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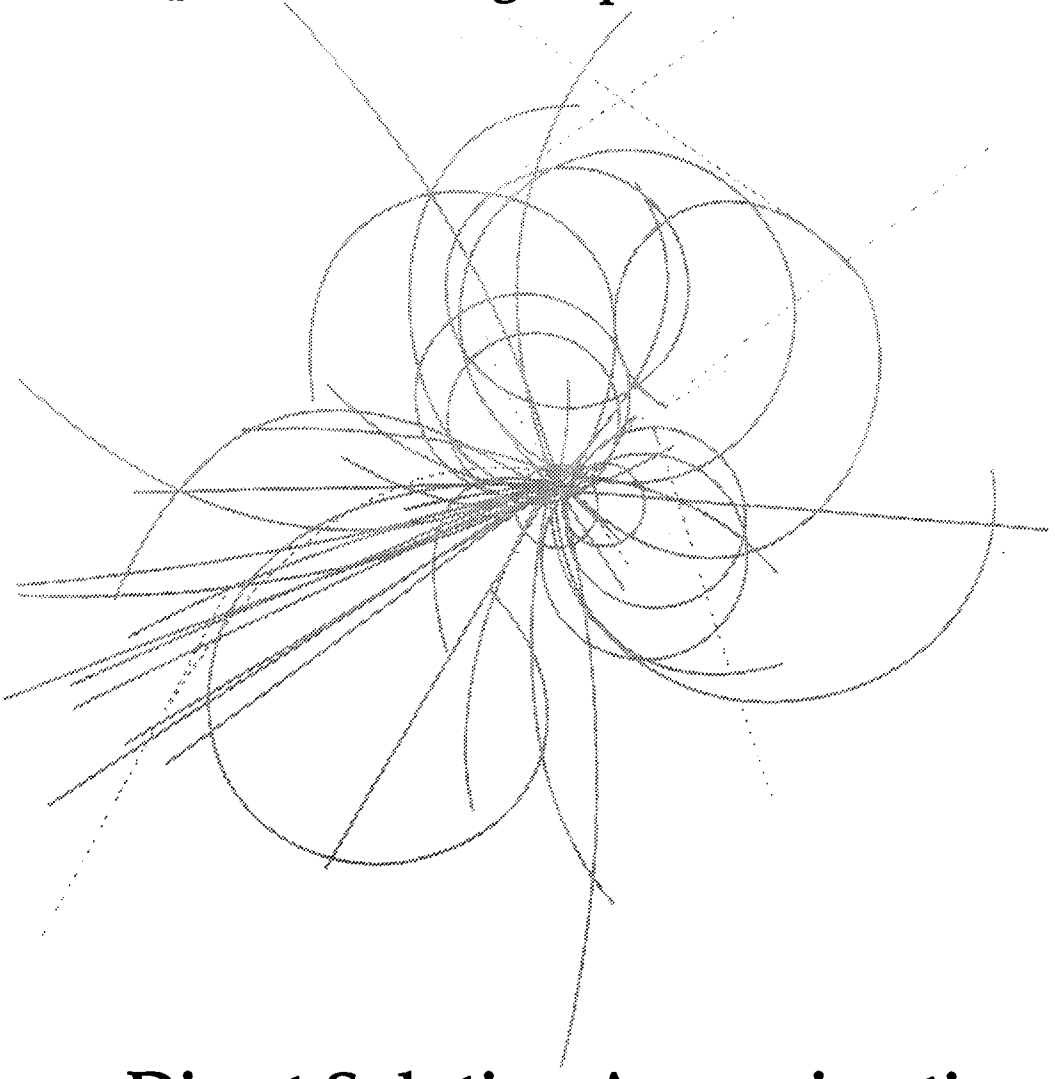

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DIRECT SOLUTION APPROXIMATION TO THE THERMAL HYDRAULIC QUENCHBACK IN SUPERCONDUCTING CABLES

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Using an approximation to the conduction fluid equations, a solution is given based on direct integration. The solution is used to establish a relationship between quench velocity and conductor length in a cable-in-conduit-type configuration.

