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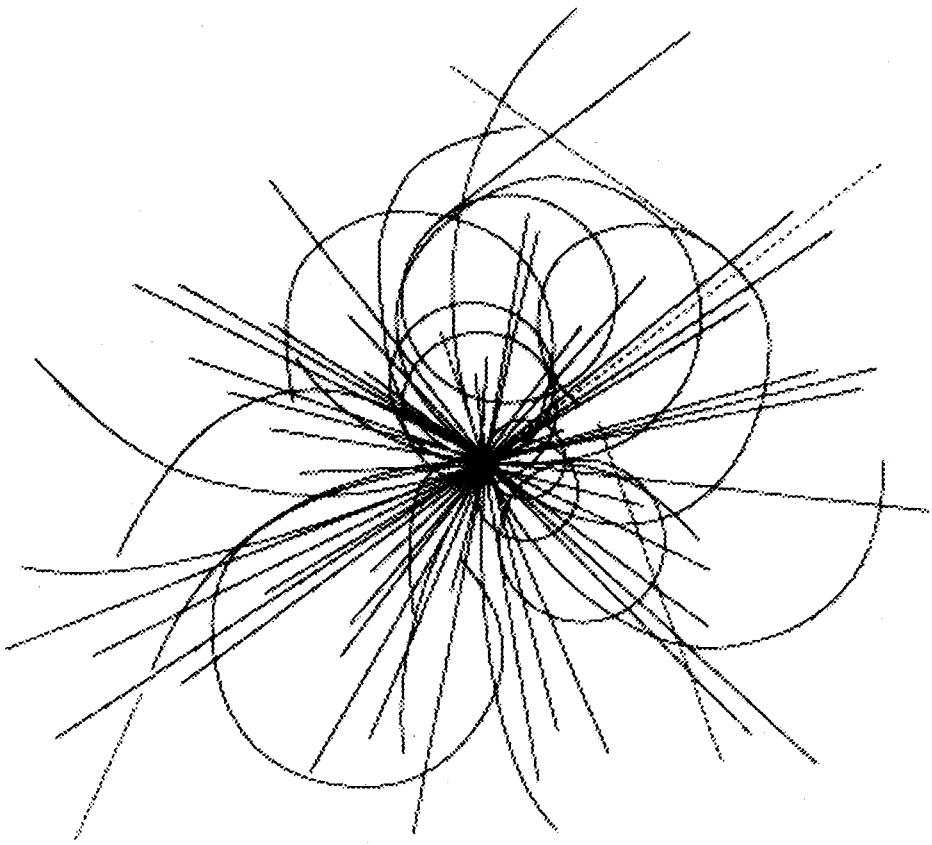

Superconducting Super Collider Laboratory

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Use of Large Dispersion to Increase the Space Charge Limit in the LEB*

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* Work performed in the Project Management Division of the Superconducting Super Collider Laboratory during a visit as Guest Scientist from April 27 to May 8, 1992.

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Abstract

The utilization of the horizontal spreading of the charge distribution by large dispersion is investigated as a means to increase the phase-space density limit due to the betatron tune shift in the Low Energy Booster (LEB). The general theory of the space-charge tune shift with dispersion is reviewed. A possible lattice design for the LEB is presented that would increase the space-charge limit by a factor of 2.3.

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1.0 INTRODUCTION

The luminosity requirements for the Superconducting Super Collider (SSC) of $\mathcal{L} = 10^{33}$ cm^{-2}s^{-1} and the design scenario that was adopted call for a very tight emittance budget. The desired invariant (normalized) *rms* emittance at the collision point is 1 π mm-mrad. Since the ion source should be capable of producing the 30 mA, 35 kV H^- beam with an emittance of 0.1 π mm-mrad, the emittance is allowed to grow by not more than a factor of 10 through the entire accelerator chain. By comparison, the emittance in the Fermilab accelerator system increases by about a factor of 40 to 50, though the upgrade program currently underway is aimed at reducing this number significantly. Specifically, the design output emittances at each accelerator station in the SSC chain are 0.2 for the Source Radio-Frequency Quadrupole (RFQ) (2.5 MeV), 0.4 for the Linear Accelerator (Linac) and transfer line (600 MeV), 0.6 for the Low Gravity Booster (LEB) (11 GeV), 0.7 for the Medium Energy Booster (MEB) (200 GeV), 0.8 for the High Energy Booster (HEB) (2 TeV) and 1.0 π mm-mrad for the Collider (20 TeV). These requirements are difficult to meet and require a precision in alignment and beam control that must be considerably better than at Fermilab. Furthermore, possibilities of improving expected performance levels must be explored to achieve the desired final emittance.

