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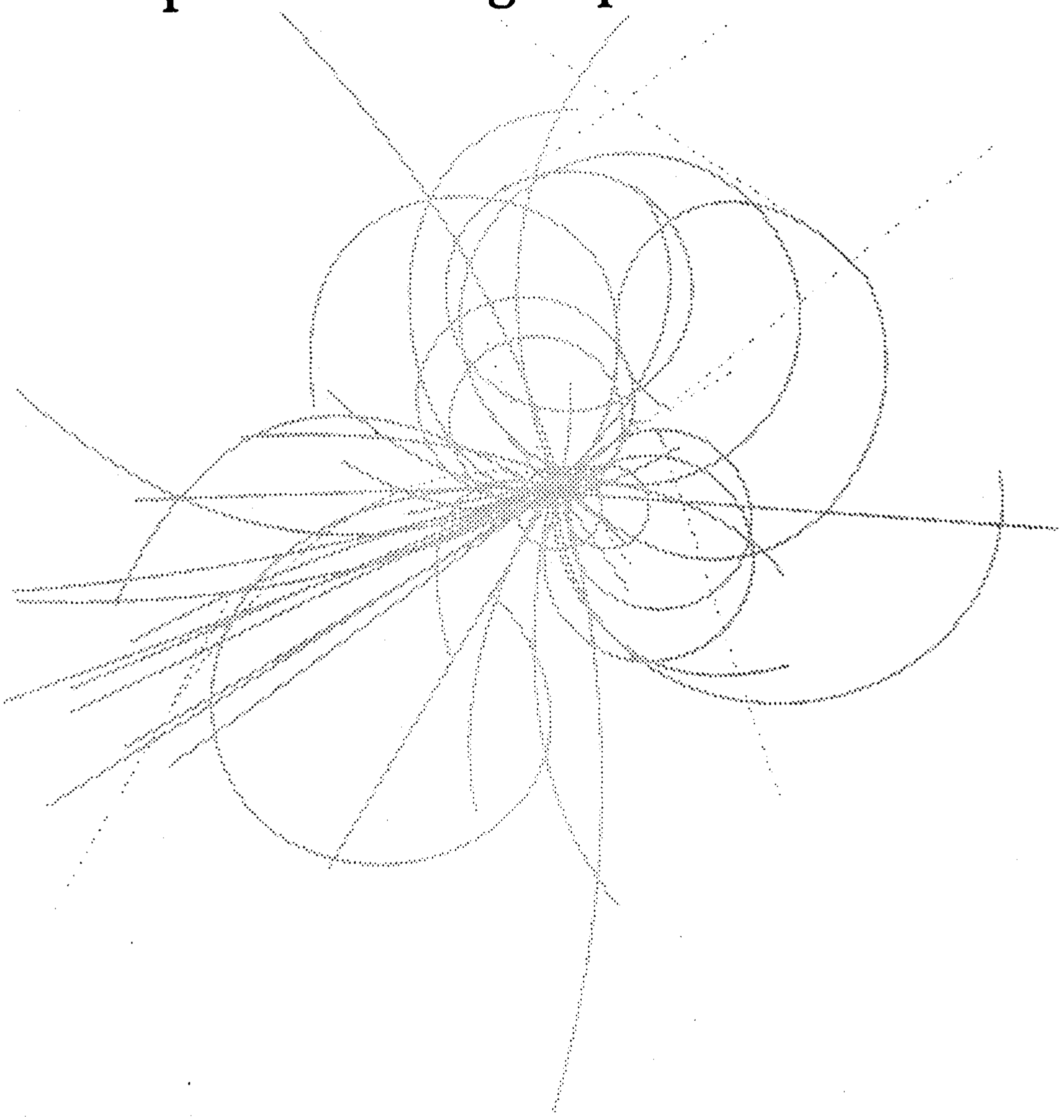

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S. Machida, G. Bourianoff, Y. Huang, and N. Mahale

Superconducting Super Collider Laboratory†
2550 Beckleymeade Avenue
Dallas, Texas 75237

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TRACKING STUDY OF HADRON COLLIDER BOOSTERS

S. Machida, G. Bourianoff, Y. Huang, and N. K. Mahale

Superconducting Super Collider Laboratory*
2550 Beckleymeade Ave.
Dallas, Texas 75237-3997, U.S.A.

