A General Control Model for Designing Beam Control Feedback Loops

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Abstract
To control the beam in the synchrotron there may be six
different primary feedback loops interacting with the beam
at a given time. Three loops are local to the rf cavity. They
are: high bandwidth cavity phase and amplitude loops
used to minimize the effects due to beam loading and a low
bandwidth cavity tuning loop. The loops global to the ring
accelerating system are: a radial loop to keep the beam on
orbit, a beam phase loop to damp the dipole synchrotron
oscillations, and a synchronization loop to essentially lock
with the succeeding machine. There are various ways in
which these loops may be designed. Designs currently in
use in operating machines are based on classical frequency
domain techniques. To apply modern feedback controllers
and study the interaction of all the feedback loops, a good
mathematical model of the beam is extremely useful. In
this paper we show the derivation of a non-linear tracking
model in terms of differential equations obtained from a
set of time varying finite difference equations. The model
compares well with the results of thin element tracking
codes.

1 INTRODUCTION
Several feedback loops, associated with a basic low-level
rf system, have to be able to bunch the beam, accelerate
without inducing unwanted coherent oscillations and thereafter
time the bunch positions relative to the next higher energy machine for synchronization. For this purpose,
a precise control of the frequency phase and amplitude of the accelerating rf signal is required. With a good
control model we would benefit a great deal while planning and configuring the feedback loops. There are several
ways the beam control loops are designed. One conceivable way is by actually measuring the transfer functions of
each parameter from the control end and then designing the appropriate stabilizing dynamics such as the propor
tional, differential or integral terms. Such an approach
has at least two drawbacks, (i) Existence of some kind
of operating machine to conduct experiments and hence
to improve the loop performance, (ii) Inaccurate measure
dments due to difficulties in considering coupling effects be
tween loops. Alternatively, by extracting the model from
the longitudinal beam dynamics, we can design the loops
more appropriately. Since almost all the practical imple
mentations of the loops are sensitive to errors, a control
model with appropriate error terms is even more useful. In
this paper we have shown the derivation of a control model
by ignoring the local cavity feedback loops, and hence will
be applicable to only low intensity machines.

2 PARTICLE TRACKING MODEL
From the control point of view, it will be useful to have
the model in differential equation form, although the ac
celeration takes place only at the cavities. However, a
discrete representation will be a starting point since it is
closer to reality. Hence a tracking code for one particle
was obtained to model the longitudinal phase oscillations
by giving an energy kick every time the particle passes
through the equivalent cavity gap. This model is shown in
Reference [1]. The betatron oscillations are decoupled in
the formulation of such models. The error introduced with
this type of approximation is very little when there is substan
tial difference between the betatron and synchrotron
frequencies. Since for beam control purpose we are in
terested in the phase of the particle with respect to the
rf signal, it is calculated by knowing the total arrival time
of the particle. Using the discrete model we tracked one
particle for the Low Energy Booster at \( t = 0 \) to 0.05 sec
with a Gaussian noise in the magnetic field errors. The
results compare quite well with the Thin Element Particle
Tracking Code by going through each magnetic lens at 5 ns
time steps. Comparisons are shown in Reference [1].

3 NON-LINEAR BEAM CONTROL
MODEL
The beam control model is derived below using the discrete
model at first for a single particle, and later we show the
model for a multiparticle case.