INTRODUCTION

When a normal zone appears in a superconducting (SC) cable, it propagates axially with a so called "quench velocity", V_q, which depends mainly on the conductor characteristics and very little on the cooling effect of helium [1]. However, if the ratio of the longitudinal to transversal dimensions of the conductor is quite large, the expansion of the heated helium, in the normal zone, may drive fluid elements far from the initial normal zone, which could induce other normal zones in the conductor because of the compression of these fluid elements and their friction with cable strands. The quench velocity would have much higher value than the pure Fourier conduction mechanism, and this is called thermohydraulic quench back (THQB) mechanism. After the numerical discovery [2] of the THQB mechanism, an analytical approximation to this phenomenon appeared [3] using the similarity method for the differential equations. Although this solution suggests high quench velocities for the THQB mechanism, an experiment [4] seems to contradict its predictions. In addition, the solution predicts much faster fluid velocities for short SC cables than for long ones which makes this solution unlikely. In this paper, a direct method of integration is used in the differential equation (derived from approximations) in order to obtain applicability limits to the solution.

APPROXIMATIONS

For one-dimensional fluid (helium) moving in a long tube pipe of diameter D and length L, satisfying the relation $D/L << 1$, and being heated up by the quenching superconductor (quench appears in the SC surounding the tube pipe at the origin, $z = 0$), the equations which govern the state of the fluid are:

The continuity equation,

$$\frac{d\rho}{dt} + \rho \frac{\partial v}{\partial z} = 0 \ , \tag{1}$$

the momentum conservation equation,

$$\rho \frac{dv}{dt} = -\frac{\partial p}{\partial z} - \rho F \ , \tag{2}$$

the mechanical energy conservation equation,

$$\rho \frac{d}{dt}(e + \frac{1}{2}v^2) = -\frac{\partial (pv)}{\partial z} + q \ , \tag{3}$$

the state equation,

$$d\rho = \frac{1}{c^2}dp - \frac{B\rho}{c_p}T \ ds \ , \tag{4}$$

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and the second law equations

$$de = Tds + d\omega \ , \tag{5}$$

In the preceding equations, c is the speed of sound, B is the voume coefficient of thermal expansion, c_p is the specific heat, T is the temperature, e is the specific internal energy, s is the specific entropy, q is the power density entering the helium, p is the pressure, ρ is the density, v is the velocity of the fluid, and F is given in terms of the Fanning friction factor, f as

$$F = \frac{2fv^2}{D} \ . \tag{6}$$

The operator d/dt is defined as

$$\frac{d}{dt} = \frac{\partial}{\partial t} + v \ \frac{\partial}{\partial z} \ . \tag{7}$$

In the model, the hot-zone (helium gas), which is located where the quench appears, is separated from the cold-zone (helium fluid) by a hot-cold interface front which has a velocity $\dot{Z}$ at its location, $z = Z$. The initial condition for any fluid element is

$$v(z, 0) = 0 \ , \tag{8}$$

and the boundary conditions (assuming an infinity cable for the moment) are

$$v(\infty, t) = 0 \tag{9a}$$

and

$$v(z = Z, t) = \dot{Z} \ . \tag{9b}$$

Using Eqs. (2) and (3), and identifying $p(\partial v/\partial z)/\rho$ with the rate of change of the external work, $d\omega/dt$, Eq. (5) is written as

$$Tds = \frac{q}{\rho} + vF \ , \tag{10}$$

where vF represents the entropy production due to the irreversible conversion by friction of kinetic energy to internal energy [3]. Using Eqs. (10) and (7) in Eq. (4), it follows

$$\frac{d\rho}{dt} = \frac{1}{c^2}\left(\frac{\partial p}{\partial t} + v \ \frac{\partial p}{\partial z}\right) - \frac{B\rho}{c_p}\left(\frac{q}{\rho} + Fv\right) \ . \tag{11}$$

