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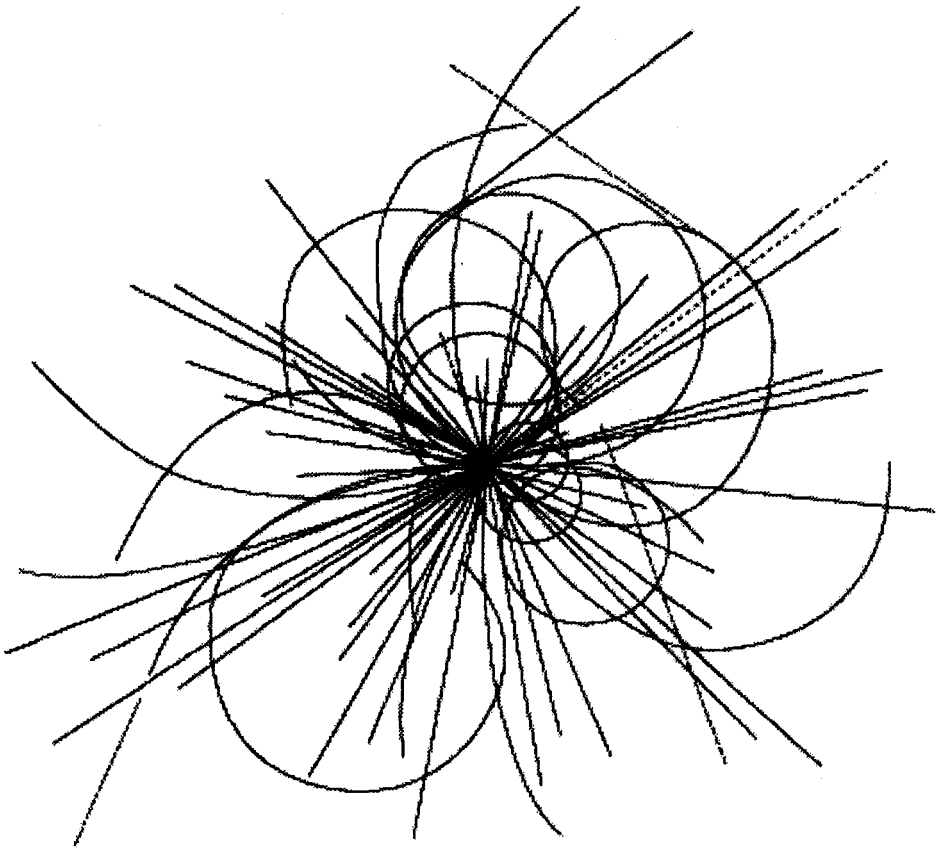

Superconducting Super Collider Laboratory

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Pumping Slots: Coupling Impedance Calculations and Estimates

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Abstract

Coupling impedances of small pumping holes in vacuum-chamber walls have been calculated at low frequencies, *i.e.*, for wavelengths large compared to a typical hole size, in terms of electric and magnetic polarizabilities of the hole. The polarizabilities can be found by solving an electro- or magnetostatic problem and are known analytically for the case of the elliptic shape of the hole in a thin wall. The present paper studies the case of pumping slots. Using results of numerical calculations and analytical approximations of polarizabilities, we give formulae for practically important estimates of slot contributions to low-frequency coupling impedances.

1.0 INTRODUCTION

To avoid beam instabilities the beam-chamber coupling impedances in modern colliders should be minimized. In doing so one tends to shield, with respect to beam electromagnetic fields, discontinuities of the vacuum chamber (vacuum boxes, bellows, *etc.*) by using metallic shields or liners. The requirements of high vacuum inside the beam pipe to ensure long beam lifetime can be satisfied using pumping holes in these shields. The number of such small discontinuities can be very large in big machines.

Two typical examples are the Large Hadron Collider (LHC) and the Superconducting Super Collider (SSC). The LHC design anticipates a thermal screen intended to shield the cold chamber walls from synchrotron radiation, with 10^7–$2 \cdot 10^8$ small holes in it for pumping (the number depends on the hole size). A similar liner for the SSC is considered as an option to provide the required beam lifetime, with the typical number of pumping holes 10^2–10^3 per meter. The large number of pumping holes contributes essentially to the total inductive impedance of the collider ring. A reasonable choice of the hole shape has to be made to minimize this contribution while satisfying vacuum and production requirements.

