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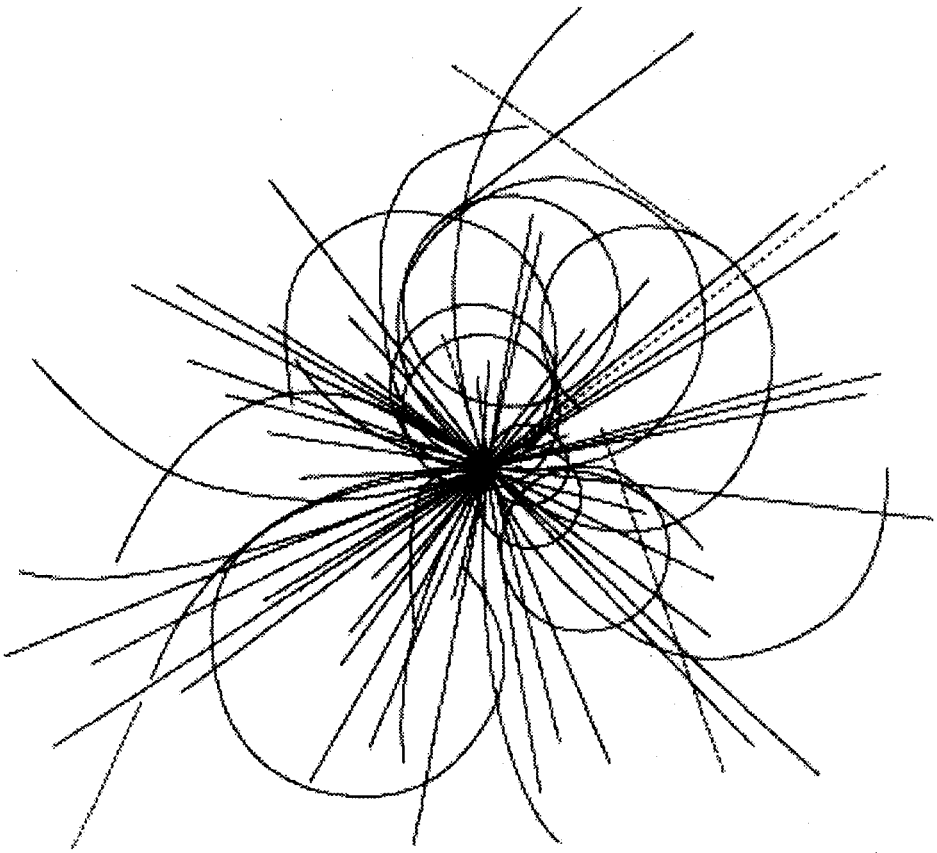

Superconducting Super Collider Laboratory

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Wakefield and Impedance Studies of a Liner Using MAFIA*

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WAKEFIELD AND IMPEDANCE STUDIES OF A LINER USING MAFIA

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ABSTRACT

The liner is a perforated beam tube that is coaxial with an outer bore tube. The 3D code MAFIA (version 3.1) is used to study the wakefields, impedances, and resonances of this structure. The short-range wakes and low-frequency (below the cutoff) impedances are in agreement with the theoretical model. The long-range wakes and high-frequency resonances are associated with the distribution of the holes (or slots). The dependence of the impedance on the size, shape, and pattern of the holes (or slots) is studied. The impact of the liner impedance on the Superconducting Super Collider impedance budget is discussed.

I. INTRODUCTION

One novel feature of the Collider at the Superconducting Super Collider (SSC) is that synchrotron radiation from the 20-TeV proton beam becomes an important issue. This radiation creates a significant heat load to the cryogenic system, and it causes a large amount of gas load from the photon-induced desorption process. The latter may result in a poor vacuum in the beam tube and an eventual short luminosity lifetime. One possible solution to the problem is to install a perforated liner inside the bore tube.[1] The principal function of the liner is to decouple the synchrotron radiation from the bore tube. The photo-desorbed gas from the inner surface of the liner would be pumped out by the bore tube (which is at 4 K) and absorbed by the cryosorber that is located in the co-axial region between the liner and the bore tube.

