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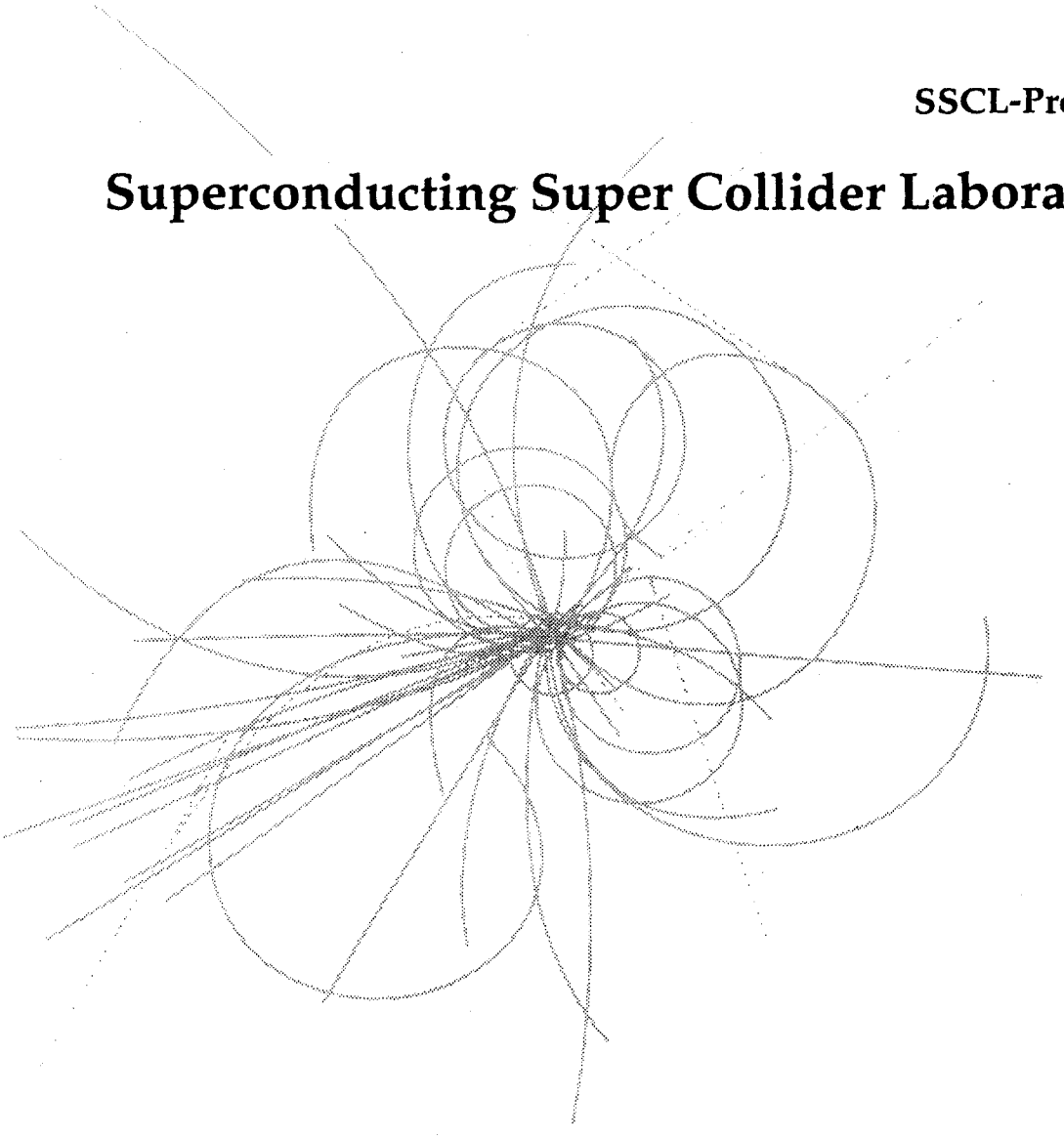

Chiral SU $(2)_L \times$ SU $(2)_R$ Liquids: A Theory of Heavy Nuclei and Neutron Stars

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May 1992

SU-ITP-92/3 SSCL-114/92
May, 1992

CHIRAL $SU(2)_L \times SU(2)_R$ LIQUIDS: A THEORY OF HEAVY NUCLEI AND NEUTRON STARS

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Submitted to *Nuclear Physics B*

ABSTRACT

The global $SU(2)_L \times SU(2)_R$ symmetries of QCD alone probably imply the existence of a liquid phase consisting of protons, neutrons and pions. Drops of this "chiral liquid" at zero external pressure emerge as saturating (the baryon number grows roughly as the volume) nontopological solitons of the low energy non-linear chiral effective Lagrangian theory of pions and nucleons, where the pions are the pseudo-Goldstone bosons resulting from the spontaneous breaking of $SU(2)_{L-R}$ by the QCD vacuum. The effective Lagrangian approach gives us control of both the tree and quantum loop levels of the theory. A crucial role is played by explicit $SU(2)_{L-R}$ breaking terms whose origin lies in quark masses; an isosinglet combination of even numbers of pions carries the primary long-range attractive force. If non-linear chiral symmetry contains such saturating nontopological soliton field configurations, they would give a chiral-symmetric explanation for the very existence of ordinary heavy nuclei, which are then to be regarded as just droplets of chiral liquid. This picture, reminiscent of the old liquid drop modelof heavy nuclei, vastly simplies the extraction of bulk nuclear properties from chiral symmetry; nucleons, treated as Fermi fluids inside a heavy nucleus, move within a huge coherent self-consistent classical pion field $< \vec{\pi} > \approx 200 - 400$ MeV. Neutron stars are then just great chunks of neutral chiral liquid held together by gravity. Quantum chiral liquids can have interesting macroscopic quantum properties such as parity doubling. these may distinguish experimentally the chiral liquid theory of heavy nuclei and neutron stars from more conventional models.

1 INTRODUCTION

The two best-known facts of low energy hadronic physics are the existence of ordinary nuclei and the validity, in scattering experiments, of chiral $SU(2)_L \times SU(2)_R$ symmetry. There is, of course, a long history of attempts to synthesize these two facts, and the idea that spontaneously broken chiral symmetry is the origin of nuclear forces is an old one.[1][2][3][4] The full power of the current algebra of modern chiral symmetry is most clear in the language of relativistic quantum field theory. Relativistic field theory was first applied to nuclear structure by T.D.Lee and G.Wick.[5] Their seminal paper on non-topological solitons focused on possible islands of nuclear stability for very large baryon number. Walecka and co-workers first applied the mathematics of relativistic non-topological solitons to the structure of ordinary nuclei resulting in their famous "$\sigma-\omega$" model[6] ; this also allows a consistent treatment of nuclear matter under extreme conditions such as in neutron stars. Until now, however, no relativistic mean field model of ordinary nuclei has successfully incorporated chiral symmetry.[6][7]*

S. Weinberg[9] has crystalized the nature of spontaneousely broken chiral symmetry: "The lesson taught by current algebra is that a symmetry like chirality does not manifest itself in linear γ_5 invariance relations, which would require the vanishing of the nucleon mass, but rather it determines relations between an arbitrary (hadronic) process $\alpha \rightarrow \beta$ and the related process $\alpha \rightarrow \beta + n\pi$ ... and ... will *not* tell us that the mass of ... (the nucleon) vanishes". Nuclear structure is precisely such a chiral soft nucleon and pion process; inside a nucleus, protons and neutrons emit and absorb an infinite number of soft pions and these pions also interact with each other in a highly non-linear way. That pions are (almost) the Goldstone bosons of the spontaneous breaking of $SU(2)_{L-R}$ allows us to specify these non-linearities *uniquely* for low-energy hadronic processes. Massless Gold-

* Although a consistent model of ordinary nuclei and chiral pseudo-Goldstone bosons based on the non-linear realization of chiral $SU(3)_L \times SU(3)_R$ symmetry with a singlet σ has appeared,[8] that model suffers from lack of predictive power precisely because the σ was taken to be a chiral singlet.

stone bosons[10] have only derivative couplings and therefore cannot interact with a static constant baryon density such as in the interior of heavy saturating nuclei. In contrast, the observed volume nuclear binding energy is $\frac{E}{B} - m_N \approx -16MeV$ (E, B, m_N are the nuclear energy and baryon number and free nucleon mass). Therefore, terms in the effective Lagrangian whose origin lies in quark masses and which break $SU(2)_{L-R}$ explicitly must be very important; these give pions mass and non-derivative couplings. The non-linear chiral interactions of point nucleons and pions result in a "classical" self-consistent mean pion field in which a large number of nucleons, treated as a Fermi fluid with Pauli exclusion in the Thomas-Fermi approximation, are trapped to form heavy nuclei: a droplet of what we will call an "$SU(2)_L \times SU(2)_R$ chiral liquid". This is precisely the mathematics of non-topological solitons in relativistic field theories[5] [11] [12] and opens the possibility that the theory may have a liquid phase and that the structure of heavy nuclei and neutron stars may then be tractible, at least at a crude level.

The plan of this paper is as follows: In Section 2 we review chiral perturbation theory, construct the effective low-energy Lagrangian and Hamiltonian and write a variational equation for the ground state of a given baryon number and third component of isospin. In Section 3 we search for a liquid phase of the theory, and, at zero external pressure, droplets of chiral liquid (non-topological solitons). We also examine some interesting macroscopic quantum properties of chiral liquids. Section 4 identifies droplets of chiral liquid with heavy nuclei. Section 5 identifies very large chunks of chiral liquid (baryon number $B \sim 10^{57}$), in which the effects of gravity must be included, as neutron stars. Section 6 gives some conclusions.

2 CHIRAL PERTURBATION THEORY

2.1 Nucleon-Pion Chiral Lagrangian from QCD

This paper examines QCD with only two light quark flavors arranged in a flavor doublet $q = \binom{u}{d}$; all effects due to heavier quarks are neglected. Left-handed chiral quarks, right-handed chiral quarks and eight gluon color gauge fields $q_L = \frac{1+\gamma_5}{2}q$, $q_R = \frac{1-\gamma_5}{2}q$ and $G_\mu = G^i_\mu \lambda^i$ (λ^i are eight Gell-Mann matrices normalized such that

$Tr\lambda^i\lambda^j = \frac{1}{2}\delta^{ij}$) are arranged into covariant derivatives $\mathcal{D}_\mu = \partial_\mu + igG_\mu$ and field strengths $G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu + ig[G_\mu, G_\nu]$ so that the Lagrangian of massless QCD

$$\mathcal{L}_{QCD}^{SU(2)_L \times SU(2)_R} = i\bar{q}_L\gamma^\mu\mathcal{D}_\mu q_L + i\bar{q}_R\gamma^\mu\mathcal{D}_\mu q_R - \frac{1}{2}Tr(G_{\mu\nu}G^{\mu\nu}) \tag{1}$$

has, in addition to local $SU(3)_{color}$ symmetry, a global continuous $SU(2)_L$ symmetry $q_L \to Lq_L$ and a global continuous $SU(2)_R$ symmetry $q_R \to Rq_R$ where L and R are unitary, independent and arbitrary (2×2 matrices in this flavor representation). The basic hypotheses of hadronic chiral symmetry are that the effective theory of hadrons will have the *same* global $SU(2)_L \times SU(2)_R$ continuous symmetries as QCD (since, unlike the axial $U(1)$ symmetry of classical QCD, global chiral symmetries are not disturbed by anomalies at the quantum loop level) and that the QCD vacuum breaks spontaneously the global continuous subsymmetry $SU(2)_{L-R}$.

The chiral algebra[9] is written in terms of the generator-charges of $SU(2)_{L-R}$ ($L \neq R$) rotations $\mathcal{Q}_{5a}$ and $SU(2)_{L+R}$ ($L = R$) isospin rotations $\mathcal{Q}_a$ with $a = 1,2,3$:

$$\begin{aligned} [\mathcal{Q}_{5a}, \mathcal{Q}_{5b}] &= i\epsilon_{abc}\mathcal{Q}_c \\ [\mathcal{Q}_a, \mathcal{Q}_{5b}] &= i\epsilon_{abc}\mathcal{Q}_{5c} \\ [\mathcal{Q}_a, \mathcal{Q}_b] &= i\epsilon_{abc}\mathcal{Q}_c \end{aligned} \tag{2}$$

In the non-linear realization of chiral symmetry, the pion π_b ($b = 1,2,3$) and nucleon $\Psi = \binom{P}{N}$ transformation rules are

$$\begin{aligned} [\mathcal{Q}_{5a}, \pi_b] &= -iF[\frac{1}{2}(1 - \frac{\vec{\pi}^2}{F^2})\delta_{ab} + \frac{\pi_a\pi_b}{F^2}] \\ [\mathcal{Q}_a, \pi_b] &= i\epsilon_{abc}\pi_c \\ [\mathcal{Q}_a, \Psi] &= -t_a\Psi \\ [\mathcal{Q}_{5a}, \Psi] &= F^{-1}\epsilon_{abc}t_b\pi_c\Psi \end{aligned} \tag{3}$$

where $F \equiv 2f_\pi$ and t_a are Hermitian isospin matices normalized so that the commutator $[t_a, t_b] = i\epsilon_{abc}t_c$. The experimental pion decay amplitude sets the spontaneous

symmetry breaking parameter $f_\pi = 93$ MeV. The chiral covariant derivatives for pions and nucleons generate nontrivial hadron dynamics

$$\begin{aligned} D_\mu \vec{\pi} &= \frac{\partial_\mu \vec{\pi}}{1 + \frac{\vec{\pi}^2}{F^2}} \\ D_\mu \Psi &= (\partial_\mu + 2iF^{-2}\epsilon_{abc} t_a \pi_b D_\mu \pi_c)\Psi \end{aligned} \tag{4}$$

and their $SU(2)_L \times SU(2)_R$ transformation properties are easily worked out

$$\begin{aligned} [\mathcal{Q}_a, D_\mu \pi_b] &= -i\epsilon_{abc} D_\mu \pi_c \\ [\mathcal{Q}_{5a}, D_\mu \pi_c] &= iF^{-1}(\delta_{ab}\delta_{cd} - \delta_{ad}\delta_{bc})\pi_c D_\mu \pi_d \\ [\mathcal{Q}_a, D_\mu \Psi] &= -t_a D_\mu \Psi \\ [\mathcal{Q}_{5a}, D_\mu \Psi] &= F^{-1}\epsilon_{abc} t_b \pi_c D_\mu \Psi \end{aligned} \tag{5}$$

These transformation laws are the most general under non-linear $SU(2)_{L-R}$ while linear under $SU(2)_{L+R}$; all others can be shown to be equivalent to these up to redefinitions of the pion fields.[13][9] The generality of the above transformations frees us from any (experimentally unjustified) prejudices as which chiral multiplet the nucleons belong.

The vacuum is hypothesized to be a complicated non-perturbative condensate of quarks and gluons and, at the hadronic level, is chosen to contain no pion expectation value. Although the effective chiral Lagrangian of nucleons and pions which follows from massless QCD is invariant under $\mathcal{Q}_{5a}$ and $\mathcal{Q}_a$ rotations the vacuum is not: $\langle 0|[\mathcal{Q}_{5a}, \pi_b]|0\rangle \sim -if_\pi \delta_{ab}$ so that while $\mathcal{Q}_a|0\rangle = 0$, $\mathcal{Q}_{5a}|0\rangle \neq 0$ and π_a are manifestly the three massless Goldstone bosons[10] of spontaneously broken $SU(2)_{L-R}$.

Of course, pions are not really massless; $m_{\pi^+} \simeq 140$ MeV. Isospin-conserving quark masses (whose origin presumably lies in weak interactions) $m_q \equiv \frac{m_u + m_d}{2} \neq 0$ are introduced into two-flavor QCD by a term

$$\mathcal{L}_{QCD}^{SU(2)_{L+R}} = -m_q(\bar{q}_L q_R + \bar{q}_R q_L) \tag{6}$$

which conserves $SU(2)_{L+R}$ isospin exactly. It breaks $SU(2)_{L-R}$ explicitly and

makes pions only *pseudo*-Goldstones.

The effective Lagrangian controlling the interactions of nucleons and pions is to be invariant under non-linear $SU(2)_{L-R}$ and linear $SU(2)_{L+R}$ rotations for $m_q = 0$ and linear $SU(2)_{L+R}$ rotations for $m_q \neq 0$ as well as parity, charge conjugation and time-reversal; it will in principle contain all terms consistent with these symmetries. Chiral perturbation theory is an expansion in inverse powers of a large chiral symmetry breaking scale Λ_{CSB} order-by-order in pion momenta and nucleon off-shell momenta. According to "naive dimensional analysis",[14] a given term has the power-counting structure (the derivative ∂ acts on the pion or nucleon fields, the n_i are integers and constants (such as γ^μ and t_a matrices) are not shown)

$$f_\pi^2 \Lambda_{CSB}^2 (\frac{\bar{\Psi}}{f_\pi \sqrt{\Lambda_{CSB}}})^{n_1} (\frac{\Psi}{f_\pi \sqrt{\Lambda_{CSB}}})^{n_1} (\frac{\pi}{f_\pi})^{n_2} (\frac{\partial}{\Lambda_{CSB}})^{n_3} (\frac{m_q}{\Lambda_{CSB}})^{n_4} \tag{7}$$

with coefficient of order one. It is here that chiral perturbation theory shows its power; it is able to control its own quantum loop structure by self-consistency. Quantum loops calculated with the effective chiral Lagrangian and an ultra-violet cut off at loop momentum Λ_{CSB} will generate all operators consistent with the symmetries of the theory, thereby renormalizing the coefficients of the operators in the original chiral Lagrangian (as well as generating important non-analytic terms). The crucial point is that chiral quantum loop corrections will also obey naive dimensional power counting[15] [14] [16] generating operators with the power counting of (7)with an additional factor of $(\frac{\Lambda_{CSB}}{4\pi f_\pi})^{2L}$ where L is the number of quantum loops in the diagram. If we now choose* the cut-off $\Lambda_{CSB} < 4\pi f_\pi = 1.17$ GeV, quantum loops will renormalize the coefficients of the chiral operators (7)to again be of order one. Thus, in chiral symmetric theories of soft hadronic processes, quantum loops are just self-consistent power counting in 4π. Typically Λ_{CSB} is chosen ≈ 600 MeV to 1 GeV. This allows the self-consistent low-energy truncation of the effective

* The chiral symmetry breaking scale Λ_{CSB} arises, like f_π, from the spontaneous breaking of $SU(2)_{L-R}$ so that it is not really an independent parameter. We can "choose" it by self-consistently changing the coefficients in (7)by numbers of order one.