One of the major bottlenecks is the space-charge tune shift in the LEB. The traditional formula for this effect can be written in the distribution-independent rms form

$$\Delta\nu = -\frac{\bar{I}R}{4I_0\tilde{\epsilon}_n\beta^2\gamma^2B_f} = -\frac{Nr_c}{8\pi\tilde{\epsilon}_n\beta\gamma^2B_f}, \tag{1}$$

where $\bar{I}$ is the average current, $I_0 = 3.1 \times 10^7$ Amperes for protons, R the average machine radius, N the number of particles in the ring, $r_c = 1.535 \times 10^{-18}$ m the classical proton radius, $\tilde{\epsilon}_n$ the normalized rms emittance, $\beta = v/c$ the relative velocity, and $\gamma = (1-\beta^2)^{-1/2}$ the relativistic energy factor. The bunching factor B_f is defined as the ratio of average current $\bar{I}$ to peak current $\hat{I}$, *i.e.*,

$$B_f = \bar{I}/\hat{I} \tag{2}$$

and has the range $0 < B_f \leq 1$, where $B_f = 1$ for the unbunched beam.

The particle distribution at high energy usually has a Gaussian shape where the linear part of the space-charge force near the center is twice as large as the rms force. One, therefore, needs to multiply the right-hand side of Eq. (1) by the factor 2 to obtain the more stringent relation

$$\Delta\nu_{core} = -\frac{N\ r_c}{4\pi\tilde{\epsilon}_n\beta\gamma^2B_f} \tag{3}$$

for the betatron tune shift in the beam core.

In the LEB design lattice the tunes are $\nu_x = 11.85$, $\nu_y = 11.80$, and to avoid the nearest half-integer resonance of 11.5 one must require that the rms tune shift be $|\Delta\nu| < 0.3$. With the LEB parameters near the injection point (600 MeV) of $\gamma = 1.64$, $\beta = 0.79$, $\tilde{\epsilon}_n = 0.4 \times 10^{-6}$ m-rad, and $B_f = 0.3$ one obtains for the intensity threshold

$$N = \frac{8\pi \times 0.4 \times 10^{-6} \times 1.64^2 \times 0.79 \times 0.3 \times 0.3}{1.535 \times 10^{-18}}$$

i.e.,

$$N = 1.25 \times 10^{12}. \tag{4a}$$

The more stringent condition (3) for the beam core reduces this number to

$$N = 0.63 \times 10^{12}. \tag{4b}$$

The desired particle number in the LEB ring for collider operation is $N = 1.1 \times 10^{12}$ which is slightly below the rms limit (4a) but above the core threshold (4b). The 3-D computer simulation run for the ideal LEB lattice with $N = 1.1 \times 10^{12}$ particles shows significant emittance growth in both the vertical and horizontal direction.[1] Unless other effects not yet clearly understood are present or the computer code is not working correctly, one would conclude that the more stringent condition (3) applies. This is to say that emittance growth occurs because the particles in the beam core encounter the 11.5 resonance. One would expect this effect to level off when the emittance $\tilde{\epsilon}_n$, the energy factor $\beta\gamma^2$ and the bunching factor B_f have changed enough so that the particles in the core no longer pass through the resonance. As long as the total emittance growth in the LEB remains within the design limit of 0.6, there should be no problem. However, the ideal lattice of the computer run cannot be realized in practice. There will undoubtedly be additional emittance growth in the actual machine, and computer runs with a nonideal lattice[1] show that this is to be expected. To make the point, suppose that the final emittance of the LEB is 0.8π mm-mrad, *i.e.*, larger than the design value of 0.6, and that this translates into a proportional increase of the final collider emittance. What would be the options to still meet the SSC luminosity requirements?

