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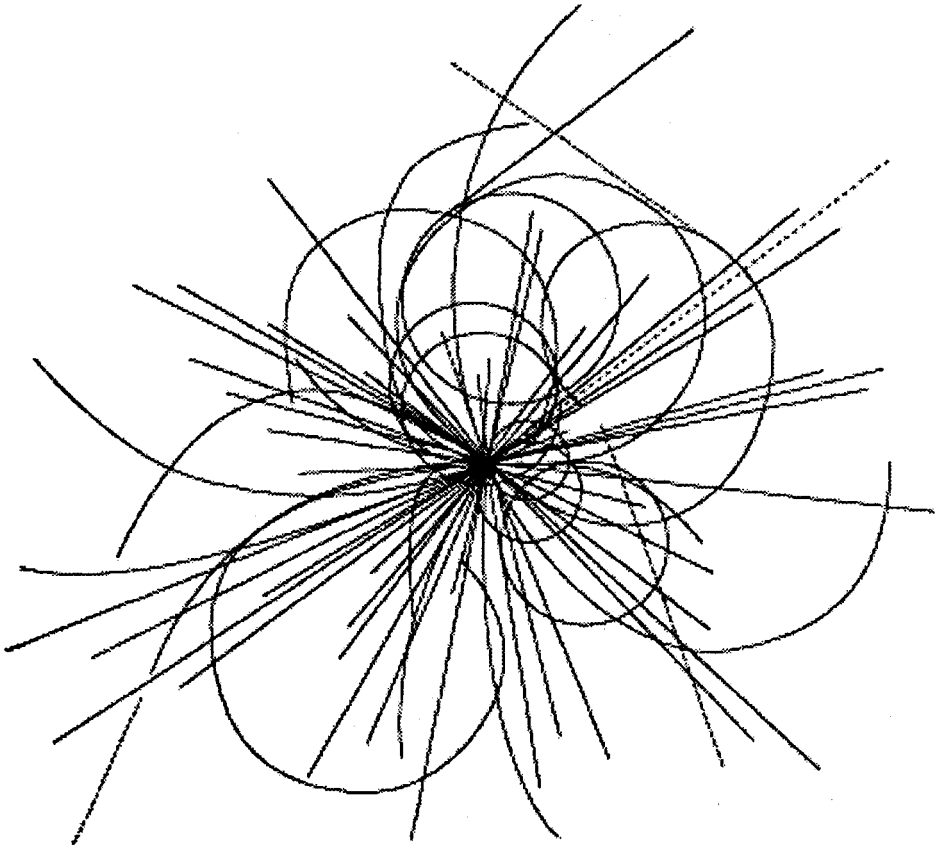

Superconducting Super Collider Laboratory

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Resonance Excitation of the SSC Beam Halo by RF Voltage Pulses

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Resonance Excitation of the SSC Beam Halo by RF Voltage Pulses

E. Tsyganov, H.-J. Shih, and A. Taratin

Abstract

We show by computer simulations that perturbation pulses in the main rf voltage, if in resonance with the synchrotron oscillations of beam halo particles, can effectively eject the particles out of the bucket without perturbing the beam core. This method can be used to place the beam halo particles onto a bent crystal for extraction from the Superconducting Super Collider.

1.0 INTRODUCTION

It has been proposed[1] to extract 20-TeV protons from the Superconducting Super Collider (SSC) using a bent crystal for fixed-target B-physics experiments. To make this extraction work, a scheme must be provided to gradually bring protons from the edge of the circulating beam onto the bent crystal. Amplitude perturbation $a(t)$ or phase perturbation $\varphi(t)$, including noise, in the main rf voltage with frequency f_{rf} and peak voltage V_0,

$$V(t) = V_0\big(1 + a(t)\big)\sin\big(2\pi f_{rf}t + \varphi(t)\big), \tag{1}$$

has been considered for diffusing beam halo particles in longitudinal phase space, which can then be intercepted by the crystal,[2–4] provided that the crystal is located in a high-dispersion region. Here we consider an amplitude perturbation in which additional voltage pulses in resonance with the synchrotron oscillations of beam halo particles are introduced to drive the particles out of the bucket. Suppose the driven halo particles are initially located in the δ-l phase space about $(\delta_p,\ l_p)$. Here $\delta = \Delta p/p_s$ is the relative momentum deviation from the synchronous momentum p_s, and l is the longitudinal displacement from the bunch center. Each time the driven halo particles complete one synchrotron oscillation the perturbing pulses are switched on with the sign of δ_p. Thus the perturbed halo particles jump onto an orbit of larger amplitude after each synchrotron oscillation. One can also consider switching on the perturbing pulses every half-synchrotron-oscillation, with their signs alternating, depending on the sign of relative momentum deviation of the driven particles. However, this will not be considered in this paper. Figure 1(a) illustrates the effect of one of the perturbing pulses in the δ-l space on the trajectory of the driven halo particles. The rf phase interval $\Delta\phi_p$, in which the perturbing pulses are turned on, is related to the pulse duration T_p by $\Delta\phi_p = 2\pi(T_p/T_{rf})$, where $T_{rf} = 1/f_{rf}$ is the oscillation period of the rf voltage. The center of this interval, ϕ_p, is determined from l_p by the relationship

$$\phi = \phi_s + \frac{2\pi h}{C_s}l, \tag{2}$$

where ϕ_s is the synchronous phase, C_s the circumference, and h the harmonic number. With respect to Figure 1(a), ϕ_p and $\Delta\phi_p$ are shown in Figure 1(b).