ABSTRACT

A simulation code **Simpsons** (previously called 6D-TEASE*T*) [1] of single- and multi-particle tracking has been developed for proton synchrotrons. The 6D phase space coordinates are calculated each time step including acceleration with an arbitrary ramping curve by integration of the rf phase. Space-charge effects are modelled by means of the Particle In Cell (PIC) method. We observed the transverse emittance growth around the injection energy of the Low Energy Booster (LEB) of the Superconducting Super Collider (SSC) with and without second harmonic rf cavities which reduce peak line density. We also employed the code to see the possible transverse emittance deterioration around the transition energy in the Medium Energy Booster (MEB) and to estimate the emittance dilution due to an injection error of the MEB.

1 INTRODUCTION

The transverse emittance preservation in the accelerator chain of high-energy colliders is essential to achieve the luminosity goal. The hadron colliders, in particular, are not expected to have enough radiation dumping so that any possible cause of the emittance growth should be suppressed throughout the machine complex. The proposed Superconducting Super Collider (SSC), for example, has three booster synchrotrons until the beam gets to the final collider ring. Although the beam current is not so high compared with the existing machines, the high brightness beam, which is a moderate current beam with small transverse emittance, brings some challenging issues in those boosters.

Space-charge effects around the injection energy of the Low Energy Booster (LEB) is one of the bottlenecks against keeping emittance small [2]. Although the injection energy of the Medium Energy Booster (MEB) is relatively high, space-charge effects are still sizable; the Laslett tune shift is about -0.08. At the transition energy of the MEB, the tune shift is even larger because of the very short bunch.

Besides space-charge effects, there are other problems which become more crucial because of the small transverse emittance; the rms emittance allowed in the SSC boosters is 0.4 to 1.0 $\pi \cdot$ mm $\cdot$ mrad. The injection error in each machine should be minimized or carefully damped in a short enough time period, otherwise the tiny mismatch could cause significant emittance dilution. The phase space distortion due to multipole error is another source which deteriorates the transverse beam quality. The power supply ripple, or more generally the time dependent error of the lattice parameters could induce the resonance crossing.

We made a simulation code especially to model these proton synchrotrons. Acceleration as well as longitudinal oscillations are included with arbitrary magnet excitation and rf voltage curve by integration of the rf phase. That makes it possible for the first time to track 6D phase space coordinates as a function of time. One typical example is the transition crossing. Space-charge effects are modelled using the Particle In Cell (PIC) method. In conjunction with space-charge effects, possible improvement by harmonic rf cavities which reduce the peak line density is checked. Strength of the magnets can be fluctuated in either a systematic or a random manner to simulate the power supply ripple.

2 THE SIMPSONS PROGRAM

2.1 6D Tracking

All the lattice elements, including rf cavities, are replaced by thin lenses [3]. The substitution of the thick lens lattice to the thin lens one, the tuning of the betatron phase and the chromaticity, and the introduction of the multipole errors and the mis-alignment are all done by the TEAPOT [4]. A particle with 6D phase space coordinates (s, p_s, x, p_x, y, p_y) is then tracked in the thin lens lattice with *time* as the independent variable. When acceleration is included, machine cycle parameters have to be also defined. The synchronous momentum $p_0(t)$ and the rf voltage $V(t)$ are read as an external table for that reason.

Within one time interval, a particle will either remain in drift space or receive one or more momentum kicks due to magnets and/or rf cavities. If there are no lattice elements in the time interval, the positions are updated and the momentum remains constant. If there are magnets in between, first the positions are updated to the magnet location, then the transverse momenta are changed.