3.1 Synchronization Model
The synchronization loop with "trip-plan" approach [2]
provides the means to phase-lock the reference bunch in
the lower energy machine with a reference bucket in the
higher energy machine. For simplicity let us consider the
bunch comprising just one particle. If \( t_i \) is the time when
the reference particle in the lower energy machine reaches
the reference point in the \( k \)th turn, then the "trip-plan" is
given by

\[(S_k)_{\text{trip-plan}} = v_k^* (t_k - \delta t_k^0) \]  

(1)

The superscript "s" is used to indicate the parameters for a non-ideal particle. If "trip" is the measured phase for a non-ideal particle, then for kth turn it is given by

\[(S_k)_{\text{actual}} = (v_k^* + \delta v_k^0) (t_k - \delta t_k^0) + \delta S_k \]  

(2)

where \(t_k\) is the actual traversal time of the particle in the lower energy machine. This can be written in the following form for a machine operating below transition:

\[t_k = t_k^0 - \sum_{n=1}^{k} \delta t_n. \]  

(3)

The error in synchronizing phase is obtained by subtracting Eq. 1 from Eq. 2. By ignoring the second order terms and converting the discrete error equation to continuous form, the phase error is written as

\[\delta S \equiv -v' \int \frac{\delta t^2}{\gamma^2} dt + \delta S_0. \]  

(4)

The deviation in time, \(\delta t\), in one traversal can be expressed in terms of radial orbit shift and the field error.

\[\delta t^2 = \eta^2 \gamma^2 \delta R \gamma^2 + \frac{1}{\gamma^2} \delta B \]  

(5)

Using Eq. 5 in Eq. 4, the phase error can be expressed in the measurable quantities

\[\phi_1 = a_{11} x_1 + a_{12} x_2 + d_{11} \delta B - a_{11} z_0 \]  

(6)

where the new variables are shown in Table 1.

### 3.2 Radial Orbit Model

If \(E_k\) is the energy in kth pass through the cavity gap, then the energy for actual particle, and a synchronous particle is given by the following difference equations [3]:

\[E_k - E_{k-1} = \epsilon(V_k^* + \delta V_k) \sin \phi_k \]  

(7)

and

\[E_k^* - E_{k-1}^* = \epsilon V_k^* \sin(\phi_k^*) \]  

(8)

where \(\delta V_k\) is given by

\[\delta V_k = \delta V_k^* + \delta V_k^* \]  

(9)

with \(\delta V_k^*\) as the control supplied to the cavity gap voltage and \(\delta V_k^*\) the error in the cavity voltage for kth turn and \(\phi_k^*\) is the particle phase for the ideal synchronous case. The quantity \(\delta V_k^*\) can be set to zero when we do not use global amplitude feedback. The energy equation in the finite difference form is in the usual way as follows:

\[\frac{E - E^*}{\nu} = \frac{\epsilon V'}{2\pi R'} \left[ \left(1 + \frac{\delta V'}{V'} \right) \sin(\phi) - \sin(\phi^*) \right] \]  

(10)

where \(E\) and \(E^*\) are assumed to be equal to the energy gain per turn of the actual particle and the synchronous particle respectively shown by Eqs. 7 and 8. If \(\delta v\) is the change in velocity from the synchronous particle, then by using Taylor Series approximation, Eq. 10 can be written as below:

\[\frac{d\delta E}{dt} = A_3 \left[ 1 + \frac{\delta V'}{V'} \sin(\phi) - \sin(\phi_0^*) \right] + A_4 \delta v \]  

(11)

where the new variables are shown in Table 1.

The particle phase, \(\phi\), can be written in terms of the nominal synchronous phase, \(\phi_0^*\), the deviation from the synchronous phase representing the synchrotron oscillations, \(\delta \phi^*\), the systematic phase error, \(\phi^s\), and also, a small phase shift, \(\phi^c\) as supplied by the controller. That is,

\[\phi = \phi_0^* + \delta \phi^* + \phi^s + \phi^c. \]  

(12)

The phase shift \(\delta \phi^*\) is included as one of the control inputs, since the radial loop can be connected to the global phase shifter after the frequency source, as in the case of Fermilab booster low level rf system. We can write the following functional relationship between energy and momentum

\[\delta E = (\delta \phi^*) \frac{dP}{dE} E'. \]  

(13)

