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An Upper Bound on Q-star Masses

DAVID A. HOCHRON

Department of Physics, Stanford University, Stanford, CA 94305

BRYAN W. LYNN

Department of Physics, Stanford University, Stanford, CA 94305 and Superconducting Super Collider Laboratory, Dallas, TX 75237-3946

STEPHEN B.SELIPSKY

Department of Physics, Boston University, Boston, MA 02215-2507

ABSTRACT

Q-stars (the gravitational generalization of Q-balls, strongly bound bulk matter that can appear in field theories of strongly interacting hadrons) are the only known compact objects consistent with the known bulk structure of nuclei and chiral symmetry that evade the Rhoades-Ruffini upper bound of $3.2M_{\odot}$. Generic bounds are quite weak: $M_{\text{Q-star}} < 400M_{\odot}$. If, however, we assume that the 1.558 ms pulsar is a Q-star, equilibrium and stability criteria of rotating fluids place a much stronger upper bound of $M_{\text{Q-star}} \leq 5.3M_{\odot}$ on such models under certain special assumptions. This has important implications for heavy compact objects such as Cygnus X-1.

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The idea [1] that bulk matter can be bound by purely strong forces has implications for cosmology [2, 3, 4, 5, 6, 7] and astrophysics [5, 8, 9, 10, 11]; various species include strange quark matter [6, 7] and baryonic matter [8, 9, 10]. Baryonic matter is particularly interesting because it offers a consistent description of the known properties of nuclei on the one hand, and exotic hadronic bulk matter with very different properties on the other hand. The bulk properties of nuclei (binding energies, charge densities, bulk compressibility, optical potential, spin observables in polarized proton scattering, *etc.*) have been simply and successfully modeled with a field theory of a baryon field Ψ, a phenomenological classical scalar field σ and a vector field V_μ [12]. Such models of ordinary nuclei can also be made compatible with the hadronic successes of chiral $SU(3)_L$ X $SU(3)_R$ symmetry [8] (baryon and pseudoscalar meson octet mass spectrum, low energy hadronic scattering, Adler-Weisberger sum rule, Goldberger-Trieman relation *etc.*) Exotic bulk matter can appear because, for certain forms of the σ-self interaction potential $U(\sigma)$, there exist large "Q-ball" solutions with hundreds of MeV binding energy per baryon. This possibility stems directly from our ignorance of the effective potential $U(\sigma)$. A crucial fact is that the bulk properties of the Q-ball solutions are virtually independent of the properties of ordinary nuclei.

This scenario has important implications for neutron stars. The generalization of Q-balls to include gravity are called Q-stars [8, 9, 10, 13, 14]. Here, Q stands for the conserved charge that stabilizes the solutions; in field theories of strongly interacting hadrons it is the baryon number. They can have large masses or high rotation rates, and should differ from conventional neutron stars in many important ways; for a large range of parameters they give counterexamples to the $3.2M_{\odot}$ upper bound on neutron star masses proposed by Rhoades

and Ruffini [15]. In fact, prior to this work, the only upper bound on Q-star masses, $M_{\text{Q-star}} \leq 400 M_\odot$, was given by the observation that white dwarfs are not Q-stars [14], thereby setting a lower limit on Q-ball densities. In this paper we bound a certain Q-star model with astrophysical data.

There is a wealth of data on neutron stars that confront any model of bulk nuclear matter, particularly if, as is widely accepted, pulsars are interpreted as rotating neutron stars. These include observed masses, population statistics, magnetic field strengths, cooling curves, rotation rates, pulsar glitches, and high space velocities.[#1] For example, the conventional interpretation of pulsar glitches as resulting from plate motion in the neutron star crust is problematic for Q-stars, for which crusts are not generic [16]. In this work, we assume that the fastest pulsar PSR 1937+214 (which is not known to glitch) is a Q-star and restrict Q-star parameters from its (presumed) rotation rate period $\tau = 1.558$ ms. We consider here for simplicity only the case of Q-stars in which baryons are massless in the interior.

Shapiro, Teukolsky, and Wasserman [19] have outlined general criteria, in light of the pulsar 1937+214 [20], period 1.588 ms, for limiting the mass and radius of the nonspinning progenitor. If the mass M of a neutron star is too low, instabilities set in for conventional neutron star models (secular and dynamical, depending on the regime and the compressibility of the star). If M is too high, there are two restrictions: the formation of destabilizing ergotoroids and gravitational collapse to a black hole. Here we examine the possible equations of state of Q-stars that are consistent with all the stability criteria.

