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Heavy hadron form factor relations for $m_c \neq \infty$ and $\alpha_s(m_c) \neq 0$

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First order power corrections to current matrix elements between heavy meson or Λ_Q baryon states are shown to vanish at the zero recoil point to all orders in QCD. Five relations among the six form factors that parametrize the semileptonic decay $\Lambda_b \to \Lambda_c e \overline{\nu}$ are also demonstrated to exist to all orders in the strong coupling at order $1/m_Q$. The $O(\alpha_s(m_c)/m_c)$ form factor relations are displayed.

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1. Introduction

In the limit where the masses of the charm and bottom quarks are taken to be infinitely greater than the QCD scale, matrix elements between hadron states containing a single heavy quark are severely constrained [1]. For example, all six form factors for the flavor changing currents which mediate $B \to D$ and $B \to D^*$ transitions are given in terms of one universal function. This so called "Isgur-Wise" function is also the form factor of the b-number current between B meson states. It is consequently normalized to unity at the maximum momentum transfers $q^2_{\max} = 0$ for $B \to B$ transitions and $q^2_{\max} = (m_B - m_D)^2$ for $B \to D$ decays.

A Heavy Quark Effective Theory (HQET) with manifest flavor and spin symmetries that lead to these normalization constraints has recently been developed [2,3]. Since the HQET is derived from QCD [4], its predictions are model independent. Moreover, corrections to results found in the infinite quark mass limit can be systematically investigated in this effective theory. Such corrections arise from QCD scaling violations which depend logarithmically upon the charm and bottom masses [5,6]. In addition, terms suppressed by inverse powers of the heavy quark masses enter at subleading order [7,8,9]. We shall refer to these deviations from the infinite mass limit as "scaling" and "power" corrections respectively.

First order power corrections to the predicted normalization of flavor changing current matrix elements between B and D or D^* states have been shown to vanish at the zero recoil point [10]. This remarkable result is often called "Luke's theorem" and holds as well for $\Lambda_b \to \Lambda_c$ transitions [11] and for an entire class of heavy hadron processes [12]. Luke's theorem was originally proved to zeroth order in the strong interactions. It consequently ruled out normalization corrections at $O(1/m_c)$ but not $O(\alpha_s/m_c)$. In this letter, we demonstrate that these latter violations are also prohibited. In fact, we show that *there are no order $1/m_c$ corrections to the zero recoil normalization of the current matrix elements to all orders in α_s.*

We then focus our attention upon the semileptonic decay $\Lambda_b \to \Lambda_c e \overline{\nu}$. This process is of considerable interest since an accurate value for the KM matrix element $|V_{cb}|$ may be determined in the future from high precision measurements of its endpoint spectrum. The transition lends itself particularly well to HQET analysis because it is tightly constrained by the heavy quark spin symmetry. Like their mesonic counterparts, the six form factors that parametrize this baryonic process are predicted at leading order in terms of a single Isgur-Wise function. Five relations among these six form factors have been found to remain after $O(1/m_c)$ power corrections are included. We extend this result to all orders in the strong coupling and then display the relations to $O(\alpha_s(m_c)/m_c)$. Such form factor relations provide a valuable means for assessing the uncertainty in future measurements of the mixing angle $|V_{cb}|$ from semileptonic Λ_b decay.

Finally, we estimate and compare the numerical sizes of the scaling and power correction expansion parameters that appear in the HQET.

2. Nonrenormalization at the zero recoil point

Finite quark mass corrections enter into the HQET in two ways. Firstly, $O(1/m_c)$ and $O(1/m_b)$ terms appear in the Lagrangian which break the theory's flavor and spin symmetries:

$$\mathcal{L}_v = \sum_{Q=c,b} \left\{ \overline{h}_v^{(Q)} (iv\cdot\mathcal{D}) h_v^{(Q)} + a_1 O_1 + a_2 O_2 \right\}. \tag{2.1}$$

