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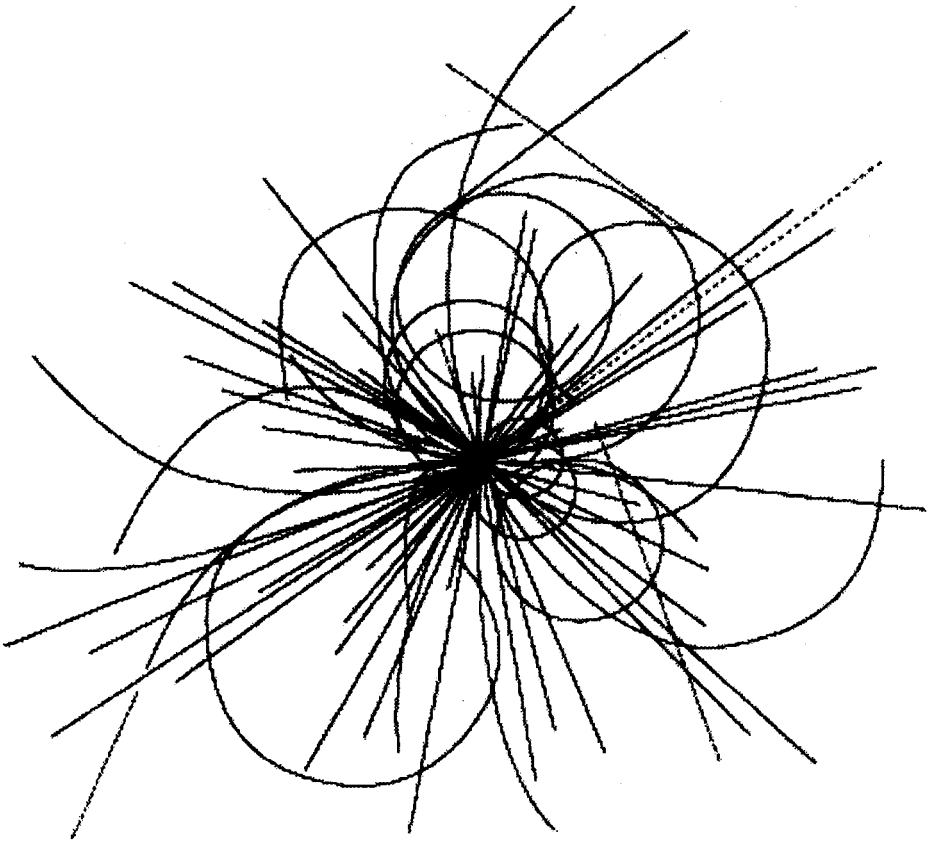

Superconducting Super Collider Laboratory

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The SSC Transverse Feedback Systems

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The SSC Transverse Feedback Systems

W. Chou and J. Peterson

Abstract

Transverse feedback systems are needed at the Superconducting Super Collider for several different reasons. The requirements of these systems are analyzed and specified. In addition to the general requirements (power, bandwidth, gain, *etc.*), specific attention is given to the noises in the systems, which need to be controlled in order to keep the emittance growth at a tolerable rate. A quantitative treatment is given to specify the allowable noise level in the feedback systems. Brief discussions of the damping process, beam decoherence, beam heating, and coherent beam instabilities are also included.

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1.0 INTRODUCTION

The Superconducting Super Collider (SSC) transverse feedback systems are designed for the following four different purposes:

1. Correction of the injection errors:
   At injection there are errors in the beam position, angle, energy, and phase. These errors can lead to beam emittance growth if not corrected. Because these errors are relatively large compared with other types of perturbations described below, the requirement of the feedback system is that it should have enough power to kick the beam back quickly to the desired orbit before any significant decoherence occurs.
2. Damping of the resistive wall instability:
   The transverse impedance of the wall of the beam tube increases as the frequency decreases. At low frequencies (around several hundred hertz), the wall impedance of the SSC is in the order of gigaohm per meter. It can lead to fast beam blowup in the transverse planes. A feedback system with a large gain is needed.
3. Damping of the coupled bunch instability driven by the rf cavities:
   The higher order modes (HOM) of the rf cavities, if not properly damped and/or stagger-tuned, may drive the beam unstable. The feedback system needs a wide bandwidth to cure this type of instability.
4. Control of possible emittance growth from the above and/or other mechanisms (*e.g.*, ground motion, coolant flow, power supply ripples, *etc.*):
   Because of the long memory of the proton beam (the radiation damping time in the SSC is about 13 h), any external perturbation will be remembered by the beam and may lead to eventual emittance growth. A feedback system that keeps the coherent motion of the beam below a certain allowable amplitude can effectively reduce the emittance growth rate.

It is seen that each purpose imposes different requirements on the feedback systems. This, of course, does not mean that a total of four systems would be required. One system may well serve several different purposes.

A special concern of the SSC is the preservation of the required small transverse beam emittance. Its design value is 1 mm-mrad (rms, normalized), which is just a fraction of the existing proton storage rings. A high noise level in the feedback system or a poor pickup resolution will lead to a fast emittance dilution, as will be discussed in detail later. As a matter of fact, this is believed to be the possible source of the emittance blowup that has been observed at both the Tevatron and SPS when the transverse feedback system is turned on. Therefore, in addition to the general requirements of a feedback system (such

as the power or kicker voltage, the bandwidth, the gain, *etc.*), one needs in particular to specify the noise level or, equivalently, the pickup resolution of the SSC feedback systems for the purpose of controlling the emittance growth.

This paper is organized in the following way. Section 2.0 is an analysis of the injection errors and their corrections by means of a feedback system. Section 3.0 discusses the continuous emittance growth due to external excitations in the absence and in the presence of a feedback system. It is followed by a discussion of the resistive wall instability in Section 4.0. The instabilities driven by the HOM are the contents of Section 5.0. The description of the SSC feedback systems and their specifications are given in Section 6.0. Finally, Section 7.0 is the summary and conclusions.

2.0 ANALYSIS OF INJECTION ERROR CORRECTIONS

2.1 Injection Process at the SSC

In the present design, there will be 8 batches injected from the High Energy Booster (HEB) into the Collider. Each batch has approximately 2000 bunches. The bunch spacing is 16.7 ns, the batch spacing 1.7 μs, and the abort gap 4.2 μs. It is the batch spacing (1.7 μs) and the injection kicker waveform ripples that determine the bandwidth requirement of the feedback system used to correct the injection errors.

2.2 Emittance Growth and Error Tolerances

2.2.1 Injection Errors and Emittance Dilution

At injection into the Collider several types of errors can lead to growth in the transverse emittances ϵ_x and ϵ_y. Errors in beam position (δx, δy), errors in angle ($\delta x'$, $\delta y'$), and errors in beam energy (δE) all produce coherent, dipole-type betatron oscillations, which, if not immediately corrected, will decohere (*i.e.*, smear out in phase space) and so increase the emittance. The function of the damper* is to reduce these coherent dipole oscillations before the emittance growth through decoherence becomes appreciable. (The decoherence mechanisms and rates will be considered in Sections 2.4 and 2.5.)