2.0 IMPEDANCE OF A SMALL HOLE

The analytical calculation of both the longitudinal and transverse coupling impedance for small holes in the perfectly conducting walls of the vacuum chamber at low frequencies has been carried out in Reference 1 for an arbitrary-shaped hole in the chamber with a circular cross section using the Bethe theory of diffraction by small holes[2] and an expansion over waveguide eigenmodes. Reference 3 gives an alternative derivation and includes effects of wall thickness. In these papers the impedance has been expressed in terms of hole polarizabilities, which are purely geometrical factors at low frequencies and can be found by solving a corresponding electro- or magnetostatic problem, *e.g.*, Reference 4. The longitudinal impedance of a hole in the chamber with the circular cross section of radius b is

$$Z(\omega) = -iZ_0 \frac{\omega}{c} \frac{(\alpha_m + \alpha_e)}{4\pi^2 b^2} , \tag{1}$$

where α_e and α_m are electric and magnetic polarizabilities of the hole, respectively. The transverse impedance of the hole is

$$\vec{Z}_\perp(\omega) = -iZ_0 \frac{\alpha_m + \alpha_e}{\pi^2 b^4} \vec{a}_h \cos(\varphi_h - \varphi_b) , \tag{2}$$

where $\vec{a}_h$ is the unit vector directed to the hole in the chamber cross section containing the hole, and φ_h and φ_b are azimuthal angles of the hole and beam in this cross section. It is worth noting that both the longitudinal and transverse impedances are proportional to

the sum of polarizabilities, $(\alpha_m + \alpha_e)$.* The generalization to an arbitrary chamber cross section, given in Reference 5, shows that the same is valid in an arbitrary chamber.

For a circular hole with radius a in a thin wall (thickness $t \ll a$),[3]

$$\alpha_m = \frac{4}{3}a^3 \ , \quad \alpha_e = -\frac{2}{3}a^3 \ , \tag{3}$$

and Eqs. (1) and (2) have a very simple form. For a thick wall, $t \geq a$, the sum $(\alpha_m + \alpha_e)$ should be multiplied by a factor 0.56 (Reference 2). In fact, for apertures in thick walls there are two different kinds of polarizabilities—internal and external—that give scattered fields on the incident and transmitted sides of the aperture, respectively. Since we are interested in fields inside the chamber, the polarizabilities cited below are the internal ones.

The low-frequency impedances given by Eqs. (1) and (2) for circular holes are in good agreement with simulations and measurements, *e.g.*, References 6 and 7.

There are also explicit analytical expressions for polarizabilities of an elliptic hole in a thin wall:[4]

$$\begin{aligned} \alpha_{m\|} &= \frac{\pi l_1^3 \varepsilon^2}{3[K(\varepsilon) - E(\varepsilon)]} \ , \\ \alpha_{m\perp} &= \frac{\pi l_1^3 \varepsilon^2 (1-\varepsilon^2)}{3[E(\varepsilon) - (1-\varepsilon^2)K(\varepsilon)]} \ , \\ \alpha_e &= -\frac{\pi l_1^3 (1-\varepsilon^2)}{3E(\varepsilon)} \ , \end{aligned} \tag{4}$$

where $\varepsilon = \sqrt{1 - l_2^2/l_1^2}$ is the eccentricity, l_1 and l_2 ($l_1 \geq l_2$) are ellipse semiaxes, and $K(\varepsilon)$ and $E(\varepsilon)$ are complete elliptic integrals of the first and second kind. The longitudinal (transverse) magnetic polarizability corresponds to the case when the exciting field is directed along the major (minor) ellipse axis. If we denote α the angle between the chamber axis and ellipse major axis, then one should substitute

$$\alpha_m = \alpha_{m\|} \sin^2 \alpha + \alpha_{m\perp} \cos^2 \alpha \tag{5}$$

in Eqs. (1) and (2) (see References 1 and 5).

* In fact, this is rather the difference because α_e and α_m have opposite signs.

The results cited are valid under the following conditions (below h is a typical hole size):

- $h \ll b$

  This means that one can neglect the curvature of the chamber wall. Certainly, h here is the transverse (with respect to the chamber axis) dimension of the hole.

- $\omega h/c \ll 1$

  The fields can be considered as equal in different hole points and the approximation of an incident plane wave can be used. As a result, the polarizabilities can be found by solving a static problem.

