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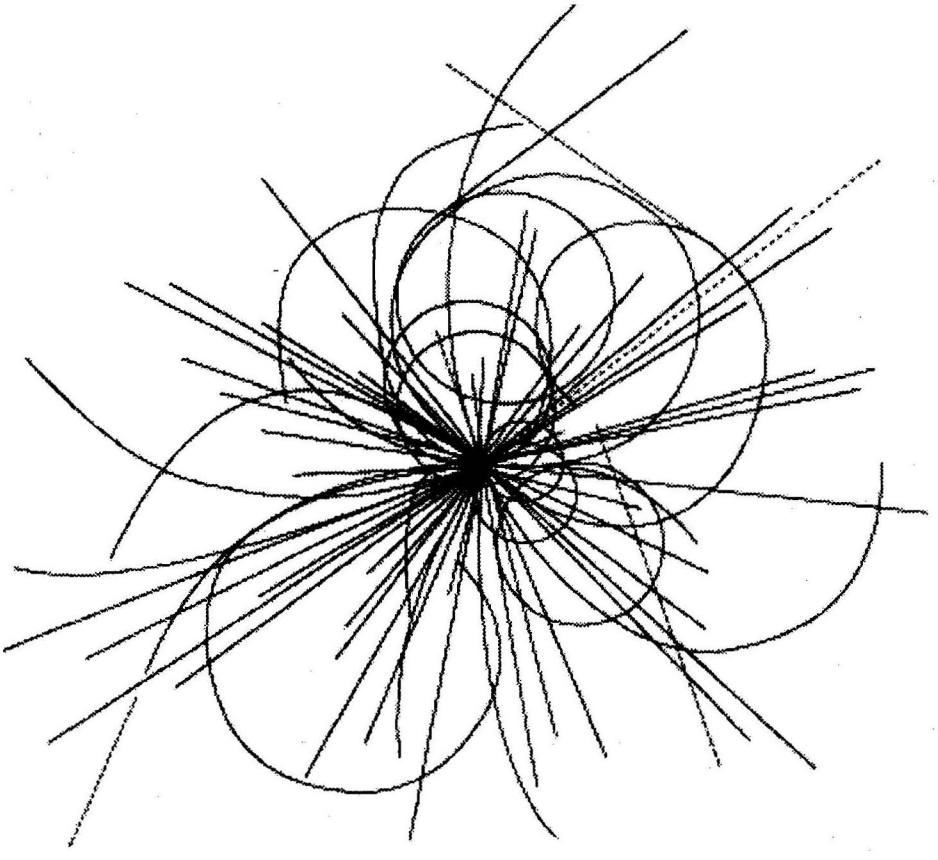

Superconducting Super Collider Laboratory

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Three Dimensional δf Simulations of Beams in the SSC*

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THREE DIMENSIONAL δf SIMULATIONS OF BEAMS IN THE SSC*

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ABSTRACT

A three dimensional δf strong-strong algorithm has been developed to apply to the study of such effects as space charge and beam-beam interaction phenomena in the Superconducting Super Collider (SSC). The algorithm is obtained from the merging of the particle tracking code Simpsons used for 3-dimensional space charge effects and a δf code. The δf method is used to follow the evolution of the non-gaussian part of the beam distribution. The advantages of this method are twofold. First, the Simpsons code utilizes a realistic accelerator model including synchrotron oscillations and energy ramping in 6-dimensional phase space with electromagnetic fields of the beams calculated using a realistic 3-dimensional field solver. Second, the beams are evolving in the fully self-consistent strong-strong sense where finite particle fluctuation noise is greatly reduced as opposed to the weak-strong models where one beam is fixed.

INTRODUCTION

In this paper we present a method for realistically simulating 3-dimensional beams in the Superconducting Super Collider (SSC). This method merges the ideas of tracking particles (Simpsons code) with the δf algorithm. There are two advantages of this approach. First, the Simpsons code utilizes a realistic accelerator model including synchrotron oscillations and energy ramping in 6-dimensional phase space with electromagnetic fields of the beams calculated using a realistic 3-dimensional field solver. Second, the δf method allows the beams to evolve in the fully self-consistent strong-strong sense with a large reduction of finite particle fluctuation noise.