The introduction of the liner brings up a list of issues that need to be studied. Among them is the additional rf impedance. There have been some theoretical studies on the hole impedance in the low-frequency region.[2] This paper reports the simulation studies using the 3D code MAFIA (version 3.1).[3] It is found that, at the frequencies below the cutoff, MAFIA results are in agreement with the theoretical model. At high frequencies, impedance resonance peaks and long-term wakes are observed in the simulations. The size, shape, and distribution of the holes (or slots) on the liner surface have significant impact on the liner impedance.

The MAFIA code runs on an IBM RS 6000/560 workstation. Statistics of the code performance are also included.

II. LOW-FREQUENCY REGION

A. Analytical Model

For some structures, such as a pipe attached to a small pillbox or a pipe with small holes on its surface (e.g., the perforated liner), the longitudinal and transverse impedances can be approximated by a pure inductance L at low frequencies (below the cutoff):

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$$Z_{||}(\omega) = i\,\omega L \tag{1}$$

$$Z_{\perp}(\omega) = i\,\frac{2c}{b^2}\,L, \tag{2}$$

in which b is the radius of the pipe, c the velocity of light. The corresponding wakefields $W_{||}$ and $W_{\perp}$ are, respectively, the derivative of the δ-function and the δ-function:[4]

$$W_{||}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega\tau}\, Z_{||}(\omega) = -L\,\delta'(\tau) \tag{3}$$

$$W_{\perp}(\tau) = \frac{-i}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega\tau}\, Z_{\perp}(\omega) = \frac{2c}{b^2}\,L\,\delta(\tau). \tag{4}$$

Letting $\tau = z/c$, one obtains

$$W_{||}(z) = -c^2\,L\,\delta'(z) \tag{5}$$

$$W_{\perp}(z) = \frac{2c^2}{b^2}\,L\,\delta(z). \tag{6}$$

For a Gaussian bunch with rms length σ, the line charge density is

$$\lambda(z) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{z^2}{2\sigma^2}}. \tag{7}$$

The longitudinal and transverse wake potentials generated by this Gaussian bunch are, respectively,

$$w_{||}(z) = \int_{-\infty}^{\infty} dz'\,\lambda(z-z')\,W_{||}(z') = \frac{c^2\,L}{\sqrt{2\pi}\sigma^3}\,z\,e^{-\frac{z^2}{2\sigma^2}} \tag{8}$$

$$w_{\perp}(z) = \int_{-\infty}^{\infty} dz'\,\lambda(z-z')W_{\perp}(z') = \frac{2c^2\,L}{\sqrt{2\pi}\sigma\,b^2}\,e^{-\frac{z^2}{2\sigma^2}}. \tag{9}$$

They are shown in Figs. 1(a) and 1(b).

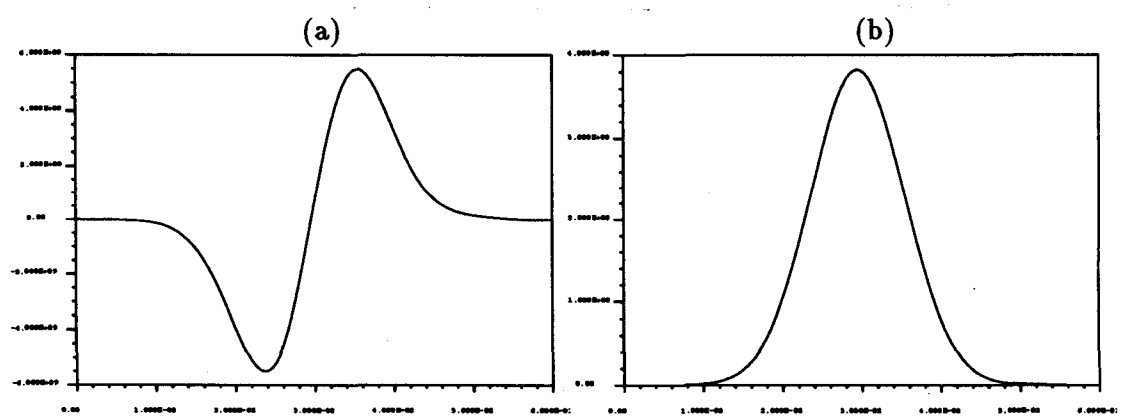

Fig. 1. The wake potentials of a purely inductive impedance for a Gaussian bunch: (a) longitudinal, (b) transverse.