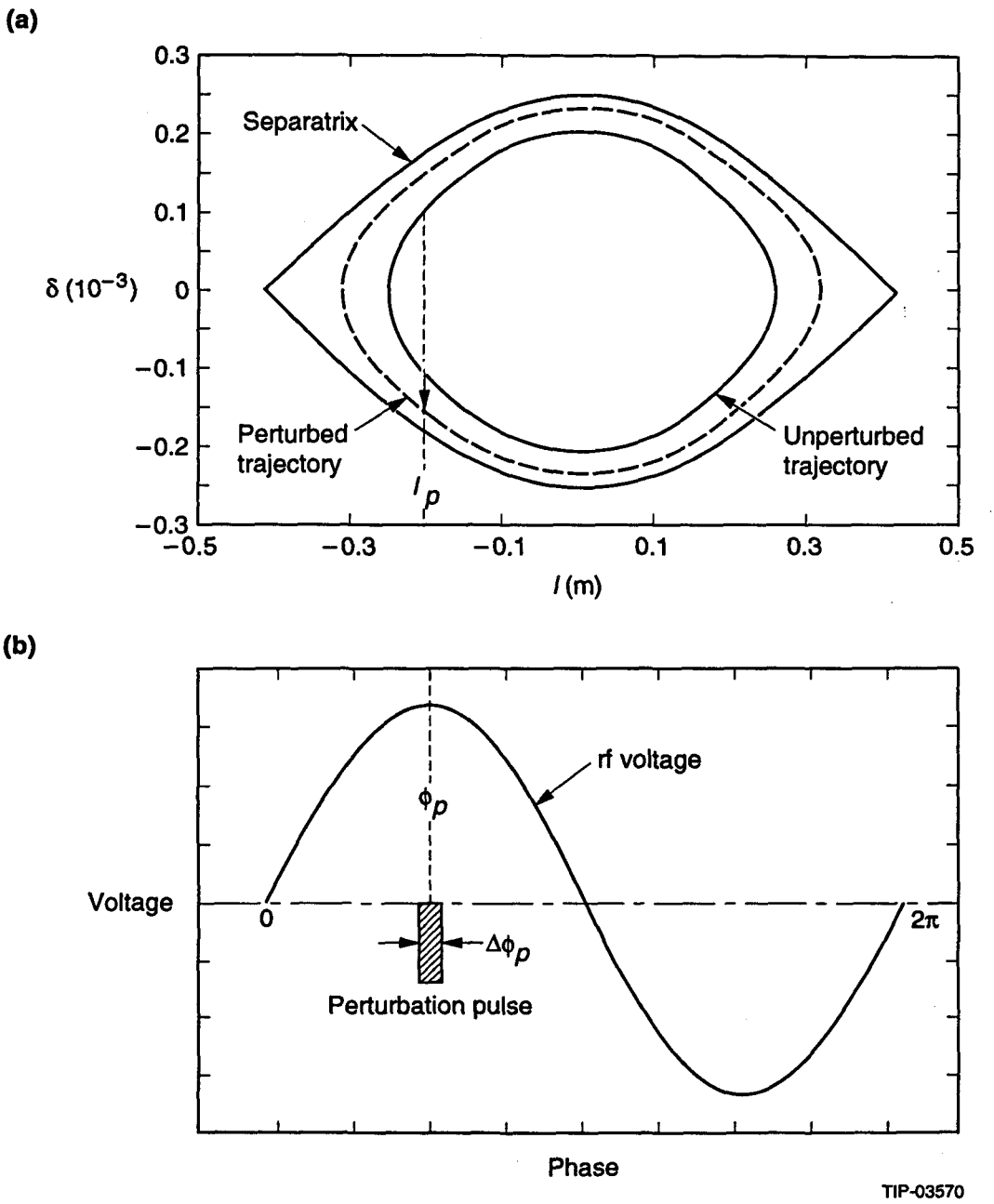

Figure 1. Schematic Picture of the Process of Ejecting Beam Halo Particles by Means of Synchronized rf Voltage Pulses. (a) Synchrotron orbits in the δ-l phase space before and after the action of a perturbation pulse. (b) Sinusoidal rf voltage and the position in rf phase of perturbation pulses. ϕ_p and l_p are related by Eq. (2).

Because the rf voltage varies sinusoidally in time, the period of a synchrotron oscillation depends on the amplitude of the oscillation. This is particularly true for beam halo particles, which have larger oscillation amplitudes. In fact, the period of a synchrotron oscillation increases as its amplitude increases. Thus, to increase monotonically the synchrotron amplitude of driven halo particles, the time interval between two successive perturbing pulses has to follow the increase of the synchrotron period. Figure 2 shows the dependence of the synchrotron period T_s on the synchrotron amplitude $\hat{\delta}$ for a stationary bucket at the SSC ($\phi_s = \pi$). In terms of the synchrotron amplitude $\hat{\psi}$, where $\psi = \phi - \phi_s$, T_s and $\hat{\delta}$ are given by[5]

$$T_s(\hat{\psi}) = T_{so}\ \frac{2}{\pi}\ K\left(\sin\frac{\hat{\psi}}{2}\right), \tag{3}$$

$$\hat{\delta}(\hat{\psi}) = \left(\frac{1}{\pi|\eta|h}\ \frac{eV}{E_s}\right)^{1/2} (1-\cos\hat{\psi})^{1/2}, \tag{4}$$

$$T_{so} = T_o\ \left(\frac{2\pi}{|\eta|h}\ \frac{E_s}{eV_0}\right)^{1/2}. \tag{5}$$

Here T_{so} is the period of small-amplitude synchrotron oscillations, T_o the revolution period of a synchronous particle, K the complete elliptic integral of the first kind, η the slip factor, and E_s the synchronous energy. At the SSC, $C_s = 87\,120$ m, $h = 104\,544$, $\eta \approx \alpha$ (momentum compaction) $= 9.1 \times 10^{-5}$, $V_0 = 20$ MV, $E_s = 20$ TeV, $T_o = 2.9 \times 10^{-4}$ s, and $f_{so} = 1/T_{so} = 4.23$ Hz.

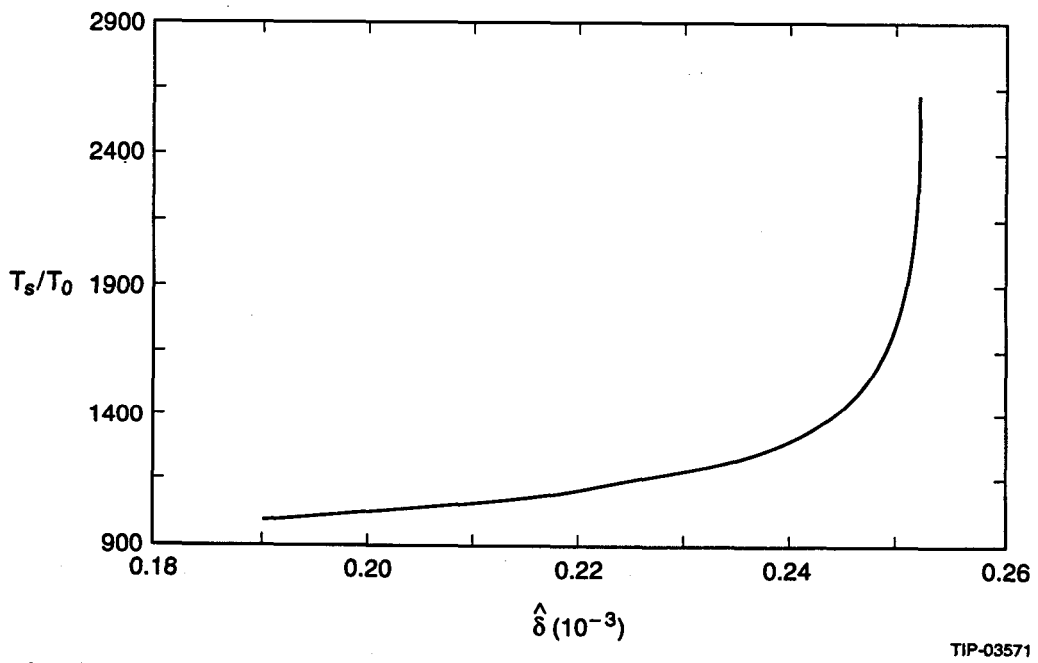

Figure 2. Synchrotron Oscillation Period, T_s, at the SSC as a Function of Relative Momentum Deviation Amplitude, $\hat{\delta}$, in Units of the Revolution Period, T_0.