If the beam optics of the injection transport line are not matched to the optics of the Collider, coherent quadrupole-type betatron oscillations are induced, which also eventually smear out and produce emittance growth.[1] However, the normal (dipole field) damper cannot correct the quadrupole type of coherent oscillation. Therefore, in this paper we shall not consider any further the effects of optical mismatches.

The magnitude of the coherent amplitude x_c due to injection position and angle errors in, say, the horizontal plane is:

* The terms "damper" and "feedback system" are used interchangeably in this paper.

$$|x_c|^2 = \delta x^2\left(1+\alpha_x^2\right) + 2\alpha_x\beta_x\delta x\delta x' + \beta_x^2\delta x'^2, \tag{1}$$

where the lattice functions α_x and β_x refer to the lattice position where δx and $\delta x'$ are measured. The eventual fractional emittance increase produced by decoherence is, for the case of small coherent amplitudes, relative to the rms beam width σ_x $(= \sqrt{\epsilon_x\beta_x})$:

$$\frac{\delta\epsilon_x}{\epsilon_x} = \frac{|x_c|^2}{2\sigma_x^2}. \tag{2}$$

For the case of an injection energy error $\delta E/E$, the resultant coherent amplitude is:

$$|x_c|^2 = \left(\frac{\delta E}{E}\right)^2 \left(D_x^2(1+\alpha_x^2) + 2\alpha_x\beta_x D_x D_x' + \beta_x^2(D_x')^2\right), \tag{3}$$

where D_x and D_x' are the dispersion function and its slope at the point of injection. Equation (3) points out the advantage of injecting into a region of zero dispersion.

2.2.2 Tolerable Injection Errors

It is of interest to estimate the magnitude of injection errors that could be considered "tolerable" in that the corresponding emittance growths are, say, less than 10%, in which case an injection damper might not be required. We will compute the injection position, angle, and energy errors that *singly* would produce a 10% emittance dilution. Consider a beam-monitor point with $\beta_x = 300$ m and $\alpha_x = 2.4$. At the Collider injection energy of 2 TeV and a normalized emittance of 1 mm-mrad, the rms beam size σ_x is 0.38 mm, and the rms beam angle σ_x' is 1.27 μrad. The tolerable coherent amplitude x_c is 0.17 mm, which could be produced either by an injection-position error δx of 0.065 mm *or* by an injection-angle error $\delta x'$ of 0.57 μrad. Since the injection area is nominally dispersion-free, let us estimate the tolerable injection energy error on the basis of an assumed spurious dispersion magnitude D_x of 1 m and a slope D_x' of 0.01. In this case the corresponding fractional injection energy error $\frac{\delta E}{E}$ that would produce a 10% emittance dilution is 0.31×10^{-4}, which is comparable to the nominal rms fractional energy width $\frac{\sigma_E}{E}$ of 1×10^{-4}. Since each of these three "tolerable" errors separately produces a 10% dilution, the actual tolerance level of each when all three are present is certainly lower, possibly by a factor of $1/\sqrt{3}$. These estimates are listed in Table 1.

Table 1. Injection Error Tolerances for $\delta\epsilon/\epsilon = 10\%$.

β_x (m)	α_x	σ_x (mm)	σ_x' (μrad)	x_c (mm)	δx (mm)	$\delta x'$ (μrad)	D_x (m)	D_x'	$\delta E/E$
300	2.4	0.38	1.27	0.17	0.065	0.57	1	0.01	0.31×10^{-4}

2.2.3 Transfer Magnet Tolerances

Errors in the strengths of the dipole magnets in the extraction system of the HEB and in the injection system of the Collider all contribute to the injection errors that we have been considering. The Lambertson septum magnets and the C-magnets (which functionally are extensions of the Lambertsons) are dc-powered, so that it is their degree of regulation with which we are concerned. The kicker magnets are pulsed, and it is the ripple on the flat-top that is of concern. Table 2 gives the number of each magnet type N_m, the length of each magnet L, the nominal field strength B, the total bend angle θ for each magnet group, the average beta-function value β for each magnet group (horizontal β for the kickers, vertical β for the Lambertsons and the C-magnets), and the fractional error $\delta\theta/\theta$ that will produce a 10% emittance dilution due to that group of magnets alone. The transfer energy is 2 TeV (which corresponds to a magnetic rigidity of 6671.3 T-m), and the normalized emittance is 1 mm-mrad.

Table 2. Transfer Magnet Tolerances for $\delta\epsilon/\epsilon = 10\%$.

Magnet Type	N_m	L (m)	B (T)	$N_m BL$ (T-m)	θ (mrad)	β (m)	$\delta\theta/\theta$
HEB kickers	6	2.625	0.125	1.97	0.30	59	4.3×10^{-3}
HEB Lambertson	7	4.86	1.1	37.5	5.6	65	2.1×10^{-4}
HEB C-magnets	3	5.0	1.1	16.5	2.5	100	3.9×10^{-4}
Coll. C-magnets	3	5.0	1.1	16.5	2.5	50	5.5×10^{-4}
Coll. Lambertson	7	4.86	1.1	37.5	5.6	100	1.7×10^{-4}
Coll. kickers	5	2.0	0.06	0.60	0.09	300	6.2×10^{-3}

For a given set of errors in the HEB extraction and the Collider injection magnets, Table 2 can be used to calculate the resultant emittance dilution in the Collider if no injection damper is used. The beam transfer system from the HEB to the Collider is being redesigned, but the design is not yet complete. The magnet system listed above was taken from the SSC Site-Specific Conceptual Design Report.[2] Although the final magnet system will undoubtedly be different in detail, the main features (numbers and strengths of magnets) are likely to be similar.[3]

If only the dc-supplies of the Lambertson septum magnets and the C-magnets exceed the tolerable levels, a low-frequency, narrow-band damper will suffice. If there is excessive wave-shape ripple on the kicker-magnet pulses, the damper bandwidth must accommodate the frequencies observed in that ripple. The dominant ripple frequency in the kicker waveform will probably be about $(2\pi\tau_r)^{-1}$, where τ_r is the rise time. For the HEB extraction kicker and the Collider injection kicker, $\tau_r = 1.1$ μsec, so that the damper bandwidth must include frequencies up to 500 kHz.

2.3 The Damping Process

Feedback damping of collective betatron oscillations proceeds as follows. The amplitude x_1 of the collective oscillation is measured by a beam position monitor at point 1. A kicker magnet at point 2, chosen to be, say, an integer-plus-one-quarter wavelengths downstream from point 1, is arranged to deflect the beam by the angle

$$\Delta x_2' = -g x_1/(\beta_1\beta_2)^{1/2}, \tag{4}$$

where β_1 and β_2 are the beta functions at points 1 and 2, and g (the "gain") is typically much less than 1. This deflection produces a change in amplitude Δx_1 at point 1 on the next turn of

$$\begin{aligned}\Delta x_1 &= \Delta x_2'(\beta_1\beta_2)^{1/2}\sin\phi_{21}\\ &= -g x_1 \sin\phi_{21},\end{aligned} \tag{5}$$

where ϕ_{21} is the relative phase between points 2 and 1. Thus, the collective amplitude is decreased in proportion to its amplitude.