Lagrangian for hadronic processes with $\approx 100 - 300$ MeV momentum transfer. After the renormalized coefficients, which now include quantum corrections, are fixed by experiments, the effective chiral Lagrangian can be examined at *tree* level[17] [9] [13] for low energy hadronic predictions.[16] Inclusion of all-orders strong quantum-loop corrections via the effective Lagrangian is crucial to the construction of chiral liquids. For the structure of heavy nuclei and neutron stars, it is a crucial difference between our approach and that of previous relativistic mean field models.

The analytic operators which follow from (1)and (7)to order Λ_{CSB} and $(\Lambda_{CSB})^0$ after spontaneous symmetry breaking are

$$\begin{gathered}\Lambda_{CSB}\bar{\Psi}\Psi,\ \ D_\mu\pi_a D^\mu\pi_a,\ \ \bar{\Psi}i\gamma^\mu D_\mu\Psi,\ \ \frac{i}{f_\pi}\bar{\Psi}\gamma^\mu\gamma^5 t_a\Psi D_\mu\pi_a \\ \frac{(\bar{\Psi}\Gamma^i\Psi)(\bar{\Psi}\Gamma_i\Psi)}{f_\pi^2},\ \ \frac{(\bar{\Psi}\Gamma^i t_a\Psi)(\bar{\Psi}\Gamma_i t_a\Psi)}{f_\pi^2}\end{gathered} \tag{8}$$

where $\Gamma^i = 1, \gamma^5, \gamma^\mu, i\gamma^\mu\gamma^5, \sigma^{\mu\nu}$. Typical of Goldstone bosons, pions couple only via their momenta in the "chiral limit" of massless quarks; this is the basis for soft pion theorems.

Quark masses break symmetry explicity rather than spontaneously. It is extremely important that (6)transforms as the fourth component of a vector, a $(\frac{1}{2},\frac{1}{2})$ representation under $SU(2)_L \times SU(2)_R$. Ward identities show that the corresponding term in the effective Lagrangian of nucleons and pions will, at the hadronic level, also transform as a $(\frac{1}{2},\frac{1}{2})$ representation of the algebra in (3)and (5)and conserve $SU(2)_{L+R}$. The analytic operators of order m_q which follow from (6)and (7)to order Λ_{CSB} and $(\Lambda_{CSB})^0$ are

$$m_q\Lambda_{CSB}f_\pi^2\frac{\frac{\vec{\pi}^2}{F^2}}{1+\frac{\vec{\pi}^2}{F^2}},\ \ m_q\bar{\Psi}\Psi\frac{\frac{\vec{\pi}^2}{F^2}}{1+\frac{\vec{\pi}^2}{F^2}} \tag{9}$$

Isospin breaking terms will also arise in QCD because $m_u \neq m_d$; among other things, a term $(m_d - m_u)(\bar{q}_L t_3 q_R + q_R t_3 q_L)$ will generate an effective hadronic term

$\pi_3\bar{\Psi}\pi_a t_a\Psi(1+\frac{\vec{\pi}^2}{F^2})^{-1}$. This paper will, except in the discussion of "parity doubling" in Section 3.5, neglect all such isospin breaking effects and regard electromagnetism as the only important source of isospin breaking.

We have neglected *for simplicity* non-analytic terms generated by quantum loops in the chiral perturbation expansion. The reader is warned: these can be important in $SU(3)_L \times SU(3)_R$[16] and may even affect our results in $SU(2)_L \times SU(2)_R$ chiral liquids, nuclear structure and neutron stars.

The effective chiral Lagrangian describing the dynamics of pions is properly constructed as a perturbation expansion in powers of pion momenta $\frac{p_\pi^\mu}{\Lambda_{CSB}} \ll 1$, and light quark masses $\frac{m_\pi}{\Lambda_{CSB}} \ll 1$. Nucleons, however, have chiral-invariant masses $m_N = 939$ MeV so that $\frac{m_N}{\Lambda_{CSB}} \approx 1$ and the appropriate expansion parameter is the off-shell momenta of (almost static) nucleons; $p_N^\mu = m_N v^\mu + k_N^\mu$ with v^μ the nucleon velocity and $\frac{k_N^\mu}{\Lambda_{CSB}} \ll 1$. The chiral perturbation expansion above uses instead nucleon four-momenta in the power counting so that terms with higher powers of nucleon time-derivatives are not suppressed and must be summed.* Assuming that the power-counting in (7)still works in the presence of such a large nucleon mass, the summation can be carried out in terms of a new nucleon field $\tilde{\Psi}(\vec{x},t) = exp(+im_N\gamma^0 t)\Psi(\vec{x},t)$[2] where $\frac{|\partial|}{\Lambda_{CSB}}\tilde{\Psi} \ll \tilde{\Psi}$. All but three of the operators which result just change the coefficients in (8)and (9)by numbers of order one and are therefore ignored. In addition, the summation gives three manifestly non-Lorentz invariant operators:

$$\frac{i}{f_\pi}\bar{\Psi}\gamma^0\gamma^5 t_a\Psi D_0\pi_a, \quad \frac{(\bar{\Psi}\gamma^0\Psi)(\bar{\Psi}\gamma_0\Psi)}{f_\pi^2}, \quad \frac{(\bar{\Psi}\gamma^0 t_a\Psi)(\bar{\Psi}\gamma_0 t_a\Psi)}{f_\pi^2} \tag{10}$$

Fermion exchange terms mix the last two sets of operators in (8)and (10).

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* More properly, non-relativistic perturbation expansions in effective field theories for the exploration of chiral heavy quark symmetries[18] , chiral baryons[2] [19] and muonium and positronium[20] avoid heavy fermion masses altogether and, for chiral-symmetric theories, have well-defined power counting similar to (7)in terms of $\frac{k_N^\mu}{\Lambda_{CSB}}$. This results in manifestly non-Lorentz-invariant operators involving two component heavy fermions which are integrated over heavy fermion velocities $\vec{v}$.

2.2 Hamiltonian and Variational Equation for the Ground State

With an effective Lagrangian $\mathcal{L}$ built from operators in (7), the local current corresponding to $SU(2)_{L-R}$ rotations

$$\mathcal{A}_a^\mu = i(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \pi_b)}[\mathcal{Q}_{5a}, \pi_b] + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi)}[\mathcal{Q}_{5a}, \Psi]) \tag{11}$$

is spontaneously broken by the vacuum and explicitly broken by quark mass terms; this is the basis for the PCAC hypothesis.[16] For our purposes, this means that, even neglecting electromagnetic breaking, the quantum states of $\mathcal{L}$ will not have good $SU(2)_{L-R}$ quantum numbers. In contrast, the isospin current generated by $SU(2)_{L+R}$ rotations

$$\mathcal{T}_a^\mu = i(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \pi_b)}[\mathcal{Q}_a, \pi_b] + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi)}[\mathcal{Q}_a, \Psi]) \tag{12}$$

is exactly conserved by strong interactions for degenerate quark masses. This means that the isospin $T_a = \int d^3x \mathcal{T}_a^0$ is a constant of the motion: $\partial_0 T_a = 0$.

The effective Hamiltonian density T^{00} and the effective Hamiltonian $H = \int d^3x T^{00}$ are constructed in the usual way from the effective Lagrangian $\mathcal{L}$; this Hamiltonian is to be examined at tree level[17] [13] [9] for predictions for processes involving physical hadrons. The ground state $|\Phi>$ of the effective Hamiltonian for fixed baryon number $B = \int d^3x \Psi^\dagger \Psi$ and third component of isospin T_3 satisfies

$$\delta(<\Phi|H - \mu_B B - \mu_3 T_3|\Phi>) \;=\; 0 \tag{13}$$

The Lagrange multipliers μ_B and μ_3 will later play the roles of chemical potentials for baryon number and third component of isospin. In order to fix the total angular momentum $J > 0$ we would need to include an appropriate chemical potential μ_J; the minimization (13)will therefore automatically find only $J = 0$ states. The isospin-squared $\vec{T}^2$ is also a qood quantum number because isospin is still an exact symmetry. Assuming that the energy of ground states of given $\vec{T}^2$ grows with $\vec{T}^2$,

the minimization (13)will therefore automatically force $\vec{T}^2$ to its minimum value $\vec{T}^2 = T_3^2 + |T_3|$ since we have included no chemical potential for it. Then, from (13)we have that the ground state energy E satisfies

$$\frac{\partial E}{\partial B}|_{T_3} = \mu_B \quad , \quad \frac{\partial E}{\partial_{T_3}}|_B = \mu_3 \tag{14}$$

It is important to realize that, subject to the applicability of the hadronic effective chiral Lagrangian, (13)and (14)include strong interaction quantum loop effects to all orders and are exact for the physical hadronic ground state $|\Phi>$ with fixed B and T_3.

Why should we believe the physics predicted by the non-linear effective low energy chiral Lagrangian constructed from (7)? The main assumption is that all dynamics of scale $\gg$ 300 MeV (e.g. explicit quark degrees of freedom or nucleon and pion form factors) can be integrated out leaving a theory of elementary point-like nucleons and pions. Imagine that we start with QCD with massless u and d quarks and $SU(2)_L \times SU(2)_R$ global symmetry. We integrate out the quark and gluon fields and assume that this gives a vacuum which breaks the symmetry spontaneously to $SU(2)_{L+R}$ and also results in confinement of low energy quarks and gluons into hadrons. The resulting low energy effective theory of hadrons has a spectrum which contains nucleons and pions as well as a set of heavy particles with masses $\geq \Lambda_{CSB}$ which include well-defined bosons such as ω, ρ, f_0, broad boson resonances, and baryons such as the Δ. At this point, the theory may or may not contain the much-disputed σ particle[21] and chiral $SU(2)_L \times SU(2)_R$ symmetry may be implemented in the linear or non-linear realization according to taste. Next, we integrate out all of the heavy particles, including the σ if it appeared, leaving us with an effective theory of nucleons and pions with highly non-local interactions. We now focus on low momentum transfer (soft) interactions $100 - 300$ MeV $\ll \Lambda_{CSB}$, treat the nucleons and pions as elementary point-like particles, and expand the non-local interactions out in powers of small momentum transfer and $SU(2)_L \times SU(2)_R$ invariant *local* interaction operators of nucleons

and pions. Pions and nucleons now transform linearly under $SU(2)_{L+R}$ and non-linearly under $SU(2)_{L-R}$ and the vacuum, now written in terms of hadrons, breaks $SU(2)_{L-R}$ spontaneously as, for example, in (3). We are then assured[17][9][13] that all representations of this theory give *identical* S-matrix elements and are related to each other by redefinitions of the pion field. Thus, the operators generated by (7)give a completely general representation of the predictions of the $SU(2)_L \times SU(2)_R$ symmetry of QCD for soft hadronic physics. They predict *only* those things and *all* of those things required from the algebra of chiral $SU(2)_L \times SU(2)_R$ currents at low energy; for soft hadronic processes, they do not forbid any of those things allowed by the chiral algebra nor do they allow anything forbidden by the chiral algebra. A simple example is the σ particle; while not required by chiral symmetry, neither is it forbidden. One simple consequence of all of this is that whether the σ exists or not doesn't matter; for the purposes of soft pion and nucleon interactions with momentum transfer $\ll m_\sigma$, the linear and non-linear sigma models give identical results when the σ is integrated out.

The next step is to calculate quantum loops with internal nucleons and pions and naive dimensional power counting. This generates important non-analytic terms as well as analytic non-local interactions among nucleons and pions. An example is the non-local two-pion-exchange interaction between two nucleons; this generates, among other things, an isoscalar spin-independent interaction mediated by a very broad resonance with peak ≈ 650 MeV.[1][7] Such non-local interactions are again expanded out in powers of momentum transfer $\ll \Lambda_{CSB}$ and $SU(2)_L \times SU(2)_R$ invariant local interactions of the form (7). The usual arguments that the full set of diagrams with a given number of quantum loops respects the symmetries (global or local) of a quantum field theory still apply.* Therefore, tree graphs generated by the effective chiral Lagrangian (7)with the chiral coefficients set by experiments contain all low energy chiral-algebraic information and a complete set

* By rescaling nucleon and pion quantum fields by an arbitrary scale α, it is easy to show[13] that the low energy chiral algebra (including explicit $SU(2)_{L-R}$ breaking as in (9)), which relates operators involving different powers of quantum fields, is satisfied to each order in α; each order in α corresponds to a specific number of quantum loops.

of $SU(2)_L \times SU(2)_R$ predictions for physical processes involving soft nucleons and pions. Similar reasoning holds for explicit $SU(2)_{L-R}$ breaking terms whose origin lies in degenerate u and d quark masses.

The phenomenological predictive success of this chiral effective Lagrangian method for soft scattering experiments and hadronic decays has been documented in great detail and includes $\pi - \pi$ and π-nucleon scattering, CVC, PCAC, Adler-Weisberger and Goldhaber-Treiman relations and, for $SU(3)_L \times SU(3)_R$, Gell-Mann-Okubo relation, baryon and meson octet masses, weak semi-leptonic decays, non-leptonic kaon and hyperon decays, etc.[16] Therefore, following empirical evidence, we will assume for the rest of this paper that the criteria for the applicability of the effective hadronic chiral Lagrangian formalism are satisfied.

If the structure of chiral liquids, heavy nuclei and neutron stars is just the result of soft pion and nucleon processes, the effective $SU(2)_L \times SU(2)_R$ chiral Lagrangian constructed from (7)should be able to predict these as well. A detailed understanding of chiral liquids and the properties of heavy nuclei and neutron stars will, however, certainly require a much more sophisticated treatment than found in this paper.† Still, the purpose of this paper is only to show that chiral symmetry could predict the existence and gross structure of chiral liquids, heavy nuclei and neutron stars; calculations will be done in the center-of-mass, where all nucleon three-momenta are small and we expect the terms in (8),(9)and(10)to be adequate.

† Power counting in non-relativistic nucleon momenta shows (see below) that higher dimension $SU(2)_L \times SU(2)_R$ terms are necessary to account for bulk nuclear properties and to ensure compatibility with scattering data. For $SU(3)_L \times SU(3)_R$ chiral theories of hadrons composed of light u, d and s quarks, the dynamical effects of the spin $S = \frac{3}{2}$ baryon decuplet must be included[19] along with the spin $S = \frac{1}{2}$ baryon octet and the pseudo-Goldstone boson octet; chiral $SU(2)_L \times SU(2)_R$ symmetry probably gives the $\Delta(1232)$, but not the $N(1440)$, a small but intellectually significant role in the structure of chiral liquids, heavy nuclei and neutron stars. Further, because operators in different $SU(3)_L \times SU(3)_R$ representations may be renormalized differently, the properties of non-strange hadrons (such as protons, neutrons and pions) may be affected by strange (quark or hadron) matrix elements[22] Still further, non-analytic terms may play a role.

3 CHIRAL LIQUIDS

It is legitimate to inquire whether the effective Lagrangian built from (7)contains a liquid phase. An "$SU(2)_L \times SU(2)_R$ chiral liquid" is defined as a statistically significant number of baryons interacting via chiral operators (7)(as well as the appropriate non-analytic terms) with an almost constant density. Like ordinary liquids, chiral liquids maintain some definite distance between constituent baryons; it costs energy both to compress the liquid and to pull its constituents apart. Although both liquids and "gases" are "fluids", liquids can survive as localized "drops" at zero external pressure. In contrast, the ground state of a Fermi gas at zero external pressure has its constituents dispersed to infinity, thereby filling any container in which the gas has been placed. Ordinary Fermi liquids can also, of course, develop macroscopic quantum properties[23] and we might hope that chiral liquids do as well.

In this Section, we will discuss chiral liquids as abstract approximate solutions $|\Phi>$ to (13). Simplifying assumptions as to the structure of chiral liquids will be guided by the observed empirical properties of ordinary heavy nuclei. We will take very seriously the physical insights of the old nuclear liquid drop model.

3.1 Classical Pions and Fermi Fluids

Chiral $SU(2)_L \times SU(2)_R$ symmetry dictates which operators appear in the effective Lagrangian $\mathcal{L}$. For simplicity, we have truncated the chiral operator expansion in $(\Lambda_{CSB})^{-n}$ keeping only $n = -1, 0$ (higher dimension $n = 1$ operators will be discussed later) and also drop all non-analytic terms. In the construction of the chiral liquid phase, we are led by the empirical facts of heavy nuclei in making our approximations. The spin of most ground-state nuclei are close to zero, hinting that spin-dependent interactions largely cancel. A good example is the chiral spin-dependent term $\frac{2ig_A}{Fg_V}\bar{\Psi}\gamma^\mu\gamma^5 t_a \Psi D_\mu \pi_a$ (with $\frac{g_A}{g_V} = 1.25$ from weak interactions). One-pion exchange arising from this term is known to lead to important tensor forces between nucleons[1] : the effective Lagrangian formulation carries the instruction to calculate only at tree level and therefore explains this

phenomenological success. Still, we judge that such spin-dependent terms cannot be responsible for the *existence* of chiral liquids, heavy nuclei or neutron stars and will not focus on them.

We choose an approximate $|\Phi>$ where spin-dependent interactions cancel.* Further, we are only interested here in static spherically symmetric drops of chiral liquid. All of the operators in (8),(9),(10)then have zero expectation values except the following two sets which we have organized into an effective Lagrangian $\mathcal{L}$:

$$\mathcal{L} = \mathcal{L}_\pi + \mathcal{L}_{\Psi\pi} \tag{15}$$

$$\mathcal{L}_\pi = -\frac{1}{2}m_\pi^2 F^2 \frac{\frac{\vec{\pi}^2}{F^2}}{1+\frac{\vec{\pi}^2}{F^2}} + \frac{1}{2}D_\mu \pi_a D^\mu \pi_a \tag{16}$$

$$\begin{aligned}\mathcal{L}_{\Psi\pi} &= 2\beta\sigma_{\pi N}\bar{\Psi}\Psi\frac{\frac{\vec{\pi}^2}{F^2}}{1+\frac{\vec{\pi}^2}{F^2}} + \bar{\Psi}(i\gamma^\mu D_\mu - m_N)\Psi \\ &- \frac{C_V^2}{2f_\pi^2}(\Psi^\dagger\Psi)(\Psi^\dagger\Psi) + \frac{C_S^2}{2f_\pi^2}(\bar{\Psi}\Psi)(\bar{\Psi}\Psi) \\ &- \frac{\bar{C}_V^2}{2f_\pi^2}(\Psi^\dagger t_a\Psi)(\Psi^\dagger t_a\Psi) + \frac{\bar{C}_S^2}{2f_\pi^2}(\bar{\Psi}t_a\Psi)(\bar{\Psi}t_a\Psi)\end{aligned} \tag{17}$$

Here, $m_N = 939$ MeV, $\sigma_{\pi N} = 60$ MeV, positive $\beta \approx 1$ from scattering data and the C's are coefficients of order one. Perhaps suprisingly, there are only four operators, all four-fermion contact terms, in (17)with unknown coefficients.