Each time the perturbing pulses are turned on, not only are the driven halo particles that are initially near (l_p, δ_p) perturbed, but all other particles having longitudinal dispacements near l_p are perturbed as well. However, over a long time scale, the perturbations on the latter particles are largely compensated. To act on halo particles only, the perturbing pulses need to be short compared to the rf period. In addition, for synchronization to be efficient, the initial region in longitudinal phase space of the driven halo particles must be small. This also requires the perturbing pulses to be short. At the SSC, $T_{rf} = T_0/h = 2.78$ ns. Therefore, we may restrict the pulse duration to be less than, say, 300 ps.

In order to drive halo particles only, the amplitude of the synchrotron orbit passing through (l_p, δ_p) should be greater than a certain value. At the SSC, the bucket half-width is $l_w = C_s/2h = 0.42$ m, and the bucket half-height is $\delta_h = (2eV_0/\pi\alpha h E_s)^{1/2} = 0.252 \times 10^{-3}$. In this study, we choose to drive the particles with $\hat{\psi} > \pi/2$. That is, the particles with $\hat{l} > l_w/2 = 0.21$ m, $\hat{\delta} > \delta_h/\sqrt{2} = 0.18 \times 10^{-3}$, or synchrotron frequency $f_s < \pi f_{so}/2K(\sqrt{2}/2) = 0.85 f_{so}$.

2.0 SINGLE RESONANCE SEQUENCE

We have considered switching on one perturbation pulse only when the driven halo particles complete a synchrotron oscillation. Thus, the time sequence of perturbing pulses is determined by the initial location (l_p, δ_p) of the driven halo particles and the pulse voltage V_p. This sequence will be called a single resonance sequence, and the turn number when the k-th perturbing pulse is turned on will be denoted by $N_p(k)$. Using computer simulation $N_p(k)$ was determined as follows. The finite-difference equations for a stationary bucket with a sinusoidal rf voltage were used to calculate unperturbed particle trajectories in the δ-l space:

$$\delta_{n+1} - \delta_n = \frac{eV}{E_s} \sin\phi_n, \tag{6}$$

$$l_{n+1} - l_n = \alpha C_s \delta_{n+1}, \tag{7}$$

where n refers to the turn number, and the variables ϕ and l are related by Eq. (2). When the longitudinal displacement of the driven particle was found within a small interval of $\Delta l_p/2$ from l_p with the sign of the initial momentum deviation δ_p, an additional increase in particle momentum, $\Delta\delta = eV_p/E_s$, was given. At this moment, the pulse number k, the turn number N_p, and the momentum deviation amplitude $\hat{\delta}$ resulting from the pulse action were registered. Figure 3(a) shows schematically the initial part of the sequence $N_p(k)$. As determined by computer simulation, the sequence $N_p(k)$ may also depend on the pulse duration T_p. In this study, we choose $\Delta l_p = 0.002$ m in determining a single resonance sequence so that the pulse duration is $T_p = \Delta l_p/c = 6.67$ ps, where c is the speed of light.

Throughout the paper, the initial location (l_p, δ_p) of halo particles to be driven by a single resonance sequence is $l_p = -0.215$ m, $\delta_p < 0$, $\hat{\delta}_p = 0.19 \times 10^{-3}$. Here $\hat{\delta}_p$ is the momentum deviation amplitude of the synchrotron orbit passing through (l_p, δ_p).

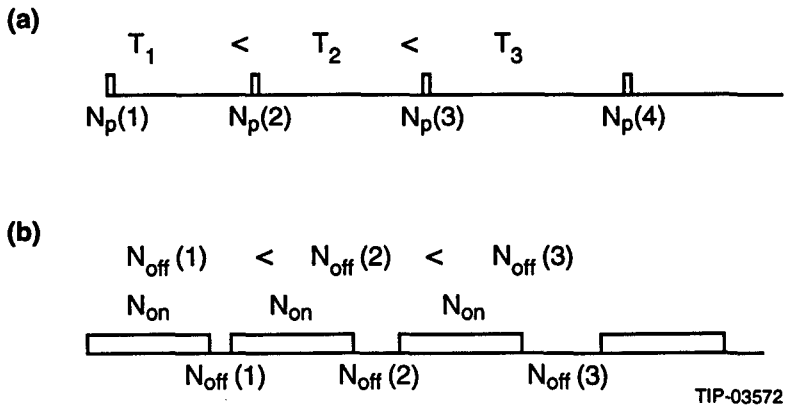

Figure 3. Schematic Representation of the Initial Part of a Perturbation Pulse Sequence. (a) Single resonance sequence. T_i is the time interval between pulses i and $i+1$. (b) Multiple resonance sequence. $N_{\text{off}}(i)$ is the number of turns without pulse action before the pulse at turn $N_p(i+1)$.

Figure 4 shows the growth of the synchrotron amplitude $\hat{\delta}$ of a halo particle driven by a single resonance sequence. The growth of the synchrotron amplitude $\hat{\delta}$ is nearly linear. The increase of momentum deviation amplitude $\Delta\hat{\delta}$ due to a single pulse action is given by

$$\Delta\hat{\delta} = \sqrt{\Delta\delta^2 + 2\Delta\delta \cdot \delta + \hat{\delta}^2} - \hat{\delta}. \tag{8}$$

Here $\delta = \delta(\psi_p, \hat{\delta})$ is the particle momentum deviation before the pulse action given by

$$\delta = -\sqrt{\hat{\delta}^2 - \delta_h^2 \sin^2\left(\frac{\psi_p}{2}\right)}, \tag{9}$$

where $\psi_p = \phi_p - \phi_s$. Figure 5 shows that the momentum amplitude increase $\Delta\hat{\delta}$ resulting from one pulse action increases as the momentum amplitude $\hat{\delta}$ increases. In light of a similar behavior of the synchrotron period, it is then possible to obtain a nearly linear growth of the synchrotron amplitude $\hat{\delta}$ in time.