One can use another expression for the longitudinal coupling impedance of the hole in the cylindrical chamber wall:[3]

$$Z(\omega) = -\frac{1}{2\pi bq} \iint_{hole} dS e^{-ikz} J_\varphi \ , \tag{6}$$

where $k = \omega/c$ and J_φ is the Fourier-harmonic of the azimuthal component of the effective "magnetic" current induced on the hole by an ultrarelativistic charge q moving along the chamber axis. For small holes at low frequencies, *i.e.*, under the conditions above, Eq. (6) can be reduced to Eq. (1). For higher frequencies and/or larger holes, one needs to solve integral equations for the effective currents instead of a static problem. That is much more difficult (see References 8 and 9).

3.0 IMPEDANCE OF A SLOT

For non-elliptical hole shapes the corresponding electrostatic problem has no exact analytical solution, since variables are not separable and there are no analytical expressions for polarizabilities. However, there are measurements and numerical results for small apertures in conducting surfaces (see Reference 10 for the bibliography) that can provide us with data on polarizabilities for simple shapes of interest, namely, rectangular and rounded-end slots.

The impedances of narrow long slots were studied[1,3] using the approximation of a slot by an elongated elliptic hole. Let us consider qualitatively the beam-induced current pattern in the wall with a slot. For a transverse slot, with its larger axis being transverse to the chamber axis, *i.e.*, angle $\alpha = \pi/2$ in Eq. (5) and $\alpha_m = \alpha_{m\|}$,* the exact shape of ends is not very essential, because such a slot cuts the current lines and produces large impedance. So, estimates from the elliptical approximation seem to be good enough. The opposite and more interesting case is a longitudinal slot, which disturbs the beam-induced current only near its ends. If we denote slot width w and length l, $w \ll l$, the typical size of the region

* Subscript $\|$ here means that the beam magnetic field is directed along the slot.

where the wall current is disturbed is of the order of w. It would be reasonable to expect that the impedance of such a slot is determined mostly by its width and the shape of its ends. The ellipse approximation is not very adequate in this case since the current pattern will be slightly distorted all along the elliptic slot. In fact, it shows the cancellation of leading terms that are proportional to l in the sum $\alpha_m + \alpha_e$, where $\alpha_m = \alpha_{m\perp}$ since angle $\alpha = 0$ for a longitudinal slot. However, the next-to-leading terms vanish when $l/w \to \infty$, while one can expect from the physical reasons mentioned above that it should be a constant.

One more argument in this direction is derived from the study of an infinitely long narrow slot. It was shown by solving integral equations for the effective magnetic current[8,9] that such a slot does not contribute to the reactive impedance. The solution has $J_\varphi = 0$, and the impedance vanishes according to Eq. (6). It seems more or less obvious from consideration of the induced current pattern. The explanation of this phenomenon in terms of static polarizabilities is that α_e and $\alpha_{m\perp}$ per unit length of slot have equal modulus and opposite signs. This can be shown for an arbitrary wall thickness.[11] The static problem is two-dimensional in this case and can be solved by conformal mapping. For further references we give here polarizabilities per unit length of an infinite slot of width w in a thin wall (thickness $t \ll w$):

$$\tilde{\alpha}_{m\perp} = -\tilde{\alpha}_e = \frac{\pi}{16} w^2 \ , \tag{7}$$

which can be found in textbooks, *e.g.*, Reference 12;* and in a very thick wall:

$$\tilde{\alpha}_{m\perp} = -\tilde{\alpha}_e = \frac{1}{2\pi} w^2 \ , \tag{8}$$

(see, for example, Reference 13).

Coming back to a longitudinal slot of large (with respect to width) but finite length, one can conclude that its central part does not contribute to the inductive impedance since it works like a piece of an infinitely long slot. We could mentally cut it out and then match two pieces of disturbed current pattern near the ends without changing the impedance. So, the impedance of a long slot as a function of its length has the form:

$$Z(l) = \kappa Z_h + Z_1(l) \ , \tag{9}$$

where κZ_h is independent of l and corrections $Z_1(l)$ vanish when $l/w \to \infty$. It is clear that the first term corresponds to a short hole and gives the contribution of slot ends. One could

* The limit of the elliptical approximation would give $\pi/24$ instead of $\pi/16$ here.

