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SUPERGRAVITY GRAND UNIFICATION, PROTON DECAY AND COSMOLOGICAL CONSTRAINTS

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ABSTRACT

Properties and experimental predictions of a broad class of supergravity grand unified models possessing an $SU(5)$-type proton decay and R parity are described. Models of this type can be described in terms of four parameters at the Gut scale in addition to those of the Standard Model i.e. m_o (universal scalar mass), $m_{1/2}$ (universal gaugino mass), A_o (cubic soft breaking parameter) and $\tan\beta =< H_2 > / < H_1 >$. Thus the 32 SUSY masses can be expressed in terms of $m_o, m_{1/2}, A_o \tan\beta$ and the as yet unknown t-quark mass m_t. Gut thresholds are examined and a simple model leads to grand unification consistent with p-decay data when $0.114 < \alpha_3(M_z) < 0.135$, in agreement with current values of $\alpha_3(M_Z)$. Proton decay is examined for the superheavy Higgs triplet mass $M_{H_3} < 10M_G(M_G \simeq 1.5 \times 10^{16}$ GeV) and squarks and gluinos lighter than 1 TeV. Throughout most of the parameter space chargino-neutralino scaling relations are predicted to hold: $2m_{\tilde{Z}_1} \cong m_{\tilde{W}_1} \cong m_{\tilde{Z}_2}, m_{\tilde{W}_1} \simeq (1/4)m_{\tilde{g}}$ (for $\mu > 0$) or $m_{\tilde{W}_1} \simeq (1/3)m_{\tilde{g}}$ (for $\mu < 0$), while $m_{\tilde{W}_2} \cong m_{\tilde{Z}_3} \cong m_{\tilde{Z}_4} >> m_{\tilde{Z}_1}$. Future proton decay experiments combined with LEP2 lead to further predictions, e.g. for the entire parameter space either proton decay should be seen at these or the $\tilde{W}_1$ seen at LEP2. Relic density constraints on the $\tilde{Z}_1$ further constrain the parameter space e.g. so that $m_t < 165$ GeV, $m_h < 105$ GeV, $m_{\tilde{W}_1} < 100$ GeV and $m_{\tilde{Z}_1} < 50$ GeV when $M_{H_3}/M_G < 6$.

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1. INTRODUCTION

Over the past two years, there has been considerable effort to deduce consequences of supergravity grand unification models[1)-7)]. This activity has been stimulated in part by the observation by several groups[8)] that unification of the coupling constants $\alpha_1 \equiv (5/3)\alpha_Y$, α_2 and α_3 appears to occur at a common value $\alpha_G \simeq 0.04$ at a scale $M_G \approx 10^{16}$ GeV if one assumes that the particle spectrum below M_G is the minimal supersymmetric one with just two Higgs doublets with the SUSY particles in the mass range $M_S \approx 10^{2-3}$ GeV. Thus, while unification fails by over 7 std. for the Standard Model mass spectrum, the SUSY mass spectrum introduces additional thresholds which allows grand unification to occur.

A second impetus to the study of supergravity models is the possibility of testing them experimentally at current or future experiments. The reason for this is due to two remarkable features of these models. First, supergravity unification allows for spontaneous breaking of supersymmetry in the "hidden" sector, something that is difficult to achieve satisfactorily in low energy global supersymmetry, and remains an important unresolved problem in superstring theory. While the physics of the hidden sector is unknown, it turns out that it can be characterized by just a few "soft breaking" parameters[9,10)]. The second important feature is that the spontaneous symmetry breaking of supersymmetry can then trigger the breaking of $SU(2) \times U(1)$[11)]. The most theoretically appealing way of doing this is by renormalization group effects[12)]. This has two immediate consequences: first qualitatively the SUSY breaking scale is related to the electroweak mass scale (as appears to be the case experimentally from grand unification analysis). More quantitatively, the renormalization group equations allow one to relate the electroweak scale to the Gut scale. As a consequence, the masses of the 32 new SUSY particles (listed in Table 1) can be determined in terms of only 4 additional Gut scale parameters, and the as yet unknown t-quark mass m_t.

Table 1. New particles predicted to exist in minimal SUSY models. For squarks and sleptons $i = 1, 2, 3$ is a generation index, a is an $SU(3)_C$ index and $\tilde{W}_i$, $\tilde{Z}_i$ are labels so that $m_i < m_j$ for $i < j$.

Name	Symbol	Type	Number
squarks	$\tilde{q}_i = (\tilde{u}_{iL}, \tilde{d}_{iL});\ \tilde{u}_{iR}, \tilde{d}_{iR}$	$j = 0$, complex	12
sleptons	$\tilde{l}_i = (\tilde{\nu}_{iL}, \tilde{e}_{iL});\ \tilde{e}_{iR}$	$j = 0$, complex	9
gluino	$\lambda^a, a = 1 \cdots 8$	$j = \frac{1}{2}$, Majorana	1
Winos (charginos)	$\tilde{W}_i;\ i = 1, 2$	$j = \frac{1}{2}$, Dirac	2
Zinos (neutralinos)	$\tilde{Z}_i,\ i = 1 \cdots 4$	$j = \frac{1}{2}$, Majorana	4
Higgs	h^o, H^o	$j = 0$, real, CP even	2
	A^o	$j = 0$, real, CP odd	1
	$H^\pm$	$j = 0$, complex	1

In principle then, if one knew the masses of 4 SUSY particles, one could predict the positions of the remaining 28 particles. Of course, no SUSY particles have yet been discovered, and so in practice what one can do is determine various allowed mass bands for SUSY particles, or mass relations, between particles. If the model possesses proton decay, existing (and future) bounds on the proton lifetime can considerably narrow these bands. Similarly, the cosmological constraint that the relic mass density of the lightest supersymmetric particle (which is stable is most models) not overclose the universe, also constrains the SUSY masses. Thus it seems possible to test these models in the relatively near future.

