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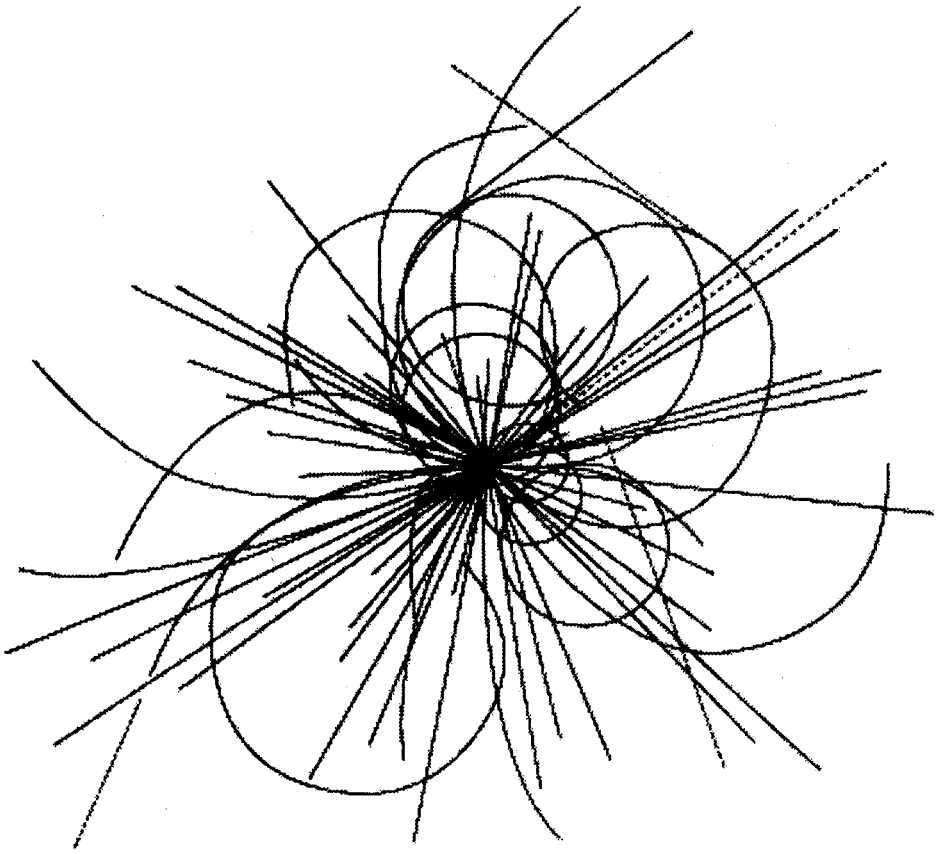

Superconducting Super Collider Laboratory

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The Simpsons Program
6-D Phase Space Tracking with Acceleration*

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THE SIMPSONS PROGRAM
6–D PHASE SPACE TRACKING WITH ACCELERATION*

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ABSTRACT

A particle tracking code, Simpsons, in 6-D phase space including energy ramping has been developed to model proton synchrotrons and storage rings. We take time as the independent variable to change machine parameters and diagnose beam quality in a quite similar way as real machines, unlike existing tracking codes for synchrotrons which advance a particle element by element. Arbitrary energy ramping and rf voltage curves as a function of time are read as an input file for defining a machine cycle. The code is used to study beam dynamics with time dependent parameters. Some of the examples from simulations of the Superconducting Super Collider (SSC) boosters are shown.

INTRODUCTION

Particle tracking codes developed so far mostly deal with either transverse or longitudinal phase space coordinates separately, or at most, track both of them but without acceleration. In a rapid cycling proton synchrotron, such as the Low Energy Booster (LEB) of the Superconducting Super Collider (SSC) which accelerates protons from 1.22 GeV/c to 12 GeV/c within 50 msec (about 26,000 turns, total), the synchrotron tune changes quite rapidly and sometimes becomes large (~ 0.1). Furthermore, multiturn injection, adiabatic capture and bunching all occur in a short time period right after the injection, usually with strong space-charge force in all directions. The Medium Energy Booster (MEB) of the SSC has the transition crossing where it is essential to observe the particle behavior in 6-D with acceleration. Coupling effects between transverse and longitudinal planes are expected to play a significant role in these machines thus, a more sophisticated tracking model with acceleration is necessary.