The damping process can be described in terms of the eigenvalues of the matrix representing these procedures, as has been done by Lebedev *et al.*,[4] but it is also of interest to derive an analytic description from the difference equations. It is convenient to use normalized coordinates, in which the collective amplitude and phase are represented by the vector η:

$$\begin{aligned}\eta &\equiv \frac{x}{\sqrt{\beta}} + i\left(\frac{\alpha x}{\sqrt{\beta}} + \sqrt{\beta}x'\right)\\ &\equiv \eta_x + i\eta_y.\end{aligned} \tag{6}$$

The feedback algorithm can be expressed as

$$\begin{aligned}\Delta\eta_{y_2} &= \sqrt{\beta_2}\Delta x_2'\\ &= -g x_1/\sqrt{\beta_1}\\ &= -g\eta_{x_1}\\ &= -g\eta_{y_2}.\end{aligned} \tag{7}$$

The amplitude $|\eta|$ is decreased in one turn by $\Delta|\eta|$:

$$\Delta|\eta| = \Delta\eta_{y_2}\cos\phi_1$$

$$= -g\,|\eta|\cos^2\phi_1, \tag{8}$$

where ϕ_1 is the betatron phase at the beam-position monitor. Also, the phase angle is changed by $\Delta\phi$:

$$\begin{aligned}\Delta\phi &= \frac{\Delta\eta_{y2}\sin\phi_1}{|\eta|} \\ &= -g\sin\phi_1\cos\phi_1.\end{aligned} \tag{9}$$

Note that the fractional decrement in the amplitude due to the kicker is monotonic and on average is equal to $g/2$, while the change in phase angle oscillates and has a zero average.

Thus, the evolution of the collective betatron amplitude can be represented by the difference equation

$$\begin{aligned}\frac{\Delta|\eta|}{\Delta n} &= -g\,|\eta|\cos^2\phi_1 \\ &= -g\,|\eta|\cos^2(2\pi\nu n+\phi_0),\end{aligned} \tag{10}$$

where n is the turn number, and ϕ_0 is the phase at the beam position monitor on the first turn. Integrating Eq. (10) up to turn N, we get

$$\begin{aligned}\ln\left(\frac{|\eta(N)|}{|\eta_0|}\right) &= -g\sum_{n=0}^{N}\cos^2(2\pi\nu n+\phi_0) \\ |\eta(N)| &= |\eta_0|\exp\left(-g\left(\frac{N}{2}+\frac{\sin\left(2\pi\nu(2N+1)\right)-\sin 2\phi_0}{4\sin 2\pi\nu}\right)\right).\end{aligned} \tag{11}$$

Thus, the collective amplitude damps as an exponential with a characteristic period of $2/g$ turns, but the exponential also has some minor wiggles.

This description of the damping process, which has ignored the effects of noise in the feedback system, is appropriate to damping of injection errors, where the amplitudes that require correction are much larger than the low noise levels required for acceptable long-time emittance growth.

2.4 Decoherence Due to Chromaticity and Momentum Spread

The rms tune spread σ_ν due to chromaticity ζ and rms momentum spread σ_p/p is

$$\sigma_\nu = \zeta\frac{\sigma_p}{p}. \tag{12}$$

Because of this tune spread each pencil element in a beam tends to stretch out in the betatron-phase coordinate of phase space. After N turns a beam element with a Gaussian

momentum spread develops a Gaussian distribution in betatron phase ϕ:[5]

$$\rho(\phi, N) = \frac{1}{\sqrt{2\pi}\,|\sigma_\phi|} \exp\left(-\phi^2/2\sigma_\phi^2\right), \tag{13}$$

where

$$\sigma_\phi = 2\zeta \frac{\sigma_p}{p} \nu_s^{-1} \sin\left(\pi \nu_s N\right) \tag{14}$$

is the rms phase width. Note that the spreading in phase is periodic in the synchrotron period ν_s^{-1} in the linear approximation. That is, if ν_s is constant, independent of synchrotron amplitude, each pencil element decoheres and then perfectly re-coheres every synchrotron period. Note also that this action is independent of betatron amplitude, so that the whole beam bunch decoheres and re-coheres as a unit in the linear approximation.

The centroid of the bunch with an initial betatron amplitude a_0 has after N turns the amplitude $a(N)$:

$$a(N) = a_0 A(N), \tag{15}$$

where the decoherence factor $A(N)$ is given by[5]

$$A(N) = \exp\left(-\sigma_\phi^2/2\right). \tag{16}$$

For the Collider at 2-TeV injection, $\sigma_p/p = 1 \times 10^{-4}$, $\nu_s = 2.2 \times 10^{-3}$, and for a residual chromaticity of 5, the linear decoherence factor oscillates between 1.0 and 0.90 at the synchrotron period of 455 turns. At 20 TeV, with $\sigma_p/p = 0.6 \times 10^{-4}$ and $\nu_s = 1.2 \times 10^{-3}$, it oscillates between 1.0 and 0.88 with a period of 830 turns. Thus, for the Collider the linear decoherence due to chromaticity varies over a range of only about 10% and so does not significantly affect the feedback requirements.

However, the perfect re-coherence of the linear case is spoiled by the non-linear variation of the synchrotron tune with synchrotron amplitude. A particle with synchrotron phase amplitude ϕ_s has a synchrotron tune given approximately by

$$\nu_s(\phi_s) = \nu_{s0}\left(1 - \frac{\phi_s^2}{16}\right), \tag{17}$$

where ν_{s0} is the synchrotron tune at zero synchrotron amplitude. For a Gaussian distribution in synchrotron phase space, the rms synchrotron tune spread is $2\nu_{s0}\left(\frac{\sigma_\phi^2}{16}\right)$. For the Collider at injection, where $\sigma_\phi = 0.41$ rad, this spread in synchrotron tune is 4.7×10^{-5}

(synchrotron oscillations per revolution), giving a mixing time in synchrotron phase space of about 2.2×10^4 turns (6.3 s). This mixing in synchrotron phase space damps out the oscillations in the linear betatron decoherence process so that the decoherence factor settles down to its average value. At 20 TeV this non-linear synchrotron mixing time is about 4.6×10^4 turns (13.4 s). Thus, in the Collider the non-linear synchrotron tune is not a significant factor in the decoherence process produced by chromaticity effects.

2.5 Decoherence Due to Non-Linear Magnetic Fields

In the Collider the betatron tune varies with amplitude because of non-linear fields in the magnet system. Simulations have shown that for typical magnetic error distributions in the lattice, the horizontal tune is well represented by

$$\nu_x = \nu_0 - \mu x^2, \tag{18}$$

where ν_0 is the tune at zero betatron amplitude, x is the betatron amplitude at a focusing quadrupole in the arcs, and μ is about 1.4×10^{-4} mm^{-2}.[6] The elementary smearing action in phase space is that a radial locus of particles transforms monotonically into a spiral.