The Dirac equation which follows from (13)and (15)is

$$\begin{aligned}0 &= [i\gamma^\mu D_\mu - m_N + 2\beta\sigma_{\pi N}\frac{\frac{\vec{\pi}^2}{F^2}}{1+\frac{\vec{\pi}^2}{F^2}} - \frac{C_V^2}{f_\pi^2}(\Psi^\dagger\Psi)\gamma^0 \\ &+ \frac{C_S^2}{f_\pi^2}(\bar{\Psi}\Psi) - \frac{\bar{C}_V^2}{f_\pi^2}(\Psi^\dagger t_a\Psi)\gamma^0 t_a + \frac{\bar{C}_S^2}{f_\pi^2}(\bar{\Psi}t_a\Psi)t_a]\Psi\end{aligned} \tag{18}$$

The first term in each of $\mathcal{L}_{\Psi\pi}$ and $\mathcal{L}_\pi$ and the third term in (18)arise from non-

* This is not strictly possible since four-fermion contact terms in (8)and (10)distinguish identical from non-identical fermions. For simplicity, we will neglect fermion exchange terms in this paper. Still, we assume that, on average, fermion contact interactions will reproduce the compressibility of chiral liquids, heavy nuclei and neutron stars.

zero quark masses in QCD operators like (6)and (9)which break axial $SU(2)_{L-R}$ explicitly and transform as a $(\frac{1}{2},\frac{1}{2})$ isosinglet representation under $SU(2)_L \times SU(2)_R$; the specific combination of even numbers of pions $\frac{\vec{\pi}^2}{F^2}(1+\frac{\vec{\pi}^2}{F^2})^{-1}$, carries the quantum numbers of the fourth component of an $SU(2)_{L-R}$ vector.[21] Particularly interesting is the first term in $\mathcal{L}_{\Psi\pi}$ and the third term in (18). Since $\beta > 0$ experimentally,[24] in the presence of pions, nucleons weigh less than when free; similarly, in the presence of nucleons, pions weigh less than when free. It then follows that explicit quark mass terms in QCD are then primarily responsible for the long-range non-derivative pion-nucleon interactions contributing to saturation in chiral liquids, ordinary heavy nuclei and neutron stars.

The success of the nuclear shell model must mean that, at least in heavy nuclei, nucleons move to good approximation in some sort of mean self-consistent spherically symmetric field; otherwise why should the angular momentum of each nucleon be an approximately good quantum number? Our solution to the variational equation (13)will therefore have a spherically symmetric "classical" pion field $(\vec{\pi}_{operator})^n|\Phi >= \vec{\pi}^n(\vec{x},t)|\Phi >$ with $\vec{\pi}(\vec{x},t)$ a c-number function and the commutator $< \Phi|[\vec{\pi}_{operator}, \partial_\mu \vec{\pi}_{operator}]|\Phi >= 0$.

Nuclear forces are known to "saturate"; the nuclear volume grows roughly as the baryon number[25] (we assume that baryon number is carried approximately by nucleons with no nucleon-anti-nucleon pairs). This saturation and the success of the Thomas-Fermi and liquid-drop models of nuclei[26] suggest not only that a large part of nuclear forces are carried by non-derivative interactions, but that the wavelength of nucleons inside a heavy nucleus is smaller than the scale over which significant spatial changes in the self-consistent mean pion field occur. These very old ideas explain the empirical success of relativistic mean field models of nuclei.[6] We will therefore treat the nucleons as a Fermi fluid, make the Thomas-Fermi approximation and search for spherically symmetric, spin-independent ground states $|\Phi >$ (for fixed values of T_3 and B) in which a self- consistent pion field traps the nucleons to form a drop of liquid or a heavy nucleus. This means that we are to

replace $\mathcal{L}_{\Psi\pi}$ with a fermion fluid with energy-momentum tensor

$$T^{\mu\nu}_{\Psi} = (\mathcal{E}_{\Psi} + P_{\Psi})U^{\mu}U^{\nu} - g^{\mu\nu}P_{\Psi} \tag{19}$$

where the fermion energy density $\mathcal{E}_{\Psi}(\pi_a)$ and fermion pressure $P_{\Psi}(\pi_a)$ depend on the classical pion field π_a but not its spatial derivatives $\vec{\nabla}\pi_a$ (Thomas-Fermi approximation). U^{μ} is the fermion fluid four-velocity. Actually, we construct two Fermi fluids, one electrically charged and one neutral; for $\Psi = \binom{P}{N}$ the time-dependence of the proton Fermi surface is $P(\vec{x},t) = P(\vec{x})exp[-i(\mu_B + \frac{1}{2}\mu_3)t]$ while for neutrons $N(\vec{x},t) = N(\vec{x})exp[-i(\mu_B - \frac{1}{2}\mu_3)t]$. Charged classical pion fields $\pi_{\pm} = \frac{\pi_1 \pm i\pi_2}{\sqrt{2}}$ have time dependence $\pi_+(\vec{x},t) = \pi_+(\vec{x})exp[-i\mu_3 t]$ with $\pi_- = (\pi_+)^{\dagger}$ while classical neutral pions are time independent. As advertized, μ_B and μ_3 play the roles of chemical potentials.

An energy-momentum tensor $T^{\mu\nu}_{\pi}$ is derivable in the usual way from the pion Lagrangian $\mathcal{L}_{\pi}$. The conservation of the total energy-momentum $\partial_{\nu}T^{\mu\nu} = 0$ with $T^{\mu\nu} = T^{\mu\nu}_{\Psi} + T^{\mu\nu}_{\pi}$ then implies (in the fluid rest frame where $U^{\mu} = (1,0,0,0)$)[27]

$$\partial_{\nu}\frac{\partial\mathcal{L}_{\pi}}{\partial(\partial_{\nu}\pi_a)} - \frac{\partial\mathcal{L}_{\pi}}{\partial\pi_a} - \frac{\partial P_{\Psi}}{\partial\pi_a} = 0 \tag{20}$$

for static $\partial_0\mathcal{E}_{\Psi} = \partial_0 P_{\Psi} = 0$. Equivalently, (20), the basic equation for chiral liquids and also the basic result of this section, follows directly from the minimization (13). Our treatment of the fermions as a fluid reproduces the naive Euler-Lagrange equations for the classical pion field from the effective Lagrangian $\mathcal{L}$ with one crucial difference; the negative of the fermion fluid pressure $-P_{\Psi}$ plays the role of a self-interaction potential for the classical pion field.[11][27]

There is a simple interpretation for (13)and (20)and (18)for the effective Lagrangian constructed from (8),(9)and (10); with the time-dependence anzatz $i\partial_0\pi_{\pm} = \pm\mu_3\pi_{\pm}$, $i\partial_0\pi_3 = 0$, $i\Psi^{\dagger}\partial_0\Psi = \mu_B\Psi^{\dagger}\Psi$, (13)and (20)may be rewritten

$$\delta < \Phi|\mathcal{S}|\Phi > \equiv \delta < \Phi|\int d^3x\mathcal{L}|\Phi > |_{time\ anzatz} = 0 \tag{21}$$

The chiral liquid or nuclear ground state configuration with fixed B and T_3 mini-

mizes the quantum effective action $\mathcal{S}$.

3.2 Isospin $T = 0$ Fermion Fluids

For zero isospin states we can ignore $\bar{C}_V^2$ and $\bar{C}_S^2$ and set $\mu_3 = 0$ so that the electric charge $Q = T_3 + \frac{1}{2}B$ is half the baryon number and the background pion field is time-independent. Following the Thomas-Fermi approximation, we construct two fermion fluids treating the fermions as free with four-momenta k_μ^P for protons and k_μ^N for neutrons. More specifically, the Thomas-Fermi approximation assumes that fermion Compton wavelengths are much smaller than the scale of spatial variations of the classical pion field so that $(i = P, N)$

$$\frac{|\vec{\nabla}\pi_a|}{F}\Psi \ll |\vec{k}^i|\Psi \tag{22}$$

For isospin $T = 0$ we may choose the classical pion field to point along the π_3 direction $\vec{\pi} = (0, 0, \pi_3)$ without loss of generality. We then form a Fermi fluid of protons and a separate Fermi fluid of neutrons; the number density of the $\vec{k}^{th}$ mode is $P_{\vec{k}}^\dagger P_{\vec{k}} = N_{\vec{k}}^\dagger N_{\vec{k}} = (2\pi)^{-3}$. The free proton states are each filled up to a position-dependent Fermi surface $\vec{k}_F^P(\vec{x})$; for $T = 0$ in a background time independent π_3 field, the neutrons are filled up to the *same* level $\vec{k}_F^N(\vec{x}) = \vec{k}_F^P(\vec{x})$. The total baryon number density is then (2 nucleon spins, 2 nucleon isospins)

$$\Psi^\dagger\Psi = 2 \cdot 2 \cdot \int^{k_F} \frac{d^3k}{(2\pi)^3} = \frac{2(k_F)^3}{3\pi^2} \tag{23}$$

for spherically symmetric $k_F^P(r) = k_F^N(r) \equiv k_F(r)$ depending only on the radial space coordinate r in the nuclear center-of-mass. The total baryon number is then $B = 4\pi \int r^2 dr \Psi^\dagger\Psi$.

The dispersion relation satisfied by a nucleon at the top of its Fermi sea which then follows from (18)is

$$(\mu_B - \frac{C_V^2}{f_\pi^2}\Psi^\dagger\Psi)^2 - k_F^2 - (m_N - \frac{C_S^2}{f_\pi^2}\bar{\Psi}\Psi - 2\beta\sigma_{\pi N}\frac{\frac{\pi_3{}^2}{F^2}}{1 + \frac{\pi_3{}^2}{F^2}})^2 = 0 \tag{24}$$

where we have used (22). It is convenient to define the reduced energy and mass

of a nucleon at the Fermi surface

$$\begin{aligned} \mu_\star &= \mu_B - \frac{C_V^2}{f_\pi^2}\Psi^\dagger\Psi \\ m_\star &= m_N - \frac{C_S^2}{f_\pi^2}\bar{\Psi}\Psi - 2\beta\sigma_{\pi N}\frac{\frac{\pi_3{}^2}{F^2}}{1+\frac{\pi_3{}^2}{F^2}} \end{aligned} \tag{25}$$

so that $\mu_\star^2 = k_F^2 + m_\star^2$. Then it is straightforward to construct the other quantities describing the Fermi fluids: The total fermion scalar density

$$\begin{aligned} \bar{\Psi}\Psi &= 2\cdot 2\cdot 4\pi\int_0^{k_F}\frac{k^2dk}{(2\pi)^3}\frac{m_\star}{(k^2+m_\star^2)^{\frac{1}{2}}} \\ &= \frac{m_\star}{\pi^2}(k_F\mu_\star - \frac{m_\star^2}{2}ln[\frac{\mu_\star+k_F}{\mu_\star-k_F}]) \end{aligned} \tag{26}$$

and fermion pressure

$$\begin{aligned} P_\Psi &= 2\cdot 2\cdot 4\pi\int_0^{k_F}\frac{k^2dk}{(2\pi)^3}\frac{k^2}{3(k^2+m_\star^2)^{\frac{1}{2}}} + \frac{C_V^2}{2f_\pi^2}(\Psi^\dagger\Psi)^2 - \frac{C_S^2}{2f_\pi^2}(\bar{\Psi}\Psi)^2 \\ &= \frac{1}{2\pi^2}(\frac{k_F^3\mu_\star}{3} - \frac{m_\star^2k_F\mu_\star}{2} + \frac{m_\star^4}{4}ln[\frac{\mu_\star+k_F}{\mu_\star-k_F}]) + \frac{C_V^2}{2f_\pi^2}(\Psi^\dagger\Psi)^2 - \frac{C_S^2}{2f_\pi^2}(\bar{\Psi}\Psi)^2 \end{aligned} \tag{27}$$

are related by the trace of the (metric weighted) fermion energy-momentum tensor to the fermion number density and energy density

$$\begin{aligned} T^\lambda_{\Psi\lambda} &= \mathcal{E}_\Psi - 3P_\Psi \\ &= (m_N - 2\beta\sigma_{\pi N}\frac{\frac{\pi_3{}^2}{F^2}}{1+\frac{\pi_3{}^2}{F^2}})\bar{\Psi}\Psi - \frac{C_V^2}{f_\pi^2}(\Psi^\dagger\Psi)^2 + \frac{C_S^2}{f_\pi^2}(\bar{\Psi}\Psi)^2 \end{aligned} \tag{28}$$

(23)through (26)must be solved self-consistently for k_F; then $\mathcal{E}_\Psi$ and P_Ψ depend upon the 3 chiral Lagrangian coefficients β, C_V^2, C_S^2 (to be fit to experiment), the baryon number chemical potential μ_B (which sets the baryon number of the solutions), and the classical background pion field $\pi^3(r)$.

3.3 $T = 0$ Nontopological Solitons: Drops of Symmetric Chiral Liquid

Isospin is exactly conserved by all of the strong interaction operators in the chiral perturbation theory (7) for degenerate u and d quarks (we are not yet considering electromagnetic isospin breaking). Since there is only one isospin $T = 0$ state, the choice of pion field $\vec{\pi} = (0, 0, \pi_3)$ in the construction of the nucleon Fermi fluids is simply a matter of convenience; any other choice can be isospin-rotated into this one. The three equations (20)then become one:

$$\frac{1}{1+\frac{\pi_3{}^2}{F^2}}\partial_\mu[\frac{\partial^\mu \pi_3}{1+\frac{\pi_3{}^2}{F^2}}] - d[P_\Psi - \frac{1}{2}m_\pi^2 F^2 \frac{\frac{\pi_3{}^2}{F^2}}{1+\frac{\pi_3{}^2}{F^2}}]/d\pi_3 = 0 \tag{29}$$

We are interested in time-independent spherically symmetric $\pi_3(r)$ and therefore introduce the transformation

$$\theta = tan^{-1}(\frac{\pi_3}{F}) \tag{30}$$

with $\frac{\pi}{2} \geq \theta \geq 0$ in order to write (20)and (29)for isospin zero states

$$\begin{aligned} (\frac{d^2}{d\tilde{r}^2} + \frac{2}{\tilde{r}}\frac{d}{d\tilde{r}})\theta &= -\frac{d\mathcal{U}}{d\theta} \\ \mathcal{U} &= F^{-4}(P_\Psi - \frac{1}{2}m_\pi^2 F^2 sin^2\theta) \\ \tilde{r} &= Fr \end{aligned} \tag{31}$$

The radial scale $\tilde{r}$ is almost in Fermis since $1fm^{-1} = 197$ MeV $= 1.06F$

We will be interested here in solutions to (31)at zero external pressure; this implies $\mathcal{U} \rightarrow 0$ as $\tilde{r} \rightarrow 0$. The ordinary differential equation (31)is easily integrated numerically but this would give us little insight into which part, if any, of the huge phase space of chiral coefficients β, C_V^2, C_S^2 and baryon number chemical potential μ_B allows the existence of saturating drops of chiral liquid and heavy nuclei. Instead, we note that rewriting P_Ψ as a function of θ rather than π_3 allows us to write (31)in the form of Newtonian potential motion; if the radial center of

mass space coordinate $\tilde{r}$ is "time", the pion field angle θ is "distance" and $\mathcal{U}$ is the "Newtonian potential". The sign of $\frac{2d\theta}{\tilde{r}d\tilde{r}}$ makes it a "time-dependent friction" which diminishes with large "time". The analogy of (31)with Newtonian potential motion then gives us an extremely powerful graphical technique to scan quickly for nontopological solitons through the (very) large phase space of parameters (three chiral coefficients and μ_B) while using all of our natural intuition about hills and valleys and the Newtonian motion of a particle through them.[27]

The chemical potential and the chiral coefficients determine the shape of $\mathcal{U}$. Demanding the existence, at zero external pressure, of saturating nontopological solitons of both finite and infinite baryon number from (31)gives a powerful constraint on the shape of $\mathcal{U}$ and thus the chiral coefficients. We seek solutions with baryon density profile $\Psi^\dagger\Psi(\tilde{r})$ similar to the solid line and pion field profile $\theta(\tilde{r})$ similar to the dashed line in Figure 1. As $\tilde{r} \to \infty$ the solution is the vacuum with $\pi_3 = \theta = 0$ and $\Psi^\dagger\Psi = 0$. The center of the drop of chiral liquid must have $\frac{d\theta}{d\tilde{r}}(\tilde{r} = 0) = 0$ because of the friction term. Heavy nuclei saturate. We will assume that drops of chiral liquid do as well; in the analogy of potential motion, the moving particle remains at rest for a long time $\tilde{r}$ at the initial position $\theta(\tilde{r} = 0)$. This suggests that, for finite baryon number solitons, the potential $\mathcal{U}$ in which the particle moves is shaped as the solid line in Figure 2; the particle starts at the diamond point $\theta(\tilde{r} = 0)$ near the top of the right-hand hill with zero initial velocity, waits there a long time while overcoming friction (this is the saturating interior (I) in Figure 1), quickly drops through the valley and climbs up the left-hand hill (this is the thin surface (S) in Figure 1), taking an infinite amount of time to finally come to rest at the top of the left-hand hill (the vacuum (V) in Figure 1). Because of dissipation of the "energy" $\frac{1}{2}(\frac{d\theta}{d\tilde{r}})^2 + \mathcal{U}$ due to friction, the particle must start above zero on the right-hand hill.