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At the location of rf cavities, a particle gains or loses its energy as

$$\Delta E = eV(t)\sin\phi_{rf}(t). \qquad (1)$$

Without acceleration, the rf phase can be replaced by

$$\phi_{rf}(t) = \omega_{rf}\cdot t + \phi_{rf}(0), \qquad (2)$$

where ω_{rf} is the constant rf frequency. With acceleration, the rf phase at time t is

$$\phi_{rf}(t) = \int_0^t \omega_{rf}(t)dt + \phi_{rf}(0), \qquad (3)$$

where $\omega_{rf}(t)$ is the "instantaneous" rf frequency and as a function of the synchronous momentum $p_0(t)$,

$$\omega_{rf}(t) = h\cdot\frac{2\pi c\left(m_0c^2/p_0(t)c\right)^2}{C\sqrt{1+\left(m_0c^2/p_0(t)c\right)^2}}, \qquad (4)$$

where h is the harmonic number and C is the machine circumference. An rf cavity with harmonic frequency can be included in the same manner as a fundamental frequency cavity.

At the transition energy, the rf phase should be shifted from $\phi_{rf,s}$ to $\pi - \phi_{rf,s}$, where $\phi_{rf,s}$ is the synchronous phase at that time. In the code, we add the phase shift $\Delta\phi = \pi - \phi_{rf,s}$ to the rf phase, the equation (3), at the specified time instantaneously.

2.2 Space-charge Calculation

We employ the Particle In Cell (PIC) method to incorporate the space-charge field[5]. Let us assume that the beam pipe has a circular cross section with radius b and the perfect conductivity $\sigma = \infty$. Under the Lorentz gauge $\frac{\partial\phi}{\partial t} + c\vec{\nabla}\cdot\vec{A} = 0$, the Maxwell equations are

$$\left(\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r}) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)\phi = -4\pi n, \qquad (5)$$

$$\left(\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r}) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)\vec{A} = -\frac{4\pi}{c}\vec{J}, \qquad (6)$$

where n and $\vec{J}$ are the charge density and the current, respectively, and the scaler potential ϕ and the vector potential $\vec{A}$ are used. We set the scaler potential $\phi \equiv 0$ at $r = b$ and invoked the ordering

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2} \sim \frac{\partial^2}{\partial z^2} \ll \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r}) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}. \qquad (7)$$

This can be justified by the typical dimension of the beam, whose transverse size is the order of 1 mm whereas the longitudinal size is the order of 1 m.

We further simplify the current density,

$$\vec{J} = J_z\vec{e}_z = \bar{v}_z n\vec{e}_z, \qquad (8)$$

where $\bar{v}_z$ is the average velocity of the beam. From the Lorentz force equation $\vec{F} = q(\vec{E} + \frac{\vec{v}}{c}\times\vec{B})$, the electromagnetic force is $\vec{F} = -\frac{q}{\gamma^2}\vec{\nabla}\phi$ and we only need to solve the scaler potential equation (5). The charge density and scaler potential are Fourier transformed in θ

$$n(r,\theta,z,t) = \sum_m n_m(r,z,t)\exp(im\theta), \qquad (9)$$

with the inverse transforms

$$n_m(r,z,t) = \frac{1}{2\pi}\int_0^{2\pi} n(r,\theta,z,t)\exp(-im\theta)d\theta, \qquad (10)$$

and same for ϕ.