The magnitudes and locations of the peaks of the wake potentials are as follows:

$$w_{\parallel}^{max(min)} = \pm \frac{c^2 L}{\sqrt{2\pi}\,\sigma^2} e^{-1/2}, \quad \text{at } z = \pm\sigma \tag{10}$$

$$w_{\perp}^{max} = \frac{2c^2 L}{\sqrt{2\pi}\,\sigma\, b^2}, \quad \text{at } z = 0. \tag{11}$$

Therefore, if the inductance L is known, then Eqs. (8)–(11) give the wake potentials for specified beam pipe radius b and rms bunch length σ.

B. MAFIA Results

The inductance of a small hole with diameter d has been worked out:[2]

$$L = \frac{Z_0}{48\pi^2 c} \frac{d^3}{b^2}, \tag{12}$$

where $Z_0 = 377\ \Omega$. Therefore, the peaks of the wake potentials of the liner are given by (all dimensions in meters):

$$w_{\parallel}^{max} = \frac{Z_0\, c}{48\pi^{5/2}\sqrt{2e}} \frac{d^3}{\sigma^2 b^2} = 5.77 \times 10^{-2} \times \frac{d^3}{\sigma^2 b^2} \quad V/nC \quad \text{(per hole)} \tag{13}$$

$$w_{\perp}^{max} = \frac{Z_0\, c}{24\pi^{5/2}\sqrt{2}} \frac{d^3}{\sigma b^4} = 1.90 \times 10^{-4} \times \frac{d^3}{\sigma\, b^4} \quad V/nC/mm \quad \text{(per hole)}. \tag{14}$$

Equations (13) and (14) can be directly compared with the MAFIA results as shown in Figs. 2(a) and 2(b).

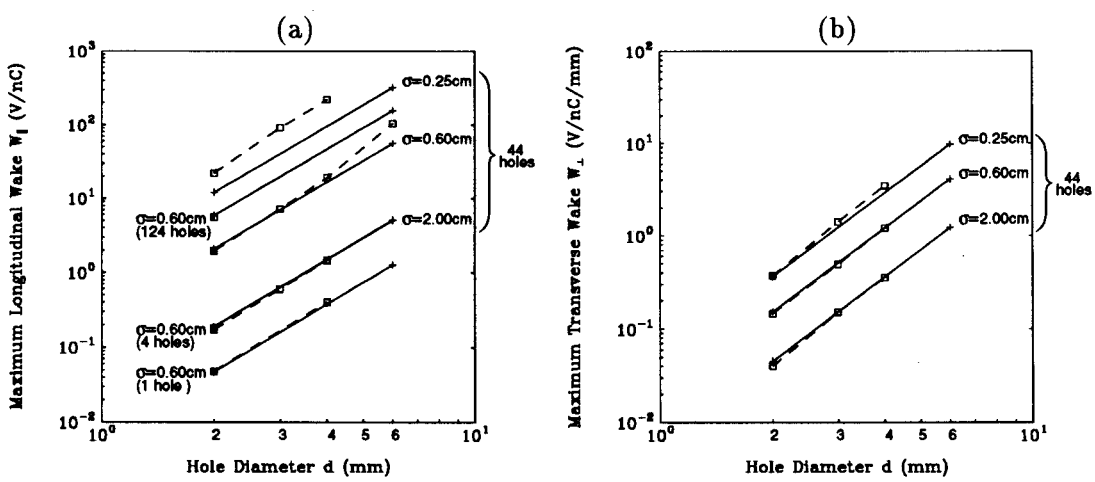

Fig. 2. The peak values of the wake potentials of a liner. The solid lines are computed using Eqs. (13)–(14). The squares and dashed lines are the MAFIA results. (a) $w_{\parallel}^{max}$, (b) $w_{\perp}^{max}$.