Since ordinary heavy nuclei saturate, it is plausible that, in the absence of electromagnetic or other isospin breaking, there should exist saturating drops of chiral liquid of arbitrarily high baryon number; for $B \to \infty$ and electric charge $Q = \frac{1}{2}B$ this will be called "infinite symmetric chiral liquid". The concept is very similar to

"infinite symmetric nuclear matter" in nuclear physics.[6] Following empirical evidence in ordinary heavy nuclei, we require infinite symmetric chiral liquid to have volume binding energy per nucleon a_{volume} with the chemical potential of infinite symmetric chiral liquid $\mu_{\infty liquid} = m_N - a_{volume}$. Further, we require the number density of infinite symmetric chiral liquid to be $\Psi^\dagger\Psi_{\infty liquid} \approx \Psi^\dagger\Psi_{nuclear\ matter} = \frac{2k^3_{F,nuclear\ matter}}{3\pi^2} \simeq 2f_\pi^3$ with $k_{F,nuclear\ matter} = 280$ MeV. In analogy with potential motion, infinite symmetric chiral liquid would correspond to a particle moving between hills of the same height, with $\mathcal{U} < 0$ in between, as in the dashed line of Figure 2; it starts at the circled point $\theta(\tilde{r} = 0) = \theta_{\infty liquid}$ and finishes at the vacuum $\theta = 0$. It is easy to see graphically in Figure 2 that, in order that drops of infinite symmetric chiral liquid exist, two saturating conditions must be met:

$$\begin{aligned} \mathcal{U}(\theta_{\infty liquid}, \mu_{\infty liquid}) &= 0 \\ \frac{d\mathcal{U}}{d\theta}(\theta_{\infty liquid}, \mu_{\infty liquid}) &= 0 \end{aligned} \tag{32}$$

For a given choice of $\theta_{\infty liquid}$ we thus have four conditions ($\mu_{\infty liquid}$, $\Psi^\dagger\Psi_{\infty liquid}$ and (32)) in the four parameters β, C_V^2, C_S^2 and $\mu_{\infty liquid}$ for infinite symmetric chiral liquid. In addition, the validity of the effective hadronic chiral Lagrangian approach also requires that the three chiral coefficients be of order one and that chiral fields not become too large; we estimate $|\pi_3| < 400$ MeV so that $0 \leq |\theta| < tan^{-1}2$.

The chemical potential μ_B is a parameter of the solutions. In order that saturating nontopological solitons of finite baryon number be energetically bound $m_N > \mu_B > \mu_{\infty liquid}$ so that, for large baryon number, the nontopological soliton energy $E < m_N B$. For $\mu_B > \mu_{\infty liquid}$, $\mathcal{U}$ must then have a smooth shape similar to the solid line of Figure 2. For saturating nontopological solitons, $B \to \infty$ and $E \to \mu_{\infty liquid} B$ as $\mu_B \to \mu_{\infty liquid}$ and the interior baryon density must become very flat. Therefore, as μ_B decreases toward $\mu_{\infty liquid}$ the initial right-hand hill must become lower while $\mathcal{U}$ interpolates between the solid and the dashed lines of Figure 2. Finally, infinite symmetric chiral liquid emerges for $\mu_B = \mu_{\infty liquid}$, as in the dashed line of Figure 2. These requirements on the shape of $\mathcal{U}$ as a function

of μ_B, along with the requirement that the surface energy of the nontopological soliton not unbind it, give further powerful constraints on the chiral coefficients.

The parameter space is overconstrained by the above considerations and we find that nontopological solitons do not exist in a large fraction of the space of chiral coefficients and baryon chemical potential. Figure 3 divides the $\mu_{\infty liquid} - \Psi^\dagger\Psi_{\infty liquid}$ plane into two regions. Using the analogy with Newtonian potential motion (31)together with the shape of $\mathcal{U}$ we have shown numerically that the shaded region contains nontopological solitons. We have also shown that the unshaded region does not; $\mathcal{U}$ is discontinuous or multivalued (as a function of k_F) there or the right-hand hill (see Figure 2) was too low so that a particle could not reach, with friction, the left hand hill at $\mathcal{U} = 0$ under potential motion. We find that if $\frac{C_S^2}{C_V^2} \geq 1.1$ no solitons exist. C_S^2 can be small or negative but this requires very large β . The reader is warned that this was a numerical study - we have no algebraic proof for most of Figure 3 - meant only to indicate the most probable region of the space of chiral coefficients in which to search for nontopological solitons. The boundary between the shaded and unshaded regions is less sharp than indicated. We have proved algebraically that there exist no nontopological solitons for $\mu_{\infty liquid} > m_N$. The analogy with Newtonian potential motion was a crucial tool in our search.

We now display results for a typical set of nontopological solitons: $a_{volume} = f_\pi, \Psi^\dagger\Psi_{\infty liquid} = .673 \times 2f_\pi^3$ and $\theta_{\infty liquid} = 1.07$ coresponding to the diamond point in Figure 3. From these values it follows that $C_V^2 = 5.14, C_S^2 = 5.16$ and $\beta = 6.66$. There is no fine tuning here; the chiral coefficients are chosen for numerical convenience. The potential $\mathcal{U}$ for infinite symmetric chiral liquid with $\mu_B = \mu_{\infty liquid}$ is displayed in the dashed line of Figure 2 with $\theta_{\infty liquid} = 1.07$ the circled point there. The potential $\mathcal{U}$ appropriate for the baryon number $B = 83$ nontopological soliton solution with $\mu_B = .999 m_N$ is displayed in the solid line of Figure 2; the interior $\theta_{B=83}(\tilde{r} = 0) = .816$ is the diamond point there.

Because we have isospin $T = 0$ in this section, the electric charge of our solitons is always $Q = \frac{1}{2}B$. The baryon number density (in units of the baryon number

density of the infinite liquid) $\frac{\Psi^\dagger\Psi(r)}{\Psi^\dagger\Psi_{\infty liquid}}$ and classical pion field $\frac{\theta_{B=83}(r)}{\theta_{\infty liquid}}$ are displayed for the baryon number $B = 83$ solution by the solid and dashed lines respectively in Figure 4. Actually, due to surface energy, this soliton is slightly unbound $\frac{E}{m_N B} = 1.02$. Figure 5 shows similar curves for the $B = 148$ solution with $\mu_B = .980 m_N$ and $\theta_{B=148}(\tilde{r} = 0) = .964$. This soliton too is slightly unbound $\frac{E}{m_N B} = 1.006$. As the solitons baryon number grows, their energy per baryon decreases. Finally, Figure 6 gives the $B = 277$ solution $\mu_B = .965 m_N$ and $\theta_{B=279}(\tilde{r} = 0) = 1.04$. This soliton is slightly bound $\frac{E}{m_N B} = .990$. The progression of solitons in Figures 4 through 6 confirm that as the baryon number increases, the solitons pion angle $\theta(\tilde{r} = 0)$ and chemical potential μ_B approach asymptotically the values for the infinite chiral liquid and the solitons become more deeply bound. It is clear from these figures that the non-linear chiral effective Lagrangian is able to sustain, at least at a crude level, drops of saturating chiral liquid. We will argue in Section 4 that the baryon density profiles of droplets of chiral liquid can be quite similar to those of ordinary heavy saturating nuclei.

It is appropriate here to check our solutions for self-consistency: The Thomas-Fermi approximation is justified since $\frac{d\pi_3}{F dr} \ll k_F \simeq 280$ MeV in most of the non-topological soliton surface. All of the chiral coefficients obey naive power counting. All fields, energies and momenta $\ll \Lambda_{CSB}$ so that the use of the effective $SU(2)_L \times SU(2)_R$ Lagrangian is justified.

We may now write an approximate formula for the energy E of isospin $T = 0$ nontopological solitons:

$$E \simeq (m_N - a_{volume})B + a_{surface} B^{\frac{2}{3}} \tag{33}$$

where the constants $a_{volume}, a_{surface} \sim f_\pi$. In order to understand this formula, we again divide the soliton into three regions as in Figure 1: the soliton interior (I), soliton surface (S) and the vacuum (V). The first term follows from the saturation conditions (32)and $\mu_{\infty liquid} = m_N - a_{volume}$ which force the soliton energy and

baryon number to grow roughly as the soliton volume V in the saturating interior (I); this ensures that $E \to \mu_{\infty liquid} B$ as $B \to \infty$.

The second term can be understood as follows: In the analogy of Newtonian potential motion, the nontopological soliton surface region (S) starts after the particle has overcome friction; (S) corresponds to the particle moving down into the valley and up the second hill to the vacuum (V). Figure 2 shows that the shape of $\mathcal{U}$ changes very little (on the natural scale of $\frac{P_*}{F^4} \approx \frac{1}{2}\frac{m_\pi^2}{F^2} = .283$) as the chemical potential μ_B is lowered to $\mu_{\infty liquid}$ and the soliton volume $V \sim B \to \infty$. Therefore, for large B, the radial profile of the soliton surface region (S) is very similar for all saturating nontopological solitons with a given set of chiral coefficients; compare the baryon density profiles (solid lines) and pion angle profiles (dashed lines) of Figures 4 through 6. The surface energy of saturating nontopological solitons is therefore essentially described by a constant surface tension;[28] since the surface area $\sim B^{\frac{2}{3}}$ for saturating solitons, the second term in (33)follows. The natural scale of $a_{surface}$, like all quantities of order $(\Lambda_{CSB})^0$ in $SU(2)_L \times SU(2)_R$ chiral symmetry, is f_π. T.D. Lee and co-workers have argued for years that the energy of nontopological solitons in certain bosonic field theories with conserved charges[29] as well as for anomalous nuclear matter[5] is given by a formula like (33).

3.4 $T > 0$ Nontopological Solitons: Drops of Non-Symmetric Chiral Liquid

Nontopological $SU(2)_L \times SU(2)_R$ solitons with $T > 0$ and $\mu_3 \neq 0$ correspond to drops of chiral liquid at zero external pressure which have $Q = T_3 + \frac{1}{2}B$ and $T_3 \neq 0$. To clarify their properties, we will, for the moment, neglect electromagnetism. For the operators (8),(9)and (10)the isospin density (12)is

$$\begin{aligned} \vec{T}^\mu &= \vec{J}^\mu + \frac{1}{1+\frac{\vec{\pi}^2}{F^2}}\vec{\pi} \times [D^\mu \vec{\pi} + 2F^{-2}\vec{\pi} \times \vec{J}^\mu + \frac{2ig_A}{Fg_V}\vec{J}^{5\mu}] \\ \vec{J}^\mu &= \bar{\Psi}\gamma^\mu \vec{t}\Psi \\ \vec{J}^{5\mu} &= \bar{\Psi}\gamma^\mu\gamma^5\vec{t}\Psi \end{aligned} \tag{34}$$

As usual, we will neglect spin-dependent $\vec{J}^{5\mu}$ here.

We next construct, at zero external pressure, spherically symmetric saturating nontopological solitons with $T \geq 0$ from the effective Lagrangian (15)and the minimization (13). Nucleon Fermi fluids are constructed as in Section 3.2 but now the chiral isospin coefficients $\bar{C}_V^2, \bar{C}_S^2$ as well as the pion field dependent terms in $D_0\Psi$ in (18)play an important role. We then need to consider two classical pion fields ($\pi_3(r)$ and $\pi_+(r)$ with $\pi_- = (\pi_+)^\dagger$) as solutions to (20). The result of (20)is that certain sets of isospin chiral coefficients allow saturation conditions analogous with (32)when a certain function of the two chemical potentials μ_B and μ_3 is zero; this gives infinite drops of non-symmetric chiral liquid with the ratio of isospin density to baryon number density $\frac{\mathcal{T}_3^0}{\Psi^\dagger\Psi}$ fixed. The classical pion field $\vec{\pi}(r)$ is then a constant $\vec{\pi}_{saturate}$; for example, the $T = 0$ nontopological solitons constructed in section 3.3 with $\mu_3 = 0$ give $\vec{\pi}_{saturate} = (\ 0, 0, F tan(\theta_{\infty liquid})\ )$ up to isospin rotations which do not change the state. Raising a certain combination of the chemical potentials slightly then gives finite saturating nontopological solitons with $T \geq 0$. For smaller baryon number, the classical pion field $\vec{\pi}(r)$ resembles the dashed line of Figure 4. For larger baryon number, the classical pion field saturates and resembles the dashed line in Figure 6. As $B \to \infty$ it resembles the dashed line of Figure 1 with $\vec{\pi} \to \vec{\pi}_{saturate}$ in the soliton interior.

Now imagine that we had included a chemical potential $\mu_{\vec{T}^2}$ for the isospin-squared $\vec{T}^2$ in (13)along with μ_3 and μ_B. $T > 0$ nontopological soliton solutions would then be described by three independent quantum numbers: baryon number B, third component of isospin T_3 and isospin-squared $\vec{T}^2$. Of course, we could just as well have quantized along the T_1 or T_2 direction so the soliton energy will not depend on T_3 but only on B and $\vec{T}^2$. An approximate formula for the nontopological soliton energy contribution due to isospin then follows: $E_{isospin} \simeq a_{\vec{T}^2}\frac{\vec{T}^2}{B}$ with $a_{\vec{T}^2} > 0$. In order to understand this effect, we focus on the term $\frac{1}{2}(D_\mu\vec{\pi})^2$ in the effective Lagrangian (15)which is the source of the term $\frac{\vec{\pi}\times D^0\vec{\pi}}{1+\frac{\vec{\pi}^2}{F^2}}$ in the isospin current (34). As the baryon number $B \sim$ the soliton volume $V \to \infty$, the effective Hamiltonian $H = \int d^3x T^{00}$ constructed from (15)gives a soliton

energy contribution

$$E_{isospin} \sim \int d^3x \frac{1}{2} \frac{(\partial^0 \vec{\pi})^2 + (\partial^i \vec{\pi})^2}{(1 + \frac{\vec{\pi}^2}{F^2})^2} \rightarrow \frac{1}{2} \frac{(\partial^0 \vec{\pi}_{saturate})^2}{(1 + \frac{\vec{\pi}^2_{saturate}}{F^2})^2} V \tag{35}$$

while the isospin contributed by this term is

$$\vec{T} \sim \int d^3x \frac{\vec{\pi} \times \partial^0 \vec{\pi}}{(1 + \frac{\vec{\pi}^2}{F^2})^2} \rightarrow \frac{\vec{\pi}_{saturate} \times \partial^0 \vec{\pi}_{saturate}}{(1 + \frac{\vec{\pi}^2_{saturate}}{F^2})^2} V \tag{36}$$

Assuming that $\partial_0(\vec{\pi}^2_{saturate}) = 0$, we then estimate the isospin contribution to the soliton energy for large B:

$$E_{isospin} \sim f_\pi [\frac{(1 + \frac{\vec{\pi}^2_{saturate}}{F^2})^2}{4\frac{\vec{\pi}^2_{saturate}}{F^2}} \frac{B}{2f_\pi^3 V}] \frac{\vec{T}^2}{B} \tag{37}$$

Since $|\vec{\pi}_{saturate}| \simeq F$ from Section 3.3 and the baryon number density $\Psi^\dagger \Psi \simeq 2f_\pi^3$, the expression in brackets is $\simeq 1$. Of course, the value of $a_{\vec{T}^2}$ for chiral nontopological solitons constructed from (15)will depend on the chiral isospin coefficients $\bar{C}_V^2$ and $\bar{C}_S^2$ but (37)establishes that $a_{\vec{T}^2} \simeq f_\pi$ if the chiral isospin coefficients obey naive power counting and are order one. Since $a_{\vec{T}^2} > 0$, the ground state for fixed T_3 will force $\vec{T}^2$ to its minimum value: $\vec{T}^2 = T_3^2 + |T_3|$.[30] Equivalently, we have set the chemical potential $\mu_{\vec{T}^2} = 0$ in (13).

We now re-introduce electromagnetic isospin breaking. The electromagnetic charge $Q = T_3 + \frac{1}{2}B$ commutes with $H, \vec{T}^2$ and T_3 so we still have symmetry under isospin rotations around the 3rd axis. For saturating nontopological soliton solutions, the contribution of electromagnetism to the energy is estimated in the usual way[31] and we have the $SU(2)_L \times SU(2)_R$ mass formula for nontopological soliton ground states:

$$\begin{aligned} E \simeq & (m_N - a_{volume})B + a_{surface}B^{\frac{2}{3}} + a_{\vec{T}^2}\frac{T_3^2 + |T_3|}{B} \\ & + a_{Coulomb}\frac{(T_3 + \frac{1}{2}B)(T_3 + \frac{1}{2}B - 1)}{B^{\frac{1}{3}}} \end{aligned} \tag{38}$$

where the chiral coefficients in (15)naturally give a_{volume}, $a_{surface}$, $a_{\vec{T}^2} \sim f_\pi$ and $a_{Coulomb} \sim .75$ MeV. The approximate dependence of μ_B and μ_3 on B and T_3 for

our nontopological solitons follows from (14). Equation (38), the basic result of this Section, follows from (13)and gives the approximate mass of a drop of chiral liquid as a function of its baryon number and third component of isospin.

3.5 Quantum Chiral Liquids

In the limit of very large baryon number, matter whose interactions come entirely from strong interaction chiral effective Lagrangian operators, such as in the chiral perturbation theory (7)supplemented by non-analytic terms, develops its own unique and interesting properties. To begin, we will examine the case in which all isospin-breaking effects, especially electromagnetic, are neglected.

Neglecting electromagnetic isospin breaking effects by setting $a_{Coulomb} = 0$ in (38), shows that as $B \rightarrow \infty$ there are an infinite number of nontopological soliton solutions with different fixed isospin $T \geq 0$ which become degenerate in energy. Together with the non-zero classical pion field , this means that isospin is spontaneously broken in the $B \rightarrow \infty$ infinite chiral liquid. Simply put, isospin is exactly conserved by an effective Lagrangian constructed from (7)but the classical pion field in our nontopological soliton solutions must choose spontaneously some value of $\vec{T}^2$ as well as some direction in isospin space in which to point. Zero-mass Goldstone bosons[10] must therefore appear in the infinite chiral liquid; as $B \rightarrow \infty$ it costs no energy to increase or decrease the isospin $\vec{T}$ or change its orientation.