even expect that Z_h is the impedance of a square (for rectangular long slots) or circular (for rounded-end ones) hole with the same width w or diameter w, and coefficient κ is of the order of 1. To check these assumptions we use data for polarizabilities of rectangular and rounded slots from References 14–16. If we denote x the slot aspect ratio, $x = w/l \leq 1$, the electric and transverse magnetic polarizabilities according to Reference 15 can be well approximated for a thin wall by the following polynomials:

for a rectangular slot:

$$\begin{aligned} \alpha_e &= -\frac{\pi}{16} w^2 l (1 - 0.5663x + 0.1398x^2), \\ \alpha_{m\perp} &= \frac{\pi}{16} w^2 l (1 + 0.3577x - 0.0356x^2) \ ; \end{aligned} \tag{10}$$

for a rounded-end slot:

$$\begin{aligned} \alpha_e &= -\frac{\pi}{16} w^2 l (1 - 0.7650x + 0.1894x^2), \\ \alpha_{m\perp} &= \frac{\pi}{16} w^2 l (1 - 0.0857x - 0.0654x^2) \ . \end{aligned} \tag{11}$$

The accuracy of these approximations is better than 1% when $0.1 \leq x \leq 1$ and a few percent for lower x. It is worth noting that leading terms in these formulae coincide with the infinitely long slot limits of Eq. (7). Now we can obtain useful formulae:

for a rectangular slot:

$$\alpha_m + \alpha_e = w^3 (0.1814 - 0.0344 w/l) \ ; \tag{12}$$

for a rounded-end slot:

$$\alpha_m + \alpha_e = w^3 (0.1334 - 0.0500 w/l) \ . \tag{13}$$

Substituting Eqs. (12) and (13) into Eqs. (1) and (2) gives the impedances of the slots.

From the equations above one can derive some conclusions of practical interest. First, the impedance of a square hole ($w = l$) is given by

$$\alpha_m + \alpha_e = 0.1470 w^3 \ . \tag{14}$$

Comparing it with $w^2/12$ for a circular hole of diameter w (which follows from Eq. (3) with $w = 2a$ or from Eq. (13) with $w = l$), we get the ratio of the impedances of a square hole and inscribed in it a circular one as

$$\frac{Z_\square}{Z_\odot} = 1.76 \ ; \tag{15}$$

or if we consider holes of the same area $A_\square = A_\odot$, as

$$\frac{(Z/A)_\square}{(Z/A)_\odot} = 1.38 \ . \tag{16}$$

Taking into account a finite wall thickness would change these ratios probably within a few percent.

The second practical result is the ratio of impedances of long slots having rectangular or rounded-end shapes and the same width. It is equal to 1.36 and is practically independent of the wall thickness.

It is also curious to compare the impedances of long slots with those of holes of the same width and how they correspond to a qualitative result of Eq. (9). From Eqs. (12) and (13) we get:

for a very long rectangular slot, $Z = 1.23 Z_\square$ and
for a very long rounded-end slot, $Z = 1.60 Z_\odot$.

Figures 1 and 2 show the impedances and the impedance/area ratio for rectangular, rounded-end, and elliptic slots with fixed width, normalized to those for the circular hole, as functions of (inverse) aspect ratio w/l for the case of a thin wall, thickness $t \ll w$. The results for a thick wall, $t \geq w$, will change slightly. For a circular hole the dependence of the polarizabilities and, hence, impedances on the wall thickness are known:[3] the hole impedances for a thick wall are 56% of those for $t = 0$. The physical explanation of this reduction is that the beam fields do not penetrate so far into the hole in a thick wall, and the field distortions and, as a result, the effective currents and polarizabilities are smaller compared to the case of a thin wall. No quantitative results on the dependence of internal polarizabilities on the wall thickness for other hole shapes are known to the author. One could expect that for a small, square hole this dependence is approximately the same as that for a circular one, perhaps within a few percent. As for long slots, the impedance reduction due to the wall thickness seems to be the same or smaller (since fields more easily penetrate into the slot) than for a hole of the same width. However, one should be careful making such predictions because in the case of long slots the resulting $\alpha_m + \alpha_e$ is a small difference of two large values, each having its own thickness dependence. Anyway, it is quite clear that results for a thin wall give an upper bound for the impedances. On the other hand, it seems reasonable to expect that Figures 1 and 2, where relative values are plotted, remain valid even for a thick wall, with an accuracy of 10–20%. This conclusion agrees with recent computer simulations for slots in a thick wall.[17]