1. One could inject a higher Linac current into the LEB allowing even some emittance increase until $N/\tilde{\epsilon}_n$ is below the tune-shift threshold. Since the luminosity scales as $\mathcal{L} \propto N^2/\tilde{\epsilon}_n$, there is always a net gain by having both N and $\tilde{\epsilon}_n$ proportionally increased. However, increasing N may not be feasible beyond limits set by synchrotron radiation, beam-beam tune shift, instabilities, *etc.*

2. One could increase the bunch length and hence the bunching factor B_f in the denominator of the tune-shift formula by use of a second-harmonic rf system. The simulation runs show that a factor 1.5 to 2 enlargement of the bunch size decreases the emittance growth by a proportional amount.[1]
3. Increasing the Linac energy from 600 MeV to 1000 MeV (or even higher) which enhances the $\beta\gamma^2$ factor in the tune-shift formula by 1.76. This works as long as the gain in $\beta\gamma^2$ is not offset by emittance growth in the addition to the Linac.

In the remaining part of this note I will discuss as a fourth option, the use of large dispersion to increase the LEB space-charge limit. First, in the next section, I will review the theory, and then, in Section 3, a preliminary test lattice design for LEB by Al Garren will be used to illustrate the effect.

2.0 REVIEW OF THEORY

The proposed method to increase the space-charge limit in circular accelerators by use of large dispersion was discussed in Reference 2. The horizontal spread of the particle distribution due to dispersion reduces the space charge forces and hence the tune shift in each direction. The effect enters as a geometry factor in the denominator of the tune shift relation, and one obtains

$$\Delta\nu_x = -\frac{Nr_c}{8\pi\tilde{\epsilon}_x\beta\gamma^2 B_f g_x}, \tag{5}$$

$$\Delta\nu_y = -\frac{Nr_c}{8\pi\tilde{\epsilon}_n y\beta\gamma^2 B_f g_y}, \tag{6}$$

where

$$g_x = \frac{(1+\Delta_D^2)^{\frac{1}{2}}}{2\Delta_\delta}[1+\Delta_\delta(1+\Delta_D^2)^{\frac{1}{2}}], \tag{7}$$

$$g_y = \frac{1}{2}\left[1+\Delta_\delta(1+\Delta_D^2)^{\frac{1}{2}}\right]. \tag{8}$$

The parameter Δ_D in Eqs. (7) and (8) defines the ratio of the horizontal rms beam width due to dispersion, δ_{xp}, to the rms width due to emittance, δ_{xb}, *i.e.*,

$$\Delta_D = \frac{\delta_{xp}}{\delta_{xb}}. \tag{9}$$

The total horizontal rms width is then

$$\delta_x = (\delta_{xb}^2 + \delta_{xp}^2)^{\frac{1}{2}} = \delta_{xb}(1+\Delta_D^2)^{\frac{1}{2}}, \tag{10}$$

where δ_{xb} is determined by the normalized rms emittance $\tilde{\epsilon}_{nx}$, the machine radius R, and the betatron tune ν_x according to the relation

$$\delta_{xb} = \left(\frac{\tilde{\epsilon}_{nx} R}{\beta\gamma\nu_x}\right)^{\frac{1}{2}}. \tag{11}$$

The other factor, Δ_δ, in Eqs. (7) and (8) enters into the theory when the tunes and/or emittances in the two directions are different, and it is defined as the ratio of the rms beam widths due to emittance

$$\Delta_\delta = \frac{\delta_{xb}}{\delta_{yb}} = \left(\frac{\tilde{\epsilon}_{nx}}{\tilde{\epsilon}_{ny}}\frac{\nu_y}{\nu_x}\right)^{\frac{1}{2}}, \tag{12}$$

with

$$\delta_{yb} = \left(\frac{\tilde{\epsilon}_{ny}\ R}{\beta\gamma\ \nu_y}\right)^{\frac{1}{2}}. \tag{13}$$

It is assumed that there is no dispersion in the y-direction so that $\delta_y = \delta_{yb}$. When the two emittances and tunes are the same, one has $\Delta_\delta = 1$.