The O_i operators are built up out of two heavy quark fields and symmetric or antisymmetric combinations of two covariant derivatives $\mathcal{D}_\mu = \partial_\mu - igA^a_\mu T^a$: [1]

$$\begin{aligned} O_1 &= \frac{1}{2m_Q} \overline{h}_v^{(Q)} (i\mathcal{D})^2 h_v^{(Q)} \\ O_2 &= \frac{g}{4m_Q} \overline{h}_v^{(Q)} \sigma^{\mu\nu} G^a_{\mu\nu} T^a h_v^{(Q)}. \end{aligned} \tag{2.2}$$

We have absorbed various numerical factors into these operators' definitions so that their tree level coefficients equal unity:

$$a_1 = a_2 = 1 + O(\alpha_s). \tag{2.3}$$

[1] A third operator $O_3 = -(1/2m_Q)\overline{h}_v^{(Q)}(iv\cdot\mathcal{D})^2 h_v^{(Q)}$ could be included with those in (2.2). However, since it can be eliminated via a nonlinear field redefinition, this operator has no effect and can be neglected without loss [9].

The Ademollo-Gatto theorem indicates that corrections to the normalization of form factors from the O_i terms in (2.1) arise only at second order in $1/m_c$ and $1/m_b$ [12,13]. The QCD corrections to the a_i coefficients in (2.3) do not upset this result.

There are also power corrections to the effective currents in the HQET which correspond to the vector and axial currents in the underlying full theory. In general, the two sets of currents are related as

$$\begin{aligned} V^\mu &= \bar{c}\gamma^\mu b \to \sum C_j^{(3)} P_j^\mu + \sum C_k^{(4)} Q_k^\mu + \cdots \\ A^\mu &= \bar{c}\gamma^\mu\gamma^5 b \to \sum C_j^{(3)\prime} P_j'^\mu + \sum C_k^{(4)\prime} Q_k'^\mu + \cdots . \end{aligned} \tag{2.4}$$

Here $P_j^{(\prime)\mu}$ and $Q_k^{(\prime)\mu}$ denote dimension three and four operators with appropriate quantum numbers while the ellipses represent higher order terms. A convenient basis for these operators is listed below:

Dimension 3 :

$$\begin{aligned} P_0^\mu &= \bar{c}_{v'}\gamma^\mu b_v & P_0'^\mu &= \bar{c}_{v'}\gamma^\mu\gamma^5 b_v \\ P_1^\mu &= \bar{c}_{v'}v^\mu b_v & P_1'^\mu &= \bar{c}_{v'}v^\mu\gamma^5 b_v \\ P_2^\mu &= \bar{c}_{v'}v'^\mu b_v & P_2'^\mu &= \bar{c}_{v'}v'^\mu\gamma^5 b_v \end{aligned} \tag{2.5a}$$

Dimension 4 :