2. CLASS OF MODELS

We specify now the class of supergravity Gut models we will consider by assuming the following:

(i) There exists a hidden sector which is a gauge singlet with respect to the physical sector gauge group G which breaks supersymmetry. This can be done by a super Higgs mechanism[13] or a gaugino condensate[14]. The superpotential W is assumed to decompose, e.g. for the super Higgs mechanism, as $W = W_{\text{phys.}}(z_a) + W_{\text{hidden}}(z)$

where $\{z_a\}$ are the physical fields and $\{z\}$ the (G singlet) hidden sector fields. The gauge hierarchy is maintained since the super Higgs fields communicate with the physical fields only gravitationally.

(ii) A Gut sector exists which breaks G to the Standard Model group at scale $Q = M_G : G \to SU(3)_C \times SU(2)_L \times U(1)_Y$. An example of this for the case $G = SU(5)$ is given by the following Gut part of the superpotential[15]:

$$\begin{aligned} W_G =& \lambda_1 \left[\frac{1}{3} Tr\Sigma^3 + \frac{1}{2} M Tr\Sigma^2 \right] + \\ & \lambda_2 \bar{H}_{2X} (\Sigma_Y^X + 3M' \delta_Y^X) H_1{}^Y \end{aligned} \tag{2.1}$$

where $\Sigma_Y^X = 24$ of $SU(5)$, $\bar{H}_{2X}$ and $H_1{}^Y$ are a $\bar{5}$ and 5 of $SU(5)$. Minimizing the effective potential one finds diag $< \Sigma_Y^X >= M(2,2,2,-3,-3)$, breaking $SU(5)$ with $M = O(M_G)$. If $M' = M + \mu_o/3\lambda_2$, $\mu_o << M$, then the color triplet parts of H_1 and $\bar{H}_2$ become superheavy, and the $SU(2)$ doublets remain light and become the two Higgs doublets of the low energy theory.

(iii) After integrating out the superheavy fields, and eliminating the super Higgs fields, the only light particles remaining below the Gut scale are those of the SUSY Standard Model with one pair of Higgs doublets.

(iv) Any super Higgs field couplings that may appear in the Kahler potential are generation independent.

Note that conditions (ii) and (iii) are just what is needed to obtain the grand unification of the coupling constants discussed in Sec. 1, while (i) and (iv) guarantees the suppression of flavor changing neutral interactions.

A general model of this type can then be described at M_G as follows[10]: There is an effective superpotential with quadratic and cubic terms $W = W^{(2)} + W^{(3)}$ given by

$$\begin{aligned} W = \mu_o H_1 H_2 + [\lambda_{ij}^{(u)} q_i H_2 u_j^C + \lambda_{ij}^{(d)} q_i H_1 d_j^C + \\ \lambda_{ij}^{(e)} l_i H_1 e_j^C], \end{aligned} \tag{2.2}$$

an effective potential given by

$$\begin{aligned} V = \{ \sum_a | \frac{\partial W}{\partial z_a} |^2 + V_D \} + [m_o^2 \sum_a z_a z_a^* + \\ (A_o W^{(3)} + B_o W^{(2)} + h.c.)] \end{aligned} \tag{2.3}$$

and a universal gaugino mass term $\mathcal{L}^{\lambda}_{\text{mass}} = -m_{1/2}\bar{\lambda}^{\alpha}\lambda^{\alpha}$. In Eq. (2.2), q_i, l_i, H_1, H_2 are $SU(2)_L$ doublets, u_i^C, d_i^C, e_i^C are conjugate singlets, V_D is the usual D term, and $\lambda^{(u)}$, $\lambda^{(d)}$, $\lambda^{(e)}$ are the usual Yukawa coupling constants. In Eq. (2.3), $m_o^2 > 0$ is a universal mass term for all scalar fields. Thus aside from the Yukawa coupling constants of the Standard Model, the theory depends on the following Gut scale parameters:

$$m_{1/2}, m_o, A_o, B_o;\ \mu_o;\ \alpha_G, M_G \tag{2.4}$$

The first four constants are the "soft-breaking" parameters that characterize supersymmetry breaking, and μ_o is the $H_1 - H_2$ mixing parameter.

3. ELECTROWEAK BREAKING

We briefly summarize next how the supergravity models give rise naturally to electroweak breaking. At $Q = M_G$, we saw in Sec. 2 that the spontaneous breaking of super symmetry gave all scalar fields a universal mass m_o where $m_o^2 > 0$. Using the renormalization group equations (RGE), each particle's mass changes due to radiative corrections as one goes to lower values of Q. The squark, slepton and H_1 (mass)^2 increase, but due to the t-quark Yukawa couplings the H_2 (mass)^2 is driven negative at the electroweak scale, as shown schematically in Fig. 1. To see this

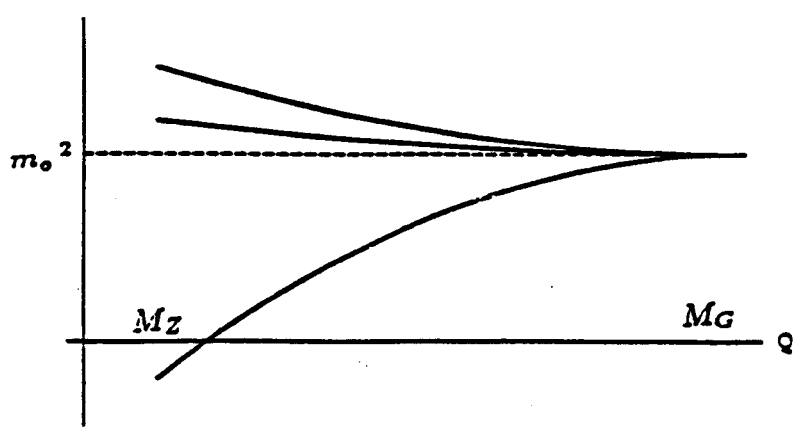

Fig. 1. Running masses in supersymmetric models as a function of the mass scale Q.