For a Gaussian betatron distribution Meller *et al.*[5] derived a decoherence factor, which for small coherent oscillations is:

$$A(N) = \left(1 + (4\pi\mu\sigma_x^2 N)^2\right)^{-1}, \tag{19}$$

where σ_x is the rms betatron width.

Note that the number of turns for the decoherence factor to equal 0.5 is $(4\pi\mu\sigma_x^2)^{-1}$. This is consistent with the expectation that the characteristic decoherence time τ_d is equal to the product of the revolution time T_0 and the inverse of the rms tune spread σ_ν, which is approximately equal to $2\mu\sigma_x^2$:

$$\tau_d = \frac{T_0}{\sigma_\nu} \tag{20}$$

$$\sigma_\nu = 2\mu\sigma_x^2. \tag{21}$$

For the Collider at 2 TeV, $(2\mu\sigma_x^2)^{-1}$ is 1.8×10^4 turns (5.1 s), and at 20 TeV, 1.8×10^5 turns (51 s).

3.0 ANALYSIS OF CONTINUOUS EMITTANCE GROWTH (BEAM HEATING)

Many external disturbances (ground motion, helium flow, air flow, cooling water flow, power supply ripples, *etc.*) may cause coherent betatron motion of the beam, which, in turn, would lead to transverse emittance growth. The growth time is usually long (in the order of hours) if these disturbances are kept under a certain level. The beam oscillation amplitude is small and the frequency low. A narrow-band, low-power feedback system is considered to control the coherent betatron oscillation to an allowable level. (Details will be discussed in the following sections.)

3.1 Continuous Emittance Growth in the Absence of Feedback

Consider the beam in a storage ring in which there is a continuous, small emittance growth ($\dot{\epsilon}_0$) due to small and random dynamic disturbances, such as quadrupole motion or power-supply jitter. These disturbances continually produce small-amplitude collective betatron oscillations that continually smear out through decoherence and so transform into emittance growth. Let the average incremental collective amplitude be a, referred to some convenient point such as a focusing quadrupole magnet in the FODO lattice of the arcs. The corresponding incremental increase in beam emittance $\delta\epsilon_0$ is (without feedback)

$$\delta\epsilon_0 = \frac{a^2}{2\beta}, \tag{22}$$

where β is the value of the betatron function at the reference point. For an average random disturbance rate $\dot{n}$, the average rate of emittance growth $\dot{\epsilon}_0$ is

$$\dot{\epsilon}_0 = \dot{n}\ \delta\epsilon_0 = \dot{n}\ \frac{a^2}{2\beta}. \tag{23}$$

Because of the random character of the disturbances, the elemental collective oscillations and their decoherence simply superpose without mutual interference. Note that in terms of these parameters, the characteristic decoherence time τ_d does *not* affect the outcome (if feedback is not applied).

This emittance growth can be expressed also in terms of an *average* collective amplitude x_{av} and the decoherence time. Since the elementary amplitudes are random, the average collective amplitude is

$$x_{av}^2 \equiv \dot{n} a^2 \tau_d, \tag{24}$$

so that the total emittance growth can be expressed also as

$$\dot{\epsilon}_0 = \frac{x_{av}^2}{2\beta} \cdot \frac{1}{\tau_d}. \tag{25}$$

3.2 Decoherence Due to Beam-Beam Interaction

A beam bunch undergoing a coherent betatron oscillation tends to smear out in betatron phase space and thus increase its emittance. The smearing action is due to the spread in betatron tune, which causes different particles to rotate at different rates so that the coherent motion decoheres.

In the Collider there are three basic sources of betatron tune spread: (1) chromaticity and momentum spread, (2) non-linear betatron motion due to non-linear magnetic fields, and (3) non-linear betatron motion due to beam-beam forces. The first two sources have been studied in the Sections 2.4 and 2.5, respectively. The third one, namely, the beam-beam tune shift, is qualitatively similar to the non-linear betatron tune shift discussed in the preceding section in that the betatron tune shift falls non-linearly with betatron amplitude over the principal part of the distribution.

There are two types of beam-beam tune shifts—the head-on and the long-range. We will consider the head-on type only in this paper. The total head-on beam-beam tune shift per collision point is

$$\xi = \frac{N_B r_p}{4\pi\epsilon_N}, \tag{26}$$

where N_B is the number of protons per bunch, r_p the classical proton radius, and ϵ_N the normalized rms beam emittance. Equation (26) gives the tune shift at zero amplitude. The dependence of the tune shift on the betatron amplitude a is[7]

$$\begin{aligned}\frac{\Delta\nu}{\xi} &= \int_0^1 du\, e^{-\alpha u}\left(I_0(\alpha u) - I_1(\alpha u)\right) \\ &= \frac{1}{\alpha}\left(1 - e^{-\alpha}\, I_0(\alpha)\right),\end{aligned} \tag{27}$$

in which I_0 and I_1 are the modified Bessel functions, and α the normalized amplitude:

$$\alpha = \frac{a^2}{4\sigma^2}. \tag{28}$$

The density distribution of a Gaussian bunch is

$$\rho(a) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{a^2}{2\sigma^2}}. \tag{29}$$

The rms tune spread of this Gaussian bunch can be obtained from the integral

$$\left(\frac{\sigma_\nu}{\xi}\right)^2 = \int_{-\infty}^{\infty} \left(\frac{\Delta\nu}{\xi}\right)^2 \rho(a)\, da. \tag{30}$$

A numerical integration has been carried out by Lebedev *et al.*[4] and the result is

$$\sigma_\nu \approx 0.2\,\xi \tag{31}$$

for each collision point. For the Collider at its nominal intensity (0.8×10^{10} protons per bunch), nominal normalized emittance (1 mm-mrad), 20-TeV beam energy, and with four collision points in operation, the total head-on beam-beam tune shift is 3.8×10^{-3}. The corresponding rms tune spread is 7.6×10^{-4}, giving a typical decoherence time of 1.3×10^3 turns (0.38 s).

Thus, we see that at 20 TeV, the tune spread due to the beam-beam effect dominates the decoherence process and determines the gain required of the feedback system.

3.3 Continuous Emittance Growth in the Presence of Feedback

Next consider the effects of applying feedback to damp the collective oscillation before decoherence has progressed very far. If on each turn the feedback system reduces the angle of the collective oscillation at the feedback kicker by the fraction g (g is the "gain", $<< 1$), the collective amplitude decreases approximately as

$$x = x_0 \exp\left(-\frac{gN}{2}\right). \tag{32}$$

Thus, the characteristic feedback damping period N_f is

$$N_f = \frac{2}{g} \text{ turns}, \tag{33}$$

and the corresponding characteristic feedback damping time τ_f is

$$\tau_f = N_f\, T_0 = \frac{2}{g}\, T_0, \tag{34}$$

where T_0 is the revolution time.