Macroparticle or Particle-in-Cell (PIC) codes typically use macroparticles to represent the entire distribution of particles. In the beam-beam interaction for the SSC, the beams consist of 10^{10} particles each. Simulating this many par-

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ticles with the PIC technique is computationally prohibitive. With the conventional PIC code 10^{10} particles are represented by only 10^3–10^4 simulation particles allowing simulation of the beam-beam interaction in a reasonable computation time. However, the fluctuation level of various quantities such as the beam density ρ in the code is much higher than that of the real beam. The fluctuation level $\delta\rho$ goes as approximately:

$$\frac{\delta\rho}{\rho} = N^{-1/2}, \tag{1}$$

where N is the number of particles. Therefore, the fluctuation level of the PIC code is about 10^3 times higher than that of the real beam. To facilitate the study of subtle effects, a one-dimensional δf code has been developed.[1] The one-dimensional δf code was used to study single particle diffusion due to the beam-beam interaction in a fully self-consistent way.[1] The particle fluctuation noise was several orders of magnitude below that of standard PIC code.[1]

In the following sections we present details on the δf algorithm in one dimension, the Simpsons code, and the full implementation in three dimensions.

δf METHOD ONE DIMENSION

The δf method follows only the fluctuating part of the distribution instead of the entire distribution. In PIC codes a finite number of particles is used to represent the Vlasov equation or Klimontovich equation. In the particular case of the beam-beam interaction:[2]

$$\frac{\partial f}{\partial s} + x'\frac{\partial f}{\partial x} - \Big(K(s)x - F_{bb}(x,s)\Big)\frac{\partial f}{\partial x'} = 0, \tag{2}$$

where $K(s)x$ is the usual magnetic guiding force of a single element and $F_{bb}(x,s)$ is the beam-beam force. The distribution function f is represented by a finite number of particles by:

$$f(x,x',s) = \sum_{i=1}^{N} \delta\Big(x - x_i(s)\Big)\delta\Big(x' - x_i'(s)\Big), \tag{3}$$

where the sum is over the number of simulation particles used. In the δf method only the perturbative part of the distribution is followed. The total distribution function $f(x,x',s)$ is decomposed into

$$f(x,x',s) = f_0(x,x',s) + \delta f(x,x',s), \tag{4}$$

where $f_0(x,x',s)$ is the steady or slowly varying part of the distribution and $\delta f(x,x',s)$ is the perturbative part. The key to this method is finding a distribution $f_0(x,x',s)$ which is close to the total distribution $f(x,x',s)$. The perturbative part $\delta f(x,x',s)$ then causes only small changes to the distribution, and thus represents only the fluctuation levels. In the particular case of the

beam-beam interaction, an analytic solution to an equation close to the original Vlasov equation can be found. For a linearized beam-beam force the Vlasov equation can be written in the form:

$$\frac{\partial f_0}{\partial s} + x'\frac{\partial f_0}{\partial x} - \Big(K(s) - F_{bb0}(s)\Big)x\frac{\partial f_0}{\partial x'} = 0, \tag{5}$$

where $F_{bb0}(s)$ is the linear portion of the beam-beam force $F_{bb}(x,s)$. The solution is a Gaussian. Subtracting the linearized equation from the total Vlasov equation

$$\frac{\partial \delta f}{\partial s} + x'\frac{\partial \delta f}{\partial x} - \Big(K(s)x - F_{bb0}(x,s)\Big)\frac{\partial \delta f}{\partial x'} = -\Big(F_{bb}(x,s) - F_{bb0}(s)x\Big)\frac{\partial f_0}{\partial x'}, \tag{6}$$

$F_{bb0}(x,s)$ is the kick from a Gaussian beam.