From a practical point of view, for example to simulate a commissioning procedure, it is desirable to have a machine simulation model which can handle time-dependent parameters such as rf frequency errors and magnet strength (including multipole errors), as arbitrary functions. We developed the code Simpsons to track multi-particles in full 6-D phase space with acceleration. The code reads external tables which specify time-dependent parameters when they are necessary. First, we will explain the detail of the simulation method. Secondly, we will show some results of the LEB and MEB of the SSC.

THE CODE SIMPSONS[1]

The code Simpsons is a multi-particle tracking program in 6-D phase space with acceleration. It takes time as the independent variable. Phase space coordinates of macro-particles are updated by a fixed time interval. Choice of the time interval depends on the physics one is studying. When the internal force among particles, such as space-charge effects, is not negligible, one needs to choose a small enough time interval because the force is a function of the coordinates themselves and self-consistency of the particle distribution and the force should be kept. When the external force, namely the force from magnetic lattice elements and rf cavities, is dominant, each macro-particle behaves independently so that a large time interval can be taken.

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The tracking is done for one cycle of a machine (it does not necessarily mean from injection to extraction), and some machine parameters have to be specified throughout a cycle in addition to the geometric lattice configuration. When the synchronous momentum and the rf voltage are constant throughout a cycle, for example the simulation of an injection porch in the MEB and the HEB, values of these constants and total real time of a cycle (or total turn numbers) are sufficient to specify a cycle. For a simulation of a machine with acceleration, one has to prepare tables which specify the strength of a bending field $\beta\varrho(t)$, and the rf voltage $V(t)$ as a function of time. In the code, these tables are interpolated to get values in a time interval. Figures 1 and 2 show these functions for the LEB.

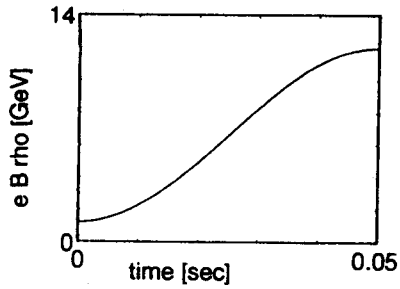

Fig. 1. $eB\varrho$ of a LEB Cycle.

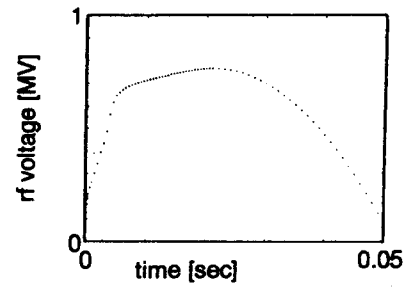

Fig. 2. Rf Voltage of a LEB Cycle.

The Simpsons itself does not have the interpretation part of the Standard Input and Command Language[2] for describing and tuning an accelerator lattice. Instead, we adopted the TEAPOT[3] as a preprogram. A lattice description file in the Standard Language is processed by the TEAPOT beforehand and replaced by a thin lens machine file. Furthermore, transverse tune and chromaticity are adjusted, multipole and misalignment errors in lattice elements are introduced, and lattice functions; β, α, η and closed orbit distortions are calculated by the TEAPOT. In this way, a thin lens machine file and a lattice function file are ready for the Simpsons as input files.

Normally, a simulation starts from creating initial 6-D phase space coordinates; s, p_s, x, p_x, y, p_y for all the macro-particles. The Gaussian distribution with a certain cut of a tail for both longitudinal and transverse directions is commonly assumed, but a parabolic and a uniform distribution for longitudinal, and a waterbag and a K-V distribution for transverse can be also selected.