The next step is to examine the quantum correlation functions to see if the chiral liquid develops super-fluidity or other super-properties. It may or it may not. The issue of whether long range quantum correlations appear is subtle. Mean-field theories such as ours are probably untrustworthy in their predictions for super-properties. Further, the presence of the classical pion field does *not* indicate that there are massless pion modes in the chiral liquid. Consider the simple case of small pion oscillations $\delta\pi_3$ along the π_3 isospin direction which move within a drop of the chiral liquid of Section 3.3. We have $\vec{\pi} = (0, 0, \pi_3)$, $\theta \equiv tan^{-1}(\frac{\pi_3}{F}) \simeq \theta_{soliton} + \delta\theta$

with $\delta\pi_3(t,\vec{x}) = F cos^{-2}\theta_{soliton} \cdot \delta\theta(t,\vec{x})$. Equation (29)is written

$$F^{-2}(-\partial_t^2 + \vec{\nabla}^2)(\theta_{soliton} + \delta\theta) \simeq -\frac{d\mathcal{U}}{d\theta}|_{\theta_{soliton}} - \frac{d^2\mathcal{U}}{d\theta^2}|_{\theta_{soliton}}\delta\theta \tag{39}$$

It follows from (31)that the mass-squared of the small $\delta\pi_3$ excitation in a droplet of chiral liquid is $-F^2\frac{d^2\mathcal{U}}{d\theta^2}|_{\theta_{soliton}}$. A pion propagating in the interior of a droplet of chiral liquid has a different mass and a different excitation spectrum than a pion propagating in the surface; this may affect pionic atoms. For the infinite chiral liquid where $\theta_{soliton} = \theta_{\infty liquid}$, the (constant) pion excitation mass gap is

$$m_\pi^2|_{\infty liquid} = -F^2\frac{d^2\mathcal{U}}{d\theta^2}|_{\theta_{\infty liquid}} > 0 \tag{40}$$

That this is positive and non-zero can be seen most easily by examination of the dashed line in Figure 2; in order that the $B \to \infty$ saturated nontopological soliton (the infinite chiral liquid) exist, the potential $\mathcal{U}$ must be a maximum at $\theta_{\infty liquid}$ (the circled point). For the $T = 0$ infinite chiral liquid of Section 3.3 whose finite baryon number solitons are displayed in Figures 4,5 and 6, we find

$$\frac{m_\pi|_{\infty liquid}}{m_\pi|_{free}} \simeq .6 \tag{41}$$

Thus, although the pion excitation mass within the infinite chiral liquid is reduced relative to its vacuum value, it is certainly not zero and there will be a mass gap in the spectrum for pion excitation. This is a crucial difference between a chiral liquid and the old idea, originally due to Migdal, of pion condensation.[32] There, quantum corrections due to the presence of nucleons result, at sufficiently high baryon number density, in a zero mass gap for pion excitation. P-wave pion condensation relies on the fourth operator in (8)and although it would persist in the limit of zero quark masses, the chiral liquid would not; for $\beta = m_\pi = 0$ the shape of $\mathcal{U}$ in (31)would not support a saturating non-topological soliton. The second term in (9)might result in S-wave pion condensation[32] (similar $SU(3)_L \times SU(3)_R$

terms might result in S-wave kaon condensation[33]) in neutron stars with a zero mass gap for the pion excitation spectrum. Since the mass gap for pion excitation in an infinite chiral liquid is not zero, it is not a pion condensate.

In the chiral liquid, $< \Phi|[\vec{\pi}_{operator}, \partial_\mu \vec{\pi}_{operator}]|\Phi >= 0$ so the classical pion field is not really composed of quantum pions but, rather, plays the role of an auxilliary field; it is to be regarded as the expectation value of two-fermion operators in the fluid. This suggests that the chiral liquid is essentially composed of free non-interacting fermions with a sharp Fermi surface k_F. For such a system with $B \to \infty$, the quantum correlation function[34]

$$< \Phi|\Psi(t,\vec{x})\bar{\Psi}(t,\vec{y})|\Phi >\sim \int^{k_F} d^3k e^{i\vec{k}\cdot(\vec{x}-\vec{y})} \sim \frac{sin\xi - \xi cos\xi}{\xi^3} \tag{42}$$

(where $\xi = k_F|\vec{x}-\vec{y}|$) does not fall off exponentially but neither is it long ranged. Rather, it is constant for small ξ and falls off as $|\vec{x}-\vec{y}|^{-2}$ for large ξ. This behavior is typical of a free non-interacting Fermi gas with a sharp Fermi surface.*

All of this implies that the infinite chiral liquid is on the borderline of having either normal properties or super-properties; chiral $SU(2)_L \times SU(2)_R$ effective Lagrangian interactions neglected in the construction of the mean-field chiral liquid will therefore determine whether it develops super-properties or other macroscopic quantum properties. An interesting example is the spin and isospin-dependent chiral axial-vector coupling of pions to nucleons arising from the fourth operator in (8). Tensor forces from one-pion exchange between nucleons within the liquid and the interaction of nucleons with the classical pion field of the liquid via this operator might cause quantum correlations in which neutrons pair with protons of opposite spin. Opposite-neutron-spin pairs would then alternate in a structure similar to a P-wave pion crystalline condensate.[32][35][1] If the infinite chiral liquid develops macrosopic quantum properties such as this or superfluidity or superconductivity,

* In the condensed matter physics literature, such quantum correlations are called "Friedel oscillations".

these may have relevance for neutron stars. Further, it would be interesting to see whether some remnant of such a super-property persists in ordinary finite heavy nuclei and is exacerbated by isospin-breaking effects such as electromagnetism.

Electromagnetism breaks isospin symmetry explicitly. When included in the infinite chiral liquid, the Goldstone bosons of spontaneous isospin breaking acquire an infinite mass gap and the third component of isospin takes the value $T_3 = -\frac{1}{2}B$ so that the liquid becomes electrically neutral. Although symmetry under rotations about the 1st and 2nd isospin directions has been lost, $\vec{T}^2, T_3$ and B still commute with the effective Hamiltonian H and are good quantum numbers.

We now prove the obvious from (38); for large $|T_3|$ and unbroken isospin, $\vec{T}^2 = T_3^2$. Our purpose is to introduce a language in which to discuss a new phenomena peculiar to chiral liquids: parity doubling. We represent possible chiral liquids by a vector in isospin space $\vec{\Pi}$ which points in some direction as in Figure 7. $\vec{\Pi}$ is to be regarded as essentially the classical pion field $\vec{\pi}(\vec{x},t)$ but includes the effects of fermion bilinears in $|\Phi>$ as well. In this representation and with the angles ϑ and ϕ defined in Figure 7, the generators of $SU(2)_{L+R}$ isospin rotations are

$$\begin{aligned} T_3 &= -\, i\frac{\partial}{\partial\phi} \\ T_\pm &= \pm\, e^{\pm i\phi}(\frac{\partial}{\partial\vartheta} \pm i\frac{cos\vartheta}{sin\vartheta}\frac{\partial}{\partial\phi}) \\ T_\pm &= T_1 \pm iT_2 \\ \vec{T}^2 &= -\, sin^{-1}\vartheta\frac{\partial}{\partial\vartheta}sin\vartheta\frac{\partial}{\partial\vartheta} + T_3^2 sin^{-2}\vartheta \end{aligned} \tag{43}$$

Electromagnetism fixes the value of T_3 by the last term in (38); we are interested in what value of $\vec{T}^2$ is taken by the chiral liquid for fixed T_3. The contribution $E_{isospin} = a_{\vec{T}^2}\frac{\vec{T}^2}{B}$ to the mass of a drop of chiral liquid is sketched as a function of the angle ϑ in the lower line of Figure 8. From the fourth equation in (43), it is clear that for fixed T_3, the ground state has no oscillations in ϑ and $E_{isospin} \sim T_3^2 sin^{-2}\vartheta$ so that the liquid takes the value $\vartheta = \frac{\pi}{2}$ and $\vec{T}^2 = T_3^2$. In the representation of Figures 7 and 8, there is a single ground state with $\Pi_3 = 0$.

The introduction of even a small explicit isospin-breaking contribution, such as the mass splitting between charged and neutral pions, now induces a dramatic effect in the chiral liquid. Consider the addition of a term (for small pion fields) which inclues electromagnetic spliting of the pions

$$\Delta H = \int d^3x \delta m \frac{m_\pi}{2}(\vec{\pi}^2 - \pi_3^2) \tag{44}$$

to the effective chiral Hamiltonian H of Section 2.2. $SU(2)_{L+R}$ is broken explicitly by the splitting of u and d quarks; $m_d - m_u \sim \delta m = m_{\pi_+} - m_{\pi_3} \simeq 4.6\ MeV$. This induces a contribution to the energy of a drop of chiral liquid $\Delta E \simeq C'\delta m V f_\pi(\vec{\Pi}^2 - \Pi_3^2) \simeq C\delta m B sin^2\vartheta$ where C, C' are order one and become constants for large baryon number. For fixed T_3, the ground state again has no oscillations in ϑ (from (43)). However, the isospin-dependent energy contribution to the drop of chiral liquid

$$E_{isospin} \simeq a_{\vec{T}^2}\frac{T_3^2}{Bsin^2\vartheta} + C\delta m B sin^2\vartheta \tag{45}$$

is minimized by

$$sin^2\hat{\vartheta} = (\frac{a_{\vec{T}^2}}{C\delta m})^{\frac{1}{2}}\frac{|T_3|}{B} \tag{46}$$

$E_{isospin}$ in (45)is sketched (much exaggerated in order to show the effect) as a function of ϑ in the upper line in Figure 8. It is clear from (46)and Figure 8 that there are now *two* values of ϑ which minimize the energy of a drop of chiral liquid: $\hat{\vartheta}$ and $\pi - \hat{\vartheta}$.

Quantum mechanically, the two configurations (they are not eigenstates) $|\vec{\Pi}_{\hat{\vartheta}}>$ and $|\vec{\Pi}_{\pi-\hat{\vartheta}}>$ corresponding to the two minima in Figure 8 are related by the parity operator $\mathcal{P}$: $\mathcal{P}|\vec{\Pi}_{\hat{\vartheta}}>=|\vec{\Pi}_{\pi-\hat{\vartheta}}>$. To see this, consider an S-wave classical pion field such as in a very large drop of chiral liquid. Since, for massless quarks, pions are the Goldstone bosons of spontaneous $SU(2)_{L-R}$ symmetry breaking, they are

pseudoscalars

$$\mathcal{P}\vec{\pi}(\vec{x},t)\mathcal{P}^{-1} = -\vec{\pi}(-\vec{x},t) \tag{47}$$

In Figure 7, the replacement $\vec{\Pi} \to -\vec{\Pi}$, is equivalent to $\vartheta \to \pi - \vartheta, \phi \to \phi + \pi$ and, for the minima in particular, $\hat{\vartheta} \to \pi - \hat{\vartheta}$ in Figure 8.

Since parity is exactly conserved by the strong and electromagnetic interactions, two parity eigenstates $|+>= \frac{1}{\sqrt{2}}(|\vec{\Pi}_{\hat{\vartheta}}> + |\vec{\Pi}_{\pi-\hat{\vartheta}}>)$ and $|->= \frac{1}{\sqrt{2}}(|\vec{\Pi}_{\hat{\vartheta}}> - |\vec{\Pi}_{\pi-\hat{\vartheta}}>)$ emerge. These two states, respectively of positive and negative parity, are split in energy because they are able to tunnel into each other (through the bump in the upper line of Figure 8) via (44). For large drops of chiral liquid, the tunneling amplitude $\sim e^{-\kappa V f_\pi^3}$ (with κ order 1) is exponentially damped with the liquid volume V. Therefore, as the baryon number of the drop $B \to \infty$, $|+>$ and $|->$ become exactly degenerate and there are *two degenerate ground states*. We call this phenomena, peculiar to chiral liquids, "parity doubling". It is essentially due to the buildup of a classical field made of pseudoscalar pions which, from (47), have nontrivial internal parity.

4 CHIRAL NUCLEI

Could the structure of heavy nuclei really be tractable from first principles rather than be a complicated morass requiring a long and unenlightening numerical solution? A vast simplification comes from the empirical knowledge that nuclei are made of nucleons (nuclei are not bags of quarks) which behave as elementary particles for the purposes of the chart of nuclides; this is the basis for our choice of an effective Lagrangian involving nucleons and pions (rather than quarks, pions and gluons[14] or just pions with topology[36]). Empirically, it is known that, within ordinary nuclei, nucleons and pions interact in a highly non-linear way and, further, the momenta transfered are all soft.

Nuclear structure is therefore just another soft pion process and is governed by the effective chiral Lagrangian (7). This means that it is also one of the few low energy hadronic processes in which we can control the strong interaction quantum

loops to all orders; this control of the quantum loop structure via the effective chiral Lagrangian is a crucial difference between the approach of this paper and previous mean field models of nuclear structure.[6][37] The long ranged interactions of pseudo Goldstone boson pions with each other and with nucleons as well as the contact nucleon-nucleon interactions are to be regarded as "fundamental", in the sense that their low energy dynamics is simple and follows from considerations of the chiral-symmetric properties of QCD alone. Nuclear structure is then analogous with the structure of many-electron atoms, where the forces between particles are known from first principles but tractibility is still doubtful.

Following the very old and (perhaps suprisingly) successful liquid drop model of heavy nuclei, we hypothesize that, at least at a crude level, ordinary saturating heavy nuclei are droplets of $SU(2)_L \times SU(2)_R$ chiral liquid which exist at zero external pressure. Indeed, all of the simplifying assumptions used in the construction of drops of chiral liquid in previous sections came from intuition gained by examination of the empirical properties of heavy nuclei. At a practical level, this constitutes a vast simplification in the chiral theory of finite nuclei. Although the liquid drop approximation is, of course, completely inappropriate for light nuclei, as the baryon number B grows the baryon number density becomes very flat because of nuclear saturation, spatial derivatives can be ignored except at the surface, and a heavy nucleus can sensibly be described as a drop of liquid which exists at zero external pressure.[31] Therefore, the paradigm of proton and neutron Fermi fluids in a huge coherent self-consistent classical mean S-wave pion field (or, equivalently, fermion bilinears) $< \Phi|\vec{\pi}|\Phi > \simeq$ 200-400 MeV, with interactions gotten entirely from an effective chiral Lagrangian of hadrons, makes the gross structure of heavy nuclei tractible. This is exactly the definition of an $SU(2)_L \times SU(2)_R$ chiral liquid.

Figure 9 shows the observed charge density distributions of doubly closed-shell nuclei with baryon number $B \geq 40$.[38] The slight dip in the experimental lines of Figure 9 is due to proton electromagnetic repulsion, an effect neglected in the calculations of Section 3.3; the slight thickness of the lines gives the experimental uncertainties. Compare the baryon density profiles (solid lines) of the nontopo-

logical solitons of Figures 4 through 6 with the electric charge densities in Figure 9. Assuming that the baryon density of ordinary heavy nuclei closely tracks the charge density, it is clear that the shape of the baryon density profile of drops of chiral liquid closely resembles that of real nuclei, although one might complain that the baryon surface region is a little thin. Beyond the shape, it is difficult to compare the magnitude of the baryon density of our isospin $T = 0$ solitons with the magnitude of the observed charge density of ordinary nuclei. Electromagnetic effects neglected in Figures 4 through 6 but included in Figure 9, force larger nuclei to have isospin $T > 0$; from this point of view, it is therefore difficult even to compare the magnitudes of the baryon densities in ${}_{20}Ca^{40}$,${}_{50}Zr^{90}$ and ${}_{82}Pb^{208}$ with each other. Assuming that all of the electric charge of heavy nuclei is carried by protons and that the neutron density distribution is $\frac{N}{Z}\times$ the charge density distribution, we find that the neutron density within the deep interior of the $B = 277$ nontopological soliton in Figure 6 is $.733\times$ the inferred neutron density in the deep interior of ${}_{82}Pb^{208}$. The proton density of this soliton is $.888\times$ that of ${}_{82}Pb^{208}$ in the deep interior. We regard the similarity of our nontopological soliton density profiles to those of real nuclei as striking. Of course, this is because of the saturation conditions (32)and because the natural density scale of chiral symmetry is f_π^3, in close agreement with the observed neutron density of heavy nuclei.

As a purely mathematical proposition, if spontaneously (and explicitly) broken chiral $SU(2)_L \times SU(2)_R$ symmetry contains saturating nontopological soliton field configurations such as we have identified with drops of chiral liquid, they are a more appropriate starting point for a perturbation expansion for nuclear structure than free-particle states because they better minimize the exact quantum energy (13)for fixed baryon number and electric charge. Perhaps more importantly, the mathematical fact of their existence would give, for the first time, a chiral symmetric explanation for the very existence of ordinary heavy nuclei in nature and, thus, for most of the periodic table of the elements.

There are, of course, problems with this picture. The natural scale of $SU(2)_L \times SU(2)_R$ chiral symmetry of nucleons and pions is f_π. Nuclear baryon number den-

sities and isospin splittings are then naturally f_π^3 and f_π. All other mass splittings are also naturally $\sim f_\pi$. From the point of view of chiral liquids composed of nucleons and pions, a central mystery of nuclear physics is why a_{volume} and $a_{surface}$ in (38)are observed to be approximately 16 and 20 MeV respectively,[31] rather than f_π. These anomalously small values result, after the inclusion of electromagnetism, in the observed nuclear binding energy per nucleon of $\simeq 8$ MeV $\ll f_\pi$ over much of the chart of nuclides. A separate theoretical disaster in (38)is avoided only if $a_{volume} < \frac{a_{\vec{T}^2}}{4}$; otherwise there would would exist neutral chunks of nuclear matter of arbitrary, even macroscopic, size.[5][11][12] It was shown in Section 3.4 that $a_{\vec{T}^2} \simeq f_\pi$ in agreement with the observed properties of real nuclei[31] so, again, a_{volume} must be anomalously small. Unfortunately, we have here provided no chiral explanation for the mystery of why nuclear binding energies are so low.*

We have found numerically isospin $T = 0$ chiral nontopological solitons whose chemical potential for the infinite chiral liquid $\mu_{\infty liquid} = m_N - 16$ MeV. These occur in the upper right hand corner of Figure 3 (near the circled point there) and naturally have baryon densities very close to that observed in large nuclei like ${}_{82}Pb^{208}$. However, we must resort to fine-tuning the cancellation of energy contributions from scalar attraction $\sim f_\pi$ and vector repulsion $\sim f_\pi$ with $\frac{|C_V^2 - C_S^2|}{C_S^2} \approx .1$ in order to set $a_{volume} = 16$ MeV and $\beta \approx 1$ (see below). The reader will have complained that $(\Psi^\dagger\Psi)^2$ and $(\bar{\Psi}\Psi)^2$ differ by terms of relative order $(\frac{k_F}{m_N})^2 \approx .1$ for non-relativistic nucleons. The next order in chiral perturbation theory, with a factor $\frac{f_\pi}{\Lambda_{CSB}}$ relative to terms we have kept, is numerically of the same size as this difference.[3] Therefore, for the case of anomalously small a_{volume}, although we have calculated self-consistently in powers of Λ_{CSB} in chiral perturbation theory, terms of order $(\Lambda_{CSB})^{-1}$ must still play a very important role in the nontopological soliton solutions. Indeed, it is self-inconsistent to neglect them.

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* An original motivation for this work was the realization that since true Goldstone bosons have only derivative couplings, pions cannot carry static long range forces to nucleons in the interior of large saturating nuclei in the chiral limit where quarks and pions are massless. Rather, pions induce static long range forces there only via explicit $SU(2)_{L-R}$ breaking terms whose QCD origin lies in quark masses. We had hoped to find some $SU(2)_L \times SU(2)_R$ reason why $a_{volume} \simeq m_{quark}$.

