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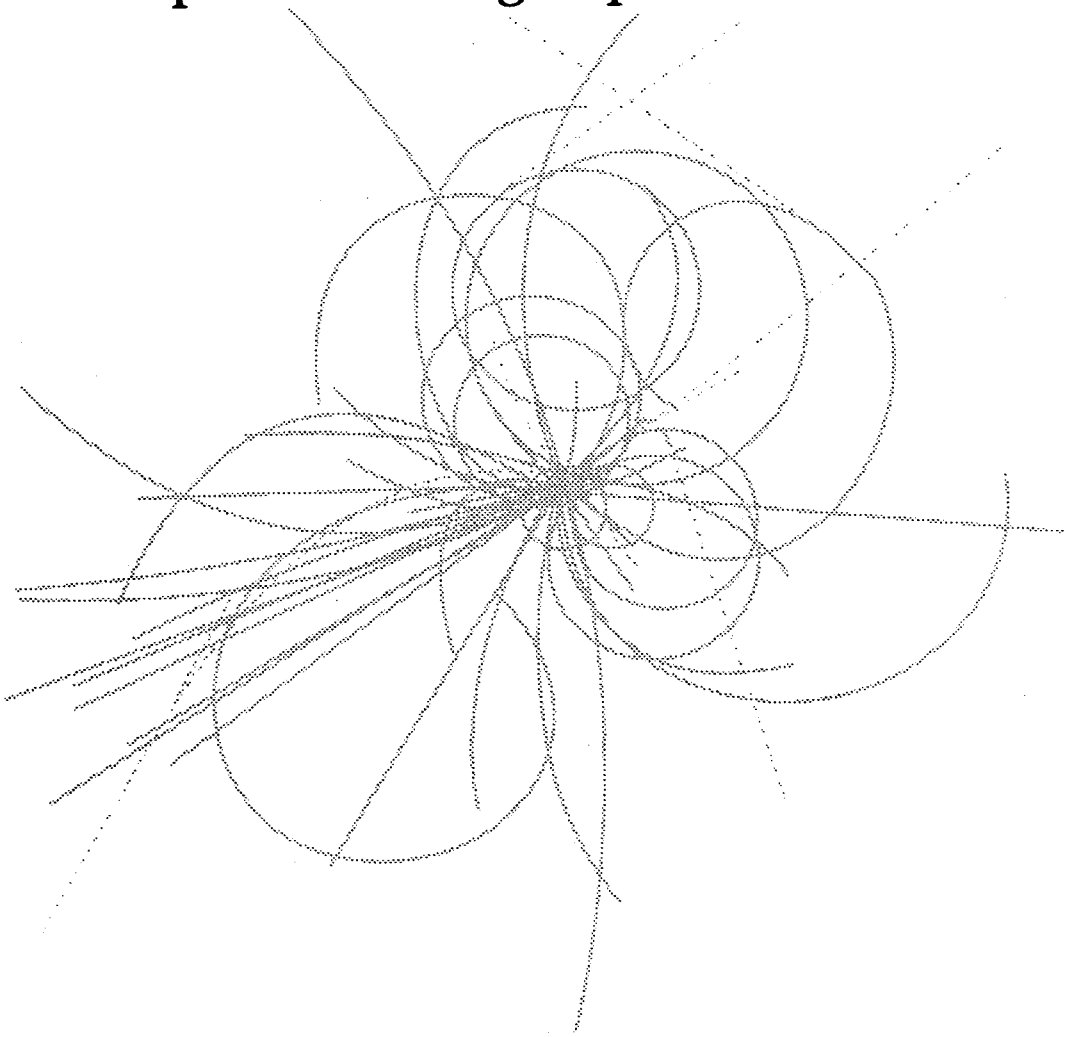
Superconducting Super Collider Laboratory



SSC Collider Dipole Ripple and Particle Motion

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1.0 OBJECTIVE

With a circumference of 87 km, the SSC main collider ring will have a revolution frequency as low as 3.4 kHz and the fractional part of the betatron oscillation frequency will be within the range of the power supply ripple frequency, $0 \sim 10\,\mathrm{kHz}$. The current ripple in the power supplies will produce a time and space variant ripple in the magnetic field of the collider ring. What occurs when the fraction betatron frequency is near the ripple frequency? There are a lot of concerns about this situation. We need to determine what the ripple effect will be.