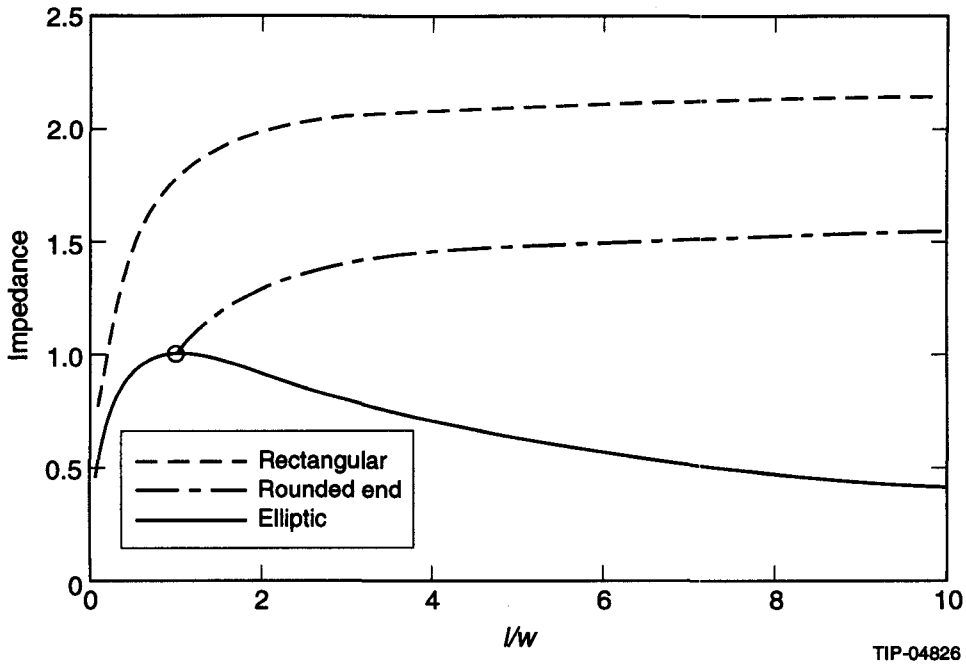

Figure 1. Slot Impedance Versus Slot Length l for Fixed Width w in Units of the Impedance of the Circular Hole with Diameter w.

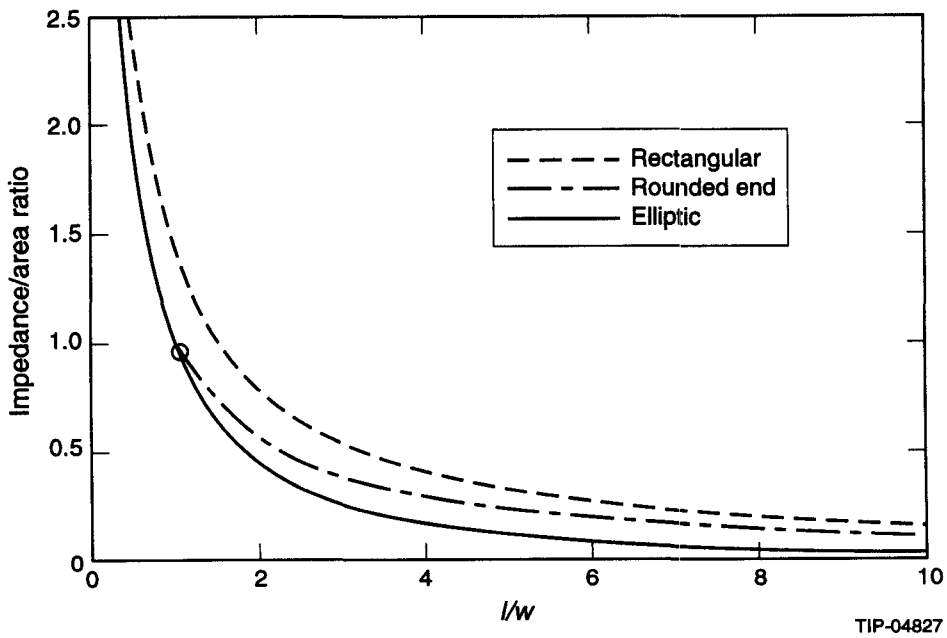

Figure 2. Impedance/Area Ratio for Slots Versus Length l for Fixed Width w (in Units of this Ratio for the Circular Hole with Diameter w).