#1 See, for example, ref. [17]. A recent review of the properties of millisecond pulsars can be found in ref. [18].

We consider the Lagrangian

$$\mathcal{L} = \bar{\Psi}[i\partial_\mu\gamma^\mu - m(\sigma) - g_V V]\Psi + \frac{1}{2}(\partial_\mu\sigma)^2 - U(\sigma) + \frac{1}{2}m_V^2 V_\mu V^\mu, \qquad (1)$$

in which Ψ is a baryon field, and σ and V are phenomenological scalar and vector fields. Serr and Walecka [21] have shown neglect of the vector field dynamics to be a good approximation for ordinary nuclei such as ^{40}Ca and ^{208}Pb in their mean-field model. This Lagrangian then gives a good mean field account of bulk nuclear properties and we examine its consequences for neutron stars. For certain choices of $U(\sigma)$ compatible with bulk nuclear properties, the stable neutron star configuration will be a Q-star rather than the metastable conventional neutron star configuration.

Spherical Q-stars in which $m(\sigma) = 0$ and $U = U_0$ in the interior have the following equation of state (EOS) [10] ($\mathcal{E}$ and P are respectively the total energy density and pressure):

$$\mathcal{E} - 3P - 4U_0 + \alpha_V(\mathcal{E} - P - 2U_0)^{\frac{3}{2}} = 0, \qquad (2)$$

in which $\alpha_V \equiv \sqrt{3}(g_V/m_V)^2/\pi$. It is convenient to define $\zeta \equiv \alpha_V U_0^{\frac{1}{2}} \frac{r}{\sqrt{3}}$, with which Q-star properties scale. For given parameters α_V and U_0, there will be a range of possible masses and radii. For low masses the density is constant, so a description in terms of an EOS is possible. The density increases as gravity becomes important, and the maximum mass occurs when $\frac{dM}{dR}$ vanishes (R is the neutron star radius). Ref. [14] summarizes many properties of Q-stars in terms of the scaling parameter ζ, and we will quote many of their results in what follows.

For any fixed ζ, there is a locus of such maximum masses ($\frac{dM}{dR} = 0$) obtained when α_V is varied. An example is shown in figure 1. Note that there is a smallest

maximum, which occurs when the scalar energy density U_0 is so large that the corresponding Q-ball is unbound. The unlabeled solid line of figure 2 shows the locus of such minimum maxima for different values of ζ; the point of maximum mass of any Q-star must lie to the right of that line.

We now seek to constrain the space of possible Q-star masses and radii from considerations of rotational stability and equilibrium assuming that the 1.558 ms pulsar is a Q-star.

There is a long history of the analysis of the maximum rotation velocity of a gravitationally bound fluid with a given EOS. The standard classical reference is by Chandrasekhar [22]. The simplest EOS is the case of incompressibility (see [17]). We will later argue that this is a good approximation for Q-stars with EOS (2). In the Newtonian regime rotation produces the MacLaurin spheroids. The angular velocity Ω and eccentricity e are related by

$$\frac{\Omega^2}{\pi G\mathcal{E}} = \frac{\sqrt{1-e^2}}{e^3}2(3-2e^2)\arcsin e - \frac{6}{e^2}(1-e^2). \tag{3}$$

The maximum Ω occurs at $e = .93$; beyond that point, an increase in angular momentum decreases Ω. Therefore, for an incompressible Newtonian fluid of energy density $\mathcal{E}$, the maximum angular velocity $\Omega_{\max}$ is given by

$$\frac{\Omega}{\sqrt{G\mathcal{E}}} \leq 1.19. \tag{4}$$

This constraint is shown in figure 2 by the dotted curve without the Xs.

Stability against gravitational radiation imposes tighter constraints. Gravitational radiation reaction causes non-axisymmetric modes of the form $e^{\pm im\phi}$ to go unstable for sufficiently high m [23]. $m = 2$ instability sets in at $e = .81$,

$m = 3$ earlier, *etc.* From (3) this means smaller critical Ω for increasing m. For example, stability of the $m = 2$ mode requires

$$\frac{\Omega}{\sqrt{G\mathcal{E}}} \leq 1.08. \tag{5}$$

Each higher mode has a longer growth time, but sets in at lower angular momentum. For sufficiently high m the instability is either damped by viscous forces or grows slowly on stellar timescales, so generally $m > 5$ need not be considered. A useful parameter for describing the onset of instability is

$$t \equiv \frac{\text{rotational kinetic energy}}{\text{gravitational binding energy}}. \tag{6}$$

The insensitivity of critical t of the fastest-growing mode to the EOS is a remarkable and useful result; *i.e.*, $m = 2$ sets in at $t = .14$, not only for incompressible stars but also for centrally condensed ones. For higher modes there is dependence on the EOS [24, 25].

