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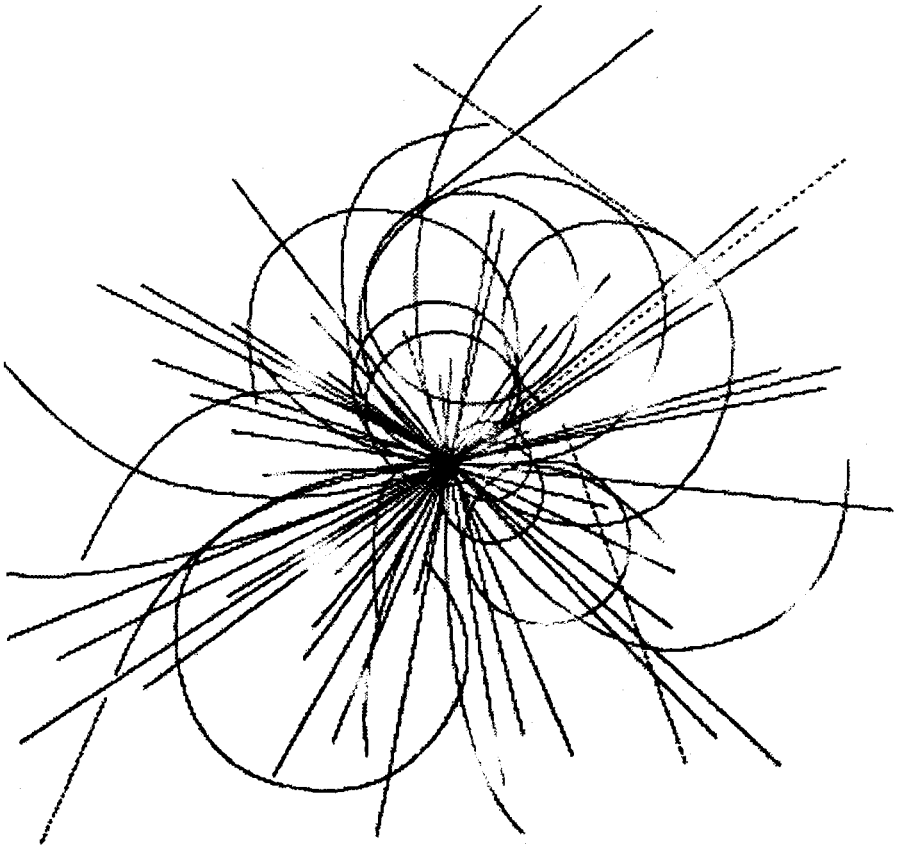

Superconducting Super Collider Laboratory

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The Nature of Dipole Field and Shimming

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The Nature of Dipole Field and Shimming

V. Thiagarajan

Abstract

Dipoles used in bending particle orbits in synchrotrons must be designed to keep the field errors (multipoles) within specified values in the region of interest called the "good field region." Specified field accuracy is usually achieved in iron core magnets by introducing a small bump called the "shim" at the tip of the pole. The field in the aperture of a dipole has been analyzed here based on a model. A method to determine the profile of the shim for a required good field region is also described. The field has been obtained through separation of variables. A method to determine random dipole errors is presented.

1.0 INTRODUCTION

Iron core dipoles are used for bending particle beams in low-energy synchrotrons such as the Superconducting Super Collider (SSC) Low Energy Booster (LEB) and the SSC Medium Energy Booster (MEB). Imperfections in the magnet, or the multipole errors will lead to undesirable results such as emittance growth, tuneshift, *etc.* The anticipated physical extent of the beam in the aperture of the magnet is usually specified through a good field region wherein the multipole errors should be designed to be below specified values. The good field region can, in principle, be increased by increasing the pole width of the magnet. This procedure results in larger volume of iron and more energy stored in the magnet. This would unnecessarily increase the cost of the magnet. The increase in the cost of power supplies for cycling magnets also could be considerable. Hence, indefinite increase in the width of the pole is wasteful. For a given pole width, the good field region can be increased considerably by introducing a small bump (or shim) in the iron near the tip of the pole.[1,2] The cost of the magnet can be minimized by a combination of minimum polewidth and optimum shim for a given good field region. The magnets are usually designed with codes like POISSON[3] or, PE2D.[4] The shim is usually introduced using empirical rules,[2] or using the code MIRT[3] which determines the shim profile for given good field region by iterating on the code POISSON. It is of interest to derive an analytical expression for the field in the aperture of the dipole and also arrive at a shim profile either analytically or, semi-analytically. This will yield useful ideas on the extent of the good field region and possible shim profiles (which, otherwise will take longer to determine if we resort to use of the codes). We will derive a semi-analytic procedure here to determine the dipole field and the shim profile for a given good field region. This method will also lead to an approximate procedure for the determination of random dipole errors.

2.0 THEORY

We will go through the necessary equations and the notation employed in this section. The governing Maxwell's equations are:

$$\nabla \cdot B = 0 \cdot 0, \tag{1}$$

$$\nabla \times H = J. \tag{2}$$

We can define a vector potential $\vec{A}$ as follows which satisfies Eq. (1).

$$B = \nabla \times A. \tag{3}$$

The magnetic field $\vec{B}$ and the magnetic intensity $\vec{H}$ can be related by the constitutive equation:

$$\vec{B} = \mu \vec{H}, \tag{4}$$

where μ is the permeability. Substituting Eqs. (3) and (4) in Eq. (2), we obtain:

$$\nabla \times \left(\frac{\nabla \times A}{\mu} \right) = J .$$

Or, expanding the left-hand side,

$$\frac{1}{\mu}\nabla(\nabla \cdot A) - \frac{1}{\mu}\nabla^2 A - (\nabla \times A) \times \nabla\left(\frac{1}{\mu}\right) = J . \tag{5}$$

For three-dimensional fields, further restriction on the vector potential $\vec{A}$ may be placed through a gauge condition. We will be dealing with two-dimensional magnetic fields in this paper and the vector potential reduces to:

$$\vec{A} = \hat{k}\, A_z(x, y) . \tag{6}$$

The permeability is a constant $= \mu_0$ for air. Therefore, Eq. (5) for air reduces to:

$$\nabla^2 A_z = -\mu_0 J . \tag{7}$$

In source free air regions Eq. (7) further reduces to:

$$\nabla^2 A_z = 0 . \tag{8}$$

Equation (3) may be rewritten using Eq. (6) as follows:

$$B_x = \frac{\partial A_z}{\partial y} , \tag{9a}$$

$$B_y = -\frac{\partial A_z}{\partial x} . \tag{9b}$$

We will be addressing the analysis of magnetic fields in resistive, iron-dominated regimes where the fields in the air will be less than 2.0 Tesla. A schematic diagram of a quadrant of a dipole is shown in Figure 1. Equation (2) can be integrated along a closed loop around the coil passing through the iron and air space by applying Stoke's theorem. This yields,

$$\int_0^h H \cdot dr \Big|_{\text{air}} + \int H \cdot dr \Big|_{\text{iron}} = NI . \tag{10}$$

The right-hand side of Eq. (10) represents the source ampereturns with I = current and N = number of turns. The left-hand side is split into two parts; the first represents the ampereturns spent in driving the magnetic flux through air and the second term on the left represents the ampereturns spent in driving the flux through the iron. For an

efficient magnet the latter term should be as small as possible. Therefore, we can define an efficiency for the magnet as follows:

$$\text{Magnet efficiency } \eta \;=\; \frac{\int_0^h H \cdot dr \Big|_{\text{air}}}{NI}. \tag{11}$$

Determination of magnetic fields in the geometry shown in Figure 1, requires the solution of Eqs. (7) and (8) with appropriate boundary conditions. The code POISSON[3] solves the problem numerically by enforcing Eq. (10) for every elementary mesh. The code PE2D[4] solves the problem using the method of finite elements. We are interested in solving the problem semi-analytically here and hence, we will resort to a transformed model.

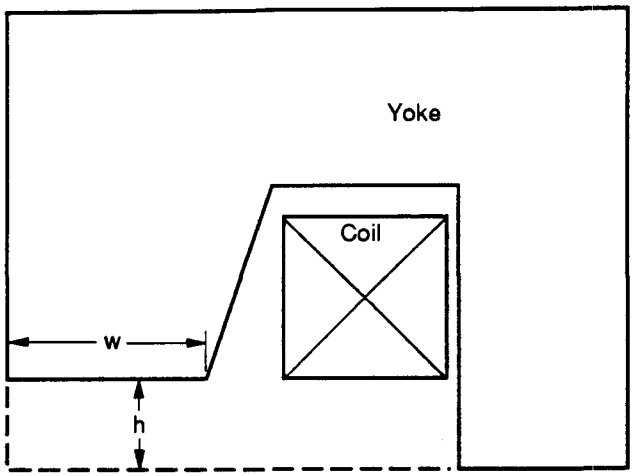

Figure 1. Dipole Schematic.

3.0 MODEL FOR ANALYSIS

Let us assume constant permeability for the iron. This assumption is just for the purpose of transforming the iron region and its effect on the solutions will be assessed later. The variation of permeability has a significant effect on the solutions through the discontinuity of tangential fields at the iron-air interface and this will definitely be included. The transformation is shown schematically in Figure 2. The air region is represented by the rectangle OABC with the height and width of the aperture being h and w. The iron region is represented by the polygon AFGHJKLBA shown in dotted lines. Since we have assumed constant permeability, the governing equations for iron and air are just the Laplace Equations (8), for which we can apply the method of conformal transformation. According to Riemann's mapping theorem, any simply-connected region can be mapped onto any other simply-connected region and an appropriate Schwartz-Christoffel transformation could be written down. Hence, we can conformally transform the iron AFGHJKLBA onto the rectangle ADEBA (shown in solid lines). We will keep the length of OAFGH equal to the

length OD = l. Our problem reduces to the solution of the Laplace equation for the air region contained in the rectangle OABC and the iron region contained in the rectangle ADEB. Once a solution is obtained, we can revert to the original geometry through an inverse transformation. The line JH in the iron maps onto the line DE in our transformation. We will ignore leakage fields and assume all the flux passes through the iron, *i.e.*, a uniform magnetic field enters through DE normally due to externally-impressed ampereturns NI, where N is the number of turns and I is the current per turn. We will also assume that all the flux passes through the air through the faces AB and OC. In our transformed model, Eq. (10) should be satisfied for any path originating from the line OC in air and ending on the line DE in iron.