The perturbative part of the distribution can be represented by a finite number of particles (characteristics):

$$\delta f(x,x',s) = \sum_{i=1}^{N} w_i \delta\Big(x - x_i(s)\Big)\delta\Big(x' - x_i'(s)\Big). \tag{7}$$

Substituting this into the equation for δf advance, we obtain:

$$\frac{dw_i}{ds} = -\frac{1}{n}\left[\Big(F_{bb}(x,s) - F_{bb0}(s)x\Big)\frac{\partial f_0}{\partial x'}\right]_i \tag{8}$$

where

$$n = \frac{N}{(\Delta x \Delta x')}. \tag{9}$$

This density n is calculated on the assumption that the particles are distributed uniformly in phase space. In the δf algorithm, x_i, x_i', and w_i are advanced. The advance of the extra term w_i increases only sightly the number of operations over the PIC method.

THE CODE SIMPSONS

The 6-D phase space coordinates; x, x', y, y', z, z' are advanced by the thin lens code Simpsons.[3] The code Simpsons uses *time* as the independent variable so that time-dependent parameters such as bending magnet field, rf voltage, power supply ripple and other noise are incorporated in the same way as in real machines. The substitution of the thick lens lattice to the thin lens one, the tuning of the betatron phase and the chromaticity, and the introduction of multipole errors and mis-alignment are done by the TEAPOT program.[4] In addition to the lattice file created by TEAPOT, the bending magnet field $B\rho(t)$ and the rf voltage $V(t)$, at least, are read as an external table to define machine cycle parameters when acceleration as well as synchrotron oscillations are simulated.

Within one time interval, a particle will either remain in drift space or receive one or more momentum kicks due to magnets and/or rf cavities. If there are no lattice elements in the time interval, the positions are updated and the momentum remains constant. If there are magnets in between, first the positions are updated to the magnet location, then the transverse momenta is changed by the thin lens kick. If there is an rf cavity in between, longitudinal and total momentum are changed. We keep track the rf phase by integrating the rf frequency with respect to time so that a particle gets proper energy gain according to the exact time when it passes through a cavity.

THREE DIMENSIONAL FIELD SOLVER[5]

The boundary condition is fixed for circular beam pipe with the perfect conductivity $\sigma = \infty$ at radius $r = b$. Under the Lorentz gauge $\frac{\partial \phi}{\partial t} + c\vec{\nabla} \cdot \vec{A} = 0$, the Maxwell equations are

$$\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\phi = -4\pi n(r,\theta,z,t), \tag{10}$$

$$\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\vec{A} = -\frac{4\pi}{c}\vec{J}(r,\theta,z,t), \tag{11}$$

where $n(\vec{x},t)$ and $\vec{J}(\vec{x},t)$ are the charge density and the current, respectively, and the scalar potential ϕ and the vector potential $\vec{A}$ are used. We set the scaler potential $\phi \equiv 0$ at $r = b$ and invoke the ordering

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2} \sim \frac{\partial^2}{\partial z^2} \ll \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}. \tag{12}$$

This can be justified by the typical dimension of the proton beam.

We further simplify the current density,

$$\vec{J} = J_z\vec{e}_z = \bar{v}_z n\vec{e}_z, \tag{13}$$

where $\bar{v}_z$ is the average velocity of the beam. From the Lorentz force equation $\vec{F} = q(\vec{E} + \frac{\vec{v}}{c} \times \vec{B})$, the electromagnetic force is $\vec{F} = -\frac{q}{\gamma^2}\vec{\nabla}\phi$ and we only need to solve the scalar potential Eq. (10). The charge density and scaler potential are Fourier transformed in θ

$$n(r,\theta,z,t) = \sum_m n_m(r,z,t)\exp(im\theta), \tag{14}$$

$$\phi(r,\theta,z,t) = \sum_m \phi_m(r,z,t)\exp(im\theta), \tag{15}$$

with the inverse transforms

$$n_m(r,z,t) = \frac{1}{2\pi}\int_0^{2\pi} n(r,\theta,z,t)\exp(-im\theta)d\theta, \tag{16}$$

$$\phi_m(r,z,t) = \frac{1}{2\pi}\int_0^{2\pi} \phi(r,\theta,z,t)\exp(-im\theta)d\theta. \tag{17}$$

For each m, the equation assumes the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi_m}{\partial r}\right) - \frac{m^2}{r^2}\phi_m = -4\pi n_m. \tag{18}$$