The results obtained allow us to easily calculate contributions from pumping holes to the impedances of the liner. Let us compare just two possible variants of pumping holes for

the SSC Collider liner: (1) the circular holes of diameter 2 mm, and (2) rounded-end slots 2×6 mm^2. Assuming machine radius $R = 13\,865$ m, liner cross section radius $b = 1.5$ cm, and liner wall thickness $t = 1$ mm, one needs some 1300 such holes or 350 slots per meter to have the pumping area 3–5% of the liner surface.[18] We neglect the mutual interaction between holes and assume additivity at low frequencies. The total impedances are shown in Table 1 in comparison with the Collider impedance budget without liner.

Table 1. Impedances of Liner Pumping Holes.

	(Z/n) / Ω	$Z_\perp$/ (MΩ/m)
Holes Ø2 mm	0.13	16
Slots 2×6 mm^2	0.05	6.4
Budget w/o liner	0.68	40

Although longer slots would give even lower low-frequency impedances, they have disadvantages at high frequencies.[6,9] At the present moment, also taking into account problems of production that practically exclude elliptic holes and of mechanical stability, the short, rounded-end slots seem to be the best choice for pumping holes in the Collider liner.

Figures 1 and 2 include also results for $x < 1$, *i.e.*, $l < w$, except the case of the rounded-end slots, where $l < w$ has no meaning. These results correspond to transverse slots with fixed length equal to w in this case, and variable width $l < w$. While the electric polarizability of such slots can be obtained from Eqs. (10) and (11) by a formal permutation $w \leftrightarrow l$, for the magnetic polarizability one should use the longitudinal one, since the beam magnetic field is directed along the major axis of such transverse slots.

The following fits from Reference 16 give the longitudinal magnetic polarizability of slots with width w and length l in a thin wall for aspect ratios in the interval $0.1 \le x = w/l \le 1$ with accuracy of the order of 1%:

for a rectangular slot:

$$\alpha_{m\|} = \frac{0.132}{\ln(1 + 0.660/x)} l^3 \; ; \tag{17}$$

for a rounded-end slot:

$$\alpha_{m\|} = \frac{0.187 + 0.052x(1-x)}{\ln(1 + 2.12/x)} l^3 \; . \tag{18}$$

For narrow transverse slots ($w/l \le 0.2$), the electric polarizability α_e is smaller than the longitudinal magnetic polarizability $\alpha_{m\|}$ by an order of magnitude and can be neglected

in calculating impedances. In this region of aspect ratios one can use an approximation of $\alpha_{m\|}$ for rectangular slots that includes thickness corrections:[19]

$$\alpha_{m\|} = \frac{\pi l^3}{24\left(\ln\frac{8l}{w} + \frac{\pi t}{2w} - \frac{7}{3}\right)}\ , \tag{19}$$

where wall thickness $t \ll l$ is assumed. It should be noted here that the approximation of a narrow transverse slot by an elliptic hole would give for $t = 0$

$$\alpha_{m\|} = \frac{\pi l^3}{24\left(\ln\frac{4l}{w} - 1\right)}\ ,$$

which is below correct values by only 20–30%.

In the case of a very "deep" slot, $t \gg l \gg w$, one can estimate $\alpha_{m\|}$ from simple energy considerations in the following way. We can equate the energy of the beam magnetic field penetrated into the deep slot, which is considered just as a waveguide, to that of magnetic dipole M immersed into unperturbed beam field H_0:

$$\frac{\mu_0}{2}\int H^2 dV = \frac{\mu_0 M H_0}{2}\ .$$

Since $M = \alpha_{m\|} H_0$, we get

$$\alpha_{m\|} = \frac{\int H^2 dV}{H_0^2}\ . \tag{20}$$

Taking into account only the dominant H_{10} mode in the rectangular waveguide $l \times w$ gives an estimate:

$$\alpha_{m\|} = \frac{w l^2}{2\pi}\ . \tag{21}$$

The following approximate expression for $\alpha_{m\|}$ of a deep narrow slot ($t, l \gg w$) is given in Reference 19:

$$\alpha_{m\|} = \frac{4l^3}{\pi^3}\left[\left(\ln\frac{8l}{w} - \frac{7}{3} + \frac{l}{w}\tanh\frac{\pi t}{2l}\right)^{-1} + \left(\ln\frac{8l}{w} - \frac{7}{3} + \frac{l}{w}\coth\frac{\pi t}{2l}\right)^{-1}\right]\ .$$

In fact, as was pointed out in Reference 19, this expression works for any wall thickness. It gives Eq. (19) when $t \ll l$ (with coefficient $4/\pi^3 = 1.290$ instead of $\pi/24 = 1.309$). When $t \gg l$, it leads to $\alpha_{m\|} = 8wl^2/\pi^3$, which is close to our estimate in Eq. (21).