Assuming now $dv/dt = 0$,

$$\frac{\partial p}{\partial z} = -\rho F \ , \tag{12}$$

in Eq. (2), and using this in Eq. (11) along with the assumption that $\rho = constant$ and $c^2 = constant$, the following expression is obtained after some rearrangements

$$\frac{\partial v}{\partial z} + \frac{1}{\rho c^2}\frac{\partial p}{\partial t} = \frac{B}{c_p}\left[\frac{q}{\rho} + \frac{4mf}{D} \ v^3\right] \ , \tag{13}$$

where m is defined as

$$m = \frac{1}{2}(1 + \frac{c_p}{Bc^2}) \ . \tag{14}$$

Consider now that the Joule power heating density does not depend on z $(\partial q/\partial z = 0)$, then using the expression

$$\frac{\partial^2 p}{\partial t \partial z} = -\frac{4\rho f v}{D}\frac{\partial v}{\partial t} \tag{15}$$

in the partial differentiation with respect to z of Eq. (13), it follows

$$\frac{\partial^2 v}{\partial z^2} = \frac{4fv}{Dc^2}\frac{\partial v}{\partial t} + \frac{3B}{c_p}(\frac{4mf}{D})v^2\frac{\partial v}{\partial z} \ . \tag{16}$$

Finally, using Eqs. (12) and (16) and rearranging terms, the following equation results

$$\frac{\partial^2 v}{\partial z^2} = (m\beta - \alpha/3)\frac{\partial v^3}{\partial z} , \tag{17}$$

where α and β are given by

$$\alpha = \frac{4f}{Dc^2} \tag{18a}$$

and

$$\beta = \frac{4Bf}{c_p D} . \tag{18b}$$

The solutions of Eq. (17) are given by the following quadrature

$$\int \frac{dv}{a(t) + (m\beta - \alpha/3)v^3} = z + b(t) , \tag{19}$$

where the functions $a(t)$ and $b(t)$ are determined by the boundary and initial conditions.

CONDITIONS AND SOLUTION

For $a(t) > 0$ and $m\beta - \alpha/3 > 0$, the solutions obtained from Eq. (19) are given by

$$\frac{1}{a(t)\xi}\left[\frac{a(t)}{m\beta - \alpha/3}\right]^{1/3} \log\frac{1+\xi}{\sqrt{1-\xi+\xi^2}} + \frac{1}{a(t)\sqrt{3}}\left[\frac{a(t)}{m\beta - \alpha/3}\right]^{1/3} \arctan\left(\frac{\sqrt{3}\,\xi}{2-\xi}\right) = z + b(t) \tag{20a}$$

where ξ is defined as

$$\xi = \left[\frac{m\beta - \alpha/3}{a(t)}\right]^{1/3} v .$$

For $m\beta - \alpha/3 < 0$ and $a(t) > 0$, or $m\beta - \alpha/3 > 0$ and $a(t) < 0$, the solution can be written as

$$\frac{1}{2|a(t)|}\left|\frac{a(t)}{m\beta - \frac{\alpha}{3}}\right|^{1/3} \log\left(\frac{\xi - 1}{\xi + 1}\right) = z + b(t) \tag{20b}$$

For $m\beta - \alpha/3 = 0$ and $a(t) \neq 0$, the solution is

$$\frac{v}{a(t)} = z + b(t) . \tag{20c}$$

And for $m\beta - \alpha/3 \neq 0$ and $a(t) = 0$, the solution is expressed as

$$\frac{-1}{2(m\beta - \alpha/3)v^2} = z + b(t) . \tag{20d}$$

It is not difficult to see that in order to satisfy the condition of Eq. (9a), the function $a(t)$ must be zero which leads one to select the solution of Eq. (20d), and applying the condition of Eq. (9b) to this, the following solution results

$$v(z,t) = \frac{\dot{Z}}{\sqrt{1 - g(z - Z)\dot{Z}^2}} , \tag{21a}$$

where g is defined as

$$g = 2(m\beta - \alpha/3) = \frac{8f}{D}\left[\frac{B}{2c_p} + \frac{1}{6c^2}\right] . \tag{21b}$$