in more detail, consider the part of the effective potential of Eq. (2.3) involving the Higgs fields:

$$V_H = m_1(t)^2 \mid H_1 \mid^2 + m_2(t)^2 \mid H_2 \mid^2 - m_3^2(t)(H_1 H_2 + h.c.) + \frac{1}{8}[g_2^2(t) + g_Y^2(t)][\mid H_1 \mid^2 - \mid H_2 \mid^2]^2 + \Delta V_1 \tag{3.1}$$

where $t = ln[M_G^2/Q^2]$ is the running parameter, $m_i^2(t) = m_{H_i}^2(t) + \mu^2(t), i = 1, 2, m_3^2(t) = -B(t)\mu(t)$ and ΔV_1 is the one loop correction. At $Q = M_G$ $(t = 0)$, the running masses obey the boundary conditions $m_i^2(0) = m_o^2 + \mu_o^2$ and $m_3^2(0) = -B_o\mu_o$. The RGE determine these parameters at all other t. One may minimize V_H with respect to the two VEVs $\sigma_{1,2} =< H_{1,2} >$ to obtain

$$\frac{1}{2}M_Z^2 = \frac{\mu_1^2 - \mu_2^2 \tan^2\beta}{\tan^2\beta - 1} \; ; \; \sin 2\beta = \frac{2m_3^2}{\mu_1^2 + \mu_2^2} \tag{3.2}$$

where $\mu_i^2 = m_i^2 + \Sigma_i, \tan\beta = v_2/v_1$ and Σ_i are the loop corrections: $\Sigma_i = \Sigma_a(-1)^{2j_a}$ $n_a[M_a(v_i)]^2 ln[M_a^2/\sqrt{e}Q^2]\frac{\partial M_a^2}{\partial v_i}$. ($M_a$ is the mass of particle a, j_a is its spin and n_a is the number of helicity states.). In practice, Eqs. (3.2) are insensitive to the value of Q^2 in the electroweak scale[16] so one may conveniently set $Q = M_Z$. Also, the loop corrections are generally small[16].

The RGE allow one to evaluate $\mu_i^2(t)$ and $m_3^2(t)$ in terms of the Gut parameters. From the boundary conditions above, one may use Eqs. (3.2) to elimate μ_o^2 in terms of M_Z and replace B_o by $\tan\beta$. One is left with the parameters

$$m_o, m_{1/2}, A_o, \tan\beta; \; \alpha_G, M_G \tag{3.3}$$

The sign of μ_o is not determined and so there are two branches: $\mu_o > 0$ and $\mu_o < 0$. Since α_G and M_G have essentially been "measured " by LEP in the grand unification analysis of Sec. 1, the theory depends on 4 + 1 constants: $m_o, m_{1/2}, A_o, \tan\beta$ and the as yet undetermined m_t. For a fixed set of these parameters, one can calculate the masses of all the SUSY particles. An example of this is given in Fig. 2

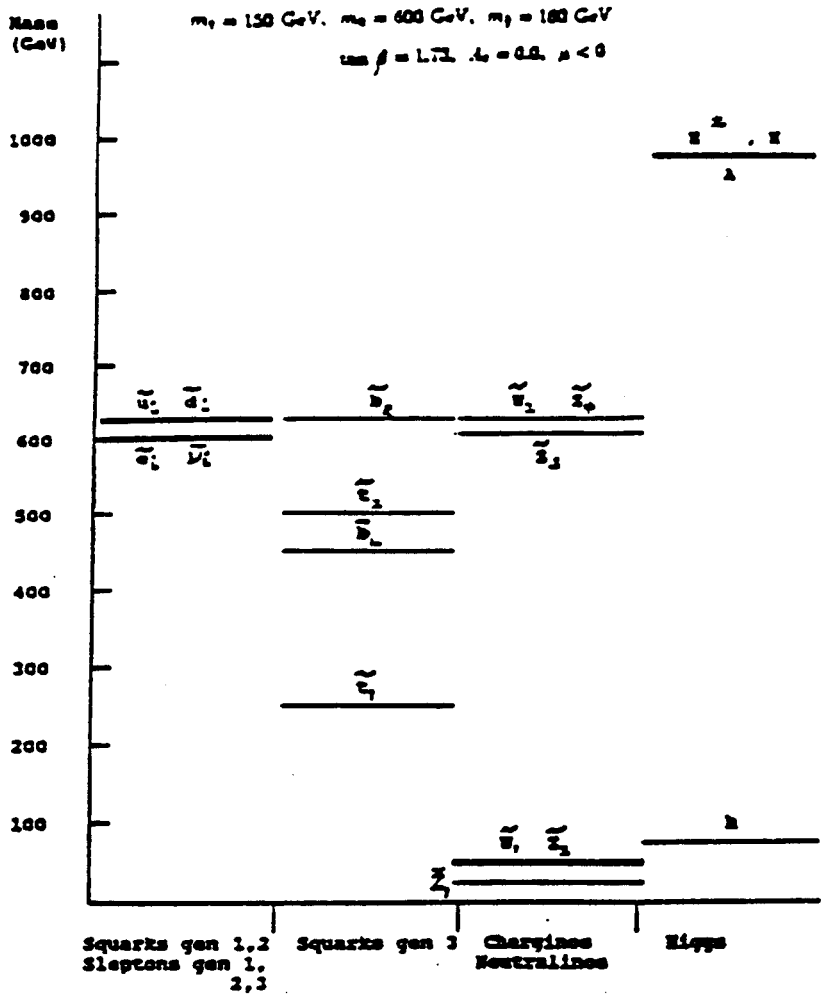

Fig. 2. The SUSY mass spectrum for $m_o = 600$ GeV, $m_{1/2} = 53$ GeV, $A_o = 0$, $\tan\beta = 1.73$, $\mu < 0$ and $m_t = 150$ GeV.

Note the mass splitting in the third generation of squarks, in the Winos and Zinos, and in the neutralinos.