Equilibrium and stability criteria have also been established for polytropes [24, 25, 26] (*i.e.*, fluids obeying an equation of state of the form $P = K\mathcal{E}^{1+\frac{1}{n}}$, where K is a constant and n is referred to as the polytropic order. $n = 0$ is the incompressible case, and the larger the order, the softer the EOS.) For general polytropes, the equilibrium limit comes not from a peak in the angular velocity, as in $n = 0$ polytropes, but from mass shedding at the equator. Whether or not secular instability sets in before mass shedding depends on the model and the relevant modes of oscillation. For an approximately incompressible (lower order polytrope) EOS, a star rotating at $t > .08$ is likely to be unstable to $m \leq 4$ modes [23, 24, 25]. In the opposite limit of extreme central condensation (the Roche

model), the limiting angular velocity due to mass shedding has been shown to be [19]:

$$\frac{\Omega}{\sqrt{G\mathcal{E}}} \leq 1.114. \tag{7}$$

Interestingly, the relativistic result is the same as the Newtonian one for the Roche model. Some of these stability criteria are displayed in figure 2.

Full general relativistic calculations offer less obvious information. The incompressible and polytropic equilibria are known [19, 27, 28], but the precise onset of secular instabilities is not. One rigorously known stability limit comes from mass shedding at the equator. Although nuclear forces bind individual baryons to Q-stars even at zero gravity, mass shedding can still occur in the form of macroscopic chunks for which the gain in surface energy is unimportant. Although the onset of secular stability has not been computed for many EOSs in the presence of strong gravitational fields, we can estimate it by assuming that the critical lines for such onsets are similar to the Newtonian result, combined with general relativistic numerical calculations of t-values for the relevant EOSs.

There is also a new purely general relativistic instability: the formation of ergotoroidal regions in which the relativistic dragging of inertial frames is so strong that all observers must rotate relative to the distant stars, and in which negative-energy perturbations grow exponentially [27, 29]. This instability does not set in for the Q-stars we consider in this paper.

We have seen that the maximum mass of a Q-star is constrained to lie in a slice of M *vs.* R space, bounded by the solid Q-star maximum mass line and one of the stability curves in figure 2. We must now address the question of compressibility to determine which curves apply. Q-star density profiles for various

values of ζ [14] are shown in figure 3; in no case can the bulk of the mass be considered centralized. The Q-stars of the highest mass also have the highest values of ζ. From figure 3, we see that they therefore have the stiffest EOSes and can be treated as approximately incompressible. Thus the incompressible mass shed curve of figure 2 gives a bound, $M_{\text{Q-star}} < 7.4 M_\odot$, which gets only slightly stronger when compressibility is taken into account.

The secular stability criterion is not known in the general relativistic case, but if as is widely believed instabilities set in for particular values of t, we can use the data of Butterworth and Ipser [27, 28, 29] to estimate a stability curve at $t = .08$. Those authors tabulated values of t_N for different rotation rates, stellar masses, and energy densities, where t_N is the relativistic generalization of our t parameter calculated by $t_N = \frac{1}{2}J\Omega/(M_0 + \frac{1}{2}J\Omega - M)$, with M the mass, J the angular momentum, and M_0 the mass in the absence of rotation and gravity. In table 1 we show, for three different values of $M_0\mathcal{E}^{1/2}$ (a measure of the strength of GR), the eccentricities and angular velocities at which $t_N = .14$ and $t_N = .08$. It is clear from the table and comparison with (5) that the effect of GR is to weaken the bound on Ω. In figure 2 we show these three values of $M_0\mathcal{E}^{1/2}$ at the $t = .08$ points A,B,C, and a curve drawn through them (using the approximate constancy of the central energy density $\mathcal{E}_c$ once Ω is fixed). Using this as a stability limit, we obtain the result $M_{\text{Q-star}} \leq 4.9 M_\odot$.