Within one time interval, a particle either remains in drift space or receives one or more momentum kicks due to magnets and/or rf cavities. If there are no lattice elements in the time interval, as is the case only with drift space, the positions are updated,

$$s_{new} = s_{old} + \frac{p_s c}{\sqrt{(m_0 c^2)^2 + (pc)^2}} \Delta t \ , \tag{1}$$

$$x_{new} = x_{old} + \frac{p_x c}{\sqrt{(m_0 c^2)^2 + (pc)^2}} \Delta t \ , \tag{2}$$

$$y_{new} = y_{old} + \frac{p_y c}{\sqrt{(m_0 c^2)^2 + (pc)^2}} \Delta t \ , \tag{3}$$

where p is the total momentum of the particle,

$$p = \sqrt{p_s^2 + p_x^2 + p_y^2} \ , \tag{4}$$

m_0 is the mass of proton, c is the speed of light, and Δt is the time interval. The momenta remain constant.

At the location of rf cavities, a particle gains or loses its energy as

$$\Delta E = eV(t) \sin \phi_{rf}(t) \ , \tag{5}$$

where $V(t)$ is the rf voltage, e is the unit charge, and $\phi_{rf}(t)$ is the rf phase at the time a particle is passing through the rf cavity. Without acceleration, the rf phase can be simply replaced by

$$\phi_{rf}(t) = \omega_{rf} \cdot t + \phi_{rf}(0) \ , \tag{6}$$

where ω_{rf} is the constant rf frequency, the product of the revolution frequency of the synchronous particle and the harmonic number h,

$$\omega_{rf} = h \cdot \omega_{rev} \ . \tag{7}$$

That is, the rf phase is a linear function of time.

The rf phase is no longer a simple linear function of time once a particle is being accelerated. With acceleration, the rf phase at time t is

$$\phi_{rf}(t) = \int_0^t \omega_{rf}(t)dt + \phi_{rf}(0) \ , \tag{8}$$

where $\omega_{rf}(t)$ is the "instantaneous" rf frequency and as a function of the bending field,

$$\omega_{rf}(t) = h \cdot \omega_{rev}(t) = h \cdot \frac{2\pi c\beta(t)}{C}$$

$$= h \cdot \frac{2\pi c\left(m_0c^2/eB\varrho(t)\right)^2}{C\sqrt{1 + \left(m_0c^2/eB\varrho(t)\right)^2}} \ , \tag{9}$$

where $\beta(t)$ is the Lorentz factor and C is the machine circumference. When a beam goes through the transition energy, the rf phase must be shifted by a proper value. In the code, we add the phase shift $\Delta\phi_{rf}$ to the rf phase at specified time instantaneously.

In either case, with and without acceleration, the total energy of a particle increases, or decreases at an rf cavity, by the amount of the Eq. (5)

$$E_{new} = E_{old} + \Delta E \ , \tag{10}$$

and the new total momentum is

$$p_{new} = \sqrt{E_{new}^2 - (m_0c^2)^2} \ . \tag{11}$$

Since only the longitudinal momentum should change at the rf cavity,

$$p_{s,new} = \sqrt{p_{new}^2 - p_{x,old}^2 - p_{y,old}^2} \ . \tag{12}$$

In summary, the most different feature of the longitudinal dynamics of the Simpsons is that it does not solve differential (or difference) equations with respect to the synchronous phase. Instead, the rf frequency is integrated to keep track of the rf phase all the time so that one can apply proper voltage when a particle is passing through an rf cavity.

If there are magnets in a time interval, first the positions are updated to the magnet location with the Eqs. (1) to (3) (Δt should be substituted by its appropriate fraction), then the transverse momenta are changed in the same way as the TEAPOT,

$$p_{x,new} = p_{x,old} - \frac{1}{1+\delta}\left(\frac{B_yL}{p_0/e}\right)p \ , \tag{13}$$

$$p_{y,new} = p_{y,old} + \frac{1}{1+\delta}\left(\frac{B_y L}{p_0/e}\right)p \ , \tag{14}$$

where p_0 is the synchronous momentum at that particular time, δ is the momentum deviation from the synchronous one

$$\delta = \frac{p - p_0}{p_0} \ , \tag{15}$$

L is the magnet length, and B_x and B_y are the magnetic fields at the particle position as the summation over multipoles.