$$\begin{aligned} Q_1^\mu &= -\frac{i}{m_c}\bar{c}_{v'}\overleftarrow{\not{\mathcal{D}}}\gamma^\mu b_v & Q_1'^\mu &= -\frac{i}{m_c}\bar{c}_{v'}\overleftarrow{\not{\mathcal{D}}}\gamma^\mu\gamma^5 b_v \\ Q_2^\mu &= \frac{i}{m_b}\bar{c}_{v'}\gamma^\mu\not{\mathcal{D}} b_v & Q_2'^\mu &= \frac{i}{m_b}\bar{c}_{v'}\gamma^\mu\gamma^5\not{\mathcal{D}} b_v \\ Q_3^\mu &= -\frac{i}{m_c}\bar{c}_{v'}v\cdot\overleftarrow{\mathcal{D}}\gamma^\mu b_v & Q_3'^\mu &= -\frac{i}{m_c}\bar{c}_{v'}v\cdot\overleftarrow{\mathcal{D}}\gamma^\mu\gamma^5 b_v \\ Q_4^\mu &= \frac{i}{m_b}\bar{c}_{v'}\gamma^\mu v'\cdot\mathcal{D} b_v & Q_4'^\mu &= \frac{i}{m_b}\bar{c}_{v'}\gamma^\mu\gamma^5 v'\cdot\mathcal{D} b_v \\ Q_5^\mu &= -\frac{i}{m_c}\bar{c}_{v'}\overleftarrow{\not{\mathcal{D}}} v^\mu b_v & Q_5'^\mu &= -\frac{i}{m_c}\bar{c}_{v'}\overleftarrow{\not{\mathcal{D}}} v^\mu\gamma^5 b_v \\ Q_6^\mu &= -\frac{i}{m_c}\bar{c}_{v'}\overleftarrow{\not{\mathcal{D}}} v'^\mu b_v & Q_6'^\mu &= -\frac{i}{m_c}\bar{c}_{v'}\overleftarrow{\not{\mathcal{D}}} v'^\mu\gamma^5 b_v \\ Q_7^\mu &= \frac{i}{m_b}\bar{c}_{v'}v^\mu\not{\mathcal{D}} b_v & Q_7'^\mu &= \frac{i}{m_b}\bar{c}_{v'}v^\mu\gamma^5\not{\mathcal{D}} b_v \\ Q_8^\mu &= \frac{i}{m_b}\bar{c}_{v'}v'^\mu\not{\mathcal{D}} b_v & Q_8'^\mu &= \frac{i}{m_b}\bar{c}_{v'}v'^\mu\gamma^5\not{\mathcal{D}} b_v \\ Q_9^\mu &= -\frac{i}{m_c}\bar{c}_{v'}\overleftarrow{\mathcal{D}}^\mu b_v & Q_9'^\mu &= -\frac{i}{m_c}\bar{c}_{v'}\overleftarrow{\mathcal{D}}^\mu\gamma^5 b_v \\ Q_{10}^\mu &= \frac{i}{m_b}\bar{c}_{v'}\mathcal{D}^\mu b_v & Q_{10}'^\mu &= \frac{i}{m_b}\bar{c}_{v'}\gamma^5\mathcal{D}^\mu b_v \\ Q_{11}^\mu &= -\frac{i}{m_c}\bar{c}_{v'}v\cdot\overleftarrow{\mathcal{D}} v^\mu b_v & Q_{11}'^\mu &= -\frac{i}{m_c}\bar{c}_{v'}v\cdot\overleftarrow{\mathcal{D}} v^\mu\gamma^5 b_v \\ Q_{12}^\mu &= -\frac{i}{m_c}\bar{c}_{v'}v\cdot\overleftarrow{\mathcal{D}} v'^\mu b_v & Q_{12}'^\mu &= -\frac{i}{m_c}\bar{c}_{v'}v\cdot\overleftarrow{\mathcal{D}} v'^\mu\gamma^5 b_v \\ Q_{13}^\mu &= \frac{i}{m_b}\bar{c}_{v'}v^\mu v'\cdot\mathcal{D} b_v & Q_{13}'^\mu &= \frac{i}{m_b}\bar{c}_{v'}v^\mu\gamma^5 v'\cdot\mathcal{D} b_v \\ Q_{14}^\mu &= \frac{i}{m_b}\bar{c}_{v'}v'^\mu v'\cdot\mathcal{D} b_v & Q_{14}'^\mu &= \frac{i}{m_b}\bar{c}_{v'}v'^\mu\gamma^5 v'\cdot\mathcal{D} b_v . \end{aligned} \tag{2.5b}$$

The operators' coefficients are determined by matching Green's functions with single current insertions in the full and effective theories. They are dimensionless functions of the strong coupling α_s, the renormalization point μ, and the quark masses m_c and m_b. Their values can be calculated perturbatively provided μ is large enough so that $\alpha_s(\mu)$ is small.

All of the effective current operator coefficients in (2.4) gain zero contribution from tree level matching except

$$\begin{aligned} C_0^{(3)} &= C_0^{(3)\prime} = 1 \\ C_1^{(4)} &= C_1^{(4)\prime} = C_2^{(4)} = C_2^{(4)\prime} = 1/2. \end{aligned}$$

In the original proof of Luke's theorem, only the operators corresponding to these nonvanishing coefficients were considered. To extend the theorem's validity to arbitrary order in α_s, one must examine the effects from all the others listed in (2.5). Therefore, consider a representative HQET matrix element of a prototype dimension four operator between heavy B and D states that both move with four-velocity v:

$$\langle \widetilde{D}(v)|\bar{c}_v i\overleftarrow{\mathcal{D}}^{\alpha}\Gamma b_v|\widetilde{B}(v)\rangle = \lambda v^{\alpha}\mathrm{Tr}\overline{M}(v)\Gamma M(v). \tag{2.6}$$