There is a question as to what degree the compressibility of the Q-star affects these bounds, $M \leq 7.4 M_\odot$ (mass shedding) and $M \leq 4.9 M_\odot$ ($m = 4$ secular stability). To answer that, we have examined $n = 1$ polytropes because the ratio of central ($\mathcal{E}_c$) to average ($\bar{\mathcal{E}}$) energy density $\mathcal{E}_c/\bar{\mathcal{E}} = 3.26$ for $n = 1$ [30], which resembles many Q-stars. In fact, Q-stars in the regime in which we are interested

should lie between $n = 0$ (incompressible) and $n = 1$. The equilibrium [31] and stability [24, 25] of $n = 1$ polytropes have been studied. From Managan *et. al.* and Butterworth we have determined mass shedding and $m = 4$ stability curves under the assumption that the critical t values persist under strong gravity. They too are shown in fig. 2. The resulting bounds are $M \leq 6.7M_\odot$ (mass shedding) and $M \leq 5.9M_\odot$ ($m = 4$ secular stability). The Q-stars in this regime are more like $n = 0$ (incompressible) than $n = 1$ polytropes, both in terms of $\mathcal{E}_c/\bar{\mathcal{E}}$ and the shape of the profile. (Polytropes have vanishing mass density at the periphery.) The $n = 1$ results merely give an indication of the way the bounds may differ from those obtained under the approximation of incompressibility. $M < 5.3M_\odot$ is probably a fair estimate of the true $m = 4$ stability limit.

There is another, independent way to limit the mass. Note that in the limiting case, $7.4M_\odot$, the model allows rotation at 1.558 ms only for high mass objects. If, on astrophysical grounds, we assume that the mass of the pulsar is on the order of $1.4M_\odot$, then the maximum possible non-rotating Q-star mass is $M_{\text{Q-star}} \leq 4.64M_\odot$ relying only on the Newtonian stability criteria.

This new $M_{\text{Q-star}} \leq 5.3M_\odot$ bound has important implications for heavy compact objects. Cygnus X-1 has a mass probably greater than $6.3M_\odot$ [32, 33]. Under the assumptions above, we would conclude that it is not a Q-star with EOS (2). A perhaps even better black hole candidate is the recurrent nova V404 Cygni [34] with a firm lower limit of $M \geq 6.26 \pm 0.31M_\odot$ and probable mass in the range $8 - 15.5M_\odot$. Thus for this object we draw the same conclusion, that it is not a Q-star with EOS (2). Similar considerations apply to the heavy compact objects LMC X-3 and A0620-00 [35].

The Rhoades-Ruffini theorem [15] states that, based on the assumptions 1) that general relativity is the right theory of long-wavelength gravity, 2) microscopic causality and 3) microscopic stability are applicable and 4) that the EOS below a density $\mathcal{E}_0 \sim$ nuclear density can be extrapolated from the properties of the interior of ^{208}Pb, *even for bulk matter with* $\sim 10^{57}$ *baryons*, there is an upper limit $M_{\text{neutron star}} \leq 3.2M_\odot$ for compact non-rotating astrophysical objects to be stable against gravitational collapse to black holes. Assumption 4) is, however, suspect; it is only necessary for the model of hadron dynamics to be compatible with the properties of ordinary nuclei and chiral $SU(3)_L$ X $SU(3)_R$ (the two best-known facts of low-energy hadronic physics). Q-stars, the only known compact objects to evade the Rhoades-Ruffini limit while still compatible with bulk nuclear properties and chiral symmetry, make use of this weaker fourth assumption and can have stable masses $M_{\text{Q-star}} >> 3.2M_\odot$. Before this work, EOS (2) gave a typical counter-example to the assertion that Cygnus X-1, V404 Cygni, LMC X-3 and A-0620-00 *must* be black holes, and it was only possible to argue that $M_{\text{Q-star}} < 400M_\odot$. In this paper we have found that, under special assumptions on the nature of Pulsar 1937+214 and the stability and equilibrium of general relativistic rotating Q-star fluids, a much stronger new limit $M_{\text{Q-star}} \leq 5.3M_\odot$ emerges for EOS (2). Similar limits almost certainly apply to all Q-stars.

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FIGURE CAPTIONS

1) The curlique is the locus of Q-stars for a particular theory (fixed α_V and U_0). The solid line is the locus of maximum masses of such curves as α_V is varied, ζ held constant. (Here $\zeta = 20$.) That locus ends when U_0 is at its maximum value for which the corresponding Q-ball is critically bound.