$$(B_y + iB_x)L = (p_0/e)\sum_{n=0}^{M}\left(\tilde{b}_n + i\tilde{a}_n\right)(x+iy)^n \ , \tag{16}$$

where $\tilde{b}_n$ and $\tilde{a}_n$ are the normal and skew multipole strengths, respectively, in the TEAPOT notation, which are related to the MAD[4] k_n by

$$\tilde{b}_n = \frac{k_n L}{n!} \ . \tag{17}$$

Since the magnitude of the total momentum should not change by magnets,

$$p_{new} = p_{old} \ , \tag{18}$$

and the longitudinal momentum is adjusted as

$$p_{s,new} = \sqrt{p_{new}^2 - p_{x,new}^2 - p_{y,new}^2} \ . \tag{19}$$

In the real machines, designed values never stay perfect and noise of all kinds of frequencies affects machine operations. Besides, one may intentionally vary some machine parameters as a function of time. For example, in low energy proton synchrotrons the operation point is set with a large fractional tune at the beginning of a cycle to accommodate the large space-charge tune shift and it is gradually pushed down for the suppression of other instabilities such as resistive wall instability. In the Simpsons, the deviation from the perfect value of the magnetic strength, the rf frequency and so on, are read as an external file tabulating values as a function of time.

EXAMPLES

We have been using the Simpsons to study several beam dynamics issues in the LEB, MEB, and HEB of the SSC. In the following section, some tracking results of the LEB and MEB will be shown to demonstrate potential capability of the code. The application to the HEB can be found elsewhere.[5]

Synchro–beta Coupling

The synchrotron tune of the LEB becomes as high as 0.05 a few milliseconds after injection and decreases slowly toward the end of a cycle. The betatron tune is around 11.85 right after the injection to accommodate the large space-charge tune shift. We have looked at the emittance growth due to synchro-beta coupling effects which are excited by rf cavities at a finite dispersion position. As a matter of fact, all the rf cavities in the LEB are planned to be located in one straight section where dispersion functions of both planes are ideally zero. We examined, however, some lattices where some of rf cavities are in an arc section with finite dispersion. In all the examined lattices, the total rf voltage is kept same.

The 100 macro-particles are tracked in a whole cycle of the LEB (50 msec). The rms $\Delta p/p$ of 100 particles is shown in Figure 3 and the estimated synchrotron tune in Figure 4. The horizontal emittance stays almost constant throughout a cycle when all four rf cavities are put in a

dispersionless (except residual ones due to errors) straight section as designed, Figure 5(a). When we put one rf cavity at the maximum horizontal dispersion position of about 3.5 m, the horizontal emittance becomes as twice as much, which mostly occurs when the synchrotron tune has the maximum value of 0.05, Figure 5(b). In both cases, the vertical emittance stays constant.

An arc section consists of four identical modules. Horizontal phase advance between each module is fixed to $0.75 \times 2\pi$. The lattice functions including dispersion repeat in each module. Instead of putting one rf cavity in one module, we tested two rf cavities in two modules. First, the two rf cavities were separated by phase advance $0.75 \times 2\pi$ (one module apart). The horizontal emittance growth becomes smaller but still exists, Figure 5(c). Second, the two rf cavities were separated by phase advance $1.50 \times 2\pi$ (two modules apart). The result shows the cancellation of synchro-beta coupling effects by means of proper phase advance, Figure 5(d).

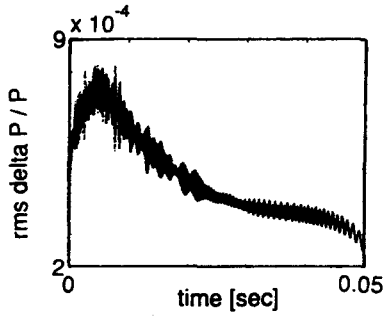

Fig. 3. Rms $\Delta p/p$ of 100 Macro-particles.

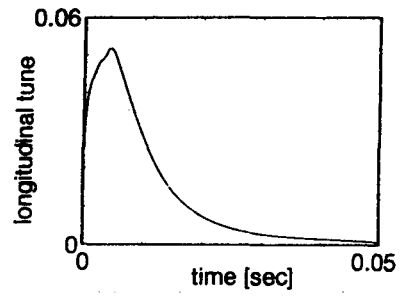

Fig. 4. Synchrotron Tune.