Note that in order for the solution to have physical meaning, the relations $z \geq Z$ and $1 - g(z - Z)\dot{Z}^2 > 0$ must be satisfied. The simplest way to satisfy the condition of Eq. (8) is to ask for Z the dependence

$$Z(t) = X_o t^n \ , \tag{22}$$

where $n > 1$ and X_o are constants. This relation is in accordance with the experiments [4]. The expression Eq. (21a) gives the velocity of the fluid elements farther apart from the hot-cold interface front. This fact, together with the preceding observation, imposes the following restriction in the space-time coordinates for the validity of the expression Eq. (21a)

$$0 \leq z - Z \leq \frac{1}{g\dot{Z}^2} \quad , t \geq 0 \tag{23}$$

i.e., the size of the region, ahead of the hot-cold interface front, where Eq. (21a) is applicable decreases inversely proportional to the square of the hot-spot interface front velocity. For $g = 0$, Eq. (21a) is valid everywhere in the fluid, and each element has the same velocity, $\dot{Z}$.

On the other hand, defining $v_L = v(L,t)$ and $v_l = v(l,t)$, one sees from Eq. (21a) that the following relation is satisfied

$$v_L^2 - v_l^2 = g\ v_L^2 v_l^2 (L - l) \ .$$

So, the fluid velocity will be higher for longer SC cables ($L > l$).

TEMPERATURE AND PRESSURE IN THE FLUID

Making use of the relation Eq. (12), the pressure rise in the fluid, at the point z and at the time t, is obtained by integrating the equation

$$p(z,t) = -\frac{2\rho f}{D} \int v^2 dz \ . \tag{24}$$

Substituting Eq. (21a) in Eq. (24) and integrating, the next expression arises

$$p(z,t) = \frac{2f\rho}{Dg} \log \left[1 - g(z - Z)\dot{Z}^2\right] + d(t) \ , \tag{25}$$

where $d(t)$ is an arbitrary function determined by the boundary conditions. Assume that at the distance $z = L$, the pressure drops to zero, then the pressure is given by

$$p(z,t) = \frac{2f\rho}{Dg} \log \left[\frac{1 - g(z - Z)\dot{Z}^2}{1 - g(L - Z)\dot{Z}^2}\right] \ . \tag{26}$$

The temperature at this point of the differential fluid element "dz" has two contributions. One is due to the pressure itself,

$$p(z,t) \left(\frac{\partial T}{\partial p}\right)_V \ , \tag{27}$$

and the other arises from the wall shear stress ($f_w = D(\rho F)/4$) at the point z,

$$\frac{2f}{c_p D} \int_0^t v^3(z,\tau) d\tau \ . \tag{28}$$

(The mass of the fluid element in consideration is $m = \pi\rho\ D^2 dz/4$, and the volume generated in the differential dz-displacement is $dV = \pi D v dz dt$). The total temperature rise is given by

$$\Delta T(z,t) = \frac{2f}{c_p D} \int_0^t v^3(z,\tau) d\tau + p(z,t) \left(\frac{\partial T}{\partial p}\right)_V \ . \tag{29}$$

THE TIME ONSET THQB AND FINISH TIME

The time at which the differential fluid elements adjacent to the hot-cold interface front reach the temperature T_{cs} (current-sharing threshold temperature) is called "The Time Onset THQB," and it is given by

$$\Delta T(z = Z, t_{on}) = (\Delta T)_{on} \ , \tag{30}$$

or using Eqs. (29), (26), and (9b), this time results from the solution of the equation

$$\frac{2fn^3X_o^3t_{on}^{3n-2}}{c_pD(3n-2)} - \left(\frac{\partial T}{\partial p}\right)_V \frac{2f\rho}{Dg} \times \log\left[1 - g(L - X_o t_{on}^n)n^2X_o^2t_{on}^{2n-2}\right] = (\Delta T)_{on} \ . \tag{31}$$