2) Various equilibrium and stability curves for objects rotating at the angular velocity of the fast pulsar. The straight solid lines are the black hole limit for non-spinning objects and the locus of termination points for varying ζ, to the right of and below which all stable Q-stars lie. The other curves are various stability limits for stars that can rotate with a period of 1.558 ms. For each type of stability considered, allowed stars lie *above* the relevant curve. The dashed curve with the Xs is the mass shed limit for incompressible stars, the Xs being data points taken from the general relativistic numerical calculations of Butterworth and Ipser. The solid curve is the $t = .08$ ($m = 4$) stability curve, drawn from the three points A, B, and C derived from the same calculations. The dot-dash curve is mass shedding for $n = 1$ polytropes, and the corresponding $m = 4$ ($t = .058$) curve is the dotted one. Coincidentally, the dotted curve also lies indistinguishably close to the Newtonian incompressible stability limit (4). The dashed curve is the Roche model for centrally condensed stars.

3) Q-star density profiles.

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TABLE 1

	$M_0\mathcal{E}^{1/2}$	$e(t_N = .08)$	$\Omega\mathcal{E}^{-1/2}(t_N = .08)$	$\Omega\mathcal{E}^{-1/2}(t_N = .14)$
A:	.077	.581	.968	1.2
B:	.16	.461	1.02	1.3
C:	.215	.295	1.05	1.35

Critical parameters for three different strengths of relativity. See text for explanations.