A related problem is the magnitude of β in (17). β comes from quark mass $SU(2)_{L-R}$ breaking; since the first term in (9)gives the estimate

$$m_q \Lambda_{CSB} \sim m_\pi^2 \tag{48}$$

we should have $\beta \sim \frac{m_\pi}{\Lambda_{CSB}}$ essentially because $m_q \simeq 9$ MeV. The experimental value $\beta \approx 1$ (with huge systematic errors[24]) seems already to be in contradiction with this. The value $\beta = 6.66$ for the drops of chiral liquid constructed in Section 3.3 and displayed in Figures 4 through 6 is even larger. It is encouraging that, for the $T = 0$ nontopological solitons with fine-tuned $a_{volume} = 16$ MeV which we have found numerically, $\beta \simeq 2$, an experimentally more reasonable value. This set of nontopological solitons with $\mu_B \geq \mu_{\infty liquid} = .983 m_N$ corresponds to the circled point in Figure 3. We worry, though, that this is still too large for naive dimensional power counting.[14] One might speculate that, for drops of chiral liquid with anomalously small values of a_{volume} in (38), terms of order $(\Lambda_{CSB})^{-1}$ will provide enough extra attraction so that β will obey naive power counting.

With the power counting (48), terms of order $(\Lambda_{CSB})^{-1}$ in the chiral perturbation theory (7)have $n_1 + n_3 + 2n_4 = 3$; there are no new explicit $SU(2)_{L-R}$ symmetry breaking $n_4 \neq 0$ terms. There are two new sets of purely chiral operators with derivatives

$$\frac{1}{\Lambda_{CSB}} \bar{\Psi}\Psi (\frac{\pi}{f_\pi})^{n_2} (\partial)^2 \ , \ \frac{1}{f_\pi^2 \Lambda_{CSB}} (\bar{\Psi}\Psi)^2 (\frac{\pi}{f_\pi})^{n_2} (\partial) \tag{49}$$

Some of these appear in Reference 3. In addition, there is a set of six-fermion operators without derivatives

$$\frac{1}{f_\pi^4 \Lambda_{CSB}} (\bar{\Psi}\Psi)^3 \tag{50}$$

As in the notation of (7), the derivative ∂ acts on nucleons or pions and all constants, like Dirac and isospin matices, are not shown in (49)and (50). Although

there are many operators with unknown coefficients in (49)and (50), very few are able to contribute to the structure of drops of spin-independent chiral liquid. For example, in the non-relativistic limit, there is a unique spin-independent isospin-independent term with no derivatives in the effective Lagrangian $\mathcal{L}$ (C_3^2 is an order one constant)

$$-\frac{C_3^2}{f_\pi^5}\frac{f_\pi}{\Lambda_{CSB}}(\Psi^\dagger\Psi)^3 \tag{51}$$

We speculate that, after inclusion of higher dimension operators, $SU(2)_L \times SU(2)_R$ chiral perturbation theory (7)will produce drops of chiral liquid which reproduce the details (charge density profiles, bulk nuclear compressibility, surface shape and energy, nuclear optical potential, collective coordinate phenomena such as giant resonances and liquid breathing modes, etc.) of bulk nuclear structure, but now with a much smaller coefficient β for the π-nucleon sigma term. a_{volume} and $a_{surface}$ will then be fine-tuned to their observed values in (38)so as to reproduce the old semi-empirical nuclear mass formula with its well known physical consequences for strong interaction nuclear ground states.[31] These include the valley of β stability of the chart of the nuclides, nuclear baryon number density profiles, the instability of lighter nuclei to fusion, the instability of very heavy nuclei to fission, and that the ground state of nucleons and pions with non-zero baryon number is ${}_{26}Fe^{56}$.

One might argue that for such phenomena the chiral symmetry itself is unimportant; that it is simply a matter of balancing attractive and repulsive forces to give saturation with the right energy and density. The counter-argument rests on the non-linearities inherent to low energy $SU(2)_L \times SU(2)_R$ chiral symmetry. Nucleon and pion scattering and decay amplitudes are sensitive to the linearized chiral Lagrangian. In contrast, nontopological solitons are notoriously non-linear objects which depend with excruciating sensitivity on the non-linearities of the effective chiral Lagrangian. Imagine now that we set all the relevant chiral coefficients in (7)by scattering and decay experiments. The bulk properties of ordinary heavy nuclei then follow entirely from and are extremely sensitive to the non-linearities

of the low energy chiral operators (7); these non-linearities, in turn, are uniquely determined by non-linear chiral $SU(2)_{L-R}$ and $SU(2)_{L+R}$ transformations obeying the algebra of (2),(3)and (5). If chiral symmetry of nucleons and pions were able to reproduce simultaneously the results of all soft nucleon and pion scattering and decay experiments as well as the *details* of the bulk structure of ordinary heavy nuclei, heavy nuclei will have been demonstrated, at least theoretically, to be of chiral origin.

The next step would then be to search experimentally for some phenomenon directly traceable to the liquid phase of non-linear $SU(2)_L \times SU(2)_R$ chiral symmetry. Ideally, certain predictions of the chiral liquid theory of bulk nuclear structure would differ qualitatively from those of more conventional formulations of nuclear structure. An example would be some remnant of parity doubling. For heavy nuclei of finite volume, the parity-doubling states $|+>$ and $|->$ of Section 3.5 will be split in energy by their tunneling amplitude; as an experimental signature, nuclear spectroscopy might be enriched. In contrast, nuclear mean field theories in which nucleons are trapped in an S-wave classical scalar isosinglet σ field[6] do not have parity doubling because $\mathcal{P}\sigma(\vec{x},t)\mathcal{P}^{-1} = +\sigma(-\vec{x},t)$; the σ is its own parity-reversed partner so there is only one non-degenerate ground state in the $B \to$ limit.

5 CHIRAL NEUTRON STARS

If heavy nuclei are droplets of $SU(2)_L \times SU(2)_R$ chiral liquid, neutron stars are great chunks of chiral liquid held together by gravitational forces. Here the paradigm should work at its best. Densities are constant over distances $\sim$ kilometers, huge on the scale of the pion Compton wavelength, and the baryon number $B \to \infty$ for the purposes of strong interaction physics. Further, aside from gravitational binding energy, momenta-transfer are small,* and nucleons and pions are the right degrees of freedom. Neutron stars are then driven by soft-pion processes and

* In this paper, we will neglect higher energy degrees of freedom such as hyperons and quarks. These may play a significant role in the deep stellar interior[39] where momenta-transfer and densities might disobey the criteria for the applicability of the $SU(2)_L \times SU(2)_R$ low energy effective Lagrangian (7).

the low energy non-linear chiral symmetric Lagrangian (7)governs the dynamics. This means that we are able to control the strong interaction quantum loops to all orders[37] [16] and extract with confidence chiral symmetric predictions as to the properties of neutron stars.

Conventional neutron star models usually make a highly nontrivial extrapolation from the experimentally observed properties of heavy nuclei with $B \sim 10^2$ to huge $B \sim 10^{57}$. But how can it be known empirically whether there is an important and qualitatively new 2000-nucleon interaction? Nothing is known from the laboratory about the properties of $B \geq 267$ systems at approximately nuclear densities and, thus, there is no empirical basis on which to extrapolate empirical knowledge from heavy nuclei to neutron stars. Here chiral $SU(2)_L \times SU(2)_R$ symmetry shows its power; the criteria for the use of the effective Lagrangian (7)include no statement about the total baryon number in a soft pion process. We may therefore use the effective Lagrangian with as much confidence in examining the properties of $B \sim 10^{57}$ at nuclear densities as in examining $B = 208$ as long as the criteria for the use of the effective Lagrangian is obeyed. In this way, chiral symmetry is a theory, rather than a model, of low energy hadron dynamics. We may have some confidence in the extrapolation from the properties of small drops of chiral liquid to the properties of huge baryon number $B \sim 10^{57}$ chunks of chiral liquid.

An immediate and crucial question is whether neutron stars have a macroscopic minimum mass. In the language of Section 3.4, the issue is whether $a_{volume} < .25 a_{\vec{T}^2}$. If not, a new phase of baryonic matter (not ordinary nuclei) appears resulting in neutral chunks of baryonic matter varying in size from $\sim 10^{-12} cm$ to several kilometers which are held together entirely by strong forces.[5][11] The density of such "baryonic Q-Balls"[11] is constant and, like heavy nuclei, they saturate. As the mass increases, they become larger and the mass $M \sim B \sim$ the volume V for small B. Eventually, first Newtonian gravity and then generally relativistic gravity become strong enough to overcome this inherent stiffness, resulting in "Q-Stars" and a generic curlique spectrum such as in the solid lines marked (a) and (b) of Figure 10.[12][8] Q-Stars are the only known compact objects able to circumvent

the widely accepted upper bound on the mass of a neutron star $M_{neutron\ star} < 3.2M_{\odot}$[40] An example appears in the curlique labelled "b" in Figure 10. Finally, though, they become so massive as to be unstable to decay into black holes, as indicated by the crosses.

Based on the observed instability of transuranic nuclei to fission and that the heaviest observed nucleus has $B = 266$,[41] we assume here that $SU(2)_L \times SU(2)_R$ chiral symmetry has no such new phase of baryonic matter. In the language of Section 3.4, $a_{\vec{T}^2} \gg a_{volume}$; the isospin energy overcomes the volume binding energy in a large drop of chiral liquid so that, at some large baryon number, electromagnetic repulsion blows the soliton apart. This, of course, is just the usual explanation for the fission of transuranic nuclei.[31] In that case, the ground state of a large number of neutrons is to be dispersed to infinity. Only when the baryon number $B > 10^{56}$ and the mass $M_{neutron\ star} > .1M_{\odot}$ are large enough so that gravitational attraction overcomes the neutron Fermi repulsion energy will a neutron star exist. A chiral neutron star is held together entirely by gravity* so that, as the mass increases, the strength of gravity increases and it becomes smaller, such as in the dashed lines of Figure 10. Chiral $SU(2)_L \times SU(2)_R$ neutron stars, whose properties are extrapolated from those of drops of chiral liquid which reproduce the bulk properties of ordinary heavy nuclei, are probably unable to evade the upper mass limit $\sim 3.2M_{\odot}$. Eventually the mass becomes large enough so as to become unstable to decay into a black hole. If there is no new baryonic matter phase, the neutron stars of chiral $SU(2)_L \times SU(2)_R$ symmetry will certainly resemble the dashed lines, rather than the solid lines, in Figure 10; they will not be Q-Stars.

The building of neutron stars is a straighforward excersize in relativistic fluid dynamics. The essential ingredients are a hadronic equation of state and general relativity; for spherically symmetric static neutron stars, the problem is reduced

* Because they would disperse to infinity in the absence of gravity, chiral neutron stars are, in a sense, more properly called a gas rather than a liquid. Still, their density is constant at a macrosopic scale so we will refer to them loosely as a liquid.

to the classic equations of Oppenheimer and Volkoff.[42][39] An estimate for the mass and radius of a neutron star follows immediately from f_π, the scale typical of chiral $SU(2)_L \times SU(2)_R$ symmetry. Since chiral neutron stars have kinetic energy density $\sim f_\pi^4$, the generally relativistic equations may be rescaled dimensionless and without large coefficients by the rescaling of the chiral neutron star mass and radius

$$\begin{aligned} M_{neutronstar} &\sim \frac{m^3_{Planck}}{(8\pi)^{\frac{1}{2}} f_\pi^2} \simeq 2M_\odot \\ R_{neutronstar} &\sim \frac{m_{Planck}}{(8\pi)^{\frac{1}{2}} f_\pi^2} \simeq 20\ km \end{aligned} \tag{52}$$

In order to do better, we have to construct the equation of state, writing the energy density $\mathcal{E} = T^{00}$ in terms of the total pressure P, but this is beyond the scope of this paper. Instead, we will outline how to do so for the effective Lagrangian (7)and the variational equation (13). We start with six particles: nucleons P and N, pions π_+, π_3, electrons e and neutrinos ν. Chemical potentials and Fermi momenta are introduced for the fermions $\mu_i, k_{F,i}$ with $i = P, N, e, \nu$ and Fermi fluids constructed in a way similar to Section 3.2. Chemical potentials for baryon number and third component of isospin μ_B, μ_3 are introduced and two classical pion fields π_+, π_3. The energy density and pressure are then constructed as functions of these 12 variables. 9 conditions are to be enforced: 1) conservation of baryon number, 2) conservation of third component of isospin (or, equivalently, electric charge), 3), 4), 5), 6) four Dirac equations, 7) weak decay equilibrium, 8) total electric charge equal zero, 9) neutrinos escape the star ($\mu_\nu = 0$) . In addition, since the classical pion field configurations vary slowly over huge distances in chiral neutron stars, (20)gives two saturation equations with $\vec{\nabla}\pi_+ = \vec{\nabla}\pi_3 = 0$ for a total of 11 conditions in 12 variables. The last one is eliminated by solving for $\mathcal{E}(P)$ and we have the equation of state. The two dashed lines in Figure 10 span the range of conventional neutron star mass versus radius relations for various models of the nucleon-nucleon force on the market. They are,respectively, the pion-condensed model π and the mean field model MF.[39] After integrating the Oppenheimer-

Volkoff equations, the $SU(2)_L \times SU(2)_R$ chiral neutron stars mass versus radius relation will resemble these and lie in their vicinity, such as we have sketched in the dotted line of Figure 10.

One could hope to find experimentally observable consequences of the peculiar quantum properties of chiral liquids in chiral neutron stars. Parity doubling could be particularly interesting there because two ground states would be available for a neutron star. After inclusion of small isospin breaking terms, the two quantum states $|+>$ and $|->$ in Section 3.5 are unable to tunnel into each other in the infinite volume limit and so are exactly degenerate. Are there then two different sorts of neutron stars? Does this lead, perhaps at the time of formation in a supernova explosion, to domain structure within a neutron star? Chiral symmetric questions such as these may allow us to add neutron stars to the list of systems to be understood entirely on the basis of chiral operators constructed according to the global symmetries of QCD as well as chiral coefficients fit from laboratory low energy scattering and decay experiments and the bulk properties of heavy nuclei.

6 CONCLUSIONS

The most powerful and successful idea in low energy hadronic physics is the validity, in scattering and decay experiments, of chiral $SU(2)_L \times SU(2)_R$ symmetry of nucleons and pions. The effective Lagrangian used there includes both tree and quantum loop levels and is unique for soft processes; it can be traced directly to the current algebra of the global symmetries of QCD.[9] [16] [13] We hypothesize that this proven theory of soft nucleons and pions contains a liquid phase and that drops of liquid could exist at zero external pressure. In addition, the liquid phase may have interesting quantum properties, such as parity doubling.

On the other hand, the best known fact of low energy strong interaction physics is the existence of ordinary nuclei. We regard the structure of ordinary heavy nuclei as simply another soft process which can be understood within the framework of strong $SU(2)_L \times SU(2)_R$ chiral symmetry. Heavy nuclei are then just droplets of chiral liquid. Indeed, the mathematical existence of saturating nontopological

soliton field configurations in broken hadronic $SU(2)_L \times SU(2)_R$ symmetry, may give a chiral explanation for the very existence of ordinary heavy nuclei. At a practical level, our treatment of heavy nuclei as nontopological soliton droplets of chiral liquid vastly simplifies the extraction of bulk nuclear properties from chiral symmetry. We have argued that chiral symmetry is able to reproduce, at least at a crude level, the existence of ordinary heavy nuclei, saturating nuclear baryon number density distributions and a semi-empirical mass formula. If heavy nuclei were indeed demonstrated to be of chiral origin, $SU(2)_L \times SU(2)_R$ symmetry of nucleons and pions would then give us access to the vast storehouse of experimental nuclear information. Trustworthy chiral predictions for neutron stars would follow. Because of the peculiar quantum properties of chiral liquids, it may be possible to distinguish experimentally the chiral liquid theory of heavy nuclei and neutron stars from more conventional models.

ACKNOWLEDGEMENTS

I thank J. Ellis and the theory group at CERN for their hospitality in the Winter of 1990 where an early attempt to make this idea realistic was made with S.B. Selipsky and E. Nardi. I thank S. Bahcall, S. Dimopoulos, S. Hanna, R.B. Laughlin, S. Meshkov, Z.E. Meziani S.B. Selipsky and V. Teplitz for interesting conversations. In addition to valuable conversations, I am indebted to L. Susskind for his suggestion that the quantum chiral liquids of Section 3.5 should develop spontaneous breaking of isospin and massless Goldstone bosons in the absence of electromagnetism, and his further suggestion that they would develop parity doubling in the presence of electromagnetism and a mass splitting between charged and neutral pions. I thank S.B. Selipsky for a seminal conversation about the equation of state of chiral neutron stars. I thank J. Thomas for many valuable physics conversations as well as clearing a path through the jungle of computer protocols and hardware. I am grateful for the hospitality of F. Gilman and the SSCL theory group during the Spring of 1992. This work was supported in part by the National Science Foundation contract NSF-PHY-89-17438 and by the Universities Research Association Inc. with Department of Energy contract DE-AC35-89ER40486.

REFERENCES

1. Nuclear physics has a long tradition in the derivation of the nucleon-nucleon force from the linear $SU(2)_L \times SU(2)_R$ σ model or the linearized non-linear σ model. Basing their work on the coupling of Goldstone boson pions to nucleons, their successes include tensor nucleon-nucleon interactions from one pion exchange, contact nucleon-nucleon interactions from two pion exchange and the long-distance properties of the deuteron. For a review see T. Ericson and W. Weise, Pions and Nuclei, Clarendon Press, Oxford, U.K. (1988).

2. Important recent work continuing the tradition of Reference 1 on the origins of nuclear forces in chiral symmetry is found in S. Weinberg, Phys. Lett. 251B(1990)288; Nuc. Phys. B357(1991)3. He has focused on effective two, three and four-nucleon interactions and very light nuclei in the chiral limit of massless quarks and pions where spin and isospin-dependent forces are expected to play a crucial role.

3. In continuing the work of Reference 2, certain higher dimension chiral operators contributing to nuclear forces can be found in the non-relativistic limit in C. Ordonez and U. van Kolk, Univ Texas, Austin preprint UTTG-01-92.

4. That nuclei are non-topological solitons of non-linear $SU(2)_L \times SU(2)_R$ chiral symmetry of pions and nucleons was first suggested by B.W. Lynn, invited lectures, South African Summer School in Physics of Strongly Interacting Matter, Capetown, South Africa, January, 1990, J. Cleymans, C.A. Engelbrecht, H. Miller and R.D. Viollier, organizers.