The rms width of the beam due to dispersion, δ_{xp}, in Eqs. (9) and (10) is defined as

$$\delta_{xp} = \tilde{D}\,\frac{\widetilde{\Delta p}}{p}, \tag{14}$$

where $\widetilde{\Delta p}/p$ is the rms momentum spread and $\tilde{D} = (\overline{D^2})^{1/2}$ is the rms average dispersion function obtained from the integral over the closed orbit

$$\overline{D^2} = \frac{1}{2\pi R}\oint D^2(s)ds. \tag{15}$$

For a regular smooth FODO lattice with no superperiods the rms average dispersion is largely determined by the horizontal betatron tune and to good approximation one has

$$\tilde{D} = \frac{R}{\nu_x^2} \tag{16}$$

so that

$$\delta_{xp} = \frac{R}{\nu_x^2}\frac{\widetilde{\Delta p}}{p}. \tag{17}$$

Using Eq. (11) for δ_{xb} one thus obtains for the dispersion parameter Δ_D in a smooth lattice the relation

$$\Delta_D = \frac{\delta_{xp}}{\delta_{xb}} = \frac{1}{\nu_x^{3/2}}\left(\frac{\beta\gamma R}{\tilde{\epsilon}_n}\right)^{\frac{1}{2}}\frac{\widetilde{\Delta p}}{p}. \tag{18}$$

This scaling relation shows that one should operate at a low tune and maximize the momentum spread of the injected beam in order to get the desired relief for the space

charge tune shift. It should be pointed out that Eq. (18) does not apply to the lattices with superperiods and large straight sections such as the LEB lattice with three superperiods. However, it still should give the general trend that dispersion increases when the tune ν_x is decreased.

The geometry parameters g_x and g_y that determine the change of the tune shift, $\Delta\nu_x$, $\Delta\nu_y$ are shown in Table 1 for different values of the parameters Δ_D and Δ_δ. The examination of the results in the table and of the equation for g_x and g_y for the case where $\tilde{\epsilon}_{nx} = \tilde{\epsilon}_{ny}$ shows the following:

1. For symmetric focusing ($\nu_x = \nu_y$, $\Delta_\delta = 1$), dispersion decreases both tune shifts; however, $g_y < g_x$, hence $|\Delta\nu_y| > |\Delta\nu_x|$ so that the space-charge limit is determined by $\Delta\nu_y$ [Eq. (6)] and hence is increased by the factor g_y. As an example, for $\Delta_D = 2$, $\Delta_\delta = 1$ one find from Table 1 that $g_y = 1.618$ which implies that due to dispersion the phase-space density $N/\tilde{\epsilon}_n$ can be increased by a factor of 1.618 compared with the case where dispersion is negligible.
2. Asymmetric focusing ($\nu_x < \nu_y$, $\Delta_\delta > 1$) further enhances the g_y factor provided that $\Delta_D > (\Delta_\delta^2 - 1)^{1/2}$. For the above example ($\Delta_D = 2$), if $\Delta_\delta = 1.5$, *i.e.*, $\nu_y = 2.25\nu_x$, one obtains $g_y = 2.177$ which is significantly higher than in the symmetric case. In the region below the limit $\Delta_D = (\Delta_\delta^2 - 1)^{1/2}$ where $g_x < g_y$, the tune shift is controlled by g_x, which is less interesting from a practical point of view.

Table 1. g_x and g_y for Different Values of $\Delta_D = \delta_{xp}/\delta_{xb}$ and $\Delta_\delta = \delta_{xb}/\delta_{yb}$.

	$\Delta_\delta = 1$		$\Delta_\delta = 1.5$		$\Delta_\delta = 2$	
Δ_D	g_x	g_y	g_x	g_y	g_x	g_y
0	1.000	1.000	0.833	1.250	0.500	1.500
0.25	1.047	1.015	0.875	1.273	0.789	1.531
0.50	1.184	1.059	0.998	1.339	0.905	1.618
0.75	1.406	1.125	1.198	1.438	1.094	1.750
1.00	1.707	1.207	1.471	1.561	1.354	1.914
1.25	2.082	1.300	1.815	1.701	1.681	2.100
1.50	2.526	1.401	2.226	1.852	2.076	2.303
1.75	3.039	1.508	2.703	2.012	2.535	2.516
2.00	3.618	1.618	3.245	2.177	3.059	2.736
2.25	4.262	1.731	3.852	2.347	3.647	2.962
2.50	4.971	1.846	4.522	2.519	4.298	3.193
2.75	5.744	1.963	5.259	2.695	5.013	3.426
3.00	6.581	2.081	6.054	2.872	5.791	3.662

3.0 APPLICATION TO LEB

The lattice for the LEB developed by the SSC designers and adopted for construction of the project is shown in Figures 1 and 2, the first figure illustrating the general layout and the second the betatron functions $\beta_x(s)$, $\beta_y(y)$ and the dispersion function $\eta_x(s)$ for one of the three superperiods. Note that $\eta_x(s)$ corresponds to our $D(s)$. The values for the tunes ν_x, ν_y, the rms average of the dispersion function $\tilde{D} = \left(\overline{D^2}\right)^{\frac{1}{2}}$ and the transition energy, γ_t, are listed in Table 2 (first column).