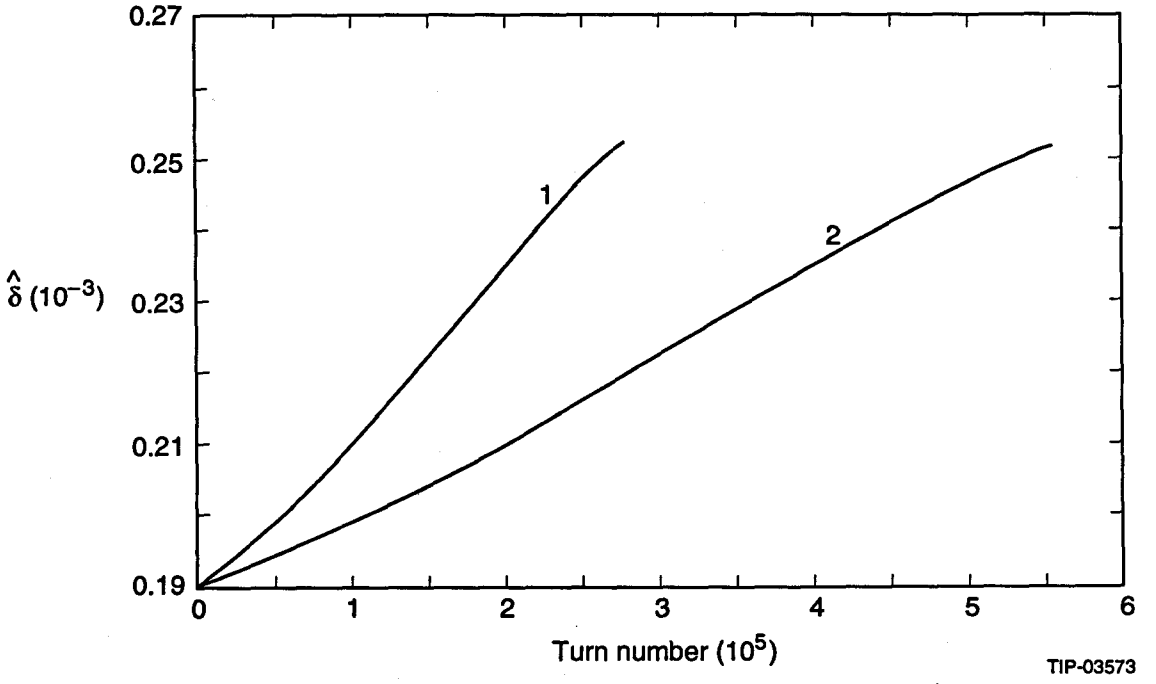

Figure 4. Relative Momentum Deviation Amplitude, $\hat{\delta}$, as a Function of Turn Number for a Halo Particle Driven by a Single Resonance Sequence. The initial location of the driven halo particle is $l_p = -0.215$ m, $\delta_p < 0$, $\hat{\delta}_p = 0.19 \times 10^{-3}$. The pulse voltage is $V_p = 10$ MV (curve 1) and $V_p = 5$ MV (curve 2).

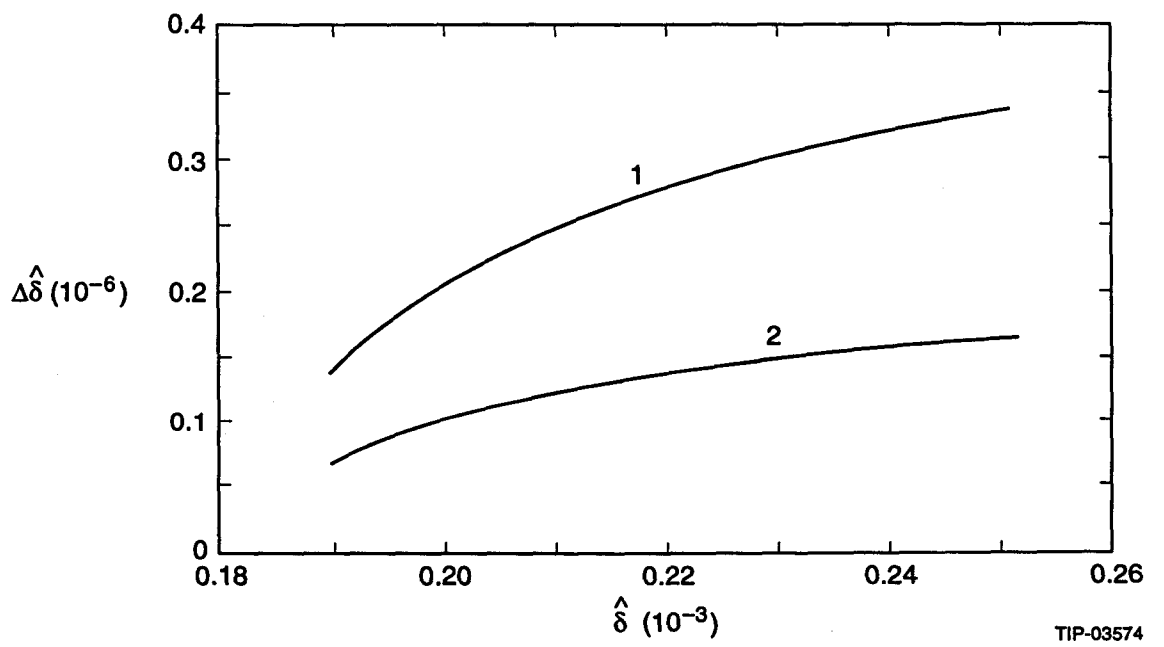

Figure 5. Increase in Relative Momentum Deviation Amplitude, $\Delta\hat{\delta}$, as a Function of Relative Momentum Deviation Amplitude, $\hat{\delta}$, for a Halo Particle Driven by a Single Resonance Sequence. The pulse voltage is $V_p = 10$ MV (curve 1) and $V_p = 5$ MV (curve 2). The other parameters are the same as in Figure 4.

3.0 MULTIPLE RESONANCE SEQUENCE

As discussed in Section 2.0, a single resonance sequence determined by the phase space point (l_p, δ_p) captures and ejects only the particles whose initial phase space locations are near that point. These particles are only a small fraction of all the particles near the synchrotron orbit passing through (l_p, δ_p). If the pulse duration is lengthened, more particles will be perturbed, but because most of the perturbed particles do not have synchrotron oscillations synchronized with the perturbing pulses, the number of ejected particles increases insignificantly.

To eject significantly more of the particles near the synchrotron orbit passing through (l_p, δ_p), we consider repeating the pulse sequence obtained in Section 2.0 for a number of subsequent turns. That is, if a single resonance sequence for driving the halo particles near (l_p, δ_p) consists of perturbing pulses at turns $N_p(1), N_p(2), N_p(3), \cdots$, the resulting sequence by repeating the pulse action for N_{on} subsequent turns is

$$\begin{aligned} &N_p(1), N_p(1)+1, N_p(1)+2, \cdots, N_p(1)+N_{on}, \\ &N_p(2), N_p(2)+1, N_p(2)+2, \cdots, N_p(2)+N_{on}, \\ &N_p(3), N_p(3)+1, N_p(3)+2, \cdots, N_p(3)+N_{on}, \\ &\cdots \end{aligned} \tag{10}$$

One can see that with such a sequence of perturbing pulses, the particles initially near (l_p, δ_p) will, as before, resonate with the pulses at turns $N_p(1), N_p(2), N_p(3), \cdots$, the particles that are initially a bit upstream of (l_p, δ_p) along the synchrotron orbit passing through (l_p, δ_p) will resonate with the pulses at turns $N_p(1)+1, N_p(2)+1, N_p(3)+1, \cdots$, the particles a bit further upstream of (l_p, δ_p) along the same orbit will resonate with the pulses at turns $N_p(1)+2, N_p(2)+2, N_p(3)+2, \cdots$, and so on. Because of the nature of multiple resonance it possesses, the new sequence (10) will be called a multiple resonance sequence. The initial part of it is shown schematically in Figure 3(b).