5. T.D.Lee and G.Wick, Phys. Rev. D9 (1974)2291

6. J.D. Walecka, Ann. Phys. 83(1974)491; Early work includes A.K. Kerman and L.D. Miller, MIT-ITP preprint 449 (1974) and R. Serber, Phys. Rev. C14 (1976)718; For an excellent review see B.D. Serot and J.D. Walecka,

Adv. Nucl. Phys. 16(1986)1. The accepted relativistic mean field models require attractive nuclear forces to be carried by a σ particle,[21] a broad scalar positive-parity resonance (decaying unobservably quickly to pions) with mass $\approx 500-600$ MeV. Since it is composite and a chiral singlet, the σ self-interaction potential $U(\sigma)$ need not obey renormalizability or unitarity conditions and therefore has an infinite number of adjustable parameters. It is precisely this inability to control $U(\sigma)$ which makes it easy[11] to make models which self-consistently contain two classes of solutions for chunks of baryon number: one class corresponding to ordinary nuclei and the other class corresponding to exotic states of large baryon number such as Lee-Wick matter. An example is the "Q-Star" model of neutron stars; stable neutron star configurations with very high masses ($M_{NeutronStar} \gg 5M_{\odot}$) can be made consistent with the known bulk properties of nuclei[11][12] (and chiral symmetry[8]), thus evading a widely accepted theoretical upper bound $M_{Neutron\ Star} \leq 3.2M_{\odot}$[40] upon which *all* present observational evidence for the existence of low mass ($M_{Black\ Hole} < 10^3 M_{\odot}$) black holes is based. Thus, while $U(\sigma)$ offers many new possibilities, the price is very high; we have lost all predictive power. Exacerbating this problem is the inability to control the quantum loop contribution to nuclear structure; in a theory of strongly interacting matter, quantum loop effects of arbitrarily high order will be large and important, so that even the phenomenological successes of present relativistic mean field models of nuclear structure will not survive the quantum loop level.[37] Further, it is worrisome that this σ particle has never been observed directly; the closest observed resonance candidate is the $f_0(975)$ which is too heavy for nuclear structure calculations. Chiral symmetry, of course, has no need of the σ.[17][9][13]

7. W.Lin and B.D.Serot, Phys. Letts. B233 (1989) 23 have shown that isoscalar spin-independent nucleon-nucleon interactions mediated by two pion exchange[1] resemble σ exchange but cannot alone lead to nuclear saturation.

8. B.W.Lynn, A.E.Nelson and N.Tetradis, Nuc. Phys. B345 (1990) 186.

9. S. Weinberg, Phys. Rev. 33 (1968) 1568. We use his representation of the chiral algebra.

10. J. Goldstone,Nov. Cim. 19(1961)15; Y.Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961)345; 124 (1961) 246; J. Goldstone, A. Salam and S. Weinberg, Phys. Rev. 127 (1962) 965.

11. S.Bahcall, B.W.Lynn and S.B.Selipsky, Nucl. Phys. B325 (1989) 606; B331 (1990) 67.

12. S. Bahcall, B.W. Lynn and S.B. Selipsky, Astrophys. J. 362 (1990) 251; B.W. Lynn, Nucl. Phys. B321 (1989) 465; S.B. Selipsky, D.C. Kennedy and B.W. Lynn, Nucl. Phys. B321 (1989) 430.

13. S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177(1968)2239; C. Callan, S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177(1968)2247

14. A. Manohar and H. Georgi, Nuc. Phys. B234 (1984)189.

15. S. Weinberg, Physica 96A (1979)327.

16. For a review of chiral symmetry, see H. Georgi, Weak Interactions and Modern Particle Theory, Benjamin-Cummings, Menlo Park, 1984; For a review of current algebra and soft pions, see S. Coleman, Aspects of Symmetry, Cambridge Univ. Press, Cambridge, 1988.

17. S. Weinberg, Phys. Rev. Letts. 18 (1967)188; J.Schwinger, Phys. Letts. 24B (1967) 473

18. N. Isgur and M.B.Wise, Phys. Letts. B232 (1989) 113; B237 (1990) 527; H.D.Politzer and M.B.Wise, Phys. Letts. B206 (1988) 681; B208 (1988)504; E.Eichten and B.Hill, Phys. Letts. B234 (1990) 511; B243 (1990) 427; B. Grinstein, Nucl. Phys. B339 (1990) 253; H.Georgi, Phys. Letts. B240 (1990) 447; M.E.Luke, Phys. Letts. B252 (1990) 447; A.F.Falk, B.Grinstein and M.E.Luke, Nuc. Phys. B357 (1991) 185; For a review see B.Grinstein, SSCL preprint 34, Feb. 1992, Ann. Rev. Nucl. Part. Sci. 42(1992)1.

19. E. Jenkins and A. Manohar, UCSD preprint PTH-91 October, 1991

20. W.E.Caswell and G.P.Lepage, Phys. Letts. B167 (1986) 437

21. J. Schwinger, Ann. Phys. 2(1957)407; J.C. Polkinghorne, Nuovo Cim. 8(1958)179; M.Gell-Mann and M. Levy, Nuovo Cimento 16 (1960) 705; In this paper, "σ particle" will specifically mean the σ' field in equations 3 and 7 of S.Weinberg in reference 17. He shows that the linear σ model is equivalent to the non-linear σ model supplemented by strong (experimentally unjustified) assumptions as to the dynamics of an $SU(2)_L \times SU(2)_R$ singlet σ'.

22. D. B. Kaplan, Phys. Letts. B275 (1992) 137

23. An introduction to this huge field can be found in E.M. Lifshitz and L. P. Pitaevskii, Vol. 9 Landau and Lifshitz Course of Theoretical Physics, Pergamon Press, Oxford, 1980.

24. J. Gasser, H. Leutwyler and M.E. Sainio, Phys. Letts. 253(1991)252 estimate a very large error on this nucleon-pion "sigma" term. Worse, the data sets are not consistent and cannot be added in quadrature; it is hoped that new low energy precision measurements, notably experiments on pionic atoms and π-nucleon scattering, can clarify the discrepancies in experimental data and give a precise determination of β. On the theoretical side, the discrepancy between the extrapolation to the Cheng-Dashen point and the naive estimate from fits to the $SU(3)_L \times SU(3)_R$ baryon octet masses has generated interest in the strangeness content of the sigma term.; see also T. Ericson, CERN-TH.6165 (1991).

25. W. Heisenberg, Z. Physik 120 (1932)1; E. Majorana, Z. Physik 82 (1933) 137.

26. An extensive bibliography can be found in W.D. Myers and W.J. Swiatecki, Ann. Phys. 55 (1969) 395; H.A. Bethe, Phys. Rev. 167 (1968) 879.

27. S. Bahcall and B.W. Lynn, Stanford Preprint SU-ITP 92/14, April, 1992

28. G.C. Wick, Nuovo Cimento 11(1934)227

29. T.D. Lee and co-workers long ago made the mathematical analogy of localized nontopological soliton solutions with Newtonian potential motion in scalar field theories with conserved Noether charges. R. Friedberg, T.D. Lee and A. Sirlin, Phys. Rev. D13 (1976) 2739; Nuc. Phys. B115 (1976)1; B115 (1976) 32; J. Frieman and B.W. Lynn, Nucl. Phys. B329(1990)1; S. Coleman, Nuc. Phys. B262 (1985) 263; J. Werle, Phys. Letts. B71 (1977) 367; T.F. Morris, Phys. Letts. B76 (1978) 337; B78 (1978) 87; A.M. Safian, S. Coleman and M. Axenides, Nucl. Phys. B298 (1988) 498; The first boson nontopological soliton which we were able to find appears in G. Rosen, J. Math. Phys. 9 (1968) 996; A theory of hadronic structure based on nontopological solitons including both the MIT and SLAC bags was given in R. Friedberg and T.D. Lee, Phys. Rev. D15 (1977) 1694; 16 (1977) 1096; The SLAC bag appears in W.A. Bardeen, M.S. Chanowitz, S.D. Drell, M. Weinstein and T.-M. Yan, Phys. Rev. D11 (1975) 1094; Boson nontopological soliton stars are discussed by T.D. Lee, Phys. Rev. D35 (1987) 3637; R. Friedberg, T.D. Lee and Y. Pang, ibid. 3640; ibid. 3658; R. Ruffini and A. Bonnazola, Phys. Rev. 187 (1969) 1767; J.D. Breit, S. Gupta and A. Zaks, Phys. Letts. B140 (1984) 329; M. Colpi, S.L. Shapiro and I. Wasserman, Phys. Rev. Letts. 57 (1986) 2485; Very large nontopological fermion soliton stars are discussed in R. Friedberg, T.D. Lee and Y. Pang, Phys. Rev. D35 (1987) 3678; For a coherent treatment of nontopological solitons see T.D. Lee, Particle Physics and Introduction to Field Theory, Harwood Academic Press., New York, 1981.

30. E.P. Wigner, Phys. Rev. 51(1937)106

31. C.F.V. Weizsacker, Z. Physik 96 (1935) 431; H.A. Bethe and R.F. Bacher, Rev. Mod. Phys. 8 (1936) 82; P.A. Seeger, Nucl. Phys. 25 (1961)1; Standard nuclear texts include J.M. Blatt and V.F. Weisskopf, Theoretical Nuclear Physics, J. Wiley, New York, 1960 and A. deShalit and H. Feshbach, Theo-

retical Nuclear Physics, Vol. 1, J. Wiley, New York, 1974.

32. A.B. Migdal, Rev. Mod. Phys. 50(1978)107.

33. D.B.Kaplan and A.Nelson, Phys. Letts. B175 (1986) 57; A.Nelson and D.B.Kaplan, Phys. Letts. B192 (1987) 193.

34. I am indebted to R.B. Laughlin for this remark.

35. T.T. Takatsuka and R. Tamagaki, Progr. Theor. Phys. 55 (1976) 624; T.T. Takatsuka, K. Tamiya, T. Tatsumi and R. Tamagaki, Progr. Theor. Phys. 59 (1978) 1933;

36. T.H.R.Skyrme, Proc. R. Soc. London A260 (1961) 127; For a review of the Skyrme model see I.Zahed and G.E.Brown, Phys. Rep. 142 (1986)1 or U.G.Meissner, Phys. Rep. 161 (1988) 1; J.J.M.Verbaarschot, Phys. Letts. B195 (1987) 450.

37. J. Bagger and S. Naculich, Phys. Re. Letts. 67 (1991) 2252; Phys. Rev. D45 (1992) 1395; S. Naculich, Johns Hopkins Preprint, JHU- TIPAC- 920014, 1992. These authors show that quantum loops destroy both classical and tree level predictions for nontopological solitons in strong interaction theories. An exception are theories governed by effective Lagrangians obeying power counting such as in (7).

38. B. Frois and C.N. Papanicolas, Ann. Rev. Nuc. Part. Phys. 37(1987)133

39. For a review of neutron star structure and dynamics, see S.L. Shapiro and S.A. Teukolsky, Black Holes, White Dwarfs and Neutron Stars, J. Wiley, New York, 1983.

40. C.E.Rhoades and R.Ruffini. Phys. Rev. Letts. 32 (1974) 324.

41. The Chart of the Nuclides, F.W. Walker, J.R. Parrington and F. Feiner, Knolls Atomic Power Laboratory, 14th edition, April 1988.

42. J.R. Oppenheimer and G.M. Volkoff, Phys. Rev. 55 (1939)374.

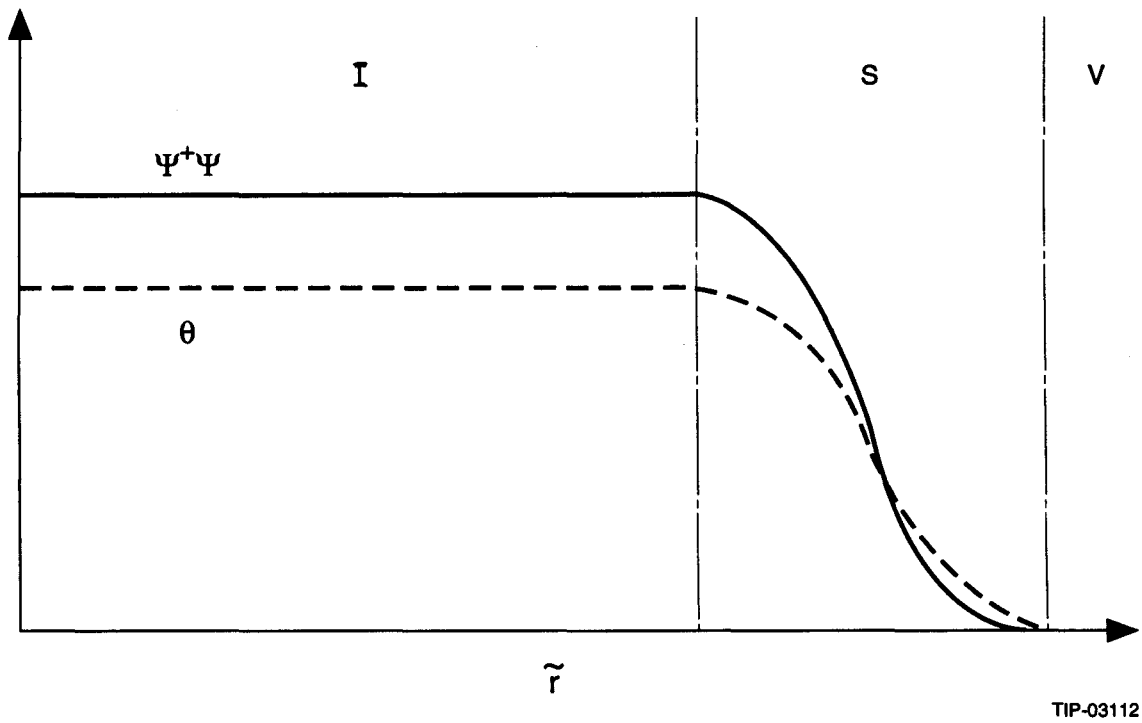


Figure 1

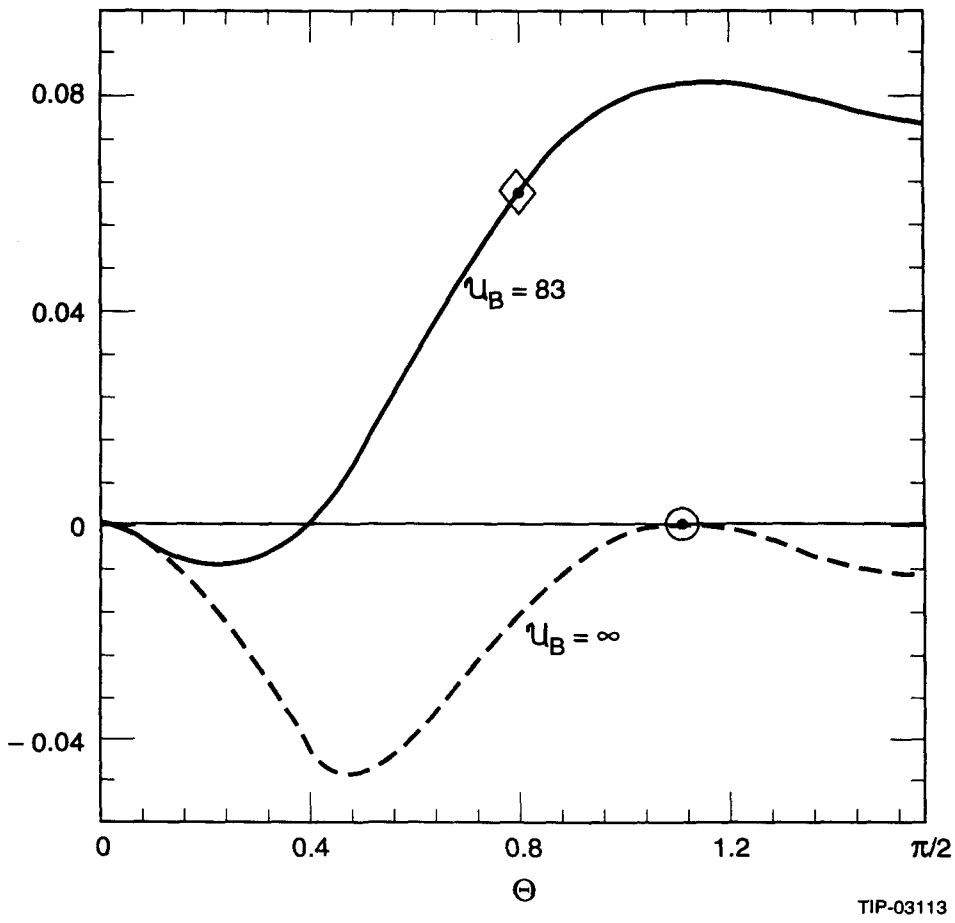
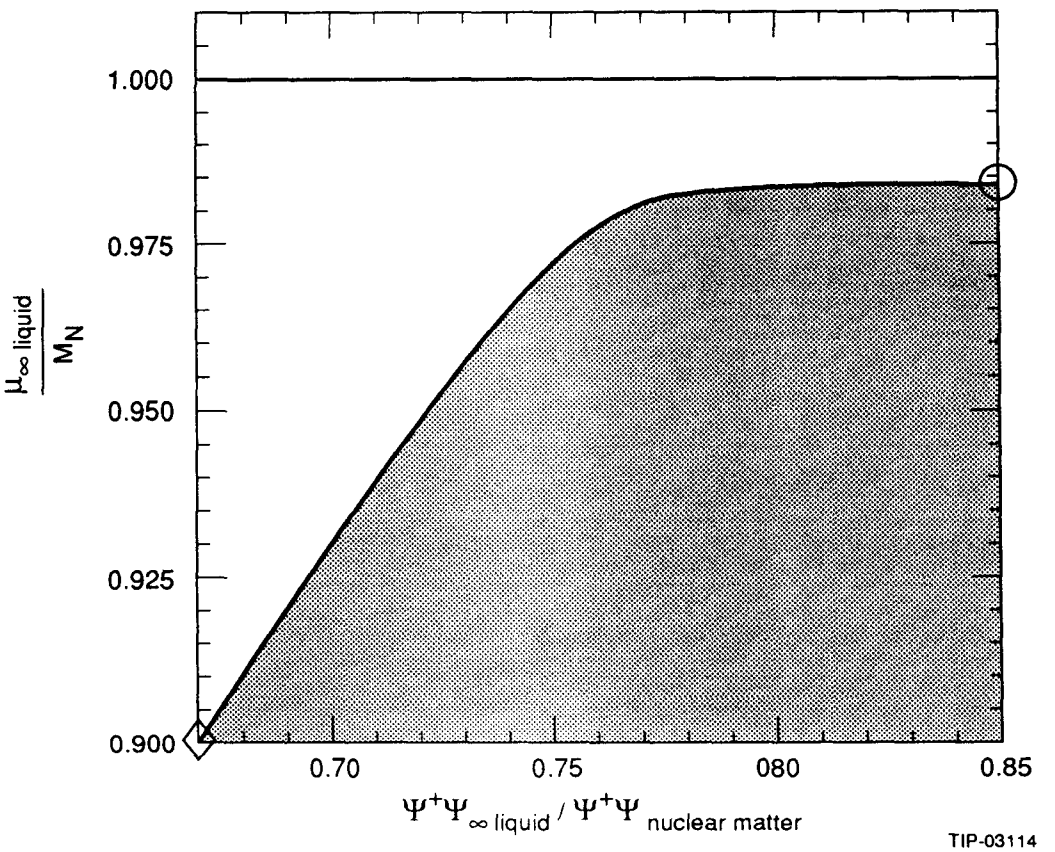