If the damping time τ_f is considerably smaller than the decoherence time τ_d, the amount of smearing, and thus the amount of emittance growth per elementary disturbance, is reduced. To estimate the amount of this reduction, we will repeatedly use two results from the paper of Meller *et al.*[5] One is Eq. (19) in Section 2.5, which gives the decoherence factor in a non-linear magnetic field. For small N it can be represented by

$$A(N) \approx 1 - (4\pi\mu\sigma_x^2 N)^2, \tag{35}$$

where σ_x is the rms beam width. The other Meller result is Eqs. (14) and (16) in Section 2.4, which give the association of the decoherence factor with the characteristic mean-

square phase spread σ_ϕ at turn N due to chromaticity decoherence. For small N, the mean-square phase spread is approximately

$$\begin{aligned}\sigma_\phi &\approx 2\pi\zeta\frac{\sigma_p}{p}N \\ &= 2\pi\ \sigma_\nu N,\end{aligned} \tag{36}$$

and the decoherence factor is approximately

$$\begin{aligned}A(N) &\approx 1-\sigma_\phi^2(N)/2 \\ &\approx 1-\left(2\pi\zeta\frac{\sigma_p}{p}N\right)^2/2,\end{aligned} \tag{37}$$

Thus, for small N and to first order, Eqs. (35) and (37) have the same dependence on N. Therefore, it is reasonable to generalize slightly and relate the mean-square decoherence phase spread σ_ϕ to the rms tune spread σ_ν generally. In other words, for the case of tune spread due to a non-linear betatron tune, we can also estimate the rms betatron phase spread similar to Eq. (36):

$$\begin{aligned}\sigma_\phi(N) &= \sqrt{2}\,4\pi\mu\sigma_x^2 N \\ &= \sqrt{2}\,2\pi\sigma_\nu N,\end{aligned} \tag{38}$$

in which σ_ν is the rms tune spread expressed in Eq. (21). There is a $\sqrt{2}$ factor difference between Eqs. (36) and (38), which is considered not important in this rough estimation. This rms spread in betatron phase is a measure of how far the decoherence process has evolved. Decoherence is practically complete when the typical spread is $\pm\pi$.

The principal question at this point is to determine the increase in the emittance for a given rms spread in betatron phase. Consider a one-dimensional analogy: Suppose that in an initial Gaussian distribution $\rho(x) = A\exp(-x^2/2\sigma_0^2)$, each point at x is spread into another Gaussian distribution $B\exp(-(x-x_1)^2/2\sigma_1^2)$. The resulting new distribution is found by folding the spreading distribution over the initial distribution:

$$\begin{aligned}\rho(x_1) &= B\int_{-\infty}^{\infty}\rho(x)\exp\left(-(x-x_1)^2/2\sigma_1^2\right)dx \\ &= C\exp\left(-x_1^2/2(\sigma_0^2+\sigma_1^2)\right),\end{aligned} \tag{39}$$

C being the normalizing constant involved. Thus, due to the spreading process, a new Gaussian distribution is formed with a mean-square width equal to the *sum* of the initial

and spreading widths. In our two-dimensional beam distribution, it is only the phase coordinate that is being spread in the decoherence process. Therefore, we can estimate that the incremental emittance growth due to the incremental oscillation amplitude a when feedback is applied is, for a perfect (noise-free) system,

$$\begin{aligned}
\delta\epsilon_f &= \delta\epsilon_0 \left(\sigma_\phi(N_f)/\pi\right)^2 \\
&= (a^2/2\beta)\,(2\sqrt{2}\sigma_\nu N_f)^2 \\
&= (a^2/2\beta)\left(2\sqrt{2}\sigma_\nu \frac{2}{g}\right)^2 \\
&= \frac{16\,a^2}{\beta}\left(\frac{T_0}{\tau_d\,g}\right)^2 \\
&= \frac{4\,a^2}{\beta}\left(\frac{\tau_f}{\tau_d}\right)^2,
\end{aligned} \tag{40}$$

in which $N_f = 2/g$ is the feedback damping period, $\tau_f = 2\,T_0/g$ the damping time, and $\tau_d = T_0/\sigma_\nu$ the decoherence time. The total emittance growth rate with feedback is

$$\begin{aligned}
\dot{\epsilon}_f &= \dot{n}\,\delta\epsilon_f \\
&= \dot{n}\,\frac{4\,a^2}{\beta}\left(\frac{\tau_f}{\tau_d}\right)^2 \\
&= \dot{\epsilon}_0 \cdot \left(\frac{2\sqrt{2}\tau_f}{\tau_d}\right)^2 \\
&= \dot{\epsilon}_0 \cdot \left(\frac{4\sqrt{2}\sigma_\nu}{g}\right)^2,
\end{aligned} \tag{41}$$

in which $\dot{\epsilon}_0$ is defined in Eq. (23). Therefore, if the feedback gain is big enough such that

$$g > 4\sqrt{2}\sigma_\nu, \tag{42}$$

then we will have

$$\dot{\epsilon}_f < \dot{\epsilon}_0, \tag{43}$$

i.e., the feedback will reduce the emittance growth rate.

3.4 Noises in the Feedback System

If there is noise in the feedback system equivalent to a beam amplitude x_N, then in addition to to collective-amplitude contributions $\dot{n}a^2$ due to machine disturbances, there

is the contribution $f_0(gx_N)^2$ due to the noise in the feedback system, where f_0 is the revolution frequency. Thus, Eq. (41) has to be modified and takes the form

$$\dot{\epsilon}_f = \left(\dot{\epsilon}_0 + \frac{f_0 g^2 x_N^2}{2\beta}\right) \cdot \left(\frac{4\sqrt{2}\sigma_\nu}{g}\right)^2. \tag{44}$$

Letting ϵ_0 be the initial emittance, then one may also define the emittance growth rate as

$$\begin{aligned} \frac{1}{\tau_\epsilon} &\equiv \frac{\dot{\epsilon}_f}{\epsilon_0} \\ &\equiv \frac{1}{\tau_{ext}} + \frac{1}{\tau_{noise}}, \end{aligned} \tag{45}$$

in which

$$\frac{1}{\tau_{ext}} = \frac{\dot{\epsilon}_0}{\epsilon_0} \cdot \left(\frac{4\sqrt{2}\sigma_\nu}{g}\right)^2 \tag{46}$$

is the emittance growth rate due to external sources, and

$$\begin{aligned} \frac{1}{\tau_{noise}} &= \frac{1}{\epsilon_0} \cdot \frac{f_0 g^2 x_N^2}{2\beta} \cdot \left(\frac{4\sqrt{2}\sigma_\nu}{g}\right)^2 \\ &= 16\, f_0 \left(\frac{x_N}{\sigma_x}\right)^2 \cdot \sigma_\nu^2 \\ &= 0.64\, f_0 \left(\frac{x_N}{\sigma_x}\right)^2 \cdot \Delta\nu^2, \end{aligned} \tag{47}$$

where we have converted the rms tune spread σ_ν to the total tune shift $\Delta\nu$ (which equals ξ times the total number of interaction points) using Eq. (31). It is interesting to note that the emittance growth rate due to the noises in the feedback system is independent of the gain. It is also interesting to compare this result with that presented in Reference 8, which was obtained by Boussard using the well established stochastic cooling theory. Eq. (47) above differs from Eq. (8) in Reference 8 only by approximately a factor of 2.