When the hole size is small and bunch length is large, the theory and simulations agree with each other. This implies that the low-frequency approximation is valid. However, when the hole becomes larger ($d \geq 4$ mm) or the bunch becomes shorter

($\sigma = 0.25$ and 0.6 cm), the simulation results appear to be larger than what the theory would predict. This probably indicates the breakdown of the low-frequency assumption.

The hole shape in the simulations is a square rather than a circle. It would thus give an inductance larger than that of a circular one as predicted by Eq. (12). On the other hand, Eq. (12) is derived from a zero-thickness liner. The finite thickness (1 mm) used in the simulations would lead to a smaller inductance.[5] It is interesting to see from Figs. 2(a) and 2(b) that these two effects seem to cancel each other, resulting in a good agreement between Eqs. (13)–(14) and the MAFIA results.

III. HIGH-FREQUENCY REGION

One interesting observation in the simulations is the long-term wakes for large holes or short bunches. These long-term wakes are potentially dangerous because they may cause coupling among successive bunches, which could be a source of multiple bunch instability and/or emittance growth. In order to understand these wakes, a fast Fourier transform (FFT) is carried out to convert the wakes to the impedance in the frequency domain.

A. Benchmark Test

Because of the lack of an appropriate analytic model of the perforated liner in the high-frequency region, the MAFIA results are at first compared with the known impedance spectra of a small pillbox. For this simple structure, the impedances obtained from several different methods (field matching,[6] TBCI,[7] and transmission line[7]) agree with each other and, therefore, can be used for a benchmark test.

Figs. 3(a) and 3(b) show the longitudinal impedance of a small pillbox obtained from the FFT of the wakes computed by MAFIA.

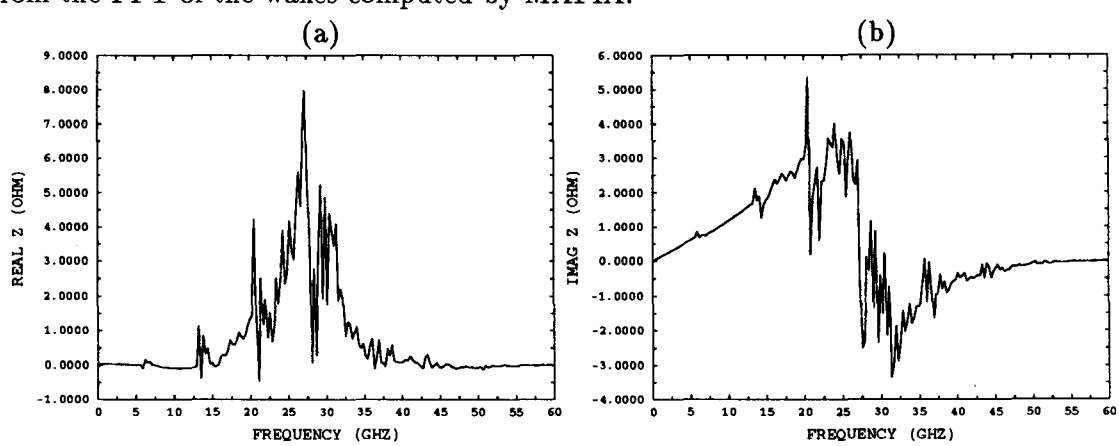

Fig. 3. The longitudinal impedance of a small pillbox obtained from MAFIA and FFT: (a) the real part, (b) the imaginary part.

Below the cutoff (which is about 5.75 GHz for a beam pipe radius of 2 cm used in the simulation), the impedance is purely inductive:

$$Z_{\|}(\omega) = i\,\omega\,\frac{\mu_0}{2\pi}\,\frac{4\,g\,\epsilon}{b}\ , \tag{15}$$

where $2g$ and 2ϵ are the width and depth of the pillbox, respectively, and $\mu_0 = 4\pi \times 10^{-7}$ H/m. The difference between Eq. (15) and the MAFIA results in Fig. 3(b) is only