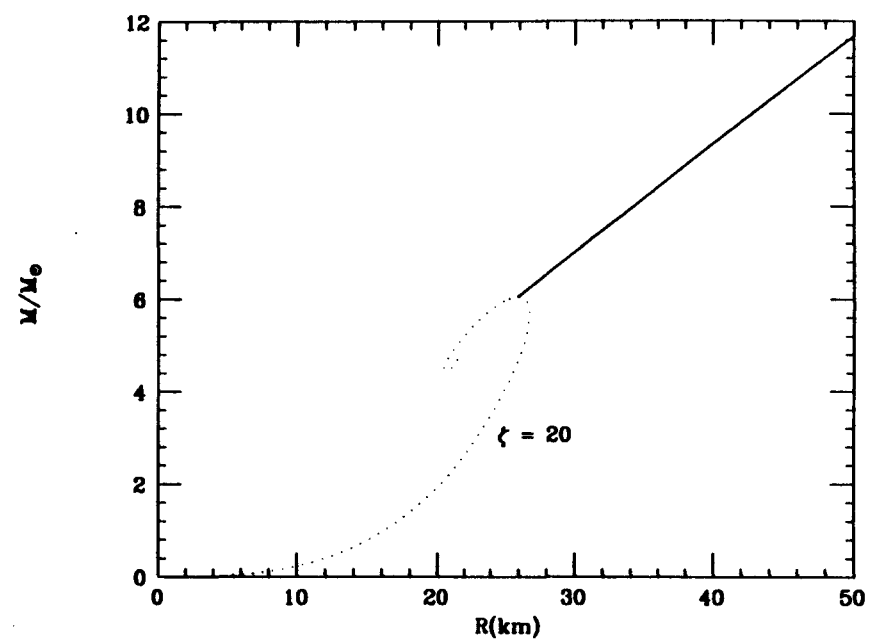

Figure 1

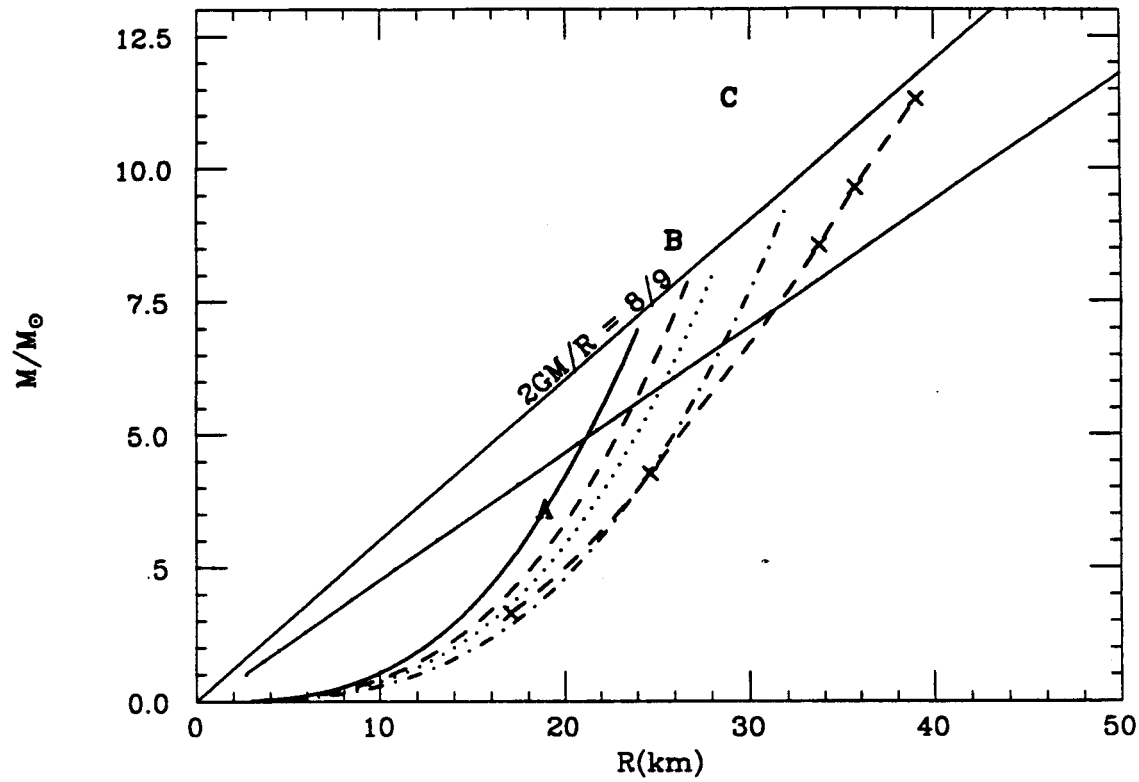

Figure 2

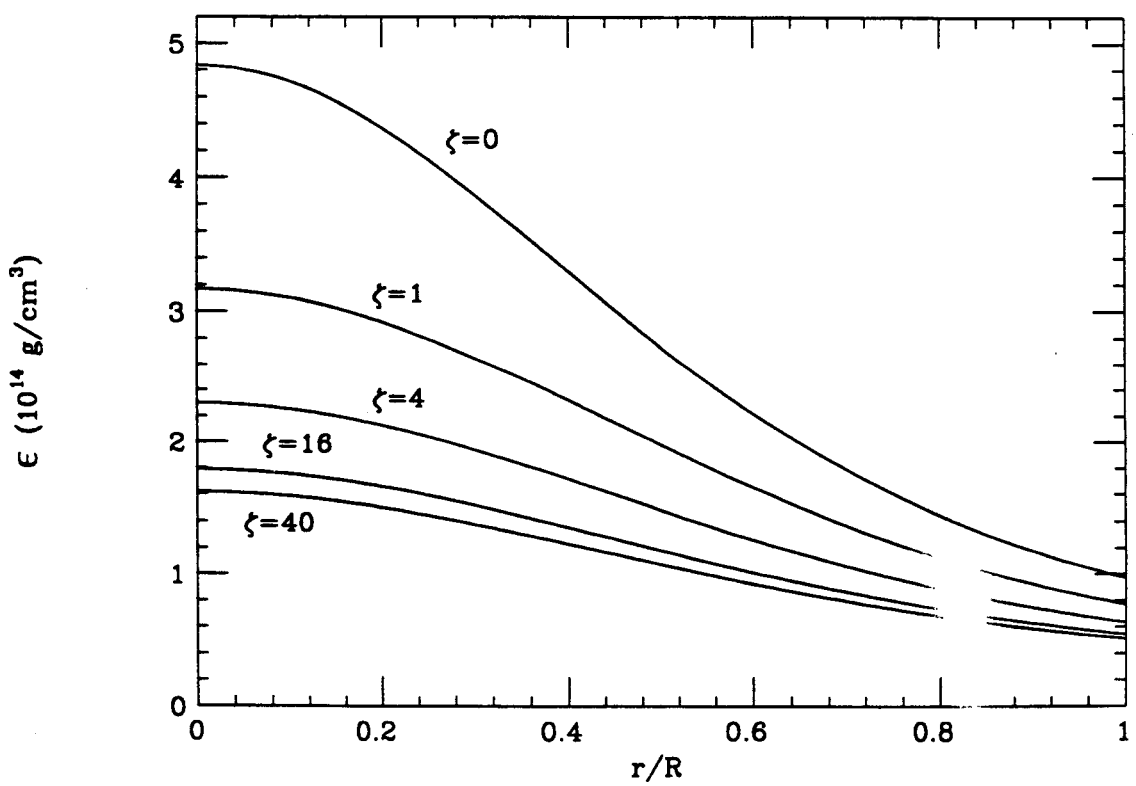

Figure 3