The time at which the entire conductor becomes normal is called "The Finish Time," and it is given by

$$\Delta T(z = L, t_f) = (\Delta T)_{on} \ . \tag{32}$$

This time is explicitly expressed as the solution of the following equation

$$\frac{2fn^3X_o^3}{c_pD}\int_0^{t_f} \frac{\tau^{3n-3}d\tau}{[1 - g(L - X_o\tau^n)n^2X_o^2\tau^{2n-2}]^{3/2}} = (\Delta T)_{on} \ . \tag{33}$$

As Figure 1 shows, for two SC cables of lengths L and l such that $L \geq l$, the finish time satisfies the relation

$$t_f(L) \leq t_f(l) \ , \tag{34}$$

so the THQB induced quench velocity never is higher for shorter SC cables.

In order for the THQB mechanism to be observed, the finish time must be smaller than the normal quench delay time, $t_q = L/V_q$, and the time of the end-quench phenomenon, t_{end}. This last time is the time taken for the device to fall off its stored energy. Then, in addition to the condition of Eq. (23), t_f must be satisfy the relation

$$\frac{L}{c} \leq t_f < min\{t_q, t_{end}\} \ . \tag{35}$$

In addition, this mechanism also depends on $(\Delta T)_{on}$ which is defined by the SC characteristics as well as its magnetic field. As a numerical example, using the following parameters [4], $n = 4/3$, $f = 0.02$, $D = 0.707mm$, $B = 0.1176\ K^{-1}$, $c_p = 8400\ J/K_gK$, $c = 131\ m/s$, $X_o = 1$, and $L = 25\ m$, it follows

$$g = 3.782 \times 10^{-3}$$

and

$$\frac{2f}{c_pD} = 6.735 \times 10^{-3}\ Ks^2m^{-3} \ .$$

Then, from Figure 1, the time to see the entire cable-in-conduit normal is higher than 5 seconds!, so a THQB mechanism is not expected here.

For $n = 4/3$, the relations of Eqs. (22) and (23) bring about the following restriction on the parameters and time

$$1 - \frac{16\ g}{9}(L - X_o t^{4/3})X_o^2t^{2/3} > 0 \ . \tag{36}$$

Therefore, given the parameters $g, L,$ andX_o, Eq. (36) may impose a restriction in the time if there is a positive real root on its polynomial associated. The possible real root is given by

$$t_{co} = \frac{\left\{3X_o\sqrt{-\frac{L^3}{27X_o^3} + \frac{81}{1024g^2X_o^6}} - \frac{9}{32gX_o^3}\right\}^{2/3} + L}{\left\{3X_o\sqrt{-L^3/27X_o^3 + 81/1024g^2X_o^6} - 9/32gX_o^3\right\}^{1/3}} \ . \tag{37}$$

If the parameters are chosen such that

$$\frac{1024}{2187}\, g^2 (L X_o)^3 > 1 \,, \tag{38}$$

the root of Eq. (37) would be a complex number. Therefore, t_f in Eq. (33) has non restrictions. On the other hand, if Eq. (38) is not satisfied, the restriction would be $t_f < t_{co}$.

However, it is possible to start selecting the parameter t_f by choosing the wanted induced THQB quench velocity ($v_{THQB} = L/t_f$). In this case, Eq. (36) imposes a restriction on the parameters which can be written, using Eq. (21b), as

$$D > \frac{128\, f}{9} \left[\frac{B}{2c_p} + \frac{1}{6c^2} \right] \left(L - X_o t_f^{4/3} \right) X_o^2 t_f^{2/3} \,. \tag{39}$$

The parameters g and X_o are criticals for the THQB to be observed as is shown on Figure 2, where all the parameters are the same as above but D and X_o.

CONCLUSION

Using the direct integration approach in the one dimensional differential equations, a restrictive fluid velocity expression was obtained which satisfies the initial and boundary conditions. This expression brings about a quench velocity v_{THQB} which increases with the length of the conductor in a quench event. However, more experimental work is required to establish and to fully understand this THQB phenomenon.