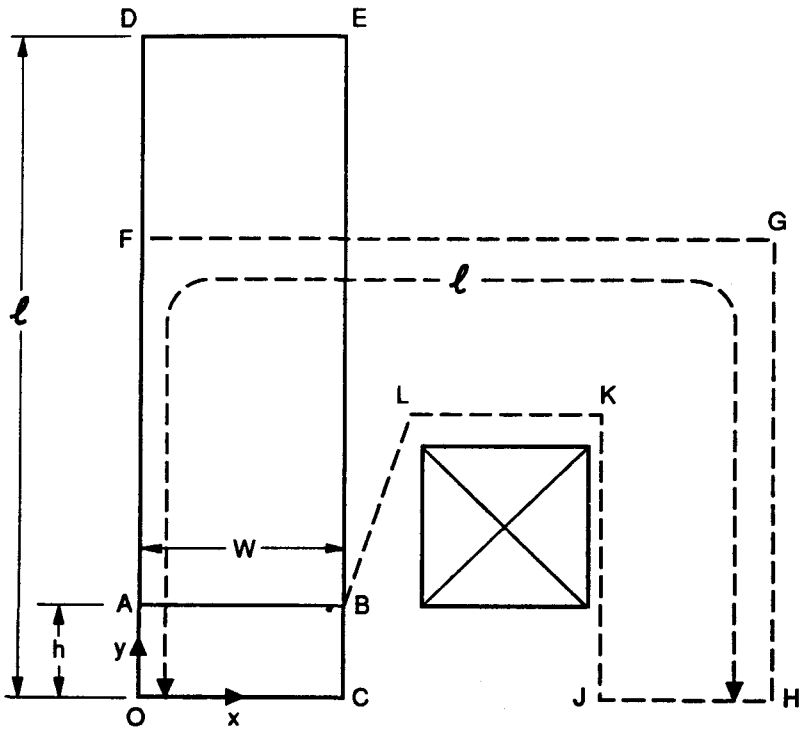

Figure 2. Transformed Model of Dipole.

4.0 BOUNDARY/INTERFACE CONDITIONS

The transformed dipole is shown in Figure 2 in solid lines. We will assume symmetry. Therefore, the field lines will enter normal to the surfaces DE and OC and parallel to the surfaces OA and AD. As a result,

$$\left.\frac{\partial A_z}{\partial y}\right|_{x=0} = \left.B_x\right|_{x=0} = 0, \tag{12}$$

$$\left.\frac{\partial A_z}{\partial y}\right|_{y=0} = \left.B_x\right|_{y=0} = 0, \tag{13}$$

$$\left.\frac{\partial A_z}{\partial y}\right|_{y=l} = \left. B_x \right|_{y=l} = 0 . \tag{14}$$

The flux entering the face DE goes through the faces AB and OC without loss. Therefore,

$$\left.\int_0^w B_y \, dx\right|_{y=0} = \left.\int_0^w B_y \, dx\right|_{y=h} = \left.\int_0^w B_y \, dx\right|_{y=l} . \tag{15}$$

In addition, Eq. (10) has to be satisfied for given ampereturns. This can be rewritten as follows:

$$\int_0^h H \cdot dy + \int_h^l H \cdot dr = NI . \tag{10a}$$

We will assume high efficiencies for the magnet (> 95%). Therefore, we can effectively neglect the second term on the left-hand side of Eq. (10a) and it simplifies to:

$$\int_0^h H \, dy = NI, \text{ for any } x . \tag{10b}$$

Further, the assumption of constant permeability for the iron which we made for the conformal transformation has no effect on the solution. Once we find a solution in the transformed geometry, we do not even need to invert it to the original geometry. The transformation helped us define a characteristic length l which we will use in the eigenvalues later.

Considering the air region OABC in Figure 2, we have prescribed symmetry conditions on the faces OA and OC and the flux passing through the face AB. Additionally, we need to prescribe boundary conditions along the face BC. If the coil were close to the face BC, it will control the field along this line. If not, the high field at the pole root (called the pole root saturation) will influence the field along BC. We will address this latter case and prescribe the field at the corner B, which may be obtained with a trial run of POISSON or PE2D. In addition we have to prescribe interface conditions at the iron-air interface AB (in Figure 2). These may be obtained by applying Eqs. (1) and (2) to a small pill box of height Δy at the interface and taking the limit as $\Delta y \rightarrow 0$. The result is:

$$\left. B_y \right|_{\text{air}} = \left. B_y \right|_{\text{iron}} \text{ at the interface}, \tag{16a}$$

$$\left.\frac{B_x}{\mu_0}\right|_{\text{air}} = \left.\frac{B_x}{\mu}\right|_{\text{iron}} \text{ at the interface}. \tag{16b}$$

5.0 FIELD ANALYSIS

The current source is outside the air (aperture) and iron regions. Therefore, the governing equation for these regions is the Laplace equation, Eq. (8). Let the solution be

$$A_z = X(x)\, Y(y) . \tag{17}$$

Substituting this into Eq. (8) and regrouping, we get:

$$\frac{X''}{X} = -\frac{Y''}{Y} = \lambda_n^2$$

where λ_n are the eigenvalues. The solution for the potential A_z is:

$$A_z = \Big(A \sinh(\lambda_n x) + B \cosh(\lambda_n x)\Big) \Big(C \sin(\lambda_n y) + D \cos(\lambda_n y)\Big)$$

where A, B, C and D are constants to be determined.

$$\frac{\partial A_z}{\partial y} = \lambda_n \Big(A \sinh(\lambda_n x) + B \cosh(\lambda_n x)\Big) \Big(C \cos(\lambda_n y) - D \sin(\lambda_n y)\Big).$$

The boundary condition Eq. (12) is satisfied if we put $B = 0$ and the boundary condition Eq. (13) is satisfied if we put $C = 0$. Further, redefining the constant and summing on the eigenvalues, we get

$$A_z = \sum_n a_n \sinh(\lambda_n x) \cos(\lambda_n y). \tag{18}$$

Also, the boundary condition Eq. (14) is satisfied with the following choice of the eigenvalues:

$$\lambda_n = \frac{2\pi n}{l} \quad n = 1, 2, 3 \ldots. \tag{19}$$

The conformal transformation helped us get a characteristic length l and define the eigenvalues given by Eq. (19), *i.e.*, we enforce the fact that the flux lines at $x = 0$ travel a length l before they encounter symmetry boundary conditions. The solution Eq. (18) along with the definition of eigenvalues, Eq. (19), is valid for both the air and iron regions. We can truncate the series solution and choose the coefficients a_n to satisfy Eq. (10b) at discrete locations and also the field at the tip of the pole. The only condition that is yet to be satisfied is the field discontinuity given by Eq. (16b). Since this is a discontinuity we need an additional series which will not violate the boundary conditions which we have satisfied so far. Let us consider the following series.

$$A_z = \sum_m b_m \sinh(\lambda_m' x) \cos(\lambda_m' y) \tag{20}$$

where,

$$\lambda_m' = \frac{\pi\,(m - 0.5)}{h} \quad m = 1, 2, 3 \ldots. \tag{21}$$

We can determine the coefficients b_m such that Eq. (16b) is satisfied at discrete points at the interface. The addition of this series does not affect the continuity of field normal to

the interface, since the B_y given by Eqs. (20) and (21) at $y = h$ is equal to zero. Therefore the solution which satisfies all the boundary/interface conditions is:

$$A_z = \sum_n a_n \sinh(\lambda_n x) \cos(\lambda_n y) + \sum_m b_m \sinh(\lambda_m' x) \cos(\lambda_m' y). \tag{22}$$