One can vary all the parameters, and in this way get allowed bands of SUSY masses. In the following, we will also impose a theoretical constraint that there will be no excessive "fine tuning" of parameters, which we will take as requiring $m_o, m_{\tilde{g}} < 1$ TeV. This also implies that squarks and gluinos lie below 1 TeV, which is also probably the upper limit for detecting these particles at the SSC or LHC.

4. PROTON DECAY

We consider here models with "$SU(5)$-type" proton decay. These are models which obey the following conditions: (i) The Gut group G contains an $SU(5)$ subgroup [or is

$SU(5)$]. (ii) The matter that remains light after G breaks to $SU(3) \times SU(2) \times U(1)$ at M_G is embedded in the usual way in the $10 + \bar{5}$ representations of the $SU(5)$ subgroup. (iii) After G breaks, there are only two light Higgs doublets which interact with matter, and these are embedded in the $5 + \bar{5}$ of the $SU(5)$ subgroup. The corresponding Higgs color triplets are assumed to become superheavy from a $M_{H_3} H_3 \bar{H}_3$ term arising after the breaking of G. (iv) There is no discrete symmetry or condition that forbids the proton decay amplitude.

Under the above conditions (which can arise in a number of models, e.g. $G = SU(5), O(10), E_6$ etc.) There is a characteristic SUSY proton decay, $p \to \bar{\nu} + K^+$, due to the exchange of the superheavy Higgsino color triplet with a model independent decay amplitude[17,18]. An example of this decay process is given in Fig. 3. Proton

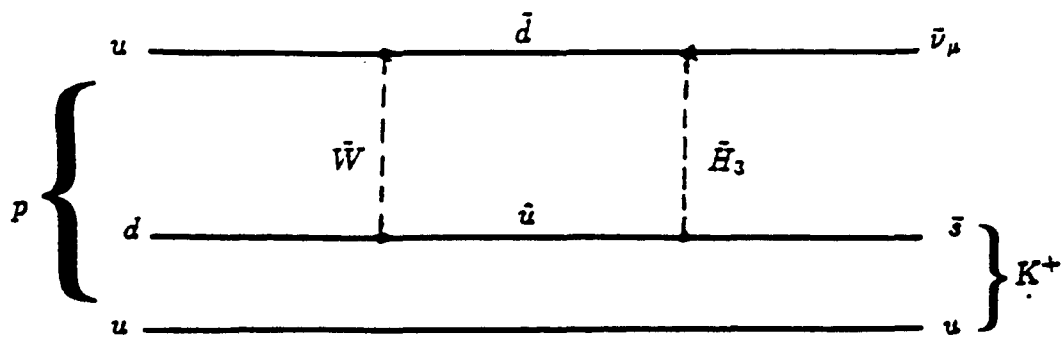

Fig. 3. One of the diagrams contributing to the $p \to \bar{n} u_\mu + K^+$ decay. The Wino ($\tilde{W}$) converts the quarks into squarks, and the baryon violating interactions occur at the $\tilde{H}_3$ vertex.

decay is a characteristic feature of supergravity grand unification models, and one must do special things to avoid it. Thus the flipped $SU(5)$ model suppresses proton decay by violating condition (2) above[19]. Models that invoke discrete symmetries to prevent p-decay from arising generally have more than one pair of light Higgs doublet s and sometimes relatively light Higgs color triplets[20]. While proton decay would be suppressed, one would expect such models to be in disagreement with the LEP grand unification data, which requires only one pair of light Higgs doublets[8].

The current experimental bound on the $p \to \bar{\nu} K^+$ mode is, from Kamiokande[21], $\tau(p \to \bar{\nu} K^+) > 1 \times 10^{32}$ yr (90% CL). However, future experiments can greatly improve on this limit and are expected to be sensitive up to $\simeq 2 \times 10^{33}$ yr for Super Kamiokande[22] and $\simeq 5 \times 10^{33}$ yr for ICARUS[23].

The total decay rate is $\Gamma(p \to \bar{\nu}K) = \Sigma_i \Gamma(p \to \bar{\nu}_i K), i = e, \mu, \tau$. The CKM matrix elements appear at the vertices of the loop integral of Fig. 3 and so all three generations can circle in the loop. Thus for a superheavy $\tilde{H}_3$, one may write[18)]

$$\Gamma(p \to \bar{\nu}K) = \text{Const}(\beta_p/M_{H_3})^2 \sum_{a,i} \mid B_{ia} \mid^2 \tag{4.1}$$

where B_{ia} is the loop amplitude of the $\bar{\nu}_i K$ mode when generation a squarks enter in the loop. (Actually, the first generation, $i = 1$ and $a = 1$, give negligible contributions.) The quantity β_p is

$$\beta_p U_L^\gamma = \varepsilon_{abc}\epsilon_{\alpha\beta} < o \mid d_{aL}^\alpha u_{bL}^\beta u_{cL}^\gamma \mid p > \tag{4.2}$$

where U_L^γ is the proton wave function. Lattice gauge calculations give[24)] $\beta_p = (5.6 \pm 0.8) \times 10^{-3}$ GeV^{-1}. The general expression for the loop amplitudes B_{ia} are complicated functions given in Ref. (18). They clearly depend on the SUSY particle $(\tilde{q}, \tilde{W}, \tilde{l})$ masses, and so an upper bound on $\Gamma(p \to \bar{\nu}K)$ will produce bounds on the SUSY masses. However, M_{H_3} also enters in Γ, and one also needs information concerning this quantity. In general one expects $M_{H_3} = O(M_G)$, and so to quantify the relation we first return to reconsider the grand unification of the coupling constants $\alpha_1, \alpha_2, \alpha_3$.