The tildes appearing on the LHS of this equation indicate that the states are evaluated in the effective theory to zeroth order in $1/m_Q$. On the RHS, the meson matrices

$$M(v) = -\frac{1+\not{v}}{2}\gamma^5$$
$$\overline{M}(v) = \gamma^5\frac{1+\not{v}}{2}$$

are contracted together in accordance with the HQET flavor and spin symmetries. After dotting both sides of (2.6) with v_α and applying the equation of motion $v\cdot\mathcal{D}c_v = 0$, one finds that the constant λ vanishes identically. Since matrix elements between $B(v)$ and $D(v')$ states of all the dimension four operators in (2.5b) can be derived from equations like (2.6), they too must vanish when $v = v'$. An analogous argument holds for $B \to D^*$ transitions.

Could the zeros in heavy meson matrix elements of the $Q_k^{(\prime)\mu}$ operators be cancelled by poles in their $C_k^{(4)(\prime)}$ coefficients? We do not believe so. Consider the analytic structure of meson form factors regarded as complex functions of the momentum transfer q^2. By examining Feynman diagrams in the underlying full QCD theory, one sees that the physical cut which starts at the maximum momentum transfer $q^2_{\max} = (m_B - m_D)^2$ originates from infrared singularities in these graphs. This infrared behavior must be reproduced by the dynamics of the effective theory and should not appear in the coefficient functions which contain only short distance information.

Therefore, since matrix elements of the dimension four operators vanish while their coefficients remain regular at $v\cdot v' = 1$, there can be no first order power corrections to the zero recoil current normalizations to all orders in QCD.

3. Form factor relations for $\Lambda_b \to \Lambda_c$ transitions

The nonrenormalization theorem discussed in the previous section for mesons applies to Λ_Q baryons as well. Vector and axial current matrix elements between Λ_b and Λ_c baryon states appear in the HQET as

$$\langle\Lambda_c(v',s')|V^\mu|\Lambda_b(v,s)\rangle = \bar{u}(v',s')[F_1(v\cdot v')\gamma^\mu + F_2(v\cdot v')v^\mu + F_3(v\cdot v')v'^\mu]u(v,s)$$
$$\langle\Lambda_c(v',s')|A^\mu|\Lambda_b(v,s)\rangle = \bar{u}(v',s')[G_1(v\cdot v')\gamma^\mu + G_2(v\cdot v')v^\mu + G_3(v\cdot v')v'^\mu]\gamma^5 u(v,s). \tag{3.1}$$

A few points about these expressions should be noted. Firstly, the Dirac spinors for the baryons' heavy quark constituents satisfy $u(v,s) = \not{v}u(v,s)$. Therefore when $v = v'$, the current matrix elements reduce to [14]

$$\langle\Lambda_c(v,s')|V^\mu|\Lambda_b(v,s)\rangle = [F_1(1) + F_2(1) + F_3(1)]\bar{u}(v,s')v^\mu u(v,s) \tag{3.2a}$$
$$\langle\Lambda_c(v,s')|A^\mu|\Lambda_b(v,s)\rangle = G_1(1)\bar{u}(v,s')\gamma^\mu\gamma^5 u(v,s). \tag{3.2b}$$

Secondly, the spin of a Λ_Q baryon comes entirely from its heavy quark in the infinite mass limit; the light spectator degrees of freedom carry zero angular momentum. The form factors F_i and G_i are consequently all determined from one universal function which is normalized at zero recoil [15]. To avoid any confusion with the Isgur-Wise function $\xi(v\cdot v')$ for heavy mesons, we will denote this universal function associated with Λ_Q baryons as $\eta(v\cdot v')$. Finally, an additional dimensionful constant $\overline{\Lambda} \approx m_{\Lambda_c} - m_c \approx m_{\Lambda_b} - m_b$ must be introduced to specify the form factors when $m_Q \neq \infty$. The parameter $\overline{\Lambda}$ may be interpreted as the baryon state's energy above the vacuum in the HQET.