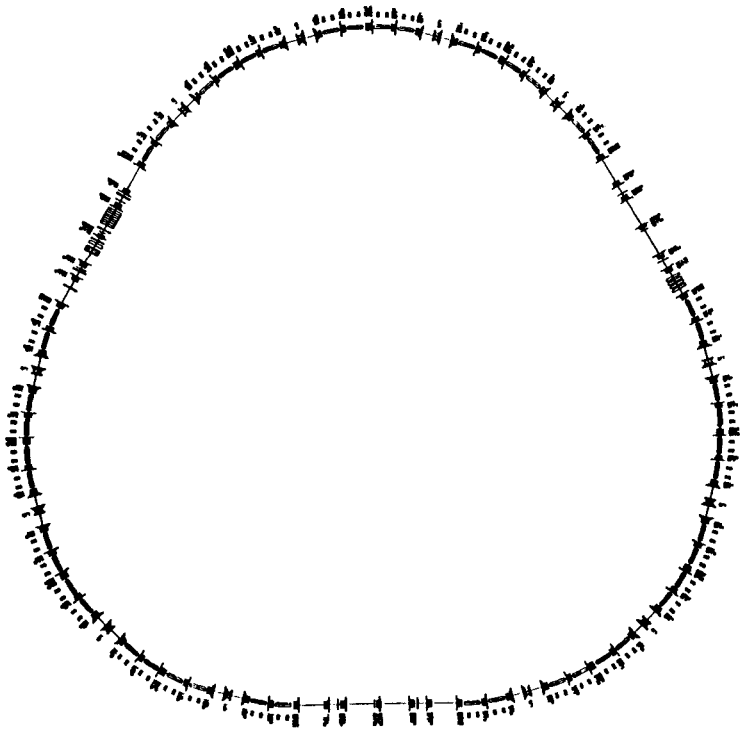

Figure 1. The Current LEB Lattice.

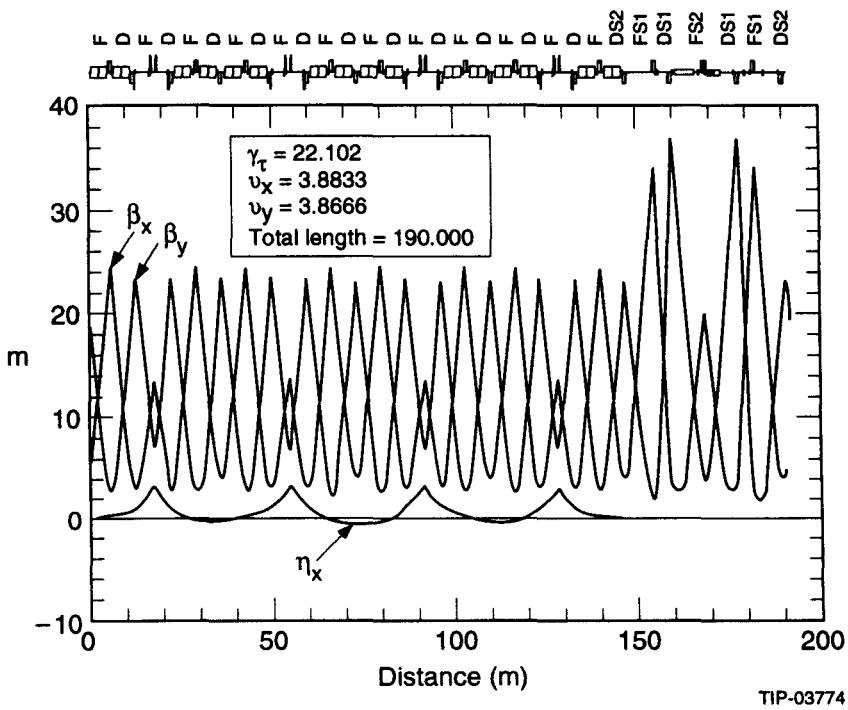

Figure 2. Betatron Functions and Dispersion ($\eta_x = D$) for the Current LEB Lattice.