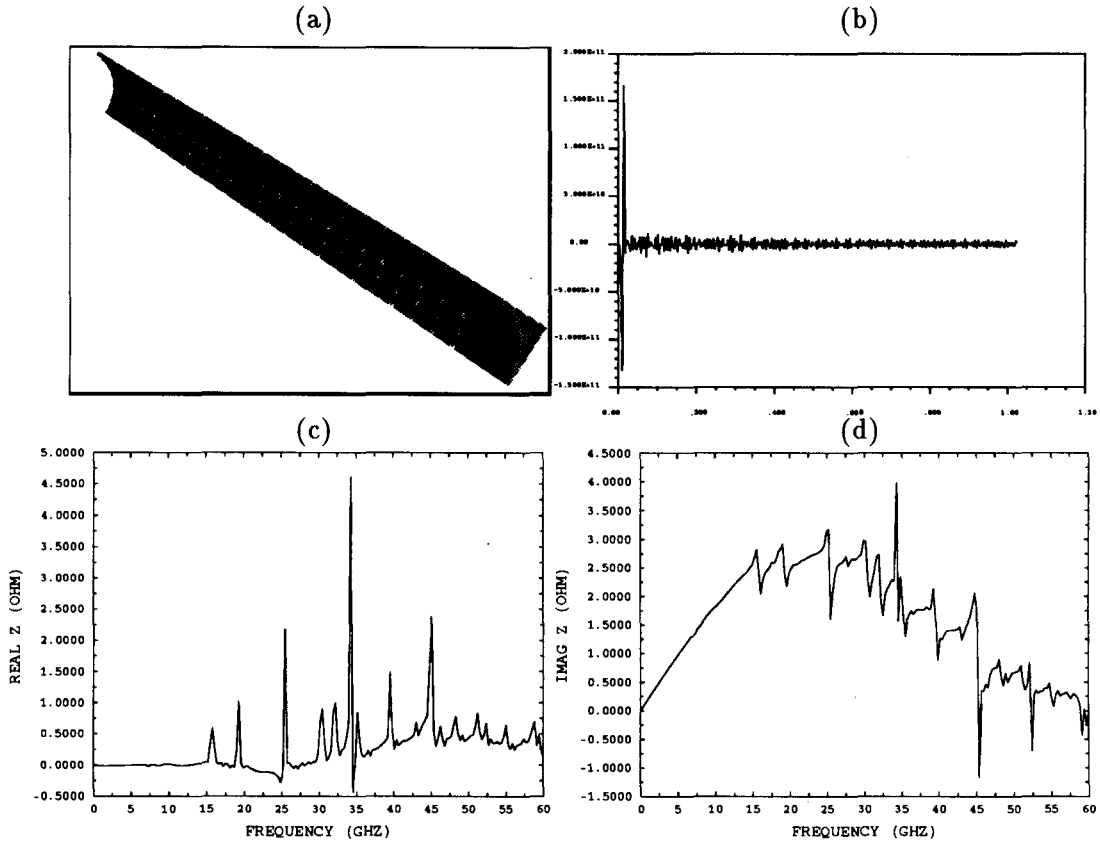

Fig. 4. (a) The liner with 420 holes of the size 2 mm × 2 mm, randomly distributed. (b) The longitudinal wake of this structure. (c) The real longitudinal impedance. (d) The imaginary longitudinal impedance.

about 3%. Above the cutoff, the impedance spectra also agree reasonably well with those in Refs. 6 and 7.

B. Periodic Distributions of Holes on a Liner

The simulations use 20 holes along the azimuthal direction of the liner, which is called a column, and 21 columns along the axial direction. The total number of holes is 420. Each hole is a 2 mm × 2 mm square. The columns are uniformly distributed with 1 cm spacing between each other. Fig. 4(a) shows the structure, and 4(b) the longitudinal wake. The long-term wake indicates the existence of resonance impedance, which is clearly seen in Figs. 4(c) and 4(d). The liner radius is 1.65 cm, which gives the longitudinal cutoff at approximately 7 GHz. Below the cutoff, the impedance is purely inductive. The analytical value is obtained from Eqs. (1) and (12):

$$Z_{\|}(\omega) = i\,\omega\,\frac{Z_0}{48\pi^2\,c}\,\frac{d^3}{b^2}\ . \tag{16}$$