It is quite clear from Figure 2 as well as from the discussion above that narrow transverse slots are bad as pumping holes due to high ratios of their impedances to pumping area. On the other hand, such slots can be a part of some more complicated discontinuities. One example of such a discontinuity is considered in Section 4.0.

4.0 ESTIMATES FOR BUTTON-TYPE BPMs

The button-type beam position monitor (BPM) usually has two or four electrodes (buttons). Each of them can be considered either as an annular narrow cut in the chamber wall (thin-wall approximation), or as a coaxial line with a very thick center conductor, connected to the beam chamber (thick-wall approximation). The choice of the approximation depends on the specific BPM design. The radius a of the cut or groove is usually much larger than its width w, typically $a/w \sim$10–20. The center conductor is connected to the chamber wall through some circuit, with a characteristic impedance Z_c. Apart from this impedance, the beam experiences the coupling impedance produced by the discontinuity (cut or groove) in the chamber wall. Let us estimate it as the impedance of a narrow (curved) slot. For this purpose one can replace the circle by the regular octagon with side $a' = \pi a/4$ and consider it now as a combination of eight narrow slots (two transverse, two longitudinal, and four tilted). Making use of Eq. (5) for the tilted slots and neglecting small contributions from the longitudinal ones yields the same impedance estimate as the impedance of four transverse slots with length a' and width w. One can now apply the results of Section 3.0 for $\alpha_{m\|}$ and neglect small α_e, which yields the following estimate:

in the thin-wall ($t \ll a$) approximation, cf. Eq. (19):

$$\alpha_m + \alpha_e = \frac{2}{3}\left(\frac{\pi}{4}\right)^4 \frac{a^3}{\ln(2\pi a/w) + \pi t/(2w) - 7/3} \; ; \tag{22}$$

in the thick-wall ($t \geq a$) approximation, cf. Eq. (21):

$$\alpha_m + \alpha_e = \frac{\pi}{8} w a^2 \; . \tag{23}$$

Substituting these expressions into Eqs. (1) and (2) gives us the estimate of the imaginary part of the coupling impedance per button for the button-type BPM.

It is interesting to compare these results with the impedance of the circular hole of radius a in a thin or thick wall. If we assume the typical dimension ratio, $a/w = 10$, and $t \ll w$, the BPM electrode impedance is 0.2 of that for the hole in a thin wall. For a thick wall the electrode-to-hole impedance ratio is approximately w/a, *i.e.*, 0.1 for $a/w = 10$.

There are results of numerical computations by means of MAFIA code for the button-type BPMs of the ATF Damping Ring.[20] With $a = 2.5$ mm, $w = 0.5$ mm, chamber radius $b = 13.5$ mm, and machine circumference $2\pi R = 147.8$ m, the reduced longitudinal impedance of the electrode was calculated as $Z/n \simeq -i2 \cdot 10^{-5}\ \Omega$. Here $n = \omega/\omega_0 = \omega R/c$ is the harmonic number of the revolution frequency ω_0. Making use of Eq. (22) with $t = 0$ would give us the estimate $Z/n \simeq -i5 \cdot 10^{-5}\ \Omega$, and the thick-wall approximation, Eq. (23),

leads to $-i3 \cdot 10^{-6}\ \Omega$. If we assume $t = w$ (the thickness value is not given in Reference 20), Eq. (22) gives the impedance $Z/n \simeq -i2 \cdot 10^{-5}\ \Omega$.

5.0 CONCLUSION

We have demonstrated applications of the general analytical expressions for the impedances of small holes in accelerator chamber walls in terms of the hole polarizabilities to the important cases of rectangular and rounded-end slots. The formulae obtained allow one to calculate easily the coupling impedances of slots at low frequencies.

As an example of application to more complicated discontinuities, the impedance of button-type beam position monitors is estimated.

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