Substituting the well known equation for the momentum change from Reference [1], and by taking the first derivative of the resulting energy equation with respect to time we obtain:

\[\frac{d\delta E}{dt} = \dot{\phi} \frac{dE}{d\phi} + \frac{dE}{dt} + \frac{d\phi}{dt} \frac{dE}{d\phi} \]  

(14)

The incremental velocity change in a given turn has a functional relationship:

\[\frac{\delta \nu}{\nu} = \frac{1}{(\gamma^2)^2} \left[ \frac{\delta R}{R^2} + \frac{\delta B}{B^2} \right]. \]  

It is substituted in Eq. 14 and then the resulting equation is compared with Eq. 11. After simplification we get the desired equation for the radial orbital deviations as follows:

\[\dot{x}_2 = a_2 x_2 + (a_23 - \dot{a}_230) \sin(x_3 + x_4) + (\dot{a}_24 - \dot{a}_240) \cos(x_3 + x_4) + \dot{a}_23 + \dot{a}_24 \delta B + \dot{a}_25 \delta B \]  

(15)

where the new variables are shown in Table 1.

### 3.3 Particle Phase Model

The discrete phase equation is well known and is written below with error and control terms

\[\phi_{k+1} = \phi_k + 2\pi(\delta f_1 + \delta f_2 + \delta f_3)(t_k + \delta t_k) + \phi^s. \]  

(16)

By substituting the equation for \(\delta t_k\) in terms of the radial orbit shift and the magnetic field errors and converting the
Table 1: Parameters of the state space model.

<table>
<thead>
<tr>
<th>Coefficients:</th>
<th>( a_{11} = \frac{\text{d}e}{\text{d}t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_{12} = -\frac{\text{d}e}{\text{d}t} )</td>
<td>( a_{21} = -\frac{\text{d}e}{\text{d}t} )</td>
</tr>
<tr>
<td>( a_{22} = -\frac{\text{d}e}{\text{d}t} )</td>
<td>( a_{31} = -\frac{\text{d}e}{\text{d}t} )</td>
</tr>
<tr>
<td>( a_{32} = -\frac{\text{d}e}{\text{d}t} )</td>
<td>( a_{41} = -\frac{\text{d}e}{\text{d}t} )</td>
</tr>
</tbody>
</table>

\( \text{Variables:} \)

\( x_1 = \delta S \)
\( x_2 = \delta R \)
\( x_3 = \delta \phi^* \)
\( x_4 = \delta \phi^* \)

\( \text{Errors:} \)

\( \delta B \quad \delta f^* \quad \delta v^* \quad \phi^* \)

Equation to continuous form the following state equation is obtained:

\[
\dot{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 0 & a_{32} & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u \quad (18)
\]

Clearly, the above equation can be written in a more general state space form as follows:

\[
\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \quad (19)
\]

where \( \mathbf{x} \) represents the state vector, \( \mathbf{A} \) represents the system matrix, \( \mathbf{B} \) represents the input matrix and \( u \) represents the control vector. Eq. 19 is known as state differential equation. With Eq. 18 several linear control combinations can be analyzed and a suitable feedback compensation can be included. The non-linear dynamical equations represented by Eq. 6, 15 and 17 can be used to design loops for large phase angle variation. However, in such cases, the single particle non-linear model will not be very accurate.

**5 MULTIPARTICLE STATE SPACE MODEL**

Let us assume that a bunch-to-bucket transfer is used at injection into the accelerator with a bunch of \( N \) particles having energy and phase spread. If \( \delta R_1, \delta R_2, \ldots, \delta R_N \) are the traversal times for each particle, then the traversal times can be written in terms of \( \delta R_1, \delta R_2, \ldots, \delta R_N \) from Eq. 5. After substituting the orbit shifts for each particle, on the average Eq. 6 can be written as,

\[
\ddot{x}_1 = a_{11}x_1 + a_{12}x_2 \quad (20)
\]

Hence we show a slight modification for the multiparticle case below.

**REFERENCES**


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