The difference between the value given in Fig. 4(d) and that given by Eq. (16) is again only a few percent, as in the case of the pillbox. The resonance peaks at high frequency

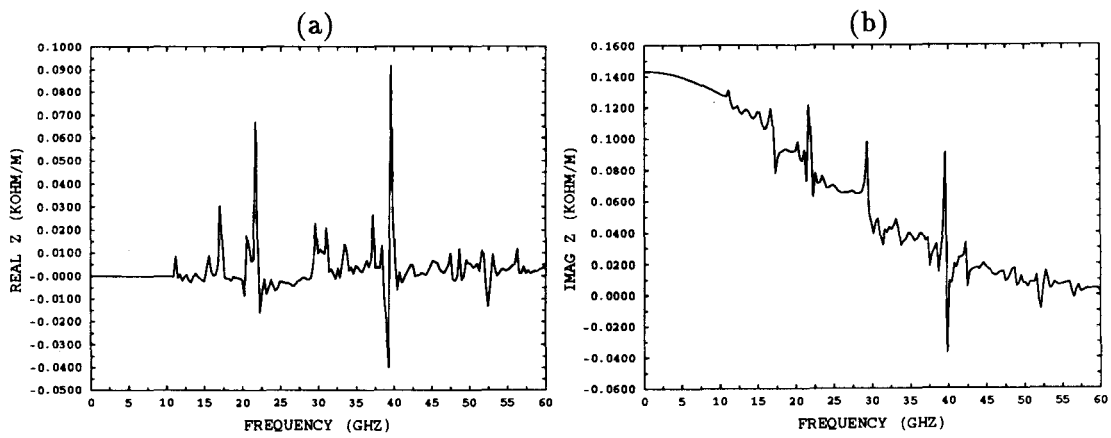

Fig. 5. The transverse impedance of the structure in Fig. 4(a): (a) the real part, (b) the imaginary part.

are associated with the periodicity of the holes. The first peak occurs near 15 GHz, which corresponds to a half wavelength of 1 cm, the periodic spacing between two columns. The transverse impedance is shown in Figs. 5(a) and 5(b).

The transverse cutoff is approximately 11 GHz. Below it, the impedance is also purely inductive. The analytical value from Eqs. (2) and (12) is

$$Z_\perp(\omega) = i\,\frac{Z_0}{24\pi^2}\,\frac{d^3}{b^4}\ . \tag{17}$$

This value agrees within a few percent with that given in Fig. 5(b) at zero frequency. But the falloff of $Im\ Z_\perp$ from dc to the cutoff is not predicted by the theoretical model. Similar to the longitudinal case, the long-term wakes and the resonance impedance at high frequencies are observed.

C. Random distribution of Holes on a Liner

One effective way to reduce the long-term wakes and resonance impedance is to destroy the periodicity of the hole distribution. For this purpose, the spacing between two neighboring holes in the axial direction is randomized. Figs. 6(a)–6(d) show the structure and the resulting longitudinal wakes and impedance. Compared with Figs. 4(a)–4(d), the short-term wakes and low-frequency impedance remain about the same (as they should, due to the additivity), whereas the long-term wakes and resonance impedance at high frequencies are greatly decreased. However, by using the same technique, the reduction in the transverse direction is less dramatic. This needs to be understood.

D. Slots on a Liner

When the holes are replaced by the slots that have the same area and have the major axis parallel to the pipe axis, the low-frequency impedances are reduced, whereas the long-term wakes (i.e., the high-frequency resonances) are enhanced because it becomes easier to resonate. This is shown in Figs. 7(a) and 7(b). Therefore, the tradeoff should be studied carefully. The short slots with rounded edges seem to be a good compromise.