The coefficients a_n and b_m can be determined to satisfy the boundary and interface conditions and also to force the fields to required values at given locations in the good field region. The series can be truncated with a convenient n and m. However, when we set about determining the coefficients numerically with Eq. (22), we encounter problems. The polewidth is normally about 2 to 3 times the magnet aperture. Therefore the term $\sinh(\lambda_m' x)$ increases steeply with m. For example it is equal to 1 for $m = 1$ and is equal to about 10^{38} for $m = 10$. Matrix solutions with such large differences in coefficients will lead to errors even with quad precision with about 32-digit accuracy. Hence, we will modify our approach as follows. The first part of the solution given by Eqs. (18) and (19) is numerically tractable. The second part of the solution given by Eqs. (20) and (21) needs to be modified as follows. Let us consider the following series for the aperture alone:

$$A_z = \sum_m b_m \sinh(\lambda_m' x) \cos(\lambda_m' y) \tag{23}$$

where,

$$\lambda_m' = \frac{2\pi m}{l}. \tag{24}$$

The arguments of sinh will be smaller than in the previous case and we can determine the b_m's to satisfy the interface discontinuity, Eq. (16b). But this series will introduce a normal field at the interface and we can cancel it out by introducing an additional series in the iron as follows:

$$A_z = \sum_k c_k \sinh(\lambda_k x) \cos\Big(\lambda_k(y-h)\Big) \tag{25}$$

where,

$$\lambda_k = \frac{2\pi k}{(l-h)}. \tag{26}$$

It should be noted that the series Eq. (25) does not introduce a tangential field at $y = h$. We can determine the coefficients c_k such that continuity of normal field is enforced. The complete solution is:

$$\begin{aligned} A_z = &\sum_n a_n \sinh(\lambda_n x) \cos(\lambda_n y) + \sum_m b_m \sinh(\lambda_m' x) \cos(\lambda_m' y) \\ &+ \sum_k c_k \sinh(\lambda_k x) \cos\Big(\lambda_k(y-h)\Big) \end{aligned} \tag{27}$$

with the λ's specified by Eqs. (19), (24) and (26). The fields B_x and B_y may be obtained from Eq. (9). They are:

$$B_x = \sum_n -a_n \lambda_n \sinh(\lambda_n x) \sin(\lambda_n y) + \sum_m -b_m \lambda'_m \sinh(\lambda'_m x) \sin(\lambda'_m y)$$
$$+ \sum_k -c_k \lambda_k \sinh(\lambda_k x) \sin\Big(\lambda_k (y-h)\Big), \tag{28}$$
$$B_y = \sum_n -a_n \lambda_n \cosh(\lambda_n x) \cos(\lambda_n y) + \sum_m -b_m \lambda'_m \cosh(\lambda'_m x) \cos(\lambda'_m y)$$
$$+ \sum_k -c_k \lambda_k \cosh(\lambda_k x) \cos\Big(\lambda_k (y-h)\Big). \tag{29}$$

It should be noted that the first series with index n is valid both for air and iron regions, the second series with index m is valid only for the air region and the third series with index k is valid only for iron. Equations (28) and (29) are the approximate analytical solutions that we seek and the coefficients may be determined numerically to satisfy all the boundary/interface conditions, good field criteria and other conditions that we may impose.

6.0 EVALUATION OF SHIMS

We will restrict our analysis to highly efficient magnets. Therefore, we will use Eq. (10b) which neglects the ampereturns required to drive the flux through the iron. With this assumption, we do not need to compute the field in the iron.

The conformal transformation helped us obtain a characteristic length l for specifying the eigenvalues to meet the boundary conditions and we can avoid an inverse transformation. The assumption of constant μ has no effect on the analysis and whatever minor effect it has, is contained in the restrictions used in obtaining Eq. (10b). We will be accounting for the variation of μ when we obtain the coefficients in Eqs. (28) and (29) to satisfy the interface conditions. Equation (10b) can be rewritten as follows:

$$\int_0^h B_y dy = \mu_0 N I. \tag{10b}$$

Or, using Eq. (29),

$$\sum_n a_n \cosh(\lambda_n x) \sin(\lambda_n h) + \sum_m b_m \cosh(\lambda_m x) \sin(\lambda_m h) = -\mu_0 N I. \tag{30}$$

If the height of the aperture h were constant with x, this will result in a field distribution. Conversely, we can specify a field distribution (say, the good field region) and determine $h(x)$ using Eq. (30). When we do so, we are compromising the solution of the Laplace's

equation to some extent, since we obtained the solution for a rectangular region, but we are varying the height of the rectangle. We find that this results in a Δh of about a mm while the height of the aperture is in tens of mm and hence, the solution is not significantly affected. We can require that h be constant for $x = 0$ to $x = x_1$ and we can vary h from x_1 to w. The resulting pole tip profile is the shim and we will call Eq. (30) the shim equation. We can differentiate both sides of Eq. (30) with respect to x and obtain the following equation:

$$\frac{dh}{dx} = -\frac{B_x}{B_y}. \tag{30a}$$

7.0 NUMERICAL PROCEDURE

We will specify the aperture height in the center h, halfwidth of the pole w, the perimeter of a quadrant of the dipole l, the field B in the center and the extent of the good field region. The objective is to calculate the coefficients a_n's, b_m's and c_k's in Eq. (29) to describe the field and also determine the shim profile. The numerical computation can be done in three stages to evaluate the coefficients a_n, b_m, c_k. In the first stage we can evaluate the coefficient a_n's as follows. The part potential corresponding to the first series in Eq. (27) is given by Eq. (18). The corresponding part fields are:

$$B_y = -\frac{\partial A_z}{\partial x} = \sum -a_n\lambda_n \cosh(\lambda_n x)\cos(\lambda_n y), \tag{18a}$$

$$B_x = \frac{\partial A_z}{\partial y} = \sum -a_n\lambda_n \sinh(\lambda_n x)\sin(\lambda_n y). \tag{18b}$$

It should be remembered that the above are part fields though we are using the same notation B_x, B_y. The ampereturns and fluxes are given by the following equations.

$$\int_0^h B_y dy = \sum -a_n \cosh(\lambda_n x)\sin(\lambda_n h), \tag{18c}$$

$$\int_0^w B_y dx = \sum -a_n \sinh(\lambda_n w)\cos(\lambda_n y). \tag{18d}$$

In order to achieve the required good field, we can enforce the following equation at a discrete number of points (say 6) on the boundary of the specified good field region, which will yield six equations.

$$B_y(x,y) = B_y(0,0). \tag{31}$$

The field B_y at $x = w$, $y = h$, may be evaluated approximately with a trial run using POISSON or PE2D and may be forced which yields the next equation:

$$B_y(w,h) = \text{given value}. \tag{32}$$

The ampereturns given by Eq. (18c) may be equated to the impressed ampereturns NI at $x = 0$ and at $x = w$, to obtain the eighth and ninth equations. The last equation may be obtained by equating the flux at $y = 0$ to the flux at $y = h$, using Eq. (18d). These ten equations may be solved by matrix inversion to obtain ten coefficients a_n, defining the part potential and part fields, in the first stage. The number of points on the boundary (or, within) of the specified good field region where Eq. (31) is enforced, depends on the problem and the required numerical accuracy of computations. Six points were found to be adequate in the sample calculations. Similarly, Eq. (18c) was enforced at only two points (at $x = 0$ and at $x = h$) and was found to be sufficient. One could enforce these equations at more points, if need be. The solution given by Eq. (18) will satisfy the interface condition Eq. (16a), but not Eq. (16b). In order to satisfy Eq. (16b), we add another series given by Eq. (23) to the solution, with part fields given by the following equations:

$$B_y = \sum -b_m \lambda'_m \cosh(\lambda'_m x) \cos(\lambda'_m y)\,, \tag{23a}$$

$$B_x = \sum -b_m \lambda'_m \sinh(\lambda'_m x) \sin(\lambda'_m y)\,. \tag{23b}$$

In order to determine the b_m's, we will begin by enforcing Eq. (16b) at a finite number of points (say 10) along the pole boundary $y = h$. The process consists of canceling out the B_x component at $y = h$ from Eq. (18b) and satisfying Eq. (16b). The result is

$$\sum -b_m \lambda'_m \sinh(\lambda'_m x) \sin(\lambda'_m y) = \left(\frac{\mu_0}{\mu} - 1\right) \sum -a_n \lambda_n \sinh(\lambda_n x) \sin(\lambda_n y)\,. \tag{33}$$

Since the fields are high near the pole tip ($x = 0$, $y = h$), it is better to choose points closer near here and space them out farther apart near $x = 0$, $y = 0$. Additionally, we need to ensure the quality of the field in the good field region. The fields given by Eqs. (23a) and (23b) constitute a perturbation to the fields given by Eqs. (18a) and (18b). It is better to confine the perturbations as close to the pole tip as possible. This is best done by enforcing $B_y = 0$ at the origin and at additional points say 6 along the y-axis (from $y = 0$ to $y = h$) and 10 along the x-axis (from $x = 0$ to $x = w$). This will give us 17 equations (making a total of 27 so far). We can integrate Eq. (23) along y and x to obtain the contribution to the ampereturns and the flux from this series similar to Eqs. (18c) and (18d). We can enforce the ampereturns to be zero at four different x's (say $x = 0$, 0.01, 0.02 and 0.03) and along with flux continuity this will yield us five more equations, making it a total of 32 so far. It is better to have control over the depth of the shim and we may require a shim thickness of δ at a given point $x = x_1$, $y = y_1$. We

can ensure this by integrating the B_y fields given by Eqs. (18a) and (23a) from $y = 0$ to $y = y_1 = (h - \delta)$ at $x = x_1$, and equating it to the impressed ampereturns, which yields

$$\int_0^{h-\delta} B_y \, dy \Big|_{x=x_1} = \sum -a_n \sinh(\lambda_n x_1) \cos(\lambda_n y_1) \; - \; \sum -b_m \sinh(\lambda'_m x_1) \cos(\lambda'_m y_1)$$
$$= \mu_0 N I \,. \tag{34}$$

Along with Eq. (32), we have 33 equations to solve for 33 b_m's. The number of equations here (33) is purely an arbitrary choice and may be varied depending on computer accuracy and the problem parameters like the pole width, *etc.* As described earlier, the solution given by Eq. (23), introduces an additional normal component of the magnetic field and in order to satisfy Eq. (16a), we introduced another series given by Eq. (25) valid only for the iron. The coefficients c_k for this series may be determined by equating B_y from this series to the B_y obtained from Eq. (23a) at discrete points along the pole boundary (say 10). The resulting equation to be solved is:

$$\sum -b_m \lambda'_m \cosh(\lambda'_m x) \cos(\lambda'_m h) \; = \sum -c_k \lambda_k \cosh(\lambda_k x) \cos(\lambda_k h) \,. \tag{35}$$

When we prescribe the shim thickness δ in the second stage, it is better to start with a low value for δ and increase it gradually in steps to reach a shim profile which will be reasonable from a fabricational point of view.

