing matrix. This mechanism may be incorporated into the study of various extensions of the Standard Model such as SUSY SU(5). The symmetry breaking at the unification scale is related to that at experimentally accessible scales by running the coefficients of the terms in the mass matrices characterized by their symmetries.

## 8. ACKNOWLEDGMENTS

The author is indebted to Peter Kaus and to Pierre Ramond for numerous important and useful discussions.

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