Table 2. Basic Parameters for the Three Lattices Being Compared.

	Current LEB	High Dispersion LEB Lattice	Smooth FODO Lattice
ν_x	11.85	3.89	5.83
ν_y	11.80	3.82	5.81
$\tilde{D}[m]$	1.15	9.16	5.00
γ_t	22.00	3.32	4.48

Two alternate lattice designs developed by Al Garren[3] for the purpose of studying the dispersion effect are shown in the next figures. Figures 3 and 4 show a high-dispersion lattice having the same footprint and length of circumference as the LEB; however, the tune is much lower, $\nu_x = 3.89$, $\nu_y = 3.82$, the dispersion much larger, $\tilde{D} = 9.16$, and one would go through transition at $\gamma_t = 3.3$ (see column 2 in Table 2). The third lattice (Figures 5 and 6) has the same length of circumference as the LEB, but is more of the standard smooth FODO type with four straight sections. This lattice has $\nu_x = 5.83$, $\nu_y = 5.81$, $\tilde{D} = 5.00$, and $\gamma_t = 4.48$ as shown in the third column of Table 2. It could be useful for comparison purposes, *i.e.*, one could see how a super-lattice structure like LEB compares with a smooth regular lattice with regard to the large dispersion effect.

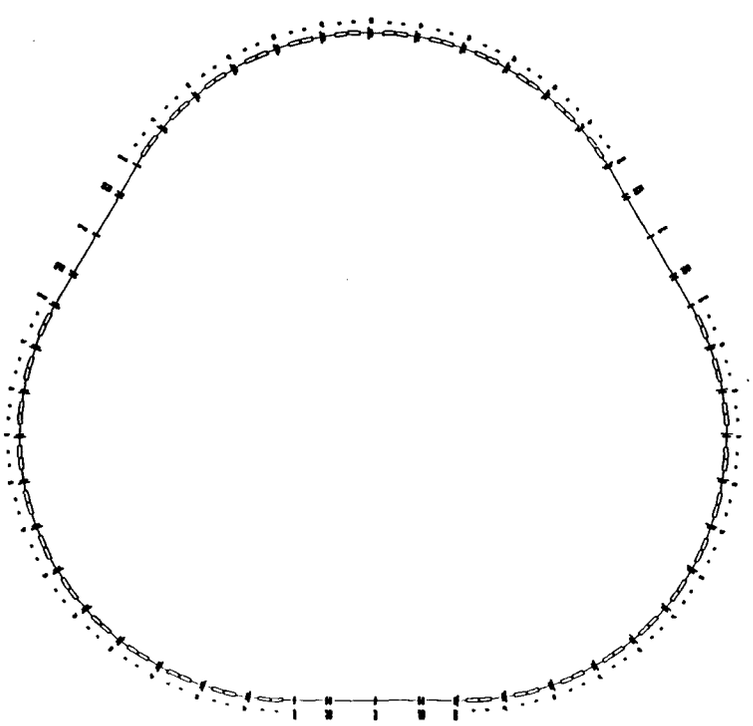

Figure 3. High Dispersion Lattice for LEB-type Configuration.