5. UNIFICATION OF COUPLING CONSTANTS

The analysis of the unification of α_1, α_2 and α_3 is complicated by the existence of two sets of thresholds that exist as one proceeds from M_Z to M_G [using the renormalization group equations (RGE)]. There are first the low energy thresholds due to the spectrum of SUSY particles at masses $\sim$ 100 GeV - 1 TeV, and second there are the superheavy Gut particles as masses $\sim M_G$ that account for the breaking of the Gut group G to $SU(3) \times SU(2) \times U(1)$. If, as a zero'th approximation, one sets all SUSY particles to a common, "average" mass M_S, and all Gut particles to M_G, then a fit to the data $\alpha_1(M_Z), \alpha_2(M_Z), \alpha_3(M_Z)$ gives

$$M_G = 10^{16.19 \pm 0.34}\text{GeV}; \; M_S = 10^{2.37 \pm 1.0} GeV \tag{5.1}$$

and $\alpha_G^{-1} = 25.7 \pm 1.7$, the errors being due to those in α_3 which we will take here as[24)]

$$\alpha_3(M_Z) = 0.118 \pm 0.007 \tag{5.2}$$

It is possible, when the Gut mass spectrum is taken into account that M_{H_3} will exceed the above value for M_G. To get some idea on how big M_{H_3} could be, consider the $SU(5)$ model for the Gut sector of Eq. (2.1). One finds, after the breaking of $SU(5)$ that $M_{H_3} = 5\lambda_2 M$, the octet and singlet component of the $\underline{24}$ have masses $M_\Sigma^{(8,3)} = 5\lambda_1 M/2$, $M_\Sigma^o = \lambda_1 M/2$, and the massive vector bosons have mass $M_V = 5gM(\alpha_G = g^2/4\pi)$. To stay within the perturbative domain we restrict $\lambda_{1,2} \leq 2$ (i.e. $\alpha_{\lambda_{1,2}} = \lambda_{1,2}^2/4\pi \lesssim 1/3$). We also limit $\lambda_{1,2} > 0.01$ (i.e. $\alpha_{\lambda_{1,2}} > 8 \times 10^{-6}$). One may now carry out the grand unification analysis including the Gut thresholds. The result for the allowed region is given in Fig. 4. We note first that grand unification implies an *upper* bound on α_3 of

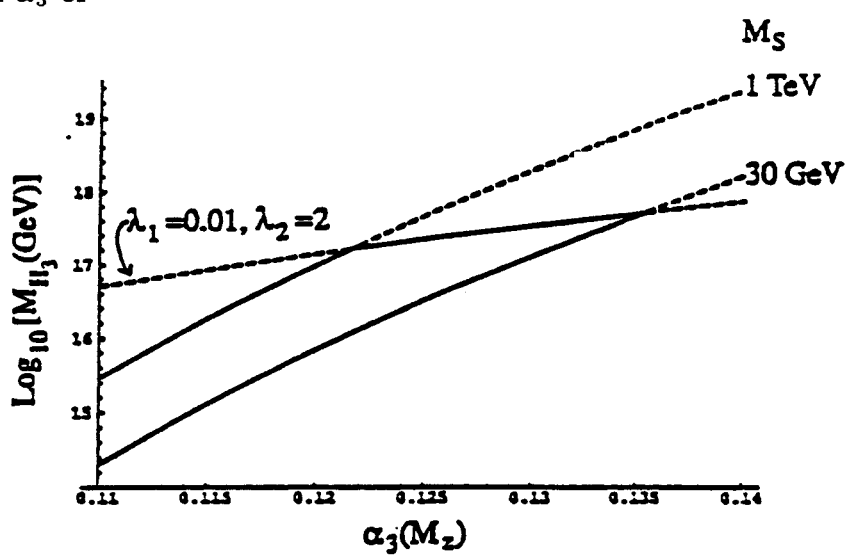

Fig. 4. Relation between Higgs triplet mass M_{H_3} and $\alpha_3(M_Z)$ required by grand unification for the Gut model of Eq. (2.1).[25] The quadrilateral region enclosed by the solid lines is the allowed region consistent with grand unification for 30 GeV $< M_S <$ 1 TeV, $\lambda_1 > 0.01, \lambda_2 < 2$.

$\alpha_3(M_Z) \lesssim 0.135$ (which is reduced for a larger value of λ_1), while the $1-\sigma$ bound of Eq. (5.2), $\alpha_3(M_Z) = 0.125$, corresponds to $M_{H_3} \simeq 2 \times 10^{17}$ GeV or $M_{H_3} \simeq 10\ M_G$. In the following, we will assume

$$3 < M_{H_3}/M_G < 10 \tag{5.3}$$

as a reasonable range for M_{H_3}.

6. SUSY MASS RELATIONS

We now examine the SUSY mass spectrum obtained by letting the parameters of the theory, $m_o, m_{1/2}, A_o, \tan\beta$ and m_t range over the entire parameter space subject only to the following constraints: (i) the SUSY masses and m_t do not violate current experimental bounds; (ii) Radiative breaking of $SU(2) \times U(1)$ occurs (i.e. solutions of Eqs. (3.2) exist); (iii) Experimental bounds on proton decay are obeyed; (iv) No excessive fine tuning occurs i.e. $m_o, m_{\tilde{g}} < 1$ TeV, and M_{H_3} is constrained by Eq. (5.3). We summarize now the consequences of the model under these conditions.

(1) We examine first the smallest value of M_{H_3}, i.e. $M_{H_3}/M_G = 3$, where proton decay is most constraining. The parameter space is limited but still sizable. One finds[1)]

$$\begin{gathered} m_o \gtrsim 500 \text{ GeV}; \; m_{\tilde{g}} \lesssim 450 \text{ GeV}; \; -1.5 \lesssim A_t/m_o \lesssim 1.5 \\ 1.1 \lesssim \tan\beta \lesssim 5 \end{gathered} \tag{6.1}$$

This implies that squarks (except perhaps $\tilde{t}_1$, the light t-squark) and probably gluinos will require the SSC and LHC to be seen. In addition, one finds the bounds $m_t < 180$ GeV and $m_h < 110$ GeV. Further[2)], for $m_t < 140$ GeV, one finds that $m_{\tilde{W}_1} < 100$ GeV whenever $m_h < 95$ GeV. Since these are the respective bounds for observing the $\tilde{W}_1$ and h particles at LEP2, one has that if $m_t < 140$ GeV, LEP2 will see either the $\tilde{W}_1$ or the h (and possibly both).