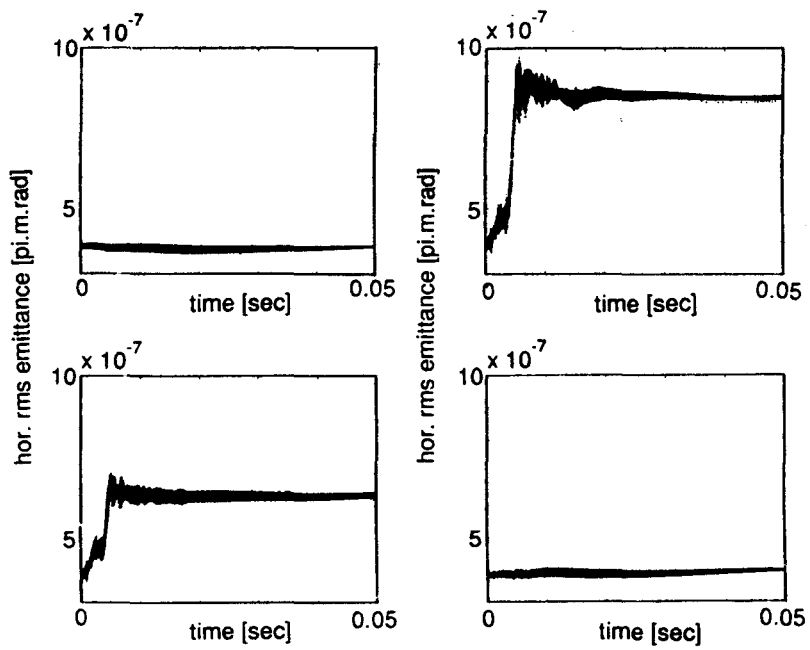

Fig. 5. Horizontal Rms Emittance; (a) 4 Rf Cavities in Dispersionless Section, (b) 1 Rf Cavity at Large Dispersion, (c) 2 Rf Cavities at Large Dispersion with Phase Difference of $0.75 \times 2\pi$, and (d) 2 Rf Cavities at Large Dispersion with Phase Difference of $1.50 \times 2\pi$.

Transition Crossing

The transition energy is defined by a lattice itself unlike a tracking program of longitudinal phase space only, in which it should be given as an input parameter. That is also true for higher order coefficients of $\Delta p/p$. Therefore, a simulation of transition crossing starts from finding transition energy itself. The revolution frequency shift is expanded as

$$\frac{\Delta f}{f} = c_0 \frac{\Delta p}{p} + c_1 \left(\frac{\Delta p}{p}\right)^2 + \dots \ . \tag{20}$$

The first coefficient c_0 is

$$c_0 = \frac{1}{\gamma^2} - \alpha_0 \ , \tag{21}$$

where γ is the Lorentz factor and α_0 is the momentum-compaction. At the transition, c_0 becomes zero and second order of $\Delta p/p$ is dominated.

To find out the transition energy of the MEB, off-momentum particles were tracked for 100 turns by scanning the synchronous momentum. When the synchronous momentum is not equal to the transition momentum, the trace of off-momentum particles draws an asymmetric curve, Figure 6(a) because the first term of Eq. (20) is not zero. At the transition energy, off-momentum particles behave symmetrically, Figure 6(b). In that MEB lattice, it turns out that a particle reaches the transition energy 0.39237 sec after acceleration starts.

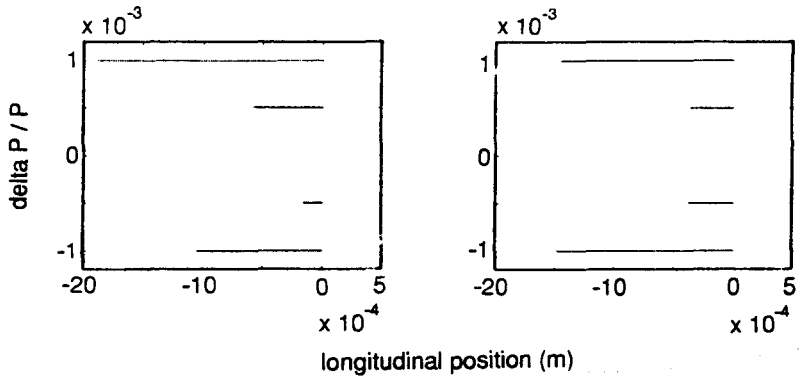

Fig. 6. Phase Space Plot of Off-momentum Particles for 100 Turns. (a) On-momentum Particle is not at the Transition Energy. (b) On-momentum Particle is at the Transition Energy.