8.0 RANDOM DIPOLE ERRORS

Manufacturing and assembly errors may lead to a perturbation of the pole face which can be described by $\Delta h(x, y)$. This will result in a perturbation of the vector potential (and hence, also in a perturbation of the fields). This perturbation is mainly due to the fact that the total magnetic flux from the iron (which is constant in our model) is driven through the aperture with heights which vary with x due to the pole perturbation, Eq. (10b). We can estimate the perturbation as follows. Let us prescribe the variations in the pole face by the following equation.

$$\Delta h \; = \; \Delta h_0 \; \sin \frac{n \pi x}{w} \,. \tag{36}$$

The index n and the amplitude Δh_0 may be varied to match the pole face variation in a given magnet. Let us consider the solution for the potential given by Eq. (18). The field B_y is given by Eq. (18a) and the integral of this field over y is given by Eq. (18c). Using the unperturbed constant value of h, we can equate the right-hand side of Eq. (18c) to $\mu_0 N I$. We can enforce this at a discrete number of points on the pole face (say 10) and obtain the coefficients for the unperturbed potential a_n for n varying from 1 to 10.

We can also use the perturbed pole face heights $h + \Delta h$ which will be a function of x and obtain ten coefficients a'_n to describe the perturbed potential. The difference in potential is a measure of the field perturbation and is given by

$$\Delta A_z = \sum (a_n - a'_n) \sinh(\lambda_n x) \cos(\lambda_n y) . \tag{37}$$

The perturbed potential may be decomposed into harmonics using the following equation:

$$\Delta A_z = \sum_{n=0}^{\infty} \left(\frac{r}{r_{\text{norm}}} \right)^n [A_n \cos(n\phi) - B_n \sin(n\phi)] . \tag{38}$$

It should be noted that we have used only the first series in Eq. (27) to determine the random dipole errors. One might use the second series also for better predictions or, we could take the view that the first series would suffice, since there will be an uncertainty in specifying the amplitude Δh_0.

9.0 RESULTS AND DISCUSSION

The complete solution for the potential and the fields is given by Eqs. (27), (28) and (29). We need to determine the coefficients numerically, to the accuracy specified or possible. The first two series with indices n, m give the solution for the air aperture region. The first and the third series with indices n, k give the solutions for the iron region. We will be addressing highly efficient magnets here and therefore, we will address the first two series and ignore the third. As described in the numerical procedure, the coefficients a_n were obtained by enforcing the ampereturns, the flux continuity and the good field region. Numerical computations show that it is possible to achieve good field over a large area of the aperture with just about ten coefficients in this series. The second series with index m was introduced to satisfy the discontinuity of tangential fields at the air-iron interface (pole face). This series causes a reduction in the area of the good field region due to the high fields at the pole tip. The shim was introduced to offset this deterioration to the extent possible and maximize the good field region. In the numerical example, 33 coefficients were needed in the series for this purpose. The computations were carried out using SUN Fortran with quad precision.

The main dipole of the SSC Low Energy Booster (LEB) with a field of 1.3 Tesla was designed[5] using the codes POISSON[3] and PE2D.[4] Tables 1, 2, and 3 list a comparison of good field regions, random dipole errors, and shim coordinates, respectively. The shim was designed using the code MIRT[3] for simultaneous optimization of the field at injection and extraction energies. The magnet has a half pole gap of 2.86 cm and a nominal half pole

width of about 7.2 cm. A code DIASOL (Appendix) was written using the present semi-analytical method and computations were done to maximize the good field at injection and extraction simultaneously. The shim profile is shown in Figure 3. The shim coordinates are shown in Table 3. We will define the good field region as that area where $\frac{\Delta B}{B} < 10^{-4}$, B being the field in the center of the magnet. The code PE2D was used to do precise calculations for the dipole with the shim designed by the present method. Due to the approximations in the analyses, we can anticipate deterioration of the good field region. The extent of the good field regions from the present method, the PE2D analysis and the POISSON/MIRT analysis[5] are shown in Table 1. The good field regions are the same at injection and extraction from the present method. The PE2D analysis shows that the good field region deteriorates considerably from that predicted by the present method and this is obviously due to the effect of the iron and the fact that we have ignored the ampereturns in the iron. Comparing the good field regions from the PE2D analysis using the present shim and the one in the LEB design[5] designed with POISSON and MIRT, we find that the present method yields a larger good field region at extraction and that one at injection is comparable. The computations with DIASOL yield solutions for the shim profile in computation times of the order of a minute, using a SUN SPARC 2 workstation; the design with MIRT takes about a couple of hours per trial.

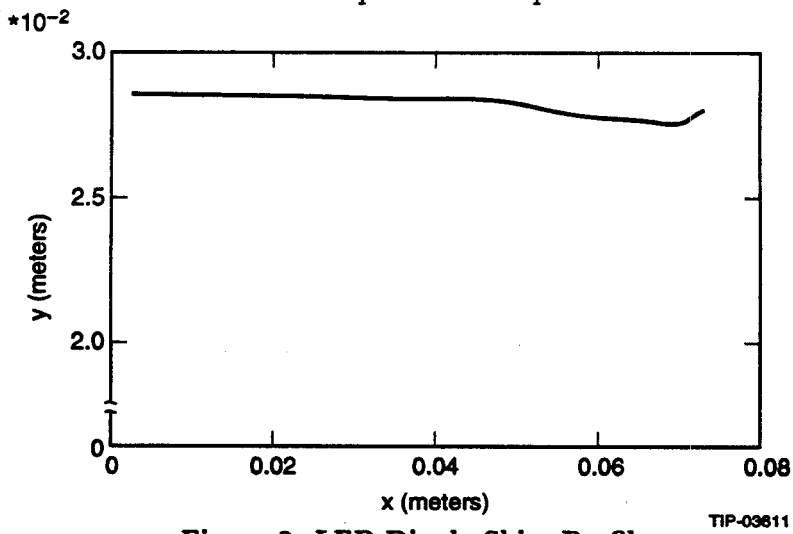

Figure 3. LEB Dipole Shim Profile.

There is scope to improve the good field region using the present method by varying the discretization coordinates of the good field region. Further, there is scope to ensure the smoothness of the variation of the profile of the shim, by imposing the thickness of the shim at chosen points. Similar computations were done for the dipole magnet for the SSC Medium Energy Booster (MEB) with a field of 1.8 Tesla. The results (not produced here) show that it is possible to achieve the required good field region at injection and

not at extraction. The reason is that the ampereturns in the iron which we have ignored play a predominant part. Random dipole errors (code DERRORA in the Appendix) were computed for chosen sample perturbations of the pole face. The results are shown in Table 2. One needs an idea of the amplitude of the perturbation in the pole face either from experience or through tests on prototypes.

In summary, we have presented a semi-analytical method to determine the shim profile for resistive dipoles with fields less than about 1.6 Tesla. This method enables faster determination of the shim profile and prediction of the good field region.

Table 1. Comparison of Good Field Regions.

A. Present Analysis	
Both Injection and Extraction	
x, mm	y, mm
38	0
41	4
38	8
33	12
18	16
14	20
14	24

B. PE2D Analysis with Present Shim			
Injection		Extraction	
x, mm	y, mm	x, mm	y, mm
32	0	18	0
34	5	20	5
22	10	40	10
23	15	25	15
29	20	26	20
13	25	29	22
14	28.6	0	25

C. PE2D Analysis with Shim from Reference 5			
Injection		Extraction	
x, mm	y, mm	x, mm	y, mm
33	0	16	0
38	5	18	5
32	10	21	10
36	15	26	15
28	20	0	20
34	25	0	25

Table 2. Random Dipole Errors.

n	a_n from Eq. (38), with $r = 0.025$, $r_{\text{norm}} = 0.0286$		
	Case I	Case II	Case III
1	0.051	0.054	0.057
3	0.083	0.088	0.093
5	0.040	0.043	0.045
7	0.009	0.01	0.01
9	0.001	0.001	0.001

Case I: $\Delta h = 0.0000254 \sin\left(\frac{0.5\pi x}{w}\right)$

Case II: $\Delta h = 0.0000254 \sin\left(\frac{1.5\pi x}{w}\right)$

Case III: $\Delta h = 0.0000254 \sin\left(\frac{2.5\pi x}{w}\right)$

Table 3. Shim Coordinates.

Number	x, mm	y, mm
1	0.0	28.60
2	34.0	28.60
3	36.0	28.59
4	38.0	28.57
5	40.0	28.56
6	46.0	28.56
7	48.0	28.52
8	50.0	28.43
9	52.0	28.32
10	54.0	28.19
11	56.0	28.09
12	58.0	28.03
13	60.0	28.00
14	62.0	27.99
15	64.0	27.95
16	66.0	27.91
17	68.0	27.88
18	70.0	27.93
19	72.0	28.17