(2) As M_{H_3}/M_G increases, the lower bound on m_o decreases and the upper bound on $m_{\tilde{g}}$ increases. Thus for $M_{H_3}/M_G \gtrsim 7$, $m_{\tilde{g}}$ can reach the maximum allowed value of 1 TeV. One will still generally expect to need the SSC or LHC to detect squarks and the gluino. (The other bounds of Eq. (6.1) also widen, through not greatly.)

(3) Over most of the allowed parameter space, for the whole range of M_{H_3} of Eq. (5.3), a remarkable set of scaling laws hold for the light charginos and neutralinos[1-3)]:

$$2m_{\tilde{Z}_1} \cong m_{\tilde{W}_1} \cong m_{\tilde{Z}_2} \tag{6.2a}$$

$$m_{\tilde{W}_1} \simeq \frac{1}{4} m_{\tilde{g}}(\mu > 0); \; m_{\tilde{W}_1} \simeq \frac{1}{3} m_{\tilde{g}}(\mu < 0) \tag{6.2b}$$

(Eqs. (6.2a) often hold to within a few percent and Eqs. (6.2b) to within 25%.) In addition, the other chargino and neutralinos are nearly degenerate and much heavier than the $\tilde{Z}_1$. Similarly, the other Higgs bosons are generally very heavy and nearly

degenerate:

$$m_{\tilde{W}_2} \cong m_{\tilde{Z}_3} \cong m_{\tilde{Z}_4} >> m_{\tilde{Z}_1} \tag{6.3a}$$

$$m_H \cong m_A \cong m_{H^\pm} >> m_h \tag{6.3b}$$

The reason for Eqs. (6.2) and (6.3a), is that generally one finds that the proton decay constraint requires $\mu^2 >> M_Z^2, \tilde{m}_2^2$ (where $\tilde{m}_2$ is the $SU(2)$ gaugino mass) while Eq. (6.3b) is a consequence of the largeness of m_o.

7. FUTURE EXPERIMENTS

One can combine the expectations from future experiments to obtain fairly stringent tests for these models. Thus Super Kamiokande expects to reach a sensitivity of[22] $\approx 2 \times 10^{33}$ yr for the $p \to \bar{\nu}K^+$ mode, while ICARUS expects to reach to[23] $\approx 5 \times 10^{33}$ yr. Figs. 5 show the maximum value of $\tau(p \to \bar{\nu}K^+)$ as a function of m_o as all other parameters are varied over the entire allowed parameter space.

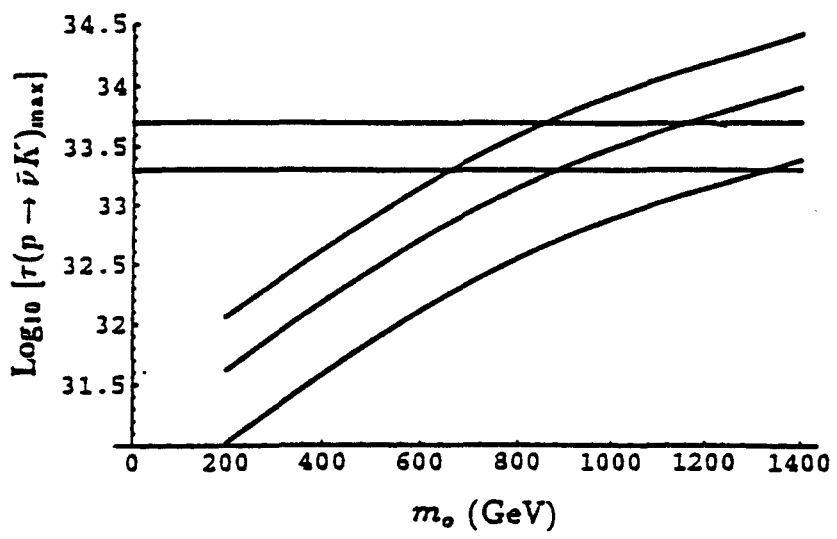

Fig. 5a. The maximum value of $\tau(p \to \bar{\nu}K^+)$ vs m_o for $m_t = 125$ GeV, $\mu < 0$. The maximum is calculated by varying all parameters except m_o over the entire allowed parameter space. The results are plotted for $M_{H_3}/M_G = 3$, 6 and 10. The lower horizontal line is the upperbound for Super Kamiokande, and the higher line is for ICARUS.

One sees that the entire domain for $m_o \lesssim 1000$ GeV is excluded by ICARUS for $M_{H_3}/M_G \lesssim 6$ if proton decay is not observed. (The same result holds for Super Kamiokande with $m_o \lesssim 800$ GeV.)

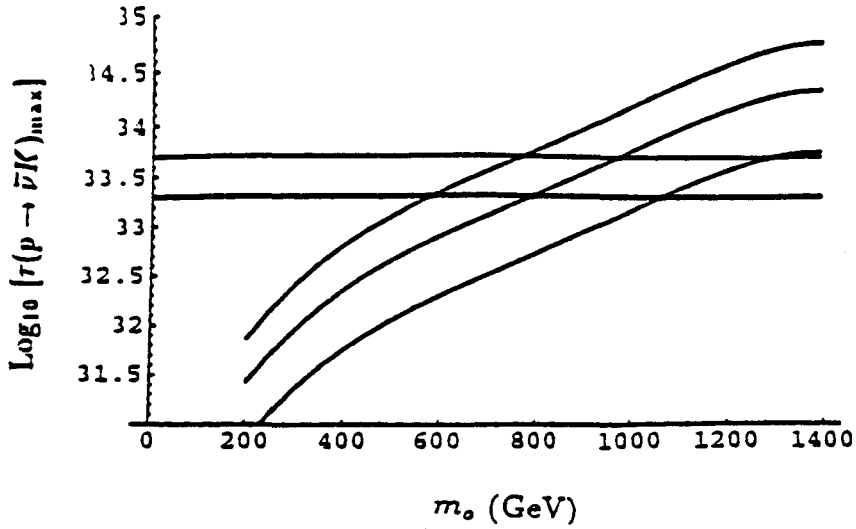

Fig. 5b. The same as Fig. 5a for $m_t = 150$ GeV, $\mu < 0$.