The solution for the equation for $m \geq 0$ is

$$\phi_m = W_m(r) - \left(\frac{r}{b}\right)^m W_m(b), \tag{19}$$

where $W_m(r)$ is

$$W_{m=0}(r) = -4\pi \int_0^r n_0(r,z,t) r' \ln \frac{r}{r'} dr', \tag{20}$$

$$W_{m\geq 1} = -\frac{2\pi r^m}{m}\int_0^r n_m(r,z,t) r'^{(1-m)} dr' - \frac{2\pi r^{-m}}{m}\int_0^r n_m(r,z,t) r'^{(1+m)} dr'. \tag{21}$$

The grids are set in the cylindrical coordinate system such that the whole charge distribution is enclosed.

δf ALGORITHM THREE DIMENSIONS

The δf algorithm can be applied to higher dimensions. The application to a storage ring with many elements is similar to the one-dimensional beam-beam interaction method. The Vlasov equation is of the form:

$$\frac{\partial f}{\partial s} + \vec{x}' \cdot \frac{\partial f}{\partial \vec{X}} + \vec{A} \cdot \frac{\partial f}{\partial \vec{X}'} = 0 \tag{22}$$

where

$$\vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad \vec{x}' = \begin{pmatrix} x' \\ y' \\ -\eta z' \end{pmatrix} \tag{23}$$

$$\frac{\partial}{\partial \vec{X}} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \qquad \frac{\partial}{\partial \vec{X}'} = \begin{pmatrix} \frac{\partial}{\partial x'} \\ \frac{\partial}{\partial y'} \\ \frac{\partial}{\partial z'} \end{pmatrix} \tag{24}$$

where $z \equiv \frac{\Delta\phi_{rf}}{2\pi}\lambda_{rf}$ and $z' \equiv \frac{\Delta(pc)}{pc}$ and

$$\vec{A} = \overleftrightarrow{Q}(s) \cdot \vec{x} + \vec{S}(\vec{x}, s) + \overrightarrow{RF}(s) + \vec{F}_{sc}(\vec{x}, s) + \vec{F}_{bb}(\vec{x}, s). \tag{25}$$

On the right-hand side of Eq. (25), $\overleftrightarrow{Q}(s)$ and $\vec{S}(\vec{x}, s)$ represent the quadrupole and other higher-order magnets, respectively, in the storage ring. For example:

$$\overleftrightarrow{Q}(s) = q(s)\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{26}$$

where $q(s)$ represents the strengths of the quadrupoles at position s. The quantity $\overrightarrow{RF}(s)$ in Eq. (25) represents the contribution from the rf cavity:

$$\overrightarrow{RF}(s) = \begin{pmatrix} 0 \\ 0 \\ \frac{f_{rf} \cdot eV}{h\beta^2 c(pc)}[\sin(\phi_s + \Delta\phi_{rf}) - \sin(\phi_s)] \end{pmatrix}. \tag{27}$$

The elements $\vec{F}_{sc}(\vec{x}, s)$ and $\vec{F}_{bb}(\vec{x}, s)$ in Eq. (25) represent the contributions from space charge and beam-beam forces, respectively.

The steady state equation again takes only the linear terms:

$$\frac{\partial f_0}{\partial s} + \vec{x}' \cdot \frac{\partial f_0}{\partial \vec{X}} + \left(\overleftrightarrow{A}_0(s) \cdot \vec{x}\right) \cdot \frac{\partial f_0}{\partial \vec{X}'} = 0, \tag{28}$$

where $\overleftrightarrow{A}_0(s)$ is the linear portion of $\overleftrightarrow{A}$:

$$\overleftrightarrow{A}_0(s) = \overleftrightarrow{Q}(s) + \overleftrightarrow{RF}_0(s) \tag{29}$$

where

$$\overleftrightarrow{RF}_0(s) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{f_{rf} \cdot eV \cos(\phi_s)}{h\beta^2 c(pc)} \frac{2\pi}{\lambda_{rf}} \end{pmatrix}. \tag{30}$$