4.0 DAMPING OF RESISTIVE WALL INSTABILITY

The wall resistivity of the beam tube may cause beam instability, which is usually of long correlation lengths and, therefore, shows no discrimination against the bunched beams from the coasting beams. The growth rate is

$$\frac{1}{\tau_w} = \frac{N_{tot}\, c\, r_p}{2\pi\gamma\, \nu_\beta} \frac{Re\, Z_\perp}{RZ_0}$$

$$= \frac{N_{tot}\, c\, r_p}{2\pi\gamma\, \nu_\beta} \frac{1}{b^3} \delta_s, \tag{48}$$

where τ_w is the growth time, N_{tot} the total number of particles, c the velocity of light, r_p the classical radius of proton, γ the relative energy of the particle, R the average machine radius, ν_β the betatron tune, Z_0 the vacuum impedance, δ_s the skin depth, b the beam tube radius, and $Re\, Z_\perp$ the real part of the transverse impedance of the wall, which is equal to

$$Z_\perp = (1+i)\, \frac{Z_0\, R\, \delta_s}{b^3}. \tag{49}$$

The skin depth is a function of the wall conductivity σ_e and frequency ω:

$$\delta_s = \sqrt{\frac{2}{\mu\sigma_e\omega}}. \tag{50}$$

It is seen that as the frequency is decreased, δ_s is increased as $\omega^{-1/2}$. So is the growth rate $1/\tau_w$. The frequency range of interest is $\omega = (n - \nu_\beta)\omega_0$, in which n is an integer and $\omega_0 = c/R$ is the angular revolution frequency, which equals $2\pi \times 3.441$ kHz for the Collider. During machine operation, the fractional part of ν_β would be scanned in order to find the optimal working point. Therefore, the lowest frequency is taken to be $0.1\omega_0$ in the estimates of the fastest growth time.

In the present design, the beam tube of the Collider is made of stainless steel that is coated in its inner surface with a thin copper layer in order to have low electrical resistivity. In this case, the thickness of the copper layer Δ is smaller than the skin depth δ_s in the frequency range mentioned above, and Eq. (48) needs to be modified.[9] With some approximations, the modified equation can be written as

$$\frac{1}{\tau_w} = \frac{N_{tot}\, c\, r_p}{2\pi\gamma\, \nu_\beta} \frac{1}{b^3} \frac{\delta_s^2}{\Delta}. \tag{51}$$

Plugging in Eq. (50) and expressing the instability in terms of the growth time, one has

$$\tau_w = \left(\frac{2\pi\gamma\, \nu_\beta\, b^3}{N_{tot}\, c\, r_p} \frac{\mu\omega}{2} \right) \sigma_e \Delta. \tag{52}$$

In other words, the growth time is proportional to the product of $\sigma_e\Delta$. The specifications of the copper layer is

$$\sigma_e\Delta \geq 2 \times 10^5 \quad 1/\Omega \quad , \tag{53}$$

in which σ_e is the conductivity of the copper layer at 4 K and 0.66 T. The nominal values are $\sigma_e = 2 \times 10^9\ \Omega^{-1}\mathrm{m}^{-1}$ (corresponding to RRR = 30), $\Delta = 0.1$ mm. This would give a

resistive wall instability growth time of 34 ms, or 117 turns during the approximately 1-h injection period.[10] At full energy (20 TeV), the growth rate would be slower due to the beam rigidity. In addition, the large betatron tune spread resulting from the beam-beam collisions may provide significant Landau damping.

This type of instability appears to be the rigid dipole mode oscillation and can be effectively damped by means of a transverse damper. The oscillation amplitude is small (a fraction of the beam size, which is 0.38 mm, rms, at injection). This means the power requirement on the damper is small. The instability frequency is very low, ranging from several hundred Hz to tens of kHz. The bandwidth requirement on the feedback will mainly be determined by the batch spacing (1.7 μs). The damper must be fast enough in order to act on each individual batch. This implies a bandwidth of about 500 kHz.

5.0 DAMPING OF COUPLED BUNCH INSTABILITIES DRIVEN BY HOM

The original candidate of the Collider rf cavity is the PEP 5-cell cavity.[2] The calculation shows that the HOM of this type of cavity would cause multiple bunch instabilities, both longitudinal and transverse. The fastest growth time would be in the order of one to several seconds, as listed in Table 3. The instability frequency could be as high as 30 MHz, which is one half of the bunch repetition frequency. The amplitude is small (a fraction of the beam size).

This kind of instability may be avoided if the single-cell type rf cavity is to be used.[11] The experiences on other machines have demonstrated that it is feasible to damp the HOM by a factor of 10 to 100 in a single-cell cavity. Thus the instability would be largely suppressed, and the need of a wide-bandwidth feedback system could be eliminated. However, because the final design of the rf cavities of the Collider has yet to be done, it is prudent to include the discussion of a wide-band feedback system that aims at damping the instabilities driven by the HOM.

Table 3. Coupled Bunch Instability Growth Time.

	Bunch Mode	At 2 TeV	At 20 TeV
Longitudinal	$m = 1$	0.9 s	5.0 s
	$m = 2$	2.2 s	16 s
Transverse	$m = 0$	8.6 s	91 s
	$m = 1$	16 s	150 s

6.0 DESCRIPTION AND SPECIFICATIONS OF THE SSC FEEDBACK SYSTEMS

6.1 Requirements of the Feedback Systems

Table 4 summarizes the requirements of the transverse dampers for the four different cases as analyzed above.

Table 4. Requirements of the Feedback Systems.

Case	Power	Bandwidth	Gain	Noise
(1) Injection Errors	Large	Small	Large	(not critical)
(2) Resistive Wall Instability	Small	Small	Large	Low
(3) Instability Driven by HOM	Small	Large	Small	Low
(4) Emittance Growth Control	Small	Small	Small	Low

6.2 Types of the Feedback Systems

In the SCDR,[2] it is suggested to use two feedback systems in each transverse plane. One system, of high power and narrow bandwidth, would be used during the injection for correcting the injection errors and for damping the resistive wall instability. Another, of low power and wide bandwidth, would be used to cure the instabilities driven by the HOM.

However, a third system may be required if the noise in the feedback were indeed to become a problem and lead to an untolerable emittance growth. This system will have low power and narrow bandwidth for control of (2) and (4) as listed in Table 4. Because the thermal noise power is proportional to the bandwidth, a narrow-band system can significantly lower its noise level.

6.3 Components of the Feedback Systems

In each transverse plane, the feedback systems consist of two pickups, three signal processors, and two kickers. The two pickups are shared by all three feedback systems as discussed above. Each feedback system has its own signal processor. One of the kickers is of high power, while another is of low power.

The advantage of the 2-pickup scheme is that its performance is independent of the betatron tune of the machine. The signals from the two pickups give both x and x' (or y and y'), from which the position of the bunch center at the kicker can be obtained.