Order $1/m_c$ power corrections to the effective vector and axial currents arising from either local dimension four $Q_k^{(\prime)\mu}$ operators in (2.5b) or time ordered products of dimension five O_i operators in (2.1) and dimension three $P_j^{(\prime)\mu}$ operators in (2.5a) were considered in ref. [11]. The time ordered products were shown to generally not contribute, and five relations among the six form factors in (3.1) were found. We now demonstrate that five relations remain even when current corrections of order $1/m_c$, $1/m_b$ and all orders in α_s are retained. We start with the identity

$$\langle\Lambda_c(v',s')|i\mathcal{D}^\alpha(\bar{c}_{v'}\Gamma b_v)|\Lambda_b(v,s)\rangle = \overline{\Lambda}\eta(v\cdot v')(v^\alpha - v'^\alpha)\bar{u}(v',s')\Gamma u(v,s)$$

which follows from the relation between momentum and derivative operators in the effective theory [3]:

$$[\mathbf{P}^\alpha, h_v^{(Q)}(x)] = -(mv^\alpha + i\mathcal{D}^\alpha)h_v^{(Q)}(x).$$

With the aid of this identity, the general matrix elements

$$\begin{aligned}\langle\Lambda_c(v',s')|\overline{c}_{v'}i\overleftarrow{\mathcal{D}}^\alpha\Gamma b_v|\Lambda_b(v,s)\rangle &= \overline{\Lambda}\eta(v\cdot v')\frac{v^\alpha - v\cdot v'v'^\alpha}{v\cdot v'+1}\,\overline{u}(v',s')\Gamma u(v,s)\\ \langle\Lambda_c(v',s')|\overline{c}_{v'}\Gamma i\mathcal{D}^\alpha b_v|\Lambda_b(v,s)\rangle &= \overline{\Lambda}\eta(v\cdot v')\frac{v\cdot v'v^\alpha - v'^\alpha}{v\cdot v'+1}\,\overline{u}(v',s')\Gamma u(v,s)\end{aligned} \tag{3.3}$$

are readily evaluated. Notice that like the meson element in (2.6), these expressions vanish for $v = v'$.

Matrix elements of all the basis operators in (2.5*b*) are fixed by those in (3.3). Since any dimension four contribution to the effective currents can be decomposed over this complete operator set, we see that Luke's theorem holds to all powers in the strong coupling. Furthermore, as no new parameters need be introduced into the current form factors, no relations among them are lost. Such relations can be determined to the order at which the effective current coefficients in (2.4) are known. We compute these coefficients assuming $m_b \gg m_c$, and we first work in an intermediate HQET with a heavy b quark but full theory c field. For simplicity, we neglect the QCD running between the bottom and charm scales which has previously been discussed in refs. [5,6,7]. We instead concentrate upon the $O(\alpha_s(m_c)/m_c)$ matching contributions to the current coefficients that arise at the charm scale boundary between the intermediate and final effective theories in which both the c and b are treated as heavy.

We match 1PI two-point Green's functions with a single vector or axial current insertion in the intermediate and final HQET's. The one-loop diagrams that enter into this matching computation are illustrated in fig. 1. The graphs contain $O(1/m_c)$ operator insertions from the Lagrangian in (2.1) and currents in (2.4). We adopt the mass independent renormalization scheme of dimensional regularization plus modified minimal subtraction to accommodate the ultraviolet infinities in these diagrams. Infrared divergences which appear after

Taylor expanding loop integrals in powers of external residual momenta can be explicitly eliminated by judiciously arranging integrand terms in the two theories into infrared safe combinations. After including the tree level $O(1/m_c)$ and $O(1/m_b)$ contributions and taking the difference between the two-point functions in the intermediate and final HQET's, we find the following c-scale matching contributions to the effective currents:

$$\begin{aligned}V^\mu = {} & P_0^\mu + \frac{1}{2}Q_1^\mu + \frac{1}{2}Q_2^\mu\\ & + \frac{1}{3}\frac{\alpha_s(m_c)}{\pi}\Big\{2(v\cdot v'+1)r\big[P_0^\mu + \frac{1}{2}Q_1^\mu\big] - 2\frac{r-1}{v\cdot v'-1}Q_3^\mu\\ & \qquad - 4r\big[P_1^\mu + \frac{1}{2}Q_5^\mu\big] - \frac{4(1-v\cdot v'r)}{v\cdot v'^2-1}Q_{11}^\mu\Big\}\end{aligned} \tag{3.4a}$$

$$\begin{aligned}A^\mu = {} & P_0'^\mu + \frac{1}{2}Q_1'^\mu + \frac{1}{2}Q_2'^\mu\\ & + \frac{1}{3}\frac{\alpha_s(m_c)}{\pi}\Big\{2(v\cdot v'-1)r\big[P_0'^\mu + \frac{1}{2}Q_1'^\mu\big] + 2\frac{r+1}{v\cdot v'+1}Q_3'^\mu\\ & \qquad - 4r\big[P_1'^\mu + \frac{1}{2}Q_5'^\mu\big] - \frac{4(1-v\cdot v'r)}{v\cdot v'^2-1}Q_{11}'^\mu\Big\}\end{aligned} \tag{3.4b}$$

where

$$r = \frac{\log(v\cdot v' + \sqrt{v\cdot v'^2-1})}{\sqrt{v\cdot v'^2-1}}\,.$$

The $O(\alpha_s(m_c))$ coefficients of the dimension three terms in these formulas are consistent with results from previous matching computations [6].

Five independent relations among the vector and axial form factors are readily derived from the currents in (3.4). We choose to express these relations as ratios relative to the first axial form factor:

$$\begin{aligned}\frac{F_1}{G_1} = {} & 1 + \Big[\frac{\overline{\Lambda}}{2m_c} + \frac{\overline{\Lambda}}{2m_b}\Big]\frac{2}{(v\cdot v'+1)}\\ & + \frac{4}{3}\frac{\alpha_s(m_c)}{\pi}r + \frac{4}{3}\frac{\alpha_s(m_c)}{\pi}\frac{\overline{\Lambda}}{2m_c}\frac{2(1+r-v\cdot v'r)}{(v\cdot v'+1)}\end{aligned} \tag{3.5a}$$

$$\begin{aligned}\frac{F_2}{G_1} = \frac{G_2}{G_1} = {} & -\frac{\overline{\Lambda}}{2m_c}\frac{2}{(v\cdot v'+1)}\\ & - \frac{4}{3}\frac{\alpha_s(m_c)}{\pi}r - \frac{4}{3}\frac{\alpha_s(m_c)}{\pi}\frac{\overline{\Lambda}}{2m_c}\frac{2(1+r-v\cdot v'r)}{(v\cdot v'+1)}\end{aligned} \tag{3.5b}$$

$$\frac{F_3}{G_1} = -\frac{G_3}{G_1} = -\frac{\overline{\Lambda}}{2m_b}\frac{2}{(v\cdot v'+1)}. \tag{3.5c}$$

As a check, we have verified that these form factor relations are renormalization scheme independent as must be the case for physical observables.

The ratios in (3.5) imply $F_1 + F_2 + F_3 = G_1$ for all values of $v{\cdot}v'$. Our enhanced version of Luke's theorem applied to eqn. (3.2) guarantees that no dimension four terms disrupt the normalization of these form factor combinations at zero recoil. Possible normalization violations from dimension three terms are also prohibited when $v = v'$ as can be readily verified in $v{\cdot}A^a = 0$ gauge. Therefore to leading order, only calculable QCD scaling corrections move the values of these form factor combinations away from unity at the zero recoil point:

$$F_1(1) + F_2(1) + F_3(1) = G_1(1) = \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25}. \tag{3.6}$$