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APPENDIX

PROGRAM DIASOL

```
c typical input
C 0.8 0.0286 0.072 3000.0 100.0
C 0.004 0.004 0.072 0.0286
C 6 1 10 16 -0.5  0.002 0.0
C 0.0 0.01
C 0.0 0.02
C 0.02 0.02
C 0.02 0.01
C 0.02 0.0
C 0.01 0.0
C 0.072 0.0286
C 0.072
C 0.0715
C 0.071
C 0.0705
C 0.07
C 0.068
C 0.066
C 0.064
C 0.06
C 0.04
C 0.0 0.004
C 0.0 0.008
C 0.0 0.012
C 0.0 0.016
C 0.0 0.02
C 0.0 0.025
C 0.004 0.0
C 0.008 0.0
C 0.012 0.0
C 0.016 0.0
C 0.025 0.0
C 0.03 0.0
C 0.04 0.0
C 0.05 0.0
C 0.06 0.0
C 0.072 0.0
C 0.072
C INPUT ENDS
C DIpole Analytical SOLution
      IMPLICIT REAL*16 (A-H,O-Z)
      DIMENSION X(300),SLA(300),SLB(300),A(300,300),R(300),RHS(300),
     1Y(300),XTA(300),YTA(300),RHS2(300),XB(300),RHS3(300),R2(300),
     2XTB(300),YTB(300),B(300)
      PAI=2.*QASIN(1.Q0)
      SMU=PAI*4.D-7
C  SMU=*MUNOT,SMUR=REL. PERMEA OF IRON,H=0.5*POLE GAP
C  AMPT=AMPERE TURNS,W=0.5*POLEWIDTH,DL=TOTAL HEIGHT WITH IRON
      READ (*,*) DL,H,W,AMPTC,SMUR
c  STEPS AND XMAX AND YMAX FOR FIELD OUTPUT
      READ (*,*) DELX,DELY,XMAX,YMAX
C NA1= NUMBER OF COEFFICIENTS IN THE FIRST  SOLUTION IN THE GOOD FIELD REGION
C NA2= NUMBER OF COEFFICIENTS IN THE FIRST SOLUTION AFTER THE GOOD FIELD REGION
C NB1= NUMBER OF COEFFICIENTS IN THE SECOND SOLUTION, WHERE BC ON BX IS FITTED
C NB2= NUMBER OF COEFFICIENTS IN THE SECOND SOLUTION, WHERE BY=0.0 IS FITTED
C PAR= COEFFICIENT FOR PEAK FIELD AT THE POLE TIP CORNER
C DB,DBA ARE THE COEFFICIENTS FOR ENFORCING BUMP SENSITIVITY
C QUITE A FEW OPTIONS TRIED ARE COMMENTED OUT AND THIS MAY LEAVE SOME INPUT
```

```
C SUPERFLUOUS
      READ (*,*) NA1,NA2,NB1,NB2,PAR,DB,DBA
      NA=NA1+NA2+3
      NB=NB1+NB2+7
C GOOD FIELD REGION, FIRST SOLUTION - X(I),Y(I) ARE SPECIFIED POINTS
      DO 8 I=1,NA1
      READ (*,*) X(I),Y(I)
8     CONTINUE
C XTA(I),YTA(I) - POINTS WHERE ELEVATED FIELD FITTED - TYPICALLY ONE POINT
      DO 10 I=1,NA2
      READ (*,*) XTA(I),YTA(I)
10    CONTINUE
C XB(I)  POINTS WHERE THE BX INTERFACE BOUNDARY CONDITION IS FITTED
      DO 11 I=1,NB1
      READ (*,*) XB(I)
11    CONTINUE
C XTB(I),YTB(I) ARE POINTS WHERE BY=0 IS FITTED IN THE SECOND SOLUTION,
C FOR GOOD FIELD REGION
      DO 15 I=1,NB2
      READ (*,*) XTB(I),YTB(I)
15    CONTINUE
C WA IS THE X CO-ORDINATE USED IN COMMENTED OUT STATEMENT, TYPICALLY =W
      READ (*,*) WA
      JBMUH=1
C      DO 95 IT=1,10
      DO 2 I=1,300
      R(I)=0.0
      DO 3 J=1,300
      A(I,J)=0.0
3     CONTINUE
2     CONTINUE
      DO 20 I=1,NA
      SLA(I)=2.0*PAI*(I)/DL
20    CONTINUE
      DO 25 I=1,NB
      SLB(I)=2.0*PAI*I/(DL)
25    CONTINUE
C      WRITE (*,*) (SLB(I),I=1,NB)
      TMU=1.-1./SMUR
C  ROW 1 OF A(I,J) TO ENFORCE AMPERE TURNS AT X=0.0
      DO 30 J=1,NA
      A(1,J)=-QSIN(SLA(J)*H)
30    CONTINUE
      R(1)=AMPTC*SMU
C  ROW 2 OF A(I,J) TO ENFORCE FLUX CONTINUITY
      DO 40 J=1,NA
      A(2,J)=-QSINH(SLA(J)*W)*(1.0-QCOS(SLA(J)*H))
40    CONTINUE
      R(2)=0.0

C ROW 3 OF A(I,J) TO ENFORCE SENSITIVITY TO DELTAH AT X=WA, OR FORCE
C BUMP THICKNESS
      R(3)=-AMPTC*SMU
      DO 42 J=1,NA
C      A(3,J)=QCOSH(SLA(J)*WA)*(QSIN(SLA(J)*H)+SLA(J)*DB*
C     1QCOS(SLA(J)*H))
      A(3,J)=QCOSH(SLA(J)*W)*QSIN(SLA(J)*H)
42    CONTINUE

C  ROWS 4 TO NA1 TO FIT FIELD ON THE ELLIPSE
```

```
      NA3P=4
      NA3F=3+NA1
      DO 12 I=NA3P,NA3F
      DO 14 J=1,NA
C234567890123456789012345678901234567890123456789012345678901234567890
      A(I,J)=(1.0-QCOSH(SLA(J)*X(I-3))*QCOS(SLA(J)*Y(I-3)))
     1*SLA(J)
14    CONTINUE
12    CONTINUE
      NA4P=NA3F+1
C  ROWS NA4 TO NA TO FIT FIELD
      DO 17 I=NA4P,NA
      IN=I-NA3F
      DO 18 J=1,NA
C234567890123456789012345678901234567890123456789012345678901234567890
      A(I,J)=-SLA(J)*(1.-PAR-QCOSH(SLA(J)*XTA(IN))*QCOS(SLA(J)*YTA(IN)))
18    CONTINUE
      R(I)=0.0
17    CONTINUE
C      DO 65 I=1,NA
C      WRITE (*,*) R(I)
C      WRITE (*,*) (A(I,J),J=1,NA) _
C65    CONTINUE
      CALL MATINP(A,R,NA,RHS)
      WRITE (*,70) (RHS(I),I=1,NA)
70    FORMAT(10(4HRHS=,D15.6))
C     PERTURBING SOLUTION AND CALCULATION OF RHS2(I)
      DO 60 I=1,NB1
      R2(I)=0.0
      BY=0.0
      DO 62 J=1,NA
      R2(I)=R2(I)-RHS(J)*SLA(J)*QSINH(SLA(J)*XB(I))*QSIN(SLA(J)*H)
      BY=BY-RHS(J)*SLA(J)*QCOSH(SLA(J)*XB(I))*QCOS(SLA(J)*H)
62    CONTINUE
      B(I)=(BY**2+R2(I)**2)**0.5
60    CONTINUE
C     WRITE (*,*) 'R2 INITIAL', (R2(I),I=1,NB1)
      DO 64 I=1,NB1
      BI=B(I)
      CALL BMUH(BI,SMU,50,SMUI,HI,JBMUH)
C.     WRITE (*,*)  'MAIN',BI,SMUI,HI
      R2(I)=-(1.0-1.0/SMUI)*R2(I)
      DO 66 J=1,NB
      A(I,J)=-SLB(J)*QSINH(SLB(J)*XB(I))*QSIN(SLB(J)*H)
66    CONTINUE
64    CONTINUE
C      WRITE (*,*) 'R2 FINAL', (R2(I),I=1,NB1)

C  GOOD FIELD REGION
      NB1P=NB1+1
      NB1F=NB1+NB2
      DO 58 I=NB1P,NB1F
      IN=I-NB1
      R2(I)=0.0
      DO 59 J=1,NB
C234567890123456789012345678901234567890123456789012345678901234567890
      A(I,J)=-SLB(J)*(1.0-QCOSH(SLB(J)*XTB(IN))*QCOS(SLB(J)*YTB(IN)))
59    CONTINUE
58    CONTINUE
```

```
C23456789012345678901234567890123456789012345678901234567890123456789012345678901234567890
C ROW NBMM (TO FIX DELTAH AT X=WA)
C       NBMM=NB-6
C       R2(NBMM)=0.0
C       DO 31 J=1,NA
C       R2(NBMM)=R2(NBMM)-RHS(J)*QCOSH(SLA(J)*WA)*QSIN(SLA(J)*H)
C      1-RHS(J)*SLA(J)*DBA*QCOSH(SLA(J)*WA)*QSIN(SLA(J)*H)
C31     CONTINUE
C       DO 32 J=1,NB