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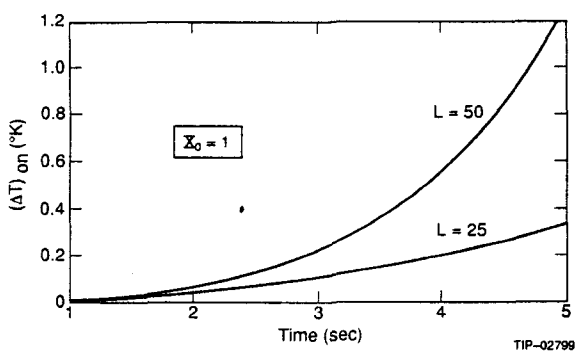

Figure 1. $(\Delta T)_{on}$ as a function of the Finish Time.

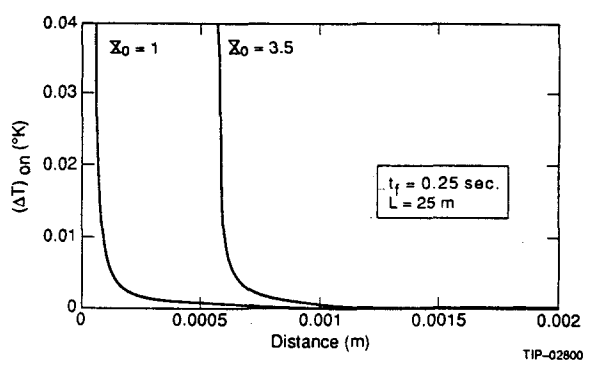

Figure 2. $(\Delta T)_{on}$ as a function of the Diameter.

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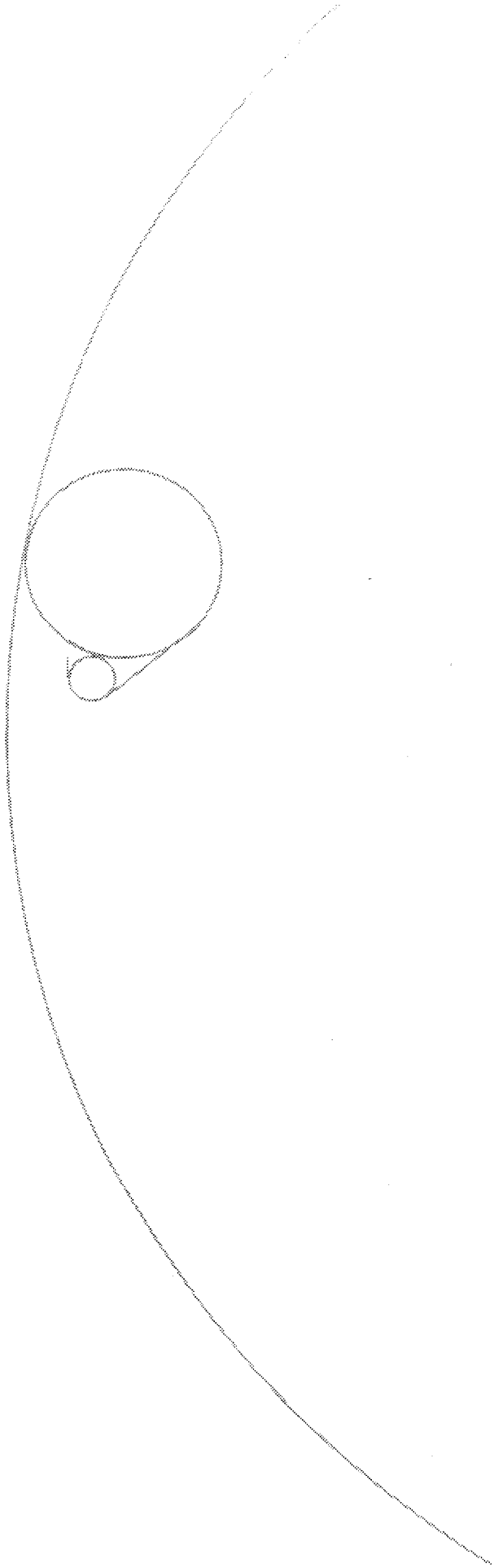
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