The pickup signal contains both the coherent betatron oscillation (which is needed) and the closed orbit signal. The latter is harmful, in particular, if a digital processor is to be used because the quantized noise problem would greatly limit the resolution of the pickups (see Section 6.7.3). One way to get rid of the closed orbit signal is to take the signals from two successive turns and subtract one from the other before the pre-amplifier.

6.4 Algorithm of the 2-Pickup, 2-Turn Scheme

Using two pickups and their readings on two consecutive turns, one can extract the closed-orbit amplitudes at the two pickups and the amplitude and phase of the collective oscillation in the beam. Let x_1 and x_2 be the two readings on turn 1, and x_3 and x_4 the readings on turn 2. If c_1 and c_2 are the amplitudes of the closed orbits at the two monitors, and ϵ the invariant emittance of the collective oscillation, the readings can be expressed as

$$\begin{aligned} x_1 &= c_1 + \sqrt{\beta_1 \epsilon} \cos \phi_1 \\ x_2 &= c_2 + \sqrt{\beta_2 \epsilon} \cos (\phi_1 + \delta_{12}) \\ x_3 &= c_1 + \sqrt{\beta_1 \epsilon} \cos (\phi_1 + 2\pi\nu) \\ x_4 &= c_2 + \sqrt{\beta_2 \epsilon} \cos (\phi_1 + 2\pi\nu + \delta_{12}), \end{aligned} \tag{54}$$

where β_1 and β_2 are the beta functions, ϕ_1 is the phase of the oscillation at the first pickup on turn 1, ν is the betatron tune, and δ_{12} is the phase difference between the two monitors. Eliminate c_1 and c_2 by forming

$$\begin{aligned} (x_3 - x_1)/\sqrt{\beta_1 \epsilon} &= \cos (\phi_1 + 2\pi\nu) - \cos \phi_1 \\ (x_4 - x_2)/\sqrt{\beta_2 \epsilon} &= \cos (\phi_1 + 2\pi\nu + \delta_{12}) - \cos (\phi_1 + \delta_{12}). \end{aligned} \tag{55}$$

Eliminate ϵ by forming

$$\frac{x_4 - x_2}{x_3 - x_1} \sqrt{\frac{\beta_1}{\beta_2}} = \frac{\cos (\phi_1 + 2\pi\nu + \delta_{12}) - \cos (\phi_1 + \delta_{12})}{\cos (\phi_1 + 2\pi\nu) - \cos \phi_1}. \tag{56}$$

Equation (56) can be solved for ϕ_1. Then Eq. (55) can be solved for ϵ. Then solve Eqs. (54) for c_1 and c_2.

This analysis has ignored the small effects of the feedback kicker between turns 1 and 2. These effects can easily be included by including the drop in ϵ and the change in phase produced by the kick:

$$\Delta\epsilon = -g \sin^2 \phi_k, \tag{57}$$

in which

$$\phi_k = \phi_1 + \delta_{1k} \tag{58}$$

is the phase at the kicker on turn 1, δ_{1k} being the phase difference between the first pickup and the kicker.

The corresponding change in phase due to the kicker is

$$\Delta\phi = -(g/2)\sin^2\phi_k. \tag{59}$$

These changes $\Delta\epsilon$ and $\Delta\phi$ can be included in the second-turn equations (54), and the analysis proceeds as indicated above. In most cases of interest, the gain g is relatively small so that these changes are not important.

6.5 Locations of the Feedback Systems

There are several possible locations for the feedback systems in the Collider—the west or east utility region, the missing dipole locations in the arcs, and the IR regions. At this moment, the feedback systems are planned to be installed in the west utility region, where the injection kickers, abort kickers, Lambertson magnets, rf cavities, *etc.*, are located. This makes the maintenance easier. (At the moment, the east utility region is not used.) The proposed locations of the two pickups and two kickers are at the points where the β-functions have relatively high values. The pickups should avoid the dispersive region. Otherwise the beam-loading-induced coherent synchrotron oscillation may cause coherent betatron motion through dispersion. A rough estimate shows that the amplitude of this kind of betatron oscillation can be at the level of several tenths of a micron, which is comparable to the required pickup resolution (see Section 6.7.2).

6.6 Power, Bandwidth, and Gain

The requirements of the power, bandwidth, and gain of the three feedback systems are listed in Table 5. The gain g and damping time $2/g$ are defined in Sections 2.3 and 3.3. X_p is the maximum betatron oscillation amplitude at the pickup, which determines the kick angle:

$$\theta = \frac{g \cdot X_p}{\sqrt{\beta_p \beta_k}}, \tag{60}$$

where β_p and β_k are the β-function at the pickup and kicker, respectively. The kick angle then determines the kicker voltage V and the kicker length L:

$$V \cdot L = \frac{\theta \cdot (\frac{E}{e}) \cdot d}{4}, \tag{61}$$

where E is the energy of the particles and d the separation of the two kicker electrodes (taken to be 3 cm). The voltage on the two electrodes is +V and −V, respectively. The power is calculated by assuming the kicker to be a 50-Ω transmission line.

Table 5. General Specifications of the SSC Transverse Dampers.

	Damper A	Damper B	Damper C
Purpose	Injection errors	Resistive wall; Emittance control	Cavity driven insta.
Gain g	0.04	0.1	0.02
Damping time $2/g$	50 turns	20 turns	100 turns
X_p	±2 mm	±100 μm	±40 μm
Kick angle θ	0.27 μrad	0.04 μrad	0.003 μrad
Kicker length L	4 m	4 m	4 m
Kicker voltage V	1 kV	150 V	150 V
Kicker power P_k	40 kW	0.9 kW	0.9 kW
Bandwidth Δf	500 kHz	500 kHz	$\geq$ 30 MHz

6.7 Noise Level Requirements

6.7.1 Allowable Coherent Betatron Oscillation Amplitude

Assume that the allowable emittance growth rate $\dot{\epsilon}_0$ is specified, then the allowable coherent beam oscillation amplitude x_{av} can be derived from Eq. (25) in the case where no feedback system is employed. For the SSC, the initial rms emittance at 20 TeV is

$$\begin{aligned} \epsilon_0 &= \epsilon_N/\gamma \\ &= 4.7 \times 10^{-11} \ \text{m-rad}, \end{aligned} \tag{62}$$

where ϵ_N is 1 mm-mrad and γ is the relativistic factor. Assume the allowable emittance-doubling time is 24 h, then the allowable average emittance growth rate is

$$\begin{aligned} \dot{\epsilon}_0 &= \epsilon_0 \ / \ (24 \ \text{h}) \\ &= 5.4 \times 10^{-16} \ \text{m-rad/s}. \end{aligned} \tag{63}$$

The total beam-beam tune shift for four interaction points is

$$\begin{aligned} \Delta\nu &= 4 \, \xi \\ &= 0.0038. \end{aligned} \tag{64}$$

The rms tune spread is

$$\sigma_\nu = 0.2 \, \Delta\nu$$

$$= 0.00076. \tag{65}$$

The decoherence time is

$$\begin{aligned} \tau_d &= T_0/\sigma_\nu \\ &= 0.38 \ \text{s}, \end{aligned} \tag{66}$$

in which the revolution time T_0 is 290 μs. Thus, from Eq. (25) we get the allowable coherent oscillation amplitude:

$$\begin{aligned} x_{av} &= (\dot{\epsilon}_0 \times 2\beta \times \tau_d)^{1/2} \\ &= 0.29 \ \mu\text{m}, \end{aligned} \tag{67}$$

in which the average beta-function is 200 m.