0.08
0.04
0
− 0.04
0
0.4
0.8
1.2
π/2
Θ
$\mathcal{U}_B = 83$
$\mathcal{U}_B = \infty$
TIP-03113

Figure 2


1.000
0.975
0.950
0.925
0.900
$\frac{\mu_{\infty\ liquid}}{M_N}$
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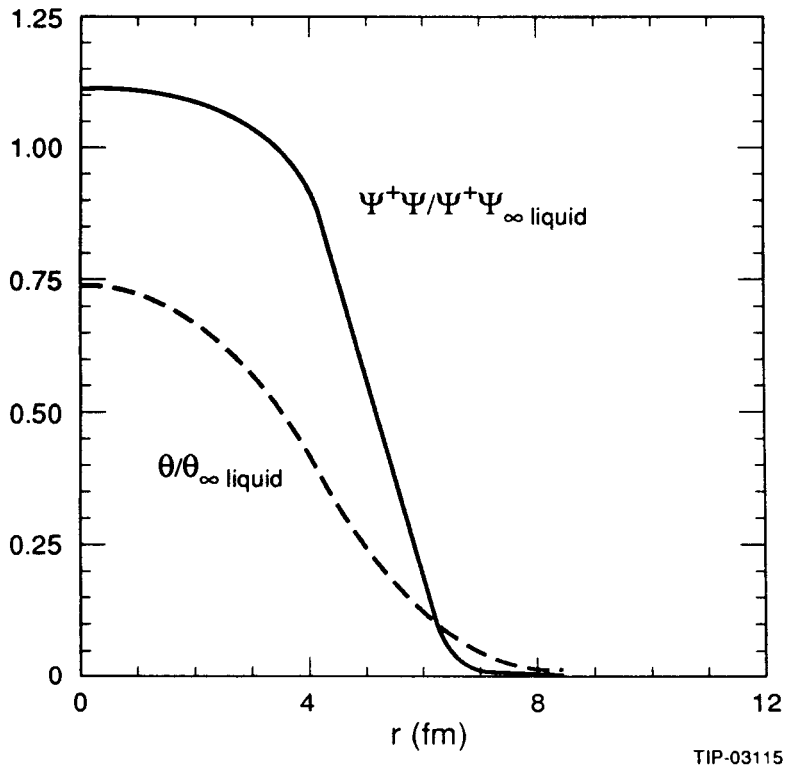
0.75
080
0.85
$\Psi^+\Psi_{\infty\ liquid} / \Psi^+\Psi_{nuclear\ matter}$
TIP-03114

Figure 3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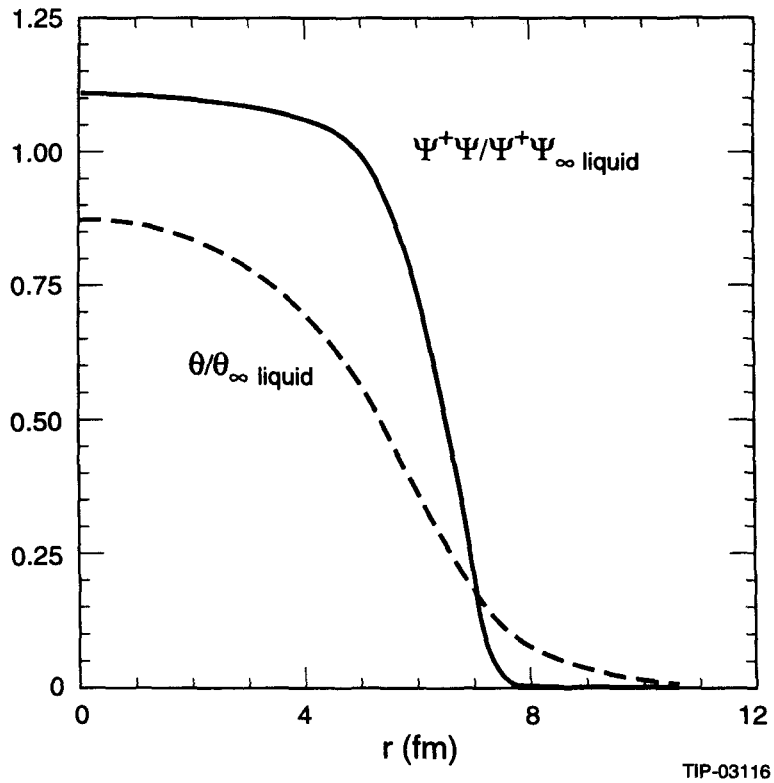
1.00
0.75
0.50
0.25
0
0
4
8
12
r (fm)
$\Psi^+\Psi/\Psi^+\Psi_{\infty\ \mathrm{liquid}}$
$\theta/\theta_{\infty\ \mathrm{liquid}}$
TIP-03115

Figure 4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1.00
0.75
0.50
0.25
0
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4
8
12
r (fm)
$\Psi^+\Psi/\Psi^+\Psi_\infty$ liquid
θ/θ_∞ liquid
TIP-03116

Figure 5

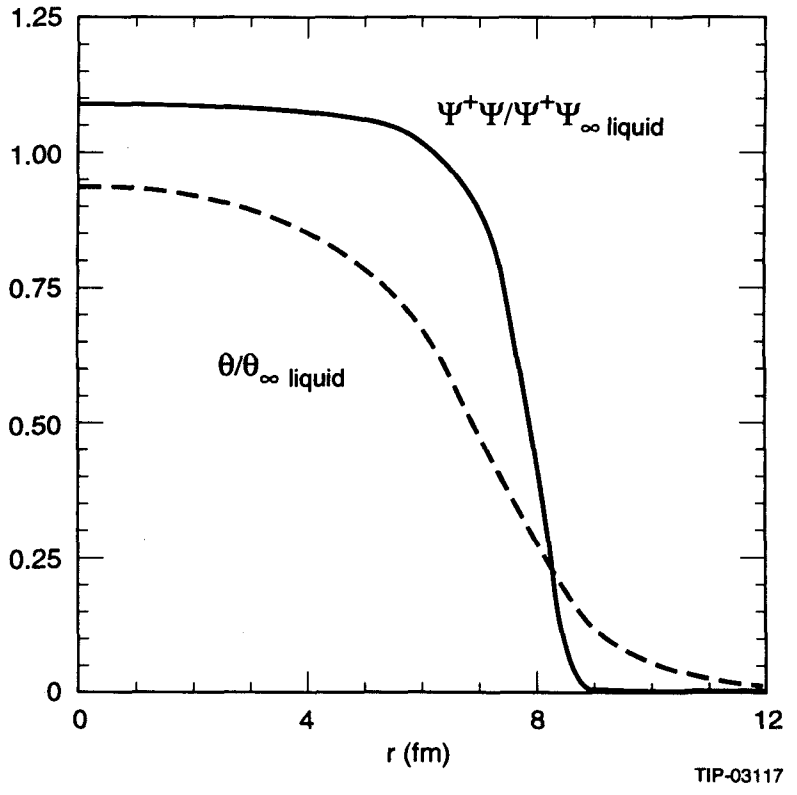
1.25
1.00
0.75
0.50
0.25
0
0
4
8
12
r (fm)
$\Psi^+\Psi/\Psi^+\Psi_\infty$ liquid
θ/θ_∞ liquid
TIP-03117

Figure 6

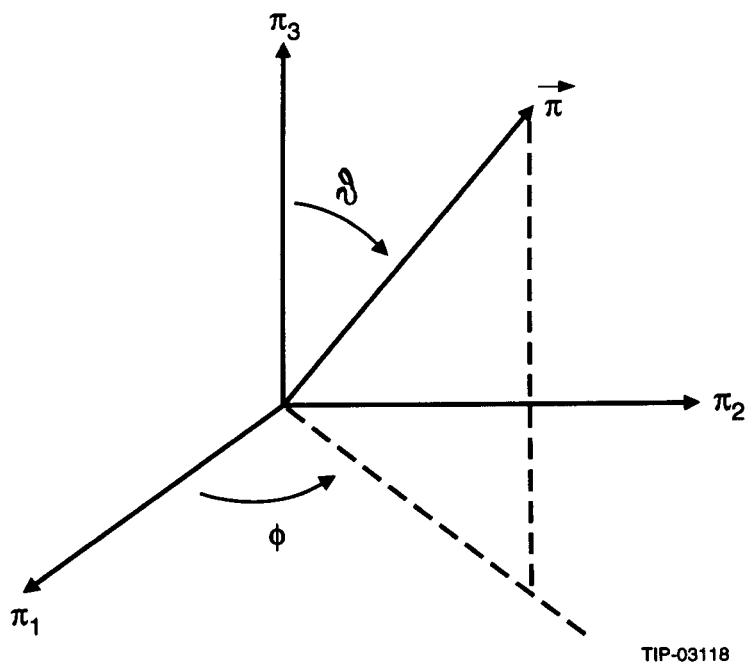

Figure 7

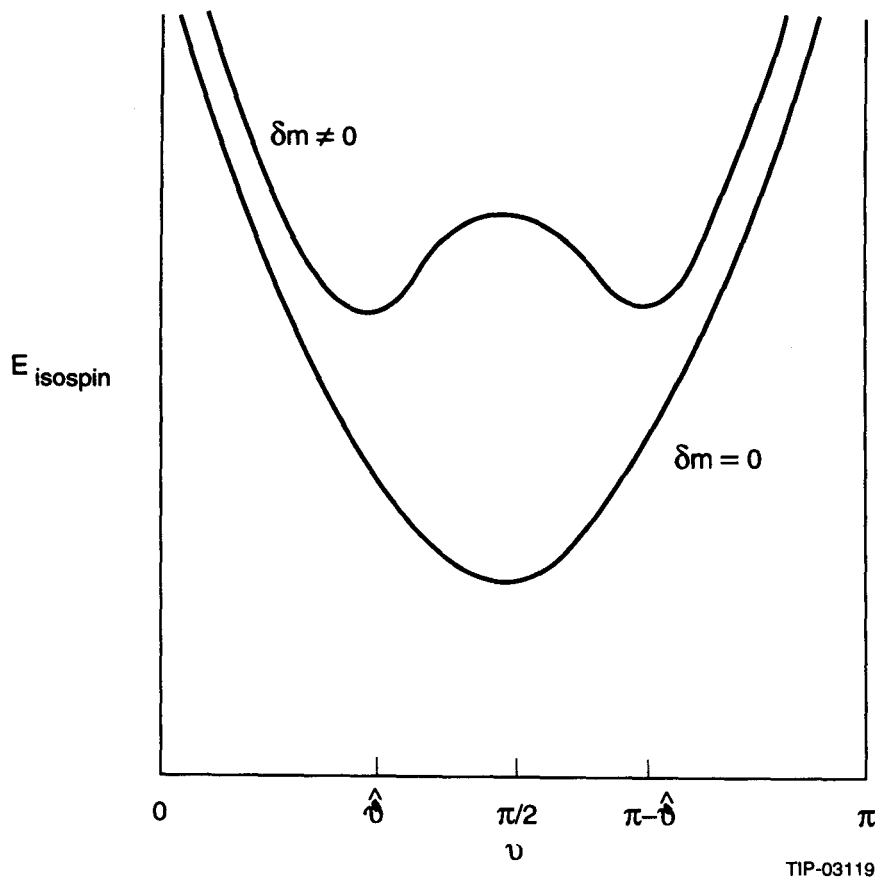

Figure 8

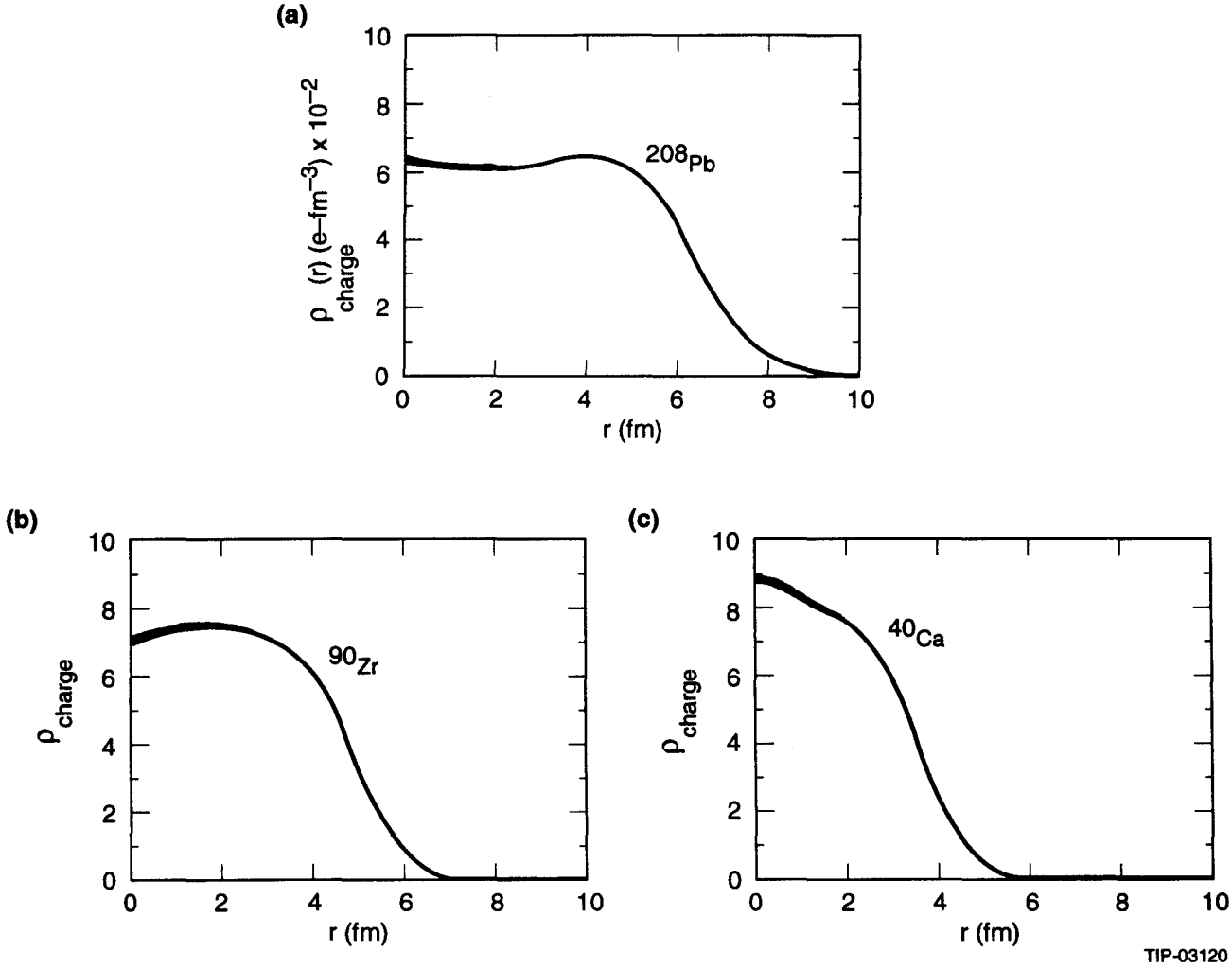
(a)
ρ_{charge} (r) (e–fm^{-3}) x 10^{-2}
^{208}Pb
r (fm)
(b)
ρ_{charge}
^{90}Zr
r (fm)
(c)
ρ_{charge}
^{40}Ca
r (fm)
TIP-03120

Figure 9

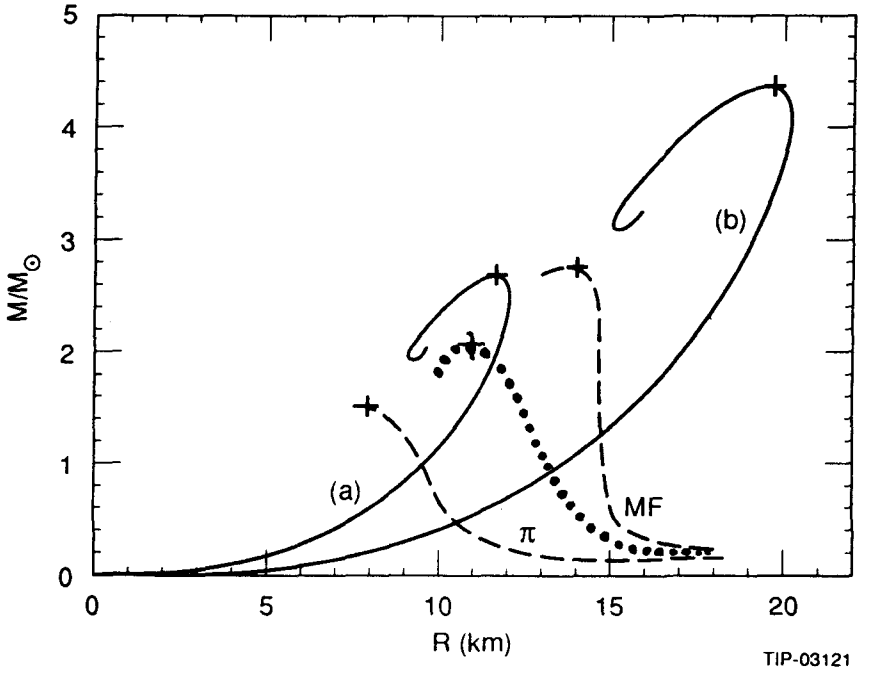

Figure 10