For each m, the equation assumes the form

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial\phi_m}{\partial r}) - \frac{m^2}{r^2}\phi_m = -4\pi n_m. \qquad (11)$$

The solution for the equation for $m \geq 0$ is

$$\phi_m = W_m(r) - (\frac{r}{b})^m W_m(b), \qquad (12)$$

where $W_m(r)$ is

$$W_{m=0}(r) = -4\pi\int_0^r n_0(r,z,t)r'\ln\frac{r}{r'}dr', \qquad (13)$$

$$W_{m\geq 1} = -\frac{2\pi r^m}{m}\int_0^r n_m(r,z,t)r'^{(1-m)}dr' - \frac{2\pi r^{-m}}{m}\int_0^r n_m(r,z,t)r'^{(1+m)}dr'. \qquad (14)$$

To solve those differential equations in the form of difference equations in a computer, the grids are set in the cylindrical coordinate system such that the whole charge distribution is enclosed. The number of grids in r,θ,z direction is increased until the electric field does not depend on them. At the same time, the minimum number of macro particles is determined such that the electric field matches the analytical one well enough for an analytically solvable distribution. The maximum mode number, m_{max}, is empirically chosen to 2 in all the following simulations.

3 EMITTANCE GROWTH BY SPACE-CHARGE

3.1 LEB Injection

The emittance evolution around the injection energy in the LEB is observed with and without second harmonic cavities. The LEB lattice with realistic multipoles except b_0 (dipole) are modelled. The bunch train from linac of 428 MHz are adiabatically captured into a 47.5 MHz booster bucket and accelerated at the machine cycle of 10 Hz. Even though the asymptotic emittance limit is not reached in both cases, Fig.1 shows the significant reduction of the emittance growth with second rf cavities which reduce peak line density.

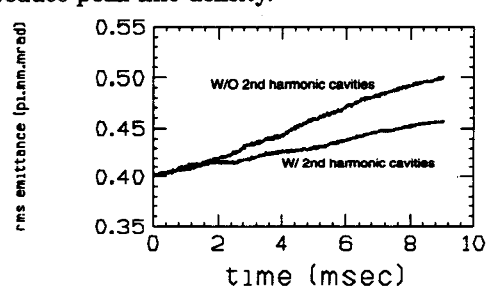

Figure 1: Horizontal emittance evolution around the LEB injection energy

3.2 MEB Transition Crossing

Since particles are tracked in the 6D phase space, the path length difference due to $\Delta p/p$ and the effect that the different $\Delta p/p$ particle has slightly different transition energy, the so-called Johnsen effect, are inherently included. Moreover, with transverse space-charge, it is possible to investigate the transition energy shift due to the transverse tune shift, the so-called Umstätter effect, as well as the direct transverse space-charge effects.

The following is a very preliminary result on the transverse emittance evolution around the transition energy. The MEB lattice in the simulation has realistic systematic and random multipole except b_0. We started the tracking about 500 turns before the transition energy, and continued until 500 turns after, namely 12 msec as total. During this period, the rf voltage increases linearly, whereas the bending magnetic fields vary quadratically. The bunching factor has its peak value of 1/80 at about 400 turns after the transition energy and decreases rapidly afterward. The maximum Laslett tune shift is -0.10. Fig. 2 shows the rms emittance evolution. No emittance growth has been observed.

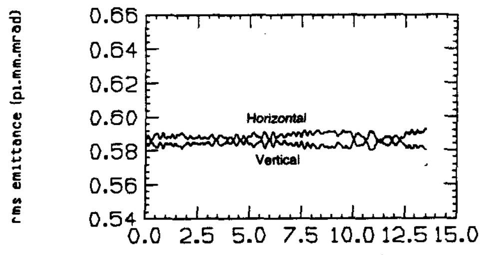

Figure 2: Horizontal and vertical emittance evolution around the MEB transition energy