C       A(NBMM,J)=QCOSH(SLB(J)*WA)*(QSIN(SLB(J)*H)+SLB(J)*DBA
C      1*QCOS(SLB(J)*H))-QSIN(SLB(J)*H)
C32     CONTINUE
C ROW NBMM TO FORCE FIELD AT (0.0,0.0) TO ZERO
C       NBMM=NB-2
C       R2(NBMM)=0.0
C       DO 41 J=1,NB
C       A(NBMM,J)=-SLB(J)
C41     CONTINUE
C ROW NBMM TO EQUATE AMPERETURNS AT X=0 AND X=GIVEN
C        NBMM=NB-1
C        R2(NBMM)=0.0
C        DO 43 J=1,NA
C        R2(NBMM)=R2(NBMM)-QSIN(SLA(J)*H)*(1.0-QCOS(SLA(J)*0.04))
C43      CONTINUE
C        DO 45 J=1,NB
C        A(NBMM,J)=QSIN(SLB(J)*H)*(1.0-QCOS(SLB(J)*0.04))
C45      CONTINUE
C ROW NBMM TO REDUCE BY AT (W,H) TO ZERO
C       NBMM=NB-2
C       R2(NBMM)=0.0
C       DO 46 J=1,NB
C       A(NBMM,J)=SLB(J)*QCOSH(SLB(J)*W)*QCOS(SLB(J)*H)
C46     CONTINUE
C ROW NBMM TO REDUCE AMPERETURNS AT X=GIVEN TO ZERO
        NBMM=NB-2
        R2(NBMM)=0.0
        DO 69 J=1,NB
        A(NBMM,J)=QCOSH(SLB(J)*0.01)*QSIN(SLB(J)*H)
69      CONTINUE
C ROW NBMM TO REDUCE AMPERETURNS AT X=GIVEN TO ZERO
        NBMM=NB-3
        R2(NBMM)=0.0
        DO 74 J=1,NB
        A(NBMM,J)=QCOSH(SLB(J)*0.02)*QSIN(SLB(J)*H)
74      CONTINUE
C ROW NBMM TO REDUCE AMPERETURNS AT X=GIVEN TO ZERO
        NBMM=NB-4
        R2(NBMM)=0.0
        DO 71 J=1,NB
        A(NBMM,J)=QCOSH(SLB(J)*0.03)*QSIN(SLB(J)*H)
71      CONTINUE
C ROW NBMM TO FORCE BY AT (X,W) TO ZERO
C        NBMM=NB-2
C        R2(NBMM)=0.0
C        DO 73 J=1,NB
C        A(NBMM,J)=SLB(J)*QCOSH(SLB(J)*W)*QCOS(SLB(J)*H)
C73      CONTINUE

C ROW NBMM TO ENFORCE ZERO AMPERE TURNS AT X=0.04
C        NBM=NB-6
```

```
C        R2(NBM)=0.0
C        DO 37 J=1,NB
C        A(NBM,J)=QCOSH(SLB(J)*0.04)*QSIN(SLB(J)*H)
C37      CONTINUE
C ROW NBM TO FORCE BY TO REQUIRED VALUE AT (0.0,0.0)
      NBM=NB-6
      R2(NBM)=0.0
      DO 371 J=1,NB
      A(NBM,J)=-SLB(J)
371   CONTINUE
C        NBM=NB-2
C        R2(NBM)=0.0
C23456789012345678901234567890123456789012345678901234567890123456789012
C IMPOSE DH/DX
C       DO 371 J=1,NA
C       R2(NBM)=R2(NBM)-RHS(J)*SLA(J)*(QSINH(SLA(J)*0.055)*QSIN(SLA(J)*
C      10.0286)-0.01*QCOSH(SLA(J)*0.066)*QCOS(SLA(J)*0.0278))
C371    CONTINUE
C       DO 372 J=1,NB
C       A(NBM,J)=SLB(J)*(QSINH(SLB(J)*0.055)*QSIN(SLB(J)*
C      10.0286)-0.01*QCOSH(SLB(J)*0.066)*QCOS(SLB(J)*0.0278))
C372    CONTINUE
C IMPOSE BUMP THICKNESS
      NBM=NB-5
      R2(NBM)=-AMPTC*SMU
      DO 373 J=1,NA
      R2(NBM)=R2(NBM)-RHS(J)*QCOSH(SLA(J)*0.06)*QSIN(SLA(J)*0.028)
373   CONTINUE
      DO 374 J=1,NB
      A(NBM,J)=QCOSH(SLB(J)*0.06)*QSIN(SLB(J)*0.028)
374   CONTINUE
C2345678901234567890
C ROW NBMM TO ENFORCE REQUIRED AMPERE TURNS AT X=0.0
        NBMM=NB-1
       R2(NBMM)=0.0
       DO 36 J=1,NB
       A(NBMM,J)=-QSIN(SLB(J)*H)
36      CONTINUE
C ROW NBM TO ENSURE ZERO FIELD AT (X=0.0,Y=0.0)
C       NBM=NB-1
C       R2(NBM)=0.0
C       DO 38 J=1,NB
C       A(NBM,J)=-SLB(J)
C38     CONTINUE
C ROW NB TO ENSURE FLUX CONTINUITY
      R2(NB)=0.0
      DO 33 J=1,NB
      A(NB,J)=-QSINH(SLB(J)*W)*(1.0-QCOS(SLB(J)*H))
33    CONTINUE
      DO 690 I=1,NB
      WRITE (*,*) R2(I)
      WRITE (*,*)(A(I,J),J=1,NB)
690    CONTINUE
      DO 691 I=1,NB
      R2(I)=R2(I)/1.Q6
      DO 692 J=1,NB
      A(I,J)=A(I,J)/1.Q6
692   CONTINUE
691   CONTINUE
```

```
      CALL MATINP(A,R2,NB,RHS2)
      WRITE (*,*) (RHS2(I),I=1,NB)


C  TOTAL AMPERE TURNS
      SAMPT=0.0
      DO 90 J=1,NB
      SAMPT=SAMPT-RHS2(J)*QSIN(SLB(J)*H)
 90   CONTINUE
      FAMPT=SMU*AMPTC
      TAMPT=SAMPT+FAMPT
      BYN=0.0
      DO 96 J=1,NA
      BYN=BYN-RHS(J)*SLA(J)
96    CONTINUE
      DO 97 J=1,NB
      BYN=BYN-RHS2(J)*SLB(J)
97    CONTINUE
      WRITE (*,*) 'BYNOT=', BYN
95    CONTINUE
      DO 61 I=1,NB1
      BXB=0.0
      DO 63 J=1,NB
      BXB=BXB-RHS2(J)*SLB(J)*QSINH(SLB(J)*XB(I))*QSIN(SLB(J)*H)
      BXR=BX/(BXB+R2(I))
63    CONTINUE
      WRITE (*,67) R2(I),BXB
67    FORMAT(/,3HBX=,D15.8,2X,4HBXB=,D15.8)
61    CONTINUE

C CALCULATION OF THE COEFFICIENTS CK FOR IRON FOR USE IN RANDOM DIPOLE ERRORS
C REDEFINE R2(I) AND A(I,J)
C       DO 700 I=1,NB1
C       R2(I)=0.0
C234567890123456789012345678901234567890123456789012345678901234567890
C       DO 705 J=1,NB
C       R2(I)=R2(I)-RHS2(J)*SLB(J)*QCOSH(SLB(J)*XB(I))*QCOS(SLB(J)*H)
C705    CONTINUE
C       DO 710 J=1,NB1
C       SLAMK=2.*PAI*J/(DL-H)
C       A(I,J)=-SLAMK*QCOSH(SLAMK*XB(I))
C710    CONTINUE
C700    CONTINUE

      CALL MATINP(A,R2,NB1,RHS3)
      WRITE (*,*) "C(K)'S FOR RANDOM DIPOLE ERRORS"
      WRITE (*,*) (RHS3(I),I=1,NB1)
      OPEN (5,FILE='rand.dat')
      DO 300 J=1,NB1
      WRITE (5,*) RHS3(J)
300   CONTINUE


C  FIELD CALCULATION AND OUTPUT
      KFLAG=1
      JFLAG=0
      YF=0.
75    CONTINUE
      XF=0.0
80    CONTINUE
```

```
      BX=0.
      BY=0.
      DO 81 J=1,NA
      BY=BY-RHS(J)*SLA(J)*QCOSH(SLA(J)*XF)*QCOS(SLA(J)*YF)
      BX=BX-RHS(J)*SLA(J)*QSINH(SLA(J)*XF)*QSIN(SLA(J)*YF)
81    CONTINUE
      BYP=0.0
      BXP=0.0
      DO 82 J=1,NB
      BYP=BYP-RHS2(J)*SLB(J)*QCOSH(SLB(J)*XF)*QCOS(SLB(J)*YF)
      BXP=BXP-RHS2(J)*SLB(J)*QSINH(SLB(J)*XF)*QSIN(SLB(J)*YF)
82    CONTINUE
      IF (KFLAG.EQ.1) BY0=BY
      IF (KFLAG.EQ.1) BYP0=BYP
      BS=(BY0-BY)/BY0
      BS=(BY0+BYP0-BY-BYP)/(BY0+BYP0)
C      WRITE (*,843) XF,YF,BY,BX
843   FORMAT(1X,3HXF=,F7.4,1X,3HYF=,F7.4,1X,3HBY=,D15.6,2X,
     13HBX=,D15.6)
C      WRITE (*,844) XF,YF,BYP,BS
844   FORMAT(1X,3HXF=,F7.4,1X,3HYF=,F7.4,1X,4HBYP=,D15.6,2X,