The number of turns for repeating the pulse action, N_{on}, should be less than the number of turns for the halo particle at $(l_p, -\delta_p)$ to move along its unperturbed orbit to (l_p, δ_p). Otherwise, compensation of the pulse action will occur for those particles that are initially upstream of $(l_p, -\delta_p)$ along the unperturbed orbit. For $l_p < 0$ and $\delta_p < 0$, or $l_p > 0$ and $\delta_p > 0$, we can have that $N_{on} > T_s(\hat{\delta}_p)/2T_o$. So, it appears that we can eject a substantial portion of the halo particles with $\hat{\delta}$ near $\hat{\delta}_p$ by means of a multiple resonance sequence.

Before we turn to the ejection efficiency of halo particles by a multiple resonance sequence, to be discussed in Section 4.0, it is interesting to note how the synchrotron amplitude of a halo particle initially at (l_p, δ_p) grows under the action of a multiple resonance sequence constructed from a single resonance sequence determined by (l_p, δ_p) and a pulse voltage. Figure 6 shows the growth of momentum deviation amplitude $\hat{\delta}$ due to a multiple resonance sequence. Again, the phase space trajectories of the halo particle were calculated using the finite-difference equations (6) and (7). The perturbing pulse would act on the particle when its phase turned out to be near ϕ_p in the interval of $\Delta\phi_p$ and when the particle turn number is in the range from $N_p(k)$ to $N_p(k) + N_{on}$. Also shown in Figure 6 is the amplitude growth due to the single resonance sequence from which the multiple resonance sequence under consideration is built. In contrast to the nearly linear growth in the single resonance case, the growth of the synchrotron amplitude is not monotonous in the multiple resonance case. For a small pulse duration (Figure 6(a)), short intervals where the amplitude grows alternate with long intervals where it does not change practically. For a large pulse duration (Figure 6(b)), in addition to the intervals of amplitude growth and intervals of constant amplitude, there are short intervals where the amplitude decreases. In any case, there is a growing trend for the synchrotron amplitude, and the average rate of the amplitude growth appears to follow the growth rate in the single resonance case.

The nonmonotonous amplitude growth in the multiple resonance case occurs because the turn numbers at which the driven halo particle comes back to the original phase ϕ_p within the interval $\Delta\phi_p$ do not coincide with the turns $N_p(1), N_p(2), N_p(3), \ldots$. The coincidence is violated because, as explained below, the driven halo particle under a multiple resonance sequence will be acted upon many times between the turns $N_p(k)$ and $N_p(k+1)$. The change of the time displacement t_r from the bunch center after one turn is given by $\Delta t_r = (T_{rf}/2\pi)\Delta\psi = hT_{rf}\alpha\delta$. Given the phase of the pulse action ϕ_p and the considered interval of momentum deviation amplitude, $\hat{\delta} = (0.19\text{–}0.25) \times 10^{-3}$, we have $\Delta t_r = (1.5\text{–}5)$ ps. Therefore, under a multiple resonance sequence, the number of times the driven particle is acted upon between the turns $N_p(k)$ and $N_p(k+1)$ is on the order of $T_p/\Delta t_r$. This is true even for very short pulses, $T_p \ll T_{rf}/4$.