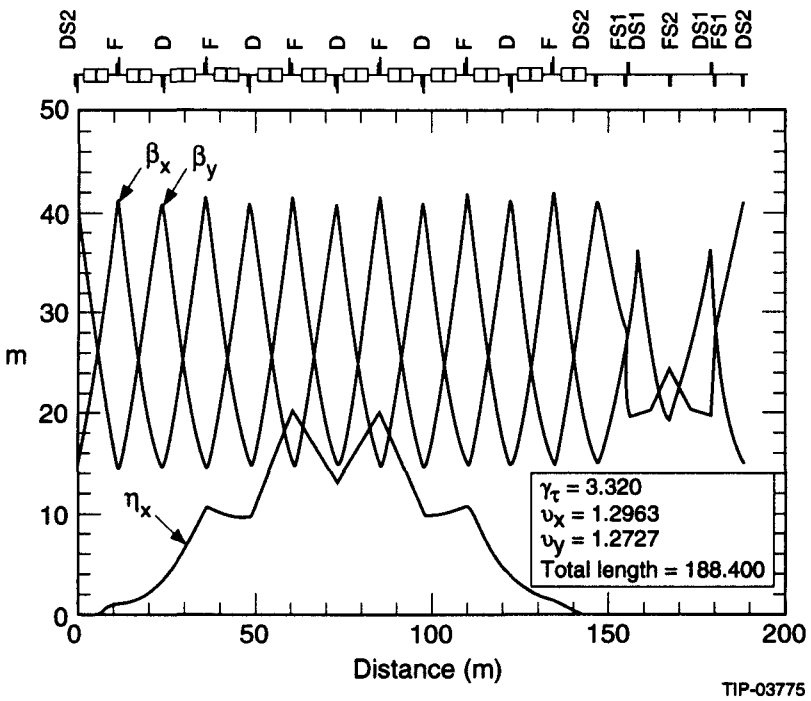

Figure 4. Betatron and Dispersion Functions for High-dispersion Lattice of the LEB-type Configuration.

Table 3 lists various parameters that have been calculated for each case with the assumptions that kinetic energy (600 MeV) and normalized emittance ($\tilde{\epsilon}_{nx} = \tilde{\epsilon}_{ny} = 0.4\pi$ mm-mrad) are the same as in the present LEB design. However, the rms momentum spread at injection from the Linac is assumed to be $\widetilde{\Delta p}/p = 10^{-3}$ which is a factor 2 higher than in the current design. The machine radius is $R = 90.72$ m in all three cases. Furthermore we assume that $\Delta_\delta = 1$ which is satisfied in all three lattices to very good approximation since the slight difference between ν_x and ν_y is negligible for our calculation.

Table 3. Parameter Values for the Three Design Lattices of Figures 1 to 6 and Table 2.

	LEB Design Lattice	LEB High-Dispersion Lattice	Smooth FODO Lattice
$\delta_{xb}[mm]$	1.538	2.679	2.188
$\delta_{xp}[mm]$	1.145	9.165	5.004
$\delta_x[mm]$	1.917	9.549	5.461
Δ_D	0.744	3.421	2.287
g_x	1.400	8.133	4.363
g_y	1.123	2.282	1.748

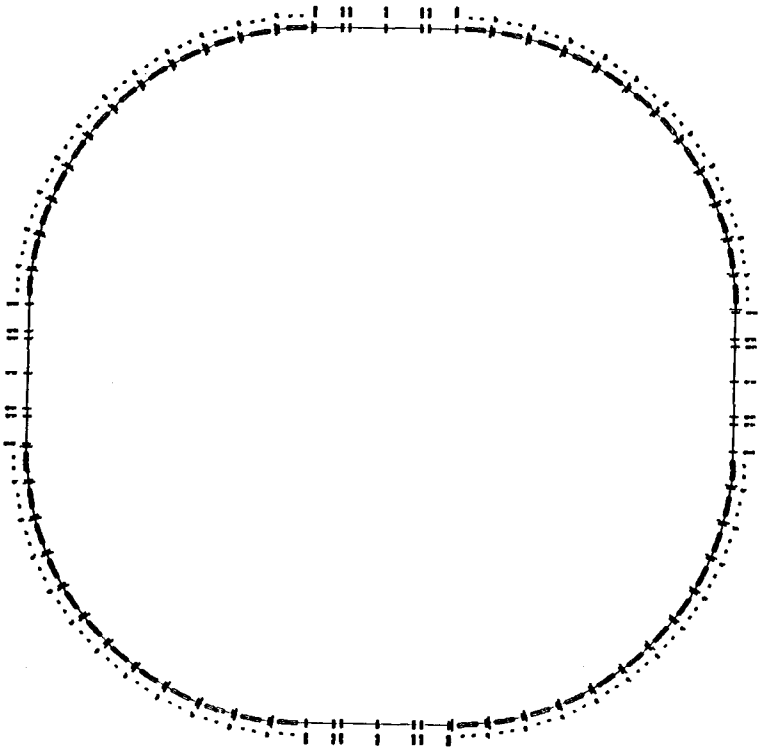

Figure 5. Smooth FODO Lattice with High Dispersion.