6.7.2 Required Pickup Resolution of the Feedback System

In the presence of a feedback system, the emittance growth rate will be reduced by a factor of $(4\sqrt{2}\sigma_\nu/g)^2$ as indicated by Eq. (41). But any feedback system will inevitably contain certain level of noises that will "heat" the beam up, *i.e.*, blow up the emittance. In order to keep the emittance growth under control, one needs to specify the allowable noise level of the feedback system or, equivalently, to specify the pickup resolution.

Again, assuming the allowable emittance growth rate is given as before, let us estimate the required pickup resolution by using Eq. (44).

(1) Assume no external excitations on the beam, *i.e.*, $\dot{\epsilon}_0 = 0$:
In this case, the emittance growth is solely due to the noises in the feedback system. Eq. (44) becomes

$$\begin{aligned} \dot{\epsilon}_f &= \frac{f_0 g^2 x_N^2}{2\beta} \cdot \left(\frac{4\sqrt{2}\sigma_\nu}{g}\right)^2 \\ &= 16 \ f_0 \frac{x_N^2}{\beta} \cdot \sigma_\nu^2, \end{aligned} \tag{68}$$

in which the revolution frequency f_0 is 3.441 kHz, and β at the pickup is assumed to be 300 m. Solving Eq. (68) for x_N, we get

$$x_N = 2.4 \ \mu\text{m}. \tag{69}$$

It is independent of the gain of the feedback system.

(2) Now assume there are external excitations that give an emittance doubling time of 0.1 h, *i.e.*, $\dot{\epsilon}_0 = \epsilon_0$ / (0.1 h):
Letting the gain $g = 0.1$ and solving Eq. (44) for x_N, we obtain

$$x_N = 1.8 \ \mu\text{m}. \tag{70}$$

6.7.3 Theoretical Limit of the Pickup Resolution

The pickup resolution is limited by the thermal noises and electronic noises. The thermal noises are determined by the product of the temperature and bandwidth. The electronic noise of an amplifier is described by a noise factor (NF).

Assuming the amplifiers have a temperature T (°K), a bandwidth Δf (Hz), and a noise factor NF (dB), then the noise power when expressed at the pickup is given by

$$P_N \text{ (W)} = k_B T \times \Delta f \times 10^{NF/10} \quad , \tag{71}$$

in which k_B is the Boltzmann constant. Sometimes the unit dBm is used. The conversion is as follows:

$$P_N \ (dBm) = 10 \log_{10} P_N \text{ (mW)} \quad . \tag{72}$$

The signal power of a pair of pickup electrodes is

$$P_S = \frac{1}{4} I^2 \left(\frac{x}{b}\right)^2 Z \sin^2\left(\frac{\omega \ell}{c}\right) F \quad , \tag{73}$$

where x is the beam displacement from the axis, b the half distance between the two electrodes, Z the characteristic impedance of the pickup, ℓ the length of the electrodes, c the velocity of light, and F the geometrical factor (which is about equal to 1 for an electrode subtended angle of 70°). If the amplifiers have an infinitely wide bandwidth, then I in Eq. (73) should be the peak current. But in a real feedback system, the bandwidth is always finite. Therefore, the value of I is approximated by the average current I_{ave} in our estimation.

By combining Eqs. (71) and (73), the theoretical limit of the pickup resolution can be obtained from

$$\Delta x = \frac{2b}{I_{ave} \sin(\frac{\omega \ell}{c})} \sqrt{\frac{k_B T \times \Delta f \times 10^{NF/10}}{Z}} \quad . \tag{74}$$

This value must be smaller than x_N given by Eq. (69) or (70) in order to avoid the emittance growth problem.

The noise level calculated in the previous sections for the SSC is summarized in Table 6.

Table 6. Specifications of Noise Level of the Feedback Systems.

Initial Emittance ϵ_0	4.7×10^{-11} m-rad
Emittance doubling time	24 h
Average emittance growth rate $\dot{\epsilon}$	5.4×10^{-16} m-rad/s
Beam-beam tune shift $\Delta\nu$	0.0038
rms tune spread σ_ν	0.00076
Decoherence time τ	0.38 s
Allowed coher oscil amp (w/o feedback) x_{av}	0.29 μm
Damping time $2/g$	20 turns
Allowed noise level x_N	2.4 μm ($\dot{\epsilon}_{ext} = 0$)
	1.8 μm ($\dot{\epsilon}_{ext} = \epsilon_0/0.1$ h)
Noise power P_N	-82 dBm (bandwidth 30 MHz)
(NF = 17 dB)	(6.2×10^{-12} W)
	-100 dBm (bandwidth 500 kHz)
	(1.0×10^{-13} W)
Pickup resolution limit Δx	0.16 μm (30 MHz, $\ell = 1.25$ m, $I = 72$ mA)
	0.02 μm (500 kHz, $\ell = 1.25$ m, $I = 72$ mA)

There are several other error sources that are not included in the above analysis but may also put a limit on the pickup resolution. These include the least significant bit (LSB) error if a digital system is used, the mechanical vibration of the pickup, the noises from the kicker power amplifier and the cable (which are usually not included in the given noise factor), *etc.*

When a digital system is used, the LSB error may become significant. As an example, the Tevatron Super-damper utilizes an 8-bit digital system for signal processing. The full scale is about 5 mm, which is determined by the residual orbit error.[12] Therefore, the maximum LSB error is

$$\Delta x_{LSB} = \frac{5 \text{ mm}}{256} \approx 20 \ \mu\text{m} \quad . \tag{75}$$

It is much larger than the theoretical limit Δx and can result in significant emittance growth. In this case, it is necessary to reduce the orbit error or to compensate the orbit error signal in order to improve the pickup resolution.

7.0 CONCLUSIONS

It is important to understand the underlying physics in designing feedback systems. In this paper, special attention is given to the analysis of the beam emittance growth and coupled bunch instabilities. The instabilities can be driven either by the wall impedance (which has a fast growth rate and a relatively narrow bandwidth) or by the HOM of the rf cavities (which usually grows slowly while crossing a broad bandwidth). The emittance growth can be caused either by the injection errors (a one-time event) or by continuous external excitations (a heating process). In the injection case, the decoherence time is determined by the chromaticity and by the nonlinear parts of the magnetic fields. In the collision case, it is determined by the beam-beam tune spread. The emittance growth time (or growth rate) is studied for both, with and without feedback systems.

Based on the above analysis, the requirements of the power, bandwidth, gain, and noise level of the feedback systems are specified. Although preliminary, these specifications will serve as useful references to guide the engineering designs of these systems.

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