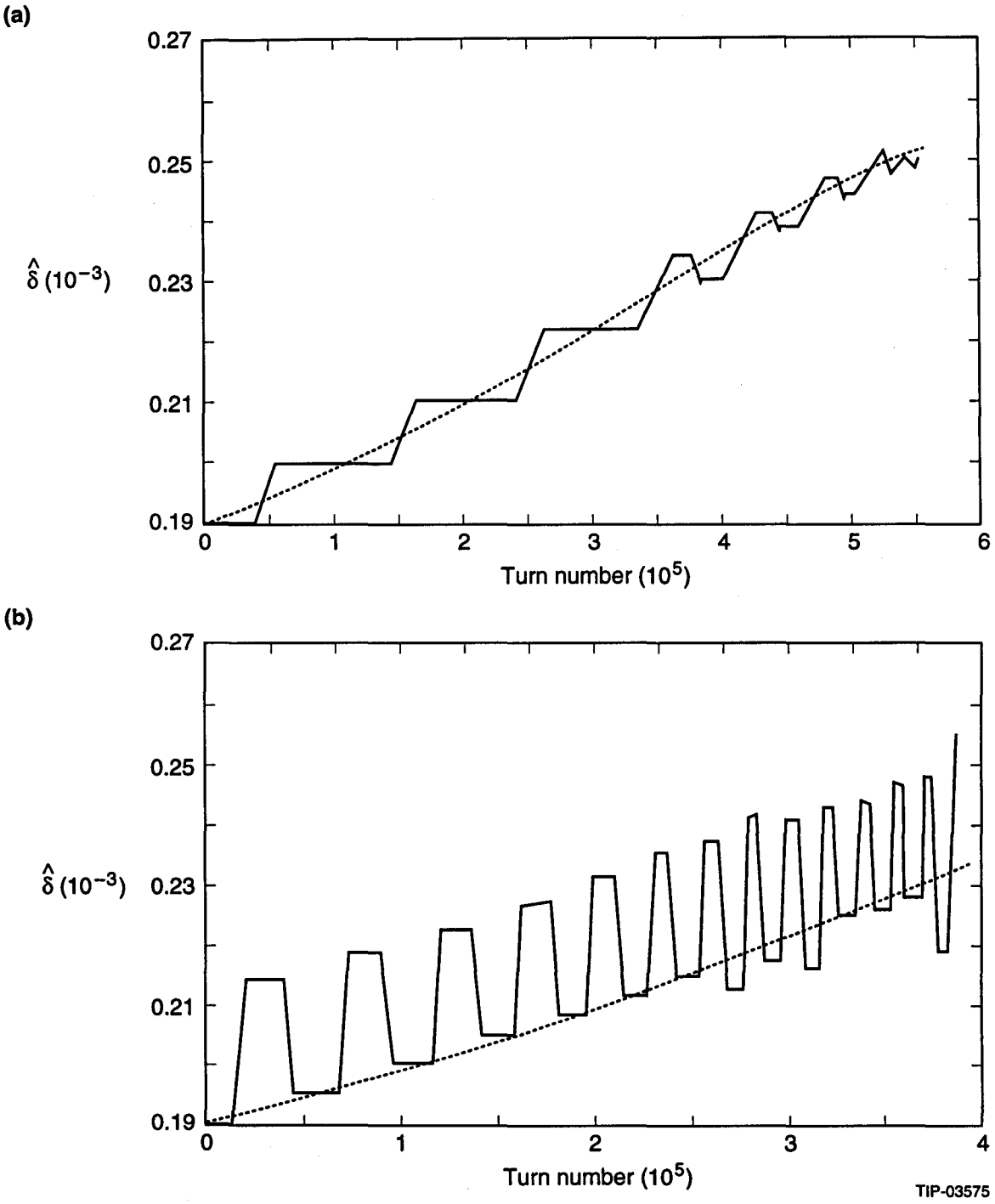

Figure 6. Relative Momentum Deviation Amplitude, $\hat{\delta}$, as a Function of Turn Number for a Halo Particle Driven by a Multiple Resonance Sequence. The pulse amplitude is $V_p = 5$ MV. Curve 2 in Figure 4 is shown here in dots. (a) The pulse duration is $T_p = 13.3$ ps (short pulses). (b) The pulse duration is $T_p = 267$ ps (long pulses).

4.0 EJECTION EFFICIENCY OF HALO PARTICLES

Consider a bunch of halo particles that are distributed uniformly in the longitudinal displacement from $-\hat{l}_p$ to $\hat{l}_p$ and in the momentum deviation amplitude from $\hat{\delta}_p - \Delta$ to $\hat{\delta}_p + \Delta$, where $\hat{l}_p$ and $\hat{\delta}_p$ are the amplitudes of the synchrotron orbit passing through the point (l_p, δ_p) and $\Delta = 10^{-6}$. One half of the halo particles have positive momentum deviation, and the other half negative. In our simulations, the initial total number of particles in the bunch is 1000. Let us now apply to the bunch a multiple resonance sequence determined by the point (l_p, δ_p), the pulse voltage V_p, and the resonance repetition number N_{on}. The distributions of the bunch halo particles in momentum deviation amplitude at different turn numbers are shown in Figure 7. For a small pulse duration (Figure 7(a)), the mean of the distribution successively shifts toward the bucket height $\delta_h = 0.252 \times 10^{-3}$. For a large pulse duration (Figure 7(b)), it shifts back and forth, but overall it moves toward δ_h.

The number of particles ejected from the bucket by a multiple resonance sequence is shown in Figure 8(a) as a function of turn number for different pulse durations. There we see that as the pulse duration increases, the halo particles leave the bucket earlier. As shown in Figure 6, the growth in momentum deviation amplitude of a halo particle due to a multiple resonance sequence oscillates with an amplitude that becomes bigger as the pulse duration increases. Furthermore, the average growth in momentum deviation amplitude due to a multiple resonance sequence is parallel to and larger than that due to a single resonance sequence by an amount that also becomes bigger as the pulse duration increases. This results in the early departure of the halo particles from the bucket when the pulse duration is large. Figure 8(b) shows that the smaller the resonance repetition number N_{on}, the lower the ejection efficiency of bunch halo particles, which is expected. On the other hand, when the resonance repetition number equals the total number of turns for the synchrotron oscillation at $\hat{\delta}_p$, *i.e.*, $N_{on} = T_s(\hat{\delta}_p)/T_0 = 965$, a slow broadening in the $\hat{\delta}$-distribution of halo particles only occurs, as in the case when the perturbing pulses act on the bunch every turn. The time it takes for the synchrotron amplitude $\hat{\delta}$ to grow to the bucket height δ_h under the action of a single resonance sequence is longer for a smaller pulse voltage V_p (Figure 4). Therefore, we see in Figure 8(c) that the time for ejecting the same percentage of halo particles is about twice as long at $V_p = 5$ MV as at $V_p = 10$ MV, even though the ejection efficiency is approximately the same (close to 90%) in both cases.