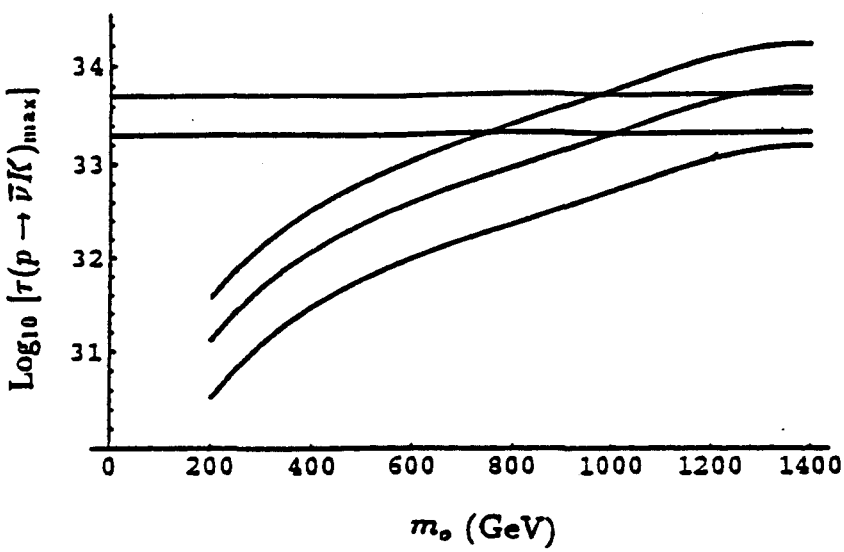

Fig. 5c. The same as Fig. 5a for $m_t = 170$ GeV, $\mu < 0$.

Fig. 6 plots the maximum value of $\tau(p \to \bar{\nu}K^+)$ for $m_o = 400$ GeV, 800 GeV and 1200 GeV as a function of m_t. This lifetime peaks at $m_t \simeq 145$ GeV. The reason for this arises from

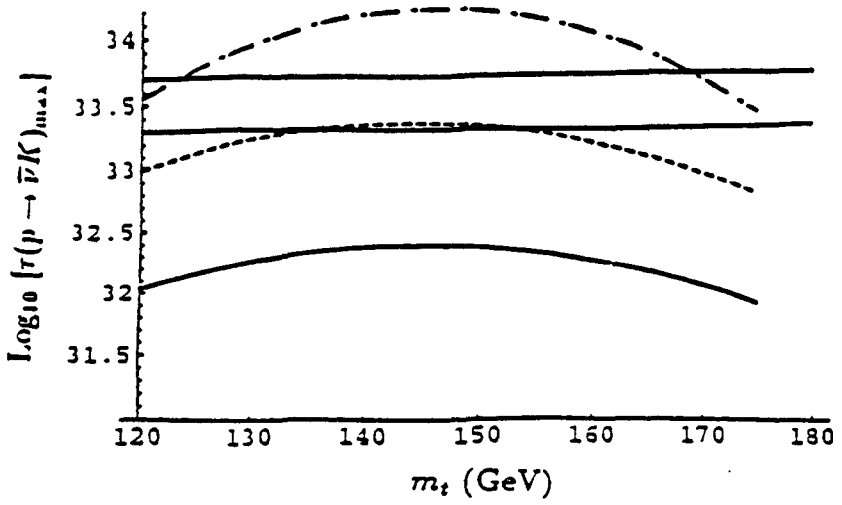

Fig. 6. Maximum value of $\tau(p \to \bar{\nu}K^+)$ vs m_t for $M_{H_3}/M_G = 6$ and $\mu < 0$. The solid line is for $m_o = 400$ GeV, the dashed line for $m_o = 800$ GeV and the dot-dashed line for $m_o = 1200$ GeV. The lower horizontal line is the upper bound that Super Kamiokande can detect, and the higher horizontal line is the upper bound for ICARUS.

two competing phenomena: As m_t increases, the off-diagonal terms of the t-squark mass matrix, $m_t(A_t m_o + \mu \text{ ctn } \beta)$, increases, reducing the $\tilde{t}_i$ mass and allowing more destructive interference between the third and second generation contributions to the loop of Fig. 3. However, for large m_t, the allowed parameter space shrinks (e.g. A_t approaches zero) reducing the off-diagonal terms again. Not e also that Fig. 6 shows that Super Kamiokande is accessible to the parameter space when $m_o \lesssim 800$ GeV and $M_{H_3}/M_G < 6$.

Fig. 7 shows the maximum value of $\tau(p \to \bar{\nu}K^+)$ as a function of m_o for $m_t = 150$

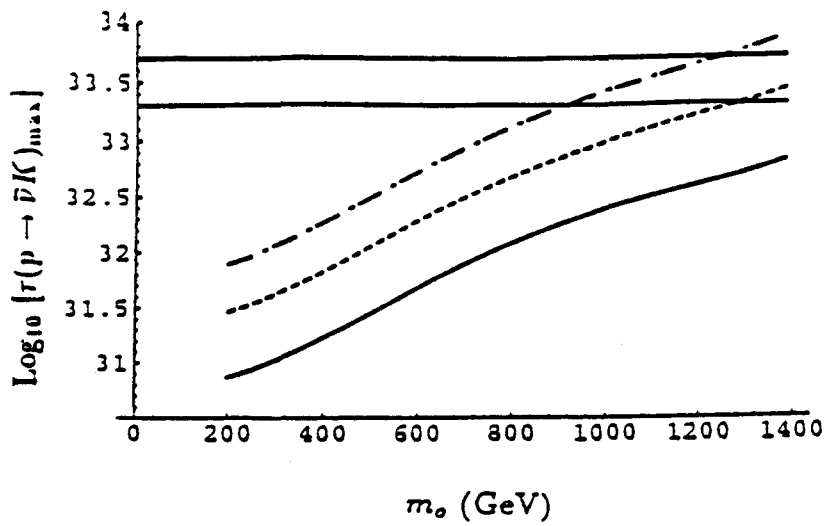

Fig. 7. Maximum value of $\tau(p \to \bar{\nu}K^+)$ vs m_o for $M_{H_3}/M_G = 3$ (solid line), $M_{H_3}/M_G = 6$ (dashed line), $M_{H_3}/M_G = 10$ (dot-dashed line) when $m_t = 150$ GeV, $\mu > 0$ and $m_{\tilde{W}_1} > 100$ GeV. The horizontal lines are as in Figs. 5,6.