So $\overleftrightarrow{A}_0(s)$ represents the contributions from the quadrupole magnet elements in the machine and the linear part of the rf force. The solution of the steady state equation is a Gaussian of the form:

$$f_0(\vec{x}, \vec{x}') = N f_{x0}(x, x') f_{y0}(y, y') f_{z0}(z, z') \tag{31}$$

$$f_{x0}(x, x') = \frac{(\gamma\beta)}{2\epsilon_x} \exp\left(-\frac{\pi A_x^2(\gamma\beta)}{2\epsilon_x}\right) \tag{32}$$

$$f_{y0}(y, y') = \frac{(\gamma\beta)}{2\epsilon_y} \exp\left(-\frac{\pi A_y^2(\gamma\beta)}{2\epsilon_y}\right) \tag{33}$$

where $\epsilon_{x,y}$ is the transverse normalized emittance, β and α are the Lorentz factor,

$$A_x^2 = \gamma_x(s)x^2 + 2\alpha_x(s)xx' + \beta_x(s)x'^2, \tag{34}$$

$$A_y^2 = \gamma_y(s)x^2 + 2\alpha_y(s)xx' + \beta_y(s)x'^2, \tag{35}$$

the $\alpha_{x,y}$ and $\beta_{x,y}$ are the Courant-Synder parameters, and:

$$f_{z0}(z, z') = \frac{\beta c\eta}{2\pi\sigma_z^2(s)\omega_s} \exp\left(-\frac{1}{2\sigma_z^2(s)}\left(z^2(s) + \left(\frac{\beta c\eta}{\omega_s}\right)^2 z'^2\right)\right) \tag{36}$$

where $\sigma_z = \sigma_{\Delta\phi_{rf}}\lambda_{rf}/2\pi$, and ω_s is the synchrotron frequency. Note that the steady state solution incorporates the linear forces and ignores coupling between x, y and z motion.

The perturbation equation can be written in the form:

$$\frac{\partial \delta f}{\partial s} + \vec{x}' \cdot \frac{\partial \delta f}{\partial \vec{X}} + \vec{A} \cdot \frac{\partial \delta f}{\partial \vec{X}'} = -\left(\vec{A} - \overleftrightarrow{A}_0(s) \cdot \vec{x}\right) \cdot \frac{\partial f_0}{\partial \vec{X}'}, \tag{37}$$

so the perturbation equation takes into account the nonlinear force terms and the coupling terms. The weight advance in the finite particle representation is of the form:

$$\frac{dw_i}{ds} = -\frac{1}{n}\left[\left(\vec{A} - \overleftrightarrow{A}_0(s) \cdot \vec{x}\right) \cdot \frac{\partial f_0}{\partial \vec{X}'}\right]_i \tag{38}$$

where

$$n = \frac{N}{(\Delta\vec{x}\Delta\vec{x}')}. \tag{39}$$

The variation of $\frac{\partial f_0}{\partial \vec{X}'}$ with s can be computed from β_x, σ_x, β_y,and σ_y. The betatron oscillation lengths $\beta(s)$ and the beam sigmas $\sigma(s)$ can be determined from the lattice function of the particular machine being studied where

$$\frac{\beta_x(s)}{\beta_{x0}} = \frac{\sigma_x^2(s)}{\sigma_{x0}^2} \tag{40}$$

$$\frac{\beta_y(s)}{\beta_{y0}} = \frac{\sigma_y^2(s)}{\sigma_{y0}^2}, \tag{41}$$

where β_{x0}, σ_{x0}^2 β_{y0}, and σ_{y0}^2 are at some initial point in the collider.

CONCLUDING REMARKS

In this paper we have presented an algorithm for realistic modeling of beams in a hadron collider such as the SSC. The code with this algorithm is expected to be a universal study tool for space-charge effects of the Low Energy Booster (LEB) to beam-beam effects of the Collider ring. The code should consist of mainly two parts. In the first part, the 3-dimensional electromagnetic fields are calculated as the sum of the steady part which is solved analytically, and the perturbative part which is computed by the field solver using macroparticles. The code Simpsons is the second part which advances 6-dimensional phase space coordinates to the next location for field calculations. The implementation of the algorithm with the full three-dimensional δf method has already been undertaken.

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