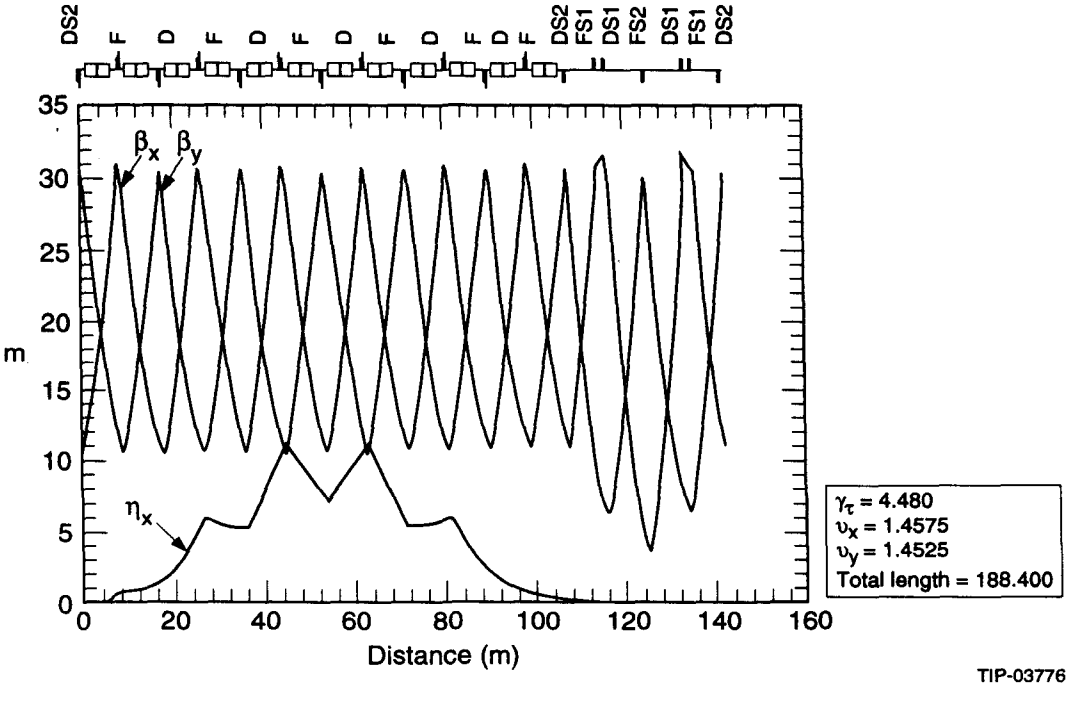

Figure 6. Betatron and Dispersion Functions for Smooth FODO Lattice with High Dispersion.

Discussion of results: The important parameter in Table 3 is g_y which shows the decrease in the vertical tune shift $|\Delta\nu_y|$, and hence the increase in the phase-space density $N/\tilde{\epsilon}_n$ due to dispersion. The results are quite significant. In the current design, if one were to increase the rms momentum spread to 10^{-3}, one could enhance the space-charge limit by about 12% ($g_y = 1.123$). The high-dispersion design for an LEB-type lattice would yield a factor of 2.3 increase in the limit, significantly more than would be achieved by increasing the Linac energy to 1,000 MeV (which yields a factor of 1.75). The smooth FODO lattice would yield a factor of 1.75 (same as the energy upgrade to 1000 MeV).

4.0 RECOMMENDATIONS

The calculations presented in this report indicate that one could expect to achieve gains in the space charge limit of the LEB by factors in the range of 1.75 to more than 2 by using a high-dispersion lattice. These results of "back-of-the-envelope" calculations are promising enough to pursue further studies. While the results of these studies may well be too late to have an effect on the current design, they could be used as a basis for a possible future upgrade of the LEB should this become desirable. One should take a careful look at the two alternate design scenarios presented here. If desirable, one could develop improved lattices that show the desired high dispersion gain for the space charge limit. The most promising designs could then be used for particle tracking and space charge simulation with Machida's code to see if the expected performance improvement with regard to the LEB space charge limit can be achieved.

REFERENCES

1. S. Machida, private communication.
2. M. Reiser, "Use of Dispersion to Increase the Space Charge Limit in Circular Accelerators," CPB Technical Report #92-003, April 7, 1992, Lab for Plasma Research, U. of Maryland (Revised Aug. 21, 1992).
3. A. Garren, private communication.