GeV, $\mu > 0$ (which is near the maximum of the Fig. 6 curves) subject to the constraint that $m_{\tilde{W}_1}$ be greater than 100 GeV (and hence not be accessible to LEP 200). The lifetime increases with increasing Wino mass, and as can be seen in Fig. 7, it implies that even for $M_{H_3}/M_G < 10$, proton decay should be accessible to ICARUS for $m_o \lesssim 1250$ GeV (and accessible to Super Kamiokande for $m_o \lesssim 950$ GeV) if the $\tilde{W}_1$ is not seen at LEP 200. Thus one or the other of these signals for this class of models should be accessible experimentally.

8. SUMMARY AND CONCLUSIONS

Supergravity grand unification models depend on relatively few additional parameters, and consequently have a good amount of predictive power. For models possessing $SU(5)$-type proton decay, the new round of planned proton decay experiments combined with LEP 200 can give strong tests of these Gut models. One finds

(i) For Gut models with $M_{H_3}/M_G < 6$ the decay mode $p \to \ barv K^+$ should be seen at ICARUS for the entire range $m_o, m_{\tilde{g}} < 1$ TeV (and be seen at Super Kamiokande for $m_o < 800$ GeV).

(ii) For $M_{H_3}/M_G < 10$ and $m_o < 1250$ GeV, $m_{\tilde{g}} < 1$ TeV, either the mode $p \to \bar{\nu}K^+$ would be seen at ICARUS or the $\tilde{W}_1$ has mass $m_{\tilde{W}_1} < 100$ GeV and hence should be observable at LEP 200. (Similarly for Super Kamiokande for $m_o < 950$ GeV).

(iii) For $M_{H_3}/M_G < 10$ and $m_o, m_{\tilde{g}} < 1$ TeV one finds that if $\tau(p \to \bar{\nu}K^+) > 1.5 \times 10^{33}$ yr, then either $m_h < 95$ GeV or $m_{\tilde{W}_1} < 100$ GeV. Thus either the h or the $\tilde{W}_1$ (and possibly both) would be observable at LEP 200. (Note that the condition $\tau > 1.5 \times 10^{33}$ GeV could be tested at both Super Kamiokande and ICARUS.)

In addition to the above, over most of the allowed parameter space, we expect the gaugino scaling relations, Eqs. (6.2), and the degeneracy relations, Eqs. (6.3), to hold . While the SSC or LHC are probably needed to see the gluino and squarks, Eqs. (6.2) allow for the possibility of detection of light gauginos and the light h Higgs at the Tevatron and LEP 200.

Models of the type we have been considering possess R parity invariance, and as a consequence, the lightest supersymmetric particle (LSP) is totally stable. The proton decay constraint implies that the LSP be the $\tilde{Z}_1$. Cosmological constraints then require that the relic density of the LSP be sufficiently small that it not over close the universe. The dominant annihilation processes in the early universe occur mainly via the s-channel h and Z poles. Recent detailed calculations show[26)] that the relic density constraint can be viewed as a bound on the allowed gluino mass region. Allowed gluino

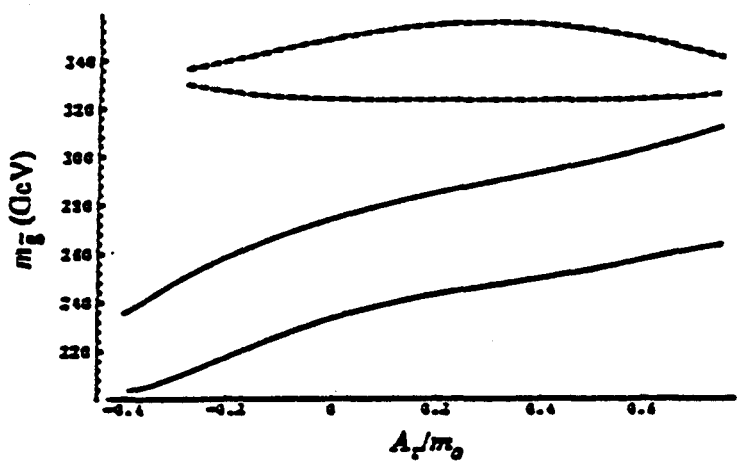

Fig. 8. Region in $m_{\tilde{g}} - A_t$ space allowed by the combined proton decay and cosmological constraints for $m_t = 125$ GeV, $m_o = 600$ GeV, $\tan\beta = 1.73, \mu > 0$ and $M_{H_3}/M_G = 6$. The lower band is due to the Higgs pole, and the upper band is due to the Z pole.

mass bands of ≈ 40 GeV arise from the h pole and ≈ 20 GeV from the Z pole. Sometimes these two regions merge giving a broad band of allowed values of $m_{\tilde{g}}$. Further, one finds $m_t \lesssim 165$ GeV, $m_h < 105$ GeV, $m_{\tilde{W}_1} < 100$ GeV and $m_{\tilde{Z}_1} < 50$ GeV for

$M_{H_3}/M_G < 6$. Thus while the cosmological constraint does indeed further limit the parameter space of supergravity grand unified models, there still remains a sizable allowed region. It should be stressed that should even one of the above considered signals be experimentally observed (e.g. a light Higgs, or proton decay) one will be able to use this new data to give even more precise predictions that could test the validity of these models.

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