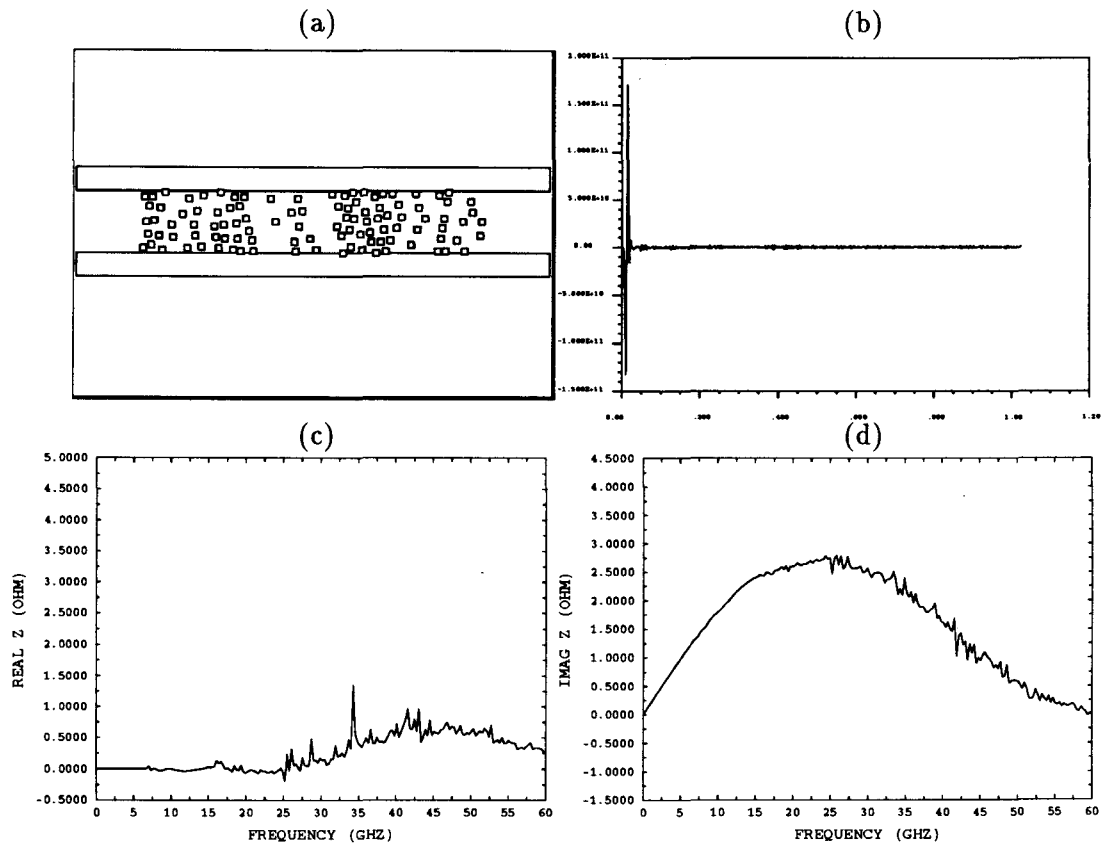

Fig. 6. (a) The liner with 420 holes of the size 2 mm × 2 mm, randomly distributed. (b) The longitudinal wake of this structure. (c) The real longitudinal impedance. (d) The imaginary longitudinal impedance.

IV. DISCUSSION

Based on the good agreement of the theory with the simulations as well as with some measurements,[8] it is believed that the low-frequency part of the impedance of a perforated liner can be estimated accurately. These impedances are plotted in Figs. 8(a) and 8(b) for three possible liner IDs. In these calculations, the area coverage of the holes is kept the same for different hole sizes, namely, 1000 holes/m for $d = 2$ mm. This is about 3%, 4%, and 5% coverage for ID = 33 mm, 24.3 mm, and 20.3 mm, respectively.

As a comparison, the impedance budget of the Collider is also plotted. It is seen that when the liner ID is 24.3 mm—as designed for the Accelerator System String Test (ASST II)—and the hole diameter 2 mm, the longitudinal impedance of the liner would be about 34% of the present budget, and the transverse impedance about 110%. This represents a significant increment of the budget. Because of a relatively large safety margin reserved in the present impedance budget (which is 6 times greater than the instability threshold impedance), this increment should not jeopardize the beam dynamics at the baseline design current of 72 mA. Rather, its main impact would be to limit the potential of beam current upgrade.