We started a simulation 0.380 sec after acceleration starts (about 1000 turns before the transition) with 1000 macro-particles. The initial longitudinal distribution is matched to the rf bucket at that time. We set the bending field increases quadratically and the rf voltage does linearly. The bunch length and $\Delta p/p$ throughout the transition crossing are shown in Figures 7 and 8. Because of unavoidable mismatch at the transition crossing, namely the different momentum particle has the different transition energy, Johnsen effects[6], the longitudinal emittance growth is observed in Figure 9 while the transverse emittance stays almost constant as shown in Figure 10 as expected.

Resonance Correction

The way of correcting betatron resonances is well-known and several harmonic correctors are designed for the SSC boosters. We used the Simpsons to check the performance of these correctors by looking at the emittance growth and beam loss due to resonance crossing taking bandwidth as a parameter.

In the LEB, the trim quadrupole strength is linearly shifted as a function of time to simulate resonance crossing. We used 100 macro-particles to calculate the evolution of the emittance. In the Simpsons, a particle loss happens if the radial amplitude of the particle is more than the beam pipe radius, which is 35 mm in the LEB. The resonance crossing simulation of the sum resonance; $\nu_x + \nu_y = 23$ is shown in Figure 11. Keeping the vertical tune constant at 11.60, the horizontal tune is scanned from 11.41 to 11.39 in 2.5 msec so that a beam crosses the sum resonance about halfway. The 50% correction of the bandwidth is not enough and a large emittance growth occurs. Figure 11(b), however, indicates the decrease of the beam loss even with the 50% correction. For the second order normal resonance; $2\nu_x = 23$, the horizontal tune is scanned from 11.51 to 11.49 in 2.5 msec and it shows that the 50% correction works well in Figure 12. There is no beam loss.

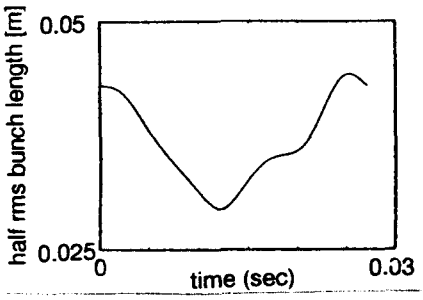

Fig. 7. Half Rms Bunch Length Around the Transition Energy Which Is At $t = 0.01237$ s.

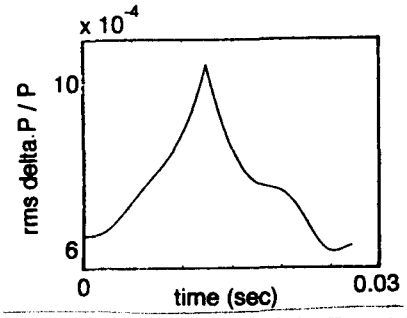

Fig. 8. Rms $\Delta p/p$ Around the Transition Energy.

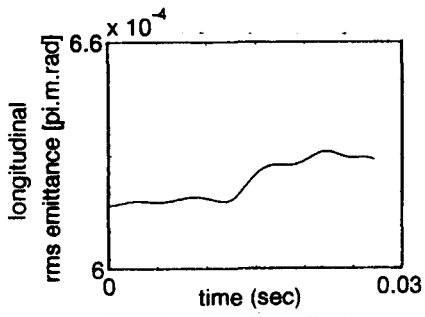

Fig. 9. Longitudinal Rms Emittance Around the Transition Energy.