     13HBS=,D15.6)
      WRITE (*,84) XF,YF,BY,BS,BX
84    FORMAT(1X,3HXF=,F7.4,1X,3HYF=,F7.4,1X,3HBY=,D15.6,2X,
     13HBS=D15.6,1X,3HBX=,D15.6)
      WRITE (*,83) XF,YF,BYP,BXP
83    FORMAT(1X,3HXF=,F7.4,1X,3HYF=,F7.4,1X,4HBYP=,D15.6,2X,
     14HBXP=,D15.6)
      KFLAG=KFLAG+1
      XF=XF+DELX
      IF(XF.LE.XMAX) GO TO 80
      YF=YF+DELY
      IF (YF.LE.YMAX) GO TO 186
      YF=YMAX
      JFLAG=JFLAG+1
186   CONTINUE
      IF (JFLAG.LT.2) GO TO 75

C  CALCULATION OF POLE PROFILE
      NAB=NA+NB
      OPEN (5,FILE='bump.dat')
      XW=W
      DXW=0.002
      K=1
110   CONTINUE
      HNOT=H
      DO 112 L=1,6
      TNR=TAMPT
      DO 100 J=1,NAB
      IF (J.GT.NA) GO TO 102
      TNR=TNR+RHS(J)*QCOSH(SLA(J)*XW)*QSIN(SLA(J)*HNOT)
C23456789012345678901234567890123456789012345678901234567890123456789 0
      GO TO 100
102   TNR=TNR+RHS2(J-NA)*QCOSH(SLB(J-NA)*XW)*QSIN(SLB(J-NA)*HNOT)
100   CONTINUE
      TNRR=TNR-SMU*AMPTCN
      SMPT=SMU*AMPTCN
      WRITE (*,205) XW,TNRR,SMPT
205   FORMAT(/,4X,3HXW=,D15.6,2X,5HTNRR=,D15.8,2X,5HSMPT=,D15.8)
      TDR=0.0
```

```
      DO 104 J=1,NAB
      IF (J.GT.NA) GO TO 106
      TDR=TDR-RHS(J)*SLA(J)*QCOSH(SLA(J)*XW)*QCOS(SLA(J)*HNOT)
      GO TO 104
106   TDR=TDR-RHS2(J-NA)*SLB(J-NA)*QCOSH(SLB(J-NA)*XW)*QCOS(SLB(J-NA)*
     1HNOT)
104   CONTINUE
      DELH=TNR/TDR
      WRITE (*,108) XW,HNOT,DELH
108   FORMAT(/,2X,3HXW=,F10.4,2X,5HHNOT=,F12.6,2X,5HDELH=,D15.6)
      HNOT=HNOT+DELH
112   CONTINUE
      WRITE (5,195) XW,HNOT
195   FORMAT(2D15.8)
      WRITE (*,109)
109   FORMAT(//)
      XW=XW-DXW
      K=K+1
      IF (XW.GT.0.002) GO TO 110

C POLE BUMP BY SECOND METHOD
      XW=0.002
      HNOT=H
      OPEN (5,FILE='bumpa.dat')
209   CONTINUE
      WRITE (5,195) XW,HNOT
      BY=0.0
      BX=0.0
      DO 210 J=1,NA
      BY=BY-RHS(J)*SLA(J)*QCOSH(SLA(J)*XW)*QCOS(SLA(J)*HNOT)
      BX=BX-RHS(J)*SLA(J)*QSINH(SLA(J)*XW)*QSIN(SLA(J)*HNOT)
210   CONTINUE
      DO 211 J=1,NB
      BY=BY-RHS2(J)*SLB(J)*QCOSH(SLB(J)*XW)*QCOS(SLB(J)*HNOT)
      BX=BX-RHS2(J)*SLB(J)*QSINH(SLB(J)*XW)*QSIN(SLB(J)*HNOT)
211   CONTINUE
      DHX=-BX/BY
      XW=XW+DXW
      HNOT=HNOT+DHX*DXW
      IF (XW.LE.W) GO TO 209
200   CONTINUE
      END
      SUBROUTINE MATINP(A,R,IMAX,RHS)
C  THIS IS A MATRIX INVERSION ROUTINE WITH PARTIAL PIVOTING
C A IS THE INPUT SQUARE MATRIX OF DIMN IMAX
C R IS THE RIGHT HAND SIDE VECTOR  RHS(IMAX) IS THE SOLUTION
      IMPLICIT REAL*16 (A-H,O-Z)
      DIMENSION A(300,300),R(300),RHS(300),B(300,300),C(300,300),
     1T(300),RT(300)
C T AND RT ARE SPARE WORKING VECTORS, B ANDC ARE SPARE WORKING ARRAYS
      DO 10 I=1,IMAX
      RT(I)=R(I)
      RHS(I)=0.0
      DO 15 J=1,IMAX
      C(I,J)=A(I,J)
      B(I,J)=0.0
 15   CONTINUE
 10   CONTINUE
      RHS(1)=RT(1)
      DO 12 J=1,IMAX
```

```
      B(1,J)=C(1,J)
 12   CONTINUE
      II=1
 20   CONTINUE
C EVALUATION OF MAX ELEMENT COLUMN II (FROM II TO IMAX)
      IIP=II+1
      TMAX=QABS(C(II,II))
      CMAX=C(I,II)
      IM=II
      DO 25 I=II,IMAX
      EMAX=QABS(C(I,II))
      IF (EMAX.LE.TMAX) GO TO 25
      CMAX=C(I,II)
      IM=I
 25   CONTINUE
C IM IS THE ROW CONTAINING THE MAXIMUM ELEMENT IN COLUMN II
C SWITCH ROWS II AND IM  INCLUDING THE RIGHT HAND SIDE
      DO 30 J=1,IMAX
      T(J)=C(II,J)
      C(II,J)=C(IM,J)
      C(IM,J)=T(J)
 30   CONTINUE
      TR=RT(II)
      RT(II)=RT(IM)
      RT(IM)=TR
C GAUSS REDUCTION
      DO 35 I=IIP,IMAX
      RHS(I)=RT(I)-RT(II)*C(I,II)/C(II,II)
      DO 37 J=1,II
      B(I,J)=0.0
 37   CONTINUE
      DO 40 J=IIP,IMAX
      B(I,J)=C(I,J)-C(II,J)*C(I,II)/C(II,II)
 40   CONTINUE
 35   CONTINUE
C234567890123456789012345678901234567890123456789012345678901234567890
      DO 28 I=1,IMAX
      RT(I)=RHS(I)
      DO 29 J=1,IMAX
      C(I,J)=B(I,J)
 29   CONTINUE
 28   CONTINUE
      II=II+1
      IF (II.EQ.IMAX) GO TO 45
      GO TO 20
 45   CONTINUE
      RHS(IMAX)=RHS(IMAX)/C(IMAX,IMAX)
      DO 50 IN=2,IMAX
      I=IMAX+1-IN
      X=0.0
      IP=I+1
      DO 70 J=IP,IMAX
      X=X+B(I,J)*RHS(J)
 70   CONTINUE
      RHS(I)=(RHS(I)-X)/B(I,I)
 50   CONTINUE
C RHS(I) IS THE SOLUTION
C CHECK
      DO 80 I=1,IMAX
      T(I)=0.0
```

```
      DO 85 J=1,IMAX
      T(I)=T(I)+A(I,J)*RHS(J)
 85   CONTINUE
 80   CONTINUE
      WRITE (*,*) "CHECK"
      WRITE (*,*) (R(I),T(I),I=1,IMAX)
      WRITE (*,*) "SOLUTION"
      WRITE (*,*) (I,RHS(I),I=1,IMAX)
      RETURN
      END

C23456789012345678901234567890123456789012345678901234567890123456789012345678901234567890
      SUBROUTINE BMUH(BFI,SMU,II,SMUI,HFI,JBMUH)
      IMPLICIT REAL*16 (A-H,O-Z)
      DIMENSION BF(50),HF(50),SMUF(50)
C      WRITE (*,*) 'SUB BMUH', BFI,SMU,II
      IF (JBMUH.NE.1) GO TO 20
      OPEN (8,FILE='BMUH.DAT')
      DO  10 I=1,II
      READ (8,*) BF(I),HF(I)
      IF (I.EQ.1) GO TO 10
      SMUF(I)=BF(I)/(HF(I)*SMU)
10    CONTINUE
      SMUF(1)=SMUF(2)
C      DO 12 I=1,II
C      WRITE (*,*) I, BF(I),HF(I),SMUF(I)
C12    CONTINUE
      JBMUH=JBMUH+1
20    CONTINUE
C   INTERPOLATION
      DO 30 I=1,II
      IF (BF(I).GT.BFI) GO TO 32
 30   CONTINUE
32    CONTINUE
C BFI LIES BETWEEN DATA SETS I AND IM=I-1
      IM=I-1
      FRAC=(BFI-BF(IM))/(BF(I)-BF(IM))
      SMUI=SMUF(IM)+(SMUF(I)-SMUF(IM))*FRAC
      HFI=BFI/(SMUI*SMU)
C      WRITE (*,*) I,IM,FRAC
C      WRITE (*,*) SMUI,HFI,JBMUH
      RETURN
      END
```