The size, shape, and distribution of the holes or slots on the liner surface have significant impact on the impedance. The rounded, short slots with random pattern

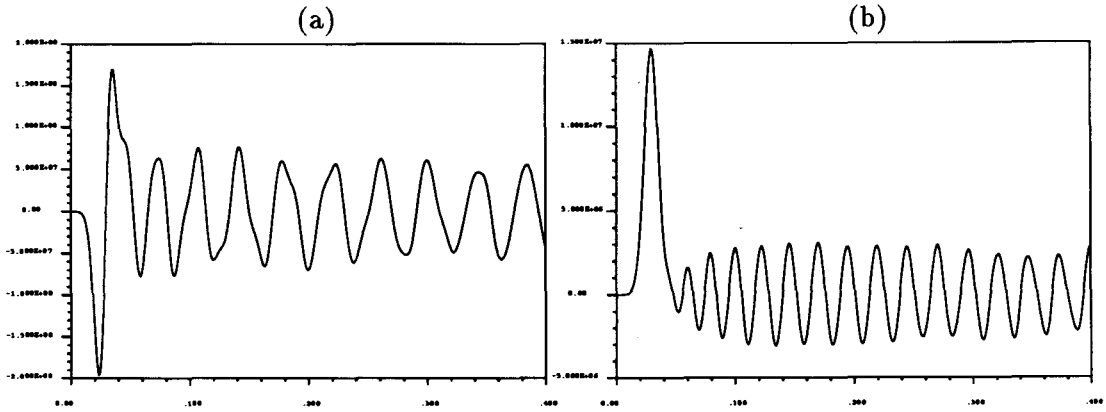

Fig. 7. The wake potentials of a liner with four long slots of the size 2 mm × 22 mm: (a) longitudinal, (b) transverse.

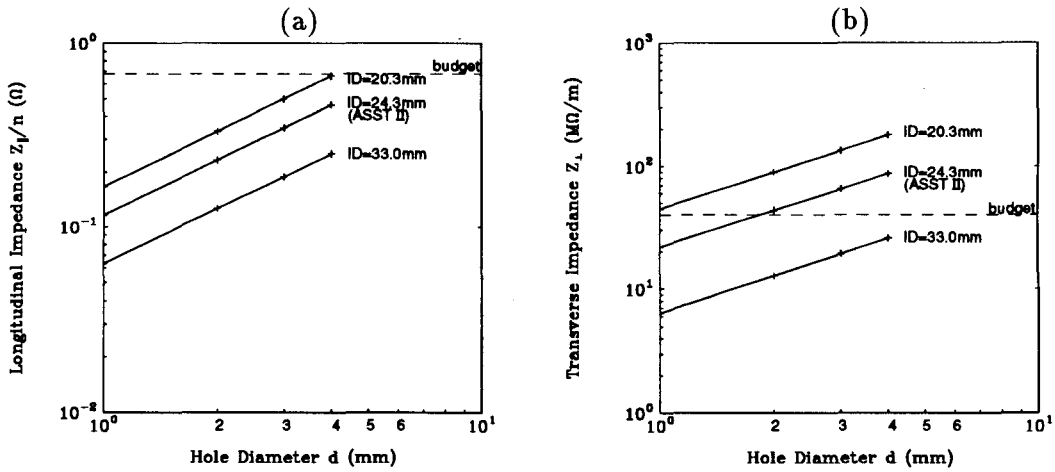

Fig. 8. The total impedance of the liner in the SSC Collider for three possible liner IDs: (a) longitudinal, (b) transverse. The dashed lines are the impedance budget.

may present the best choice to minimize the impedance while meeting the vacuum pumping requirements.

The code MAFIA (version 3.1) runs on an IBM 560 workstation with 128 MB RAM. It is a virtual memory machine. The maximum number of mesh points in the simulations can reach approximately 1.8 million. When this number is exceeded, the CPU usage would be greatly degraded. A typical job calculating approximately 10^6 mesh points and 10^3 wake points takes approximately 4 h CPU time. The same job would take approximately 7 h CPU time on the NERSC Cray.

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