To conclude, we estimate the numerical sizes of the expansion parameters that enter into HQET computations when the bottom and charm quarks are sequentially treated as heavy and the running between them is neglected. Such calculations are organized as perturbative expansions in $\overline{\Lambda}/2m_c$, $\alpha_s(m_c)/\pi$, $\overline{\Lambda}/2m_b$ and $\alpha_s(m_b)/\pi$. Assuming the reasonable values $m_b = 4.5$ GeV, $m_c = 1.5$ GeV, $\overline{\Lambda} = 0.5$ GeV and $\Lambda^{(3)}_{\rm QCD} = 0.2$ GeV and using the leading log approximation for the strong interaction fine structure constant, we find that the charm scale parameters $\overline{\Lambda}/2m_c = 0.17$ and $\alpha_s(m_c)/\pi = 0.11$ are of comparable magnitude. Their squares $(\overline{\Lambda}/2m_c)^2 = 0.03$, $(\alpha_s(m_c)/\pi)\overline{\Lambda}/2m_c = 0.02$ and $(\alpha_s(m_c)/\pi)^2 = 0.01$ are not much smaller than the bottom scale expansion parameters $\alpha_s(m_b)/\pi = 0.07$ and $\overline{\Lambda}/2m_b = 0.05$. Further corrections lie below the 1 % level. The uncertainty in the relations (3.5) and (3.6) is therefore dominated by second order $(\overline{\Lambda}/2m_c)^2$ power corrections. Such terms are comparable in size to the order $(\alpha_s(m_c)/\pi)\overline{\Lambda}/2m_c$ contributions that we have considered here.

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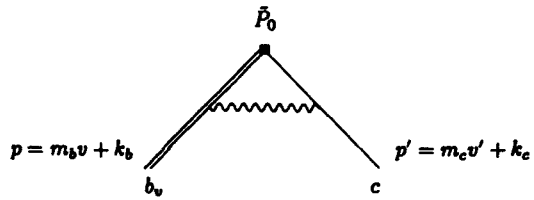
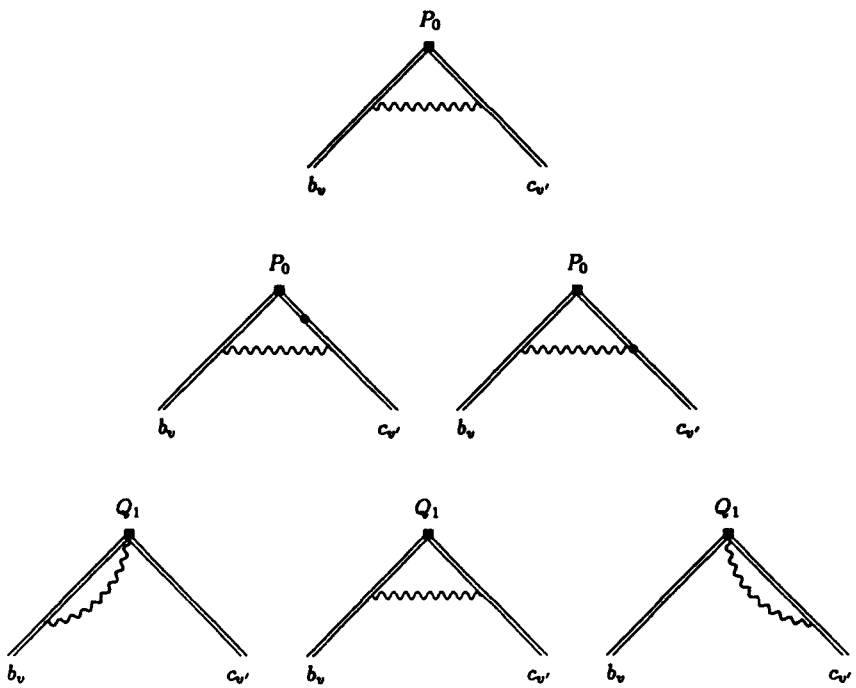
Intermediate HQET graph for $m_c < \mu < m_b$:



Final HQET graphs for $\mu < m_c$:



One-loop Feynman diagrams in the intermediate and final HQET's whose difference determines the $O(\alpha_s(m_c))$ matching contributions to the $C_j^{(3)^{(\prime)}}$ and $C_k^{(4)^{(\prime)}}$ coefficients in eqn. (2.4). Solid boxes denote $P_0^{(\prime)}$ and $Q_1^{(\prime)}$ current operators while solid dots represent O_1 and O_2 Lagrangian operator insertions.