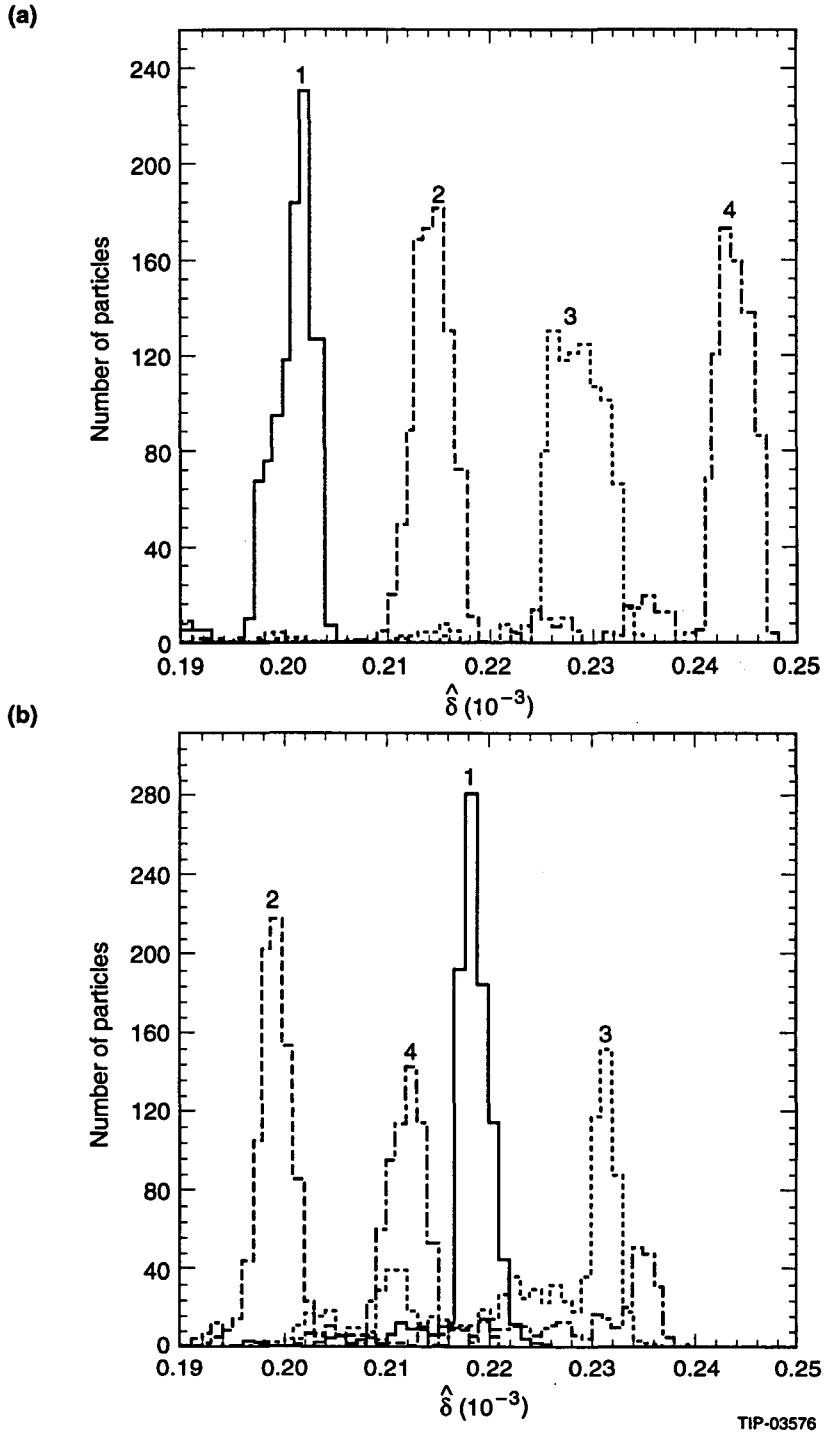

Figure 7. Distribution of Halo Particles in Relative Momentum Deviation Amplitude, $\hat{\delta}$, at Different Turn Numbers Under the Action of a Multiple Resonance Sequence. The initial distribution consists of 1000 particles uniformly distributed in the interval of $\hat{\delta}$ from $\hat{\delta}_p - \Delta$ to $\hat{\delta}_p + \Delta$, with $\hat{\delta}_p = 0.19 \times 10^{-3}$ and $\Delta = 10^{-6}$. The pulse voltage is $V_p = 5$ MV. The resonance repetition number is $N_{on} = 840$. (a) The pulse duration is $T_p = 13.3$ ps. Histograms 1, 2, 3, and 4 are at 1.2×10^5, 2.4×10^5, 3.6×10^5, and 4.8×10^5 turns, respectively. (b) The pulse duration is $T_p = 267$ ps. Histograms 1, 2, 3, and 4 are at 0.8×10^5, 1.0×10^5, 2.0×10^5, and 2.2×10^5 turns, respectively.

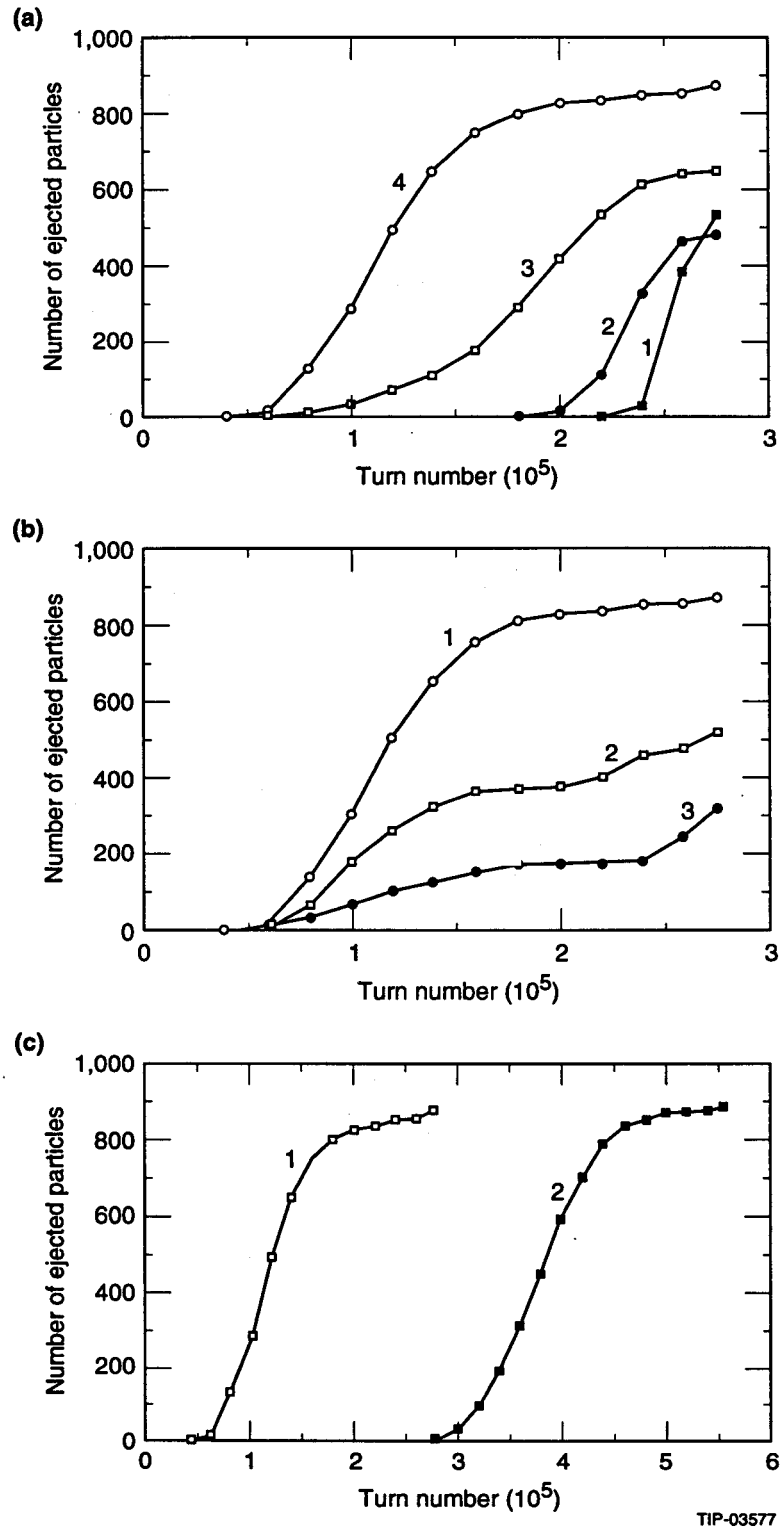

Figure 8. Number of Particles That Have Left the Bucket as a Function of Turn Number Under the Action of a Multiple Resonance Sequence. The initial distribution of halo particles is the same as in Figure 7. (a) The pulse voltage is $V_p = 10$ MV. The resonance repetition number is $N_{on} = 840$. Curves 1, 2, 3, and 4 are for pulse durations T_p of 33.4, 66.7, 133.4, 267 ps, respectively. (b) The pulse voltage is $V_p = 10$ MV. The pulse duration is $T_p = 267$ ps. Curves 1, 2, and 3 are for resonance repetition numbers of 840, 400, and 200, respectively. (c) The pulse duration is $T_p = 267$ ps. The resonance repetition number is $N_{on} = 840$. The pulse voltage is $V_p = 10$ MV (curve 1) and $V_p = 5$ MV (curve 2).