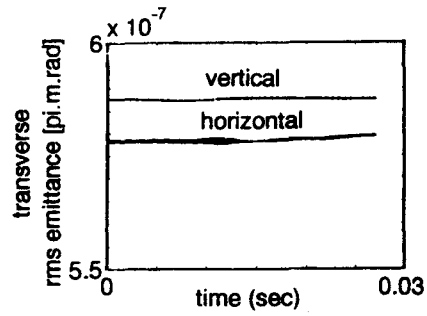

Fig. 10. Transverse Rms Emittance Around the Transition Energy.

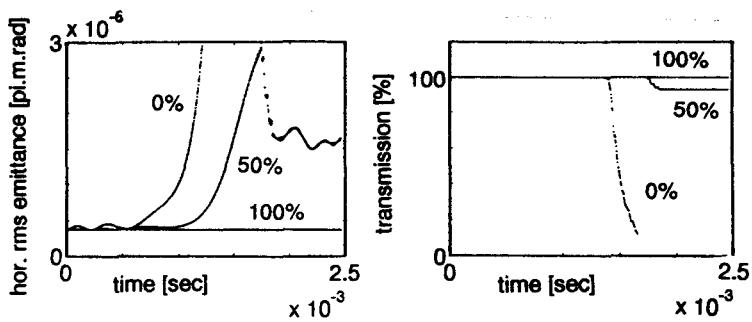

Fig. 11. (a) Horizontal Rms Emittance Growth Due To Resonance Crossing of $\nu_x + \nu_y = 23$ Which Occurs at About $t = 0.1$ milliseconds. (b) Beam Loss Due To Resonance Crossing.

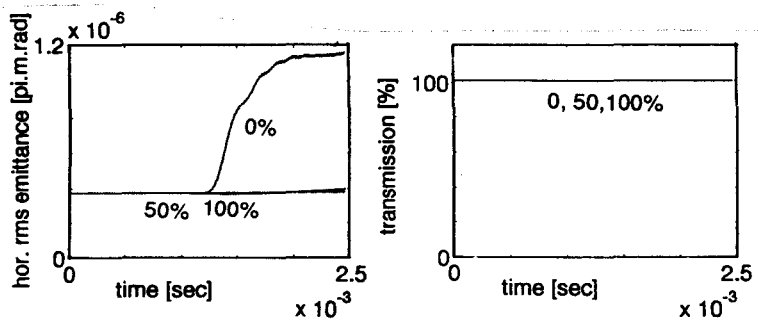

Fig. 12. (a) Horizontal Rms Emittance Growth Due To Resonance Crossing of $2\nu_x = 23$ Which Occurs at About $t = 0.1$ milliseconds. (b) Beam Loss Due To Resonance Crossing.

SPACE-CHARGE EFFECTS

There are two methods of calculating space-charge effects in the Simpsons. One employs macro-particles for calculation of charge distribution and emittance. The electromagnetic space-charge field is solved self-consistently in 3-D configuration space according to macro-particle distribution which is updated every time interval. A more detailed calculation method and some results can be found elsewhere.[7] The other, which is a simplified version, uses macro-particles only for emittance calculation and the space-charge field is fixed throughout a simulation. An initial charge distribution is assumed to be Gaussian in 3-D configuration space and we use the analytical formula of the transverse space-charge field for that distribution.[8] Nevertheless, an evolution of the bunching factor due to the rf voltage envelope, adiabatic damping of transverse beam size, and cancellation of an electric field by a magnetic field depending on the Lorentz factor γ are included. Once the distribution evolves into non-Gaussian or the emittance grows significantly, the approximation which assumes constant distribution function and emittance is not appropriate. In that sense, the latter for space-charge calculation is only appropriate to see the growth rate in a quick way, not to predict asymptotic emittance.

Space-charge effects are one of the crucial beam dynamics issues for the emittance preservation especially in the LEB and MEB. In each stage of the tracking studies we have mentioned above, we should include the effects in a proper way and that is certainly one of the goals of the code Simpsons. Some other methods to incorporate space-charge effects are also under development.[9] Although the simulation of the LEB with space-charge effects becomes almost possible in an established way, the transition crossing simulation with space-charge effects, especially of the transverse direction, seems to need further development.

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