4 EMITTANCE DILUTION AT INJECTION

The injection error is a major source of the emittance dilution. Especially in the MEB, the tune spread due to space-charge effects $\Delta\nu$, about -0.08, was considered as a leading factor of the dilution and the damping system should be designed in a time scale of $1/\Delta\nu$. When the beam is injected off center from the closed orbit, however, the tune shift as a function of particle amplitude seems different. Let us calculate it in the simplest case. A Gaussian distribution beam with a circular cross section has the electromagnetic field $E_r = \frac{\lambda}{2\pi\epsilon_0}\frac{1-\exp(-\frac{r^2}{2\sigma^2})}{r}$, where λ is the line density and σ is the rms beam size. We assumed the coasting beam. The Hamiltonian involving that space-charge term is

$$H = \Delta k(\frac{r^2}{2} - \frac{r^4}{16\sigma^2}), \tag{15}$$

where $\Delta k = \frac{r_p\lambda}{\sigma^2\beta^2\gamma^3}$, that is a coefficient which corresponds to the linear Laslett tune shift and r_p is the classical proton radius, and we took up to octupole term only. When there is an injection error of X_e in the horizontal direction, the Hamiltonian becomes

$$H_1 = \Delta k\Big[\frac{(\sqrt{2J_x\beta_x}\cos\phi_x - X_e\cos\psi_x)^2}{2} - \frac{(\sqrt{2J_x\beta_x}\cos\phi_x - X_e\cos\psi_x)^4}{16\sigma^2}\Big], \tag{16}$$

where we change the coordinate to the action J_x and angle ϕ_x variable and ψ_x is the angle of the beam center in the horizontal phase space. The tune shift of the particle becomes

$$\Delta\nu_x = -\frac{1}{4\pi}\int_C \Delta k\beta_x\Bigg[\Big(1 - \frac{X_e}{\sqrt{2J_x\beta_x}}\Big) - \frac{3}{16\sigma^2}\frac{(\sqrt{2J_x\beta_x} - X_e)^3}{\sqrt{2J_x\beta_x}}\Bigg]ds, \tag{17}$$

where we looked at particles with $\psi_x \sim \phi_x$. The last equation indicates that the tune shift at the center of the beam is zero where the charge density is maximum. This implies that the dilution of the beam could be slower than the time scale of $1/\Delta\nu$, where $\Delta\nu$ is the maximum tune shift without the injection error. The simulation actually confirms the prediction as shown in Fig. 3, where $\Delta\nu_x = -0.08$ (without injection error), $X_e = 2$ mm, and $\sigma = 1.5$ mm.

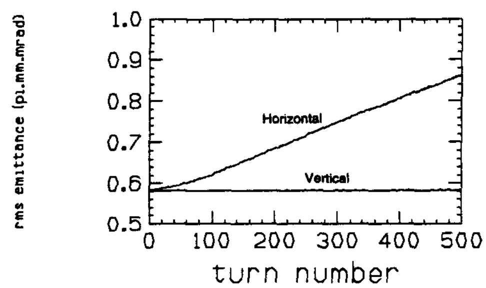

Figure 3: Emittance dilution due to the injection error in the MEB

5 SUMMARY

We have demonstrated the transverse beam emittance evolution in a few critical stages of the SSC boosters. Space-charge effects cause the emittance growth at the LEB injection, but it can be reduced by second harmonic rf cavities. In view of our limited simulation results, the transition crossing in the MEB does not affect the transverse beam quality. The emittance dilution at the MEB injection seems to be much slower than expected by the maximum tune spread without the injection error.

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[3] More detailed description of the code can be found in Machida, S., "6D Phase Space Tracking with Acceleration: Modeling Low Energy Proton Synchrotrons (I)," SSCL-570, SSC Laboratory, March 1992.

[4] Schachinger, L. and Talman, R., "TEAPOT: A Thin-Element Accelerator Program for Optics and Tracking," *Particle Accelerators*, Vol.22, pp.35–56, 1987. The latest version of the TEAPOT can do the 6D phase space tracking with acceleration in a different way from the **Simpsons**.

[5] The original formalism and approximation in this section was derived by M.L.Sloan. Sloan, M.L. "Calculation of Electromagnetic Self Forces for LEB Parameter Beams," March 1991, unpublished.