Until now we have considered only the halo particles with initial momentum amplitudes near $\hat{\delta}_p$. However, the ejection efficiency for a multiple resonance sequence can be affected if the particles whose initial momentum amplitudes are not near $\hat{\delta}_p$ are taken into account. Our computer simulation shows that these particles are not captured by the multiple resonance sequence with short perturbing pulses. Only a broadening in the $\hat{\delta}$ -distribution occurs. With long perturbing pulses, the multiple resonance sequence is also effective in ejecting these particles. This is shown in Figure 9(a), where we present the ejection history of the halo particles with initial momentum amplitudes near $\hat{\delta} = 0.21 \times 10^{-3}$ (curve 2) and $\hat{\delta} = 0.23 \times 10^{-3}$ (curve 3). Presented in Figure 9(b) is the ejection history of all the halo particles whose initial momentum amplitudes are in the range of $\hat{\delta} = (0.19\text{–}0.25) \times 10^{-3}$. Again the initial distribution consists of 1000 particles uniformly distributed in phase ψ and in momentum deviation amplitude $\hat{\delta}$. We see that the ejection efficiency of these halo particles by a multiple resonance sequence is quite high (about 95%) for $T_p = 267$ ps and $V_p = 10$ MV (curve 1). It decreases as the pulse voltage or the pulse duration decreases (curves 2 and 3).

5.0 CONCLUSION

Our computer simulations show that by synchronizing a sequence of rf-voltage perturbation pulses with the synchrotron oscillations of beam halo particles, we can eject the particles effectively from the bucket onto a crystal deflector for extraction from the SSC. The ejection efficiency and rate may be regulated by the pulse voltage, the pulse duration, and the resonance repetition number. Moreover, it is possible to act either on a single bunch or on a few bunches simultaneously. Although we have dealt with large amplitude halo particles in the simulations, one can, in principle, position the perturbation pulses (in rf phase) near the edge of the beam regardless of the amplitude of particles there. By gradually shifting the perturbation pulses from the edge of the beam toward the beam center (for a small interval of rf phase, of course), one can eject particles only from the tail of the beam without perturbing the core. Thus, there are two important requirements for the scheme of resonance excitation by rf voltage pulses: (1) the positioning of perturbation pulses can be done accurately without too much jittering, and (2) the pulse edge must be very sharp. The technical feasibility of this scheme is now under investigation.

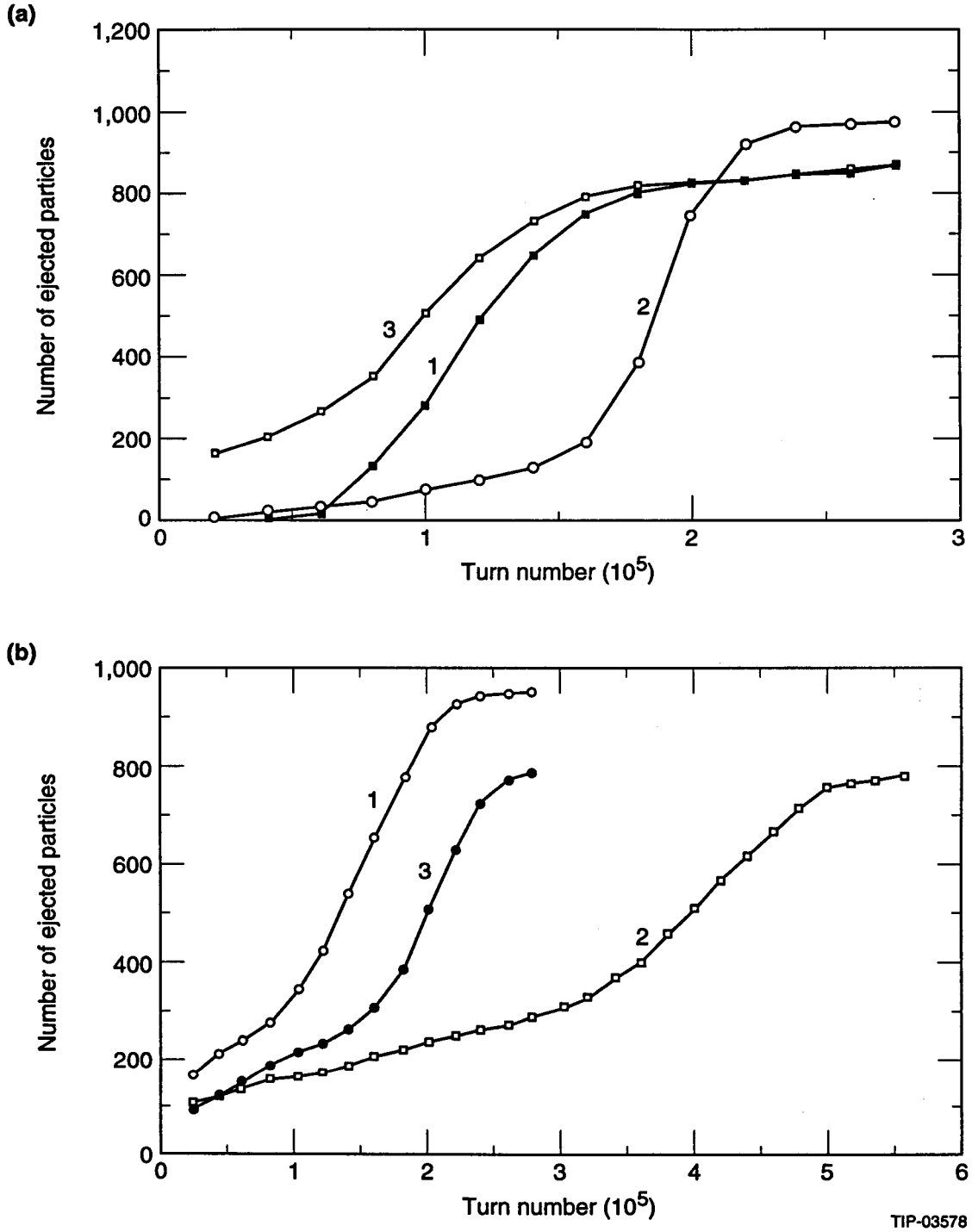

Figure 9. Same as in Figure 8 Except for the Initial Distribution of Halo Particles in $\hat{\delta}$. (a) The initial distribution is near 0.19×10^{-3} (curve 1), 0.21×10^{-3} (curve 2), and 0.23×10^{-3} (curve 3). The other parameters are $T_p = 267$ ps, $V_p = 10$ MV, and $N_{on} = 840$. (b) The initial distribution is in the range of $(0.19\text{–}0.25) \times 10^{-3}$. The other parameters are $T_p = 267$ ps, $V_p = 10$ MV, $N_{on} = 840$ (curve 1); $T_p = 267$ ps, $V_p = 5$ MV, $N_{on} = 840$ (curve 2); and $T_p = 133.4$ ps, $V_P = 10$ MV, $N_{on} = 840$ (curve 3).

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