PROGRAM DERRORA

```
      IMPLICIT REAL*16 (A-H,O-Z)
      DIMENSION A(300,300),R2(300),X(300),RHS2(300),SLA(300),
     1RHS(300),BY(500)
c  READ NUMBER OF TERMS NEEDED
      WRITE (*,40)
 40   FORMAT(/,8X,'INPUT N - THE NUMBER OF COEFFICIENTS NEEDED',/)
      READ (*,*) N
      WRITE (*,*) N
C  READ NUMBER OF POINTS AND ANGLE
      WRITE (*,52)
 52   FORMAT(/,8X,'INPUT M AND THETA -NO. OF POINTS AND ANGLE
     10 TO 360 DEGREES',/)
      READ (*,*) M,THETA
      WRITE (*,*) M,THETA
C  READ AND WRITE RADIUS 'R' AT WHICH POTENTIALS ARE EVALUATED
C  AND RADIUS OF NORMALIZATION 'RN'.
      WRITE (*,53)
 53   FORMAT(/,4X,'INPUT R, RN - RADIUS AT WHICH POTENTIALS ARE')
      WRITE (*,55)
 55   FORMAT(4X, 'EVALUATED AND RADIUS OF NORMALIZATION')
      READ (*,*) R,RN
      WRITE (*,*) R,RN
C  READ AND WRITE DELTA,KB (POLE FACE PERTURBATION PARAMETERS)
C  READ AND WRITE H , W AND DL (HALF POLE GP AND POLE HALF-WIDTH,TOTAL LENGTH)
C23456789012345678912345678901234567890123456789012345678901234567890123456789012
      WRITE (*,64)
64    FORMAT(/,8X,'INPUT DELTA,CKB (POLE FACE PERTURBATION PARAMETERS)')
      READ (*,*) DELTA,CKB
      WRITE (*,*) DELTA,CKB
      WRITE (*,66)
66    FORMAT(/,8X,'INPUT H,W,DL,AMPTC  POLE HALF DEPTH, HALF WIDTH,
     1AMPERETURNS C')
      READ (*,*) H,W,DL,AMPTC
      WRITE (*,*) H,W,DL,AMPTC

C   COMPUTE POTENTIALS BY

      PAI=2.0*QASIN(1.Q0)
      CONV=PAI/180.0
      DTHETA=THETA*CONV/(M-1)
      SMU=PAI*4.0Q-7
      WRITE (*,56)
56    FORMAT(/,4X,'INPUT JX (NO. OF X S WHERE DELTA AZ IS TO BE FITTED')
      READ (*,*) JX
      WRITE (*,*) JX
      WRITE (*,57)
57    FORMAT(/,4X,'INPUT X S, JX NOS. AT Y=H')
      READ (*,*) (X(I), I=1,JX)
      WRITE (*,*) (X(I),I=1,JX)

      DO 75 I=1,JX
      SLA(I)=2.*PAI*I/DL
75    CONTINUE
      DO 201 I=1,JX
      R2(I)=AMPTC*SMU
      DO 203 J=1,JX
      A(I,J)=-QCOSH(SLA(J)*X(I))*QSIN(SLA(J)*H)
203   CONTINUE
201   CONTINUE
```

```
      CALL MATINP(A,R2,JX,RHS)

C     CALCULATION OF RHS2 FOR PERTURBED POTENTIAL
      DO 205 I=1,JX
      R2(I)=AMPTC*SMU
      HP=H+DELTA*QSIN(CKB*PAI*X(I)/W)
      DO 207 J=1,JX
      A(I,J)=-QCOSH(SLA(J)*X(I))*QSIN(SLA(J)*HP)
207   CONTINUE
205   CONTINUE
      CALL MATINP(A,R2,JX,RHS2)

      DO 209 I=1,JX
      CO=RHS2(I)-RHS(I)
      WRITE (*,*) I,CO
209   CONTINUE

C COMPUTATION OF PERTURBED POTENTIALS
      BETA=0.0
      DO 76 I=1,M
      XP=R*QCOS(BETA)
      YP=R*QSIN(BETA)
      SUM=0.0
      DO 78 J=1,JX
      SUM=SUM+RHS2(J)*QSINH(SLA(J)*XP)*QCOS(SLA(J)*YP)
78    CONTINUE
      BY(I)=SUM
      WRITE (*,*) I,XP,YP,BY(I)
      BETA=BETA+DTHETA
76    CONTINUE
C      GO TO 200

C  MIRROR IMAGES OF INPUT DATA
      MP=M-1
      LC=360./THETA
      WRITE (*,30) LC
 30   FORMAT(//,8X,3HLC=,I3)
      IF (LC.EQ.1) GO TO 34
      DO 28 L=2,LC
      IFLAG=(M-1)*(L-1)+1
      MBY=1
      BFLAG=QABS(BY(IFLAG))
      IF(BFLAG.LT.1.Q-8) MBY=-1
      DO 26 I=1,MP
      LA=L-2
      LB=(L-1)*M
      BY(I+LB-LA)=MBY*BY(LB-LA-I)
 26   CONTINUE
 28   CONTINUE
      K=MP+LB-LA
 34   IF (LC.EQ.1) K=M
      WRITE (*,32) K
 32   FORMAT (/,4X,'NUMBER OF INTERPOLATED POINTS=',I3)
      DO 12 I=1,K
      WRITE (*,14) I,BY(I)
 12   CONTINUE
 14   FORMAT(/,5X,I3,5X,9HB OR POT=,D15.9)
      FACM=1.0/(2.*PAI)
      WRITE (*,44) R,RN
 44   FORMAT(/,18X,'COEFFICIENTS EVALUATED WITH R=',F 10.5,3X,
```

```
     1'AND RN=',F10.5)
      WRITE (*,38)
 38   FORMAT(/,25X,' HARMONIC COEFFICIENTS   ',/)
      WRITE (*,39)
 39   FORMAT(/,10X,'FOR POISSON POTENTIAL',25X,'FOR CDR FIELD',/)
      KM=K-1
      DO 18 L=1,N
      SUM=0.0
      SUMS=0.0
      LM=L-1
      FLM=FLOAT(LM)
      DO 16 I=1,KM
      ALPHA=(I-1)*DTHETA
      ALB=ALPHA+DTHETA
      APR=(BY(I+1)-BY(I))/DTHETA
      IF (LM.NE.0) GO TO 60
      SUM=SUM+(BY(I+1)+BY(I))*DTHETA/(2.0*FACM)
      SUMS=0.
      GO TO 16
60    CONTINUE
      SUM=SUM+(BY(I)-APR*ALPHA)*(QSIN(FLM*ALB)-QSIN(FLM*ALPHA))/FLM
     1+(APR/FLM)*(ALB*QSIN(FLM*ALB)-ALPHA*QSIN(FLM*ALPHA))+(APR/
     2FLM**2)*(QCOS(FLM*ALB)-QCOS(FLM*ALPHA))
C234567890123456789012345678901234567890123456789012345678901234567890
      SUMS=SUMS+(BY(I)-APR*ALPHA)*(QCOS(FLM*ALB)-QCOS(FLM*ALPHA))/FLM
     1+(APR/FLM)*(ALB*QCOS(FLM*ALB)-ALPHA*QCOS(FLM*ALPHA))-(APR/
     2FLM**2)*(QSIN(FLM*ALB)-QSIN(FLM*ALPHA))
 16   CONTINUE
      FACA=(RN/R)**FLM
      SUM=SUM*FACA/PAI
      SUMS=-SUMS*FACA/PAI
      WRITE (*,20) LM,SUM,SUMS
 20   FORMAT (/,8X,2HN=,I3,4X,5HA(N)=,E10.4,4X,5HB(N)=,E10.4)
      IF (L.EQ.1.OR.L.EQ.N) GO TO 18
      FACB=1.0/RN**(FLM)
      SUMB=-LM*SUM*1.0Q-4/FACA
      SUMC=-LM*SUMS*1.0Q-4/FACA
      LS=L-2
      WRITE(*,46) LS,SUMB,SUMC
 46   FORMAT(/,40X,2HN=,I3,2X,5HB(N)=,E10.4,2X,5HA(N)=,E10.4)
 18   CONTINUE
200   CONTINUE
      END

      SUBROUTINE MATINP(A,R,IMAX,RHS)
C  THIS IS A MATRIX INVERSION ROUTINE WITH PARTIAL PIVOTING
C A IS THE INPUT SQUARE MATRIX OF DIMN IMAX
C R IS THE RIGHT HAND SIDE VECTOR  RHS(IMAX) IS THE SOLUTION
      IMPLICIT REAL*16 (A-H,O-Z)
      DIMENSION A(300,300),R(300),RHS(300),B(300,300),C(300,300),
     1T(300),RT(300)
C T AND RT ARE SPARE WORKING VECTORS, B ANDC ARE SPARE WORKING ARRAYS
      DO 10 I=1,IMAX
      RT(I)=R(I)
      RHS(I)=0.0
      DO 15 J=1,IMAX
      C(I,J)=A(I,J)
      B(I,J)=0.0
 15   CONTINUE
 10   CONTINUE
```

```
      RHS(1)=RT(1)
      DO 12 J=1,IMAX
      B(1,J)=C(1,J)
 12   CONTINUE
      II=1
 20   CONTINUE
C EVALUATION OF MAX ELEMENT COLUMN II (FROM II TO IMAX)
      IIP=II+1
      TMAX=QABS(C(II,II))
      CMAX=C(I,II)
      IM=II
      DO 25 I=II,IMAX
      EMAX=QABS(C(I,II))
      IF (EMAX.LE.TMAX) GO TO 25
      CMAX=C(I,II)
      IM=I
 25   CONTINUE
C IM IS THE ROW CONTAINING THE MAXIMUM ELEMENT IN COLUMN II
C SWITCH ROWS II AND IM  INCLUDING THE RIGHT HAND SIDE
      DO 30 J=1,IMAX
      T(J)=C(II,J)
      C(II,J)=C(IM,J)
      C(IM,J)=T(J)
 30   CONTINUE
      TR=RT(II)
      RT(II)=RT(IM)
      RT(IM)=TR
C GAUSS REDUCTION
      DO 35 I=IIP,IMAX
      RHS(I)=RT(I)-RT(II)*C(I,II)/C(II,II)
      DO 37 J=1,II
      B(I,J)=0.0
 37   CONTINUE
      DO 40 J=IIP,IMAX
      B(I,J)=C(I,J)-C(II,J)*C(I,II)/C(II,II)
 40   CONTINUE
 35   CONTINUE
C23456789012345678901234567890123456789012345678901234567890123456789012345678901234567890
      DO 28 I=1,IMAX
      RT(I)=RHS(I)
      DO 29 J=1,IMAX
      C(I,J)=B(I,J)
 29   CONTINUE
 28   CONTINUE
      II=II+1
      IF (II.EQ.IMAX) GO TO 45
      GO TO 20
 45   CONTINUE
      RHS(IMAX)=RHS(IMAX)/C(IMAX,IMAX)
      DO 50 IN=2,IMAX
      I=IMAX+1-IN
      X=0.0
      IP=I+1
      DO 70 J=IP,IMAX
      X=X+B(I,J)*RHS(J)
 70   CONTINUE
      RHS(I)=(RHS(I)-X)/B(I,I)
 50   CONTINUE
C RHS(I) IS THE SOLUTION
C CHECK
```

```
      DO 80 I=1,IMAX
      T(I)=0.0
      DO 85 J=1,IMAX
      T(I)=T(I)+A(I,J)*RHS(J)
85    CONTINUE
80    CONTINUE
      WRITE (*,*) "CHECK"
      WRITE (*,*) (R(I),T(I),I=1,IMAX)
      WRITE (*,*) "SOLUTION"
      WRITE (*,*) (I,RHS(I),I=1,IMAX)
      RETURN
      END
```