Some experiments done at Fermilab showed that when the betatron frequency was near the ripple frequency, the beam was immediately lost.[8,9] However, experiments at CERN didn't show this effect.[2] Why did experiments with different machines indicate different results? Does the geometric location of the power supplies influence the ripple dependence? How does ripple really affect the beam dynamics? How much tolerance, in frequency and amplitude, will the SSC collider ring have?

Ripple exists in both the quadrupole field and the dipole field. The ripple in the quadrupole field modulates the focusing and defocusing forces. As a result the tune is modulated around the designed operating point. When the ripple amplitude is large enough to shake the tune across the resonant lines, some particles become "lost." As a result of this process, particles gradually exit the beam tube. Such particle loss is a long term effect and the simulation results[2,4,5,7] show that the dynamic aperture is highly dependent on the ripple amplitude. The higher the ripple amplitude, the smaller the beam dynamic aperture. The frequency dependence is not very obvious. Tracking studies show that the frequency dependence only appears after very long time periods. The lower the frequency, the smaller the dynamic aperture.

The ripple in the dipole field, on the other hand, modulates the centripetal force. This force is in the same direction as the horizontal focusing and defocusing forces; therefore, its effect will be coupled into the horizontal betatron oscillation. This report gives a preliminary analysis of the dipole ripple effect on the betatron oscillation.

2.0 DIPOLE RIPPLE IN HILL'S EQUATION

The following discussion assumes that the design trajectory is a planar closed curve. Figure 1 shows the collider ring reference orbit, where ρ is the local curvature, $\hat{s}$, $\hat{x}$, and $\hat{y}$ are the three unit vectors that describe the particle position. The particle's transverse motion is described by Hill's equations:[1]

$$\frac{d^2x}{ds^2} - \frac{\rho + x}{\rho^2} = -\frac{B_y}{(B_\rho)}\left[1 + \frac{x}{\rho}\right]^2 \tag{1}$$

$$\frac{d^2y}{ds^2} = -\frac{B_x}{(B_\rho)}\left[1+\frac{y}{\rho}\right]^2 \tag{2}$$

where B_x and B_y represent the x and y-components of the magnetic field, and B_ρ represents the magnetic rigidity, $B_\rho = \frac{P}{e}$.

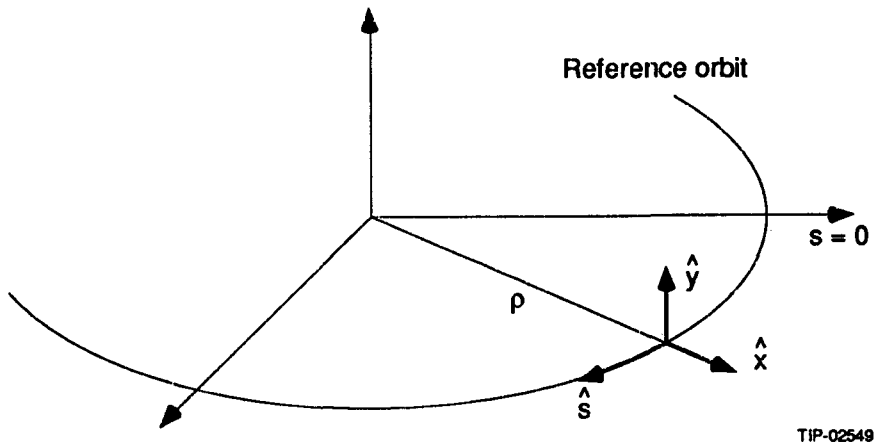

Figure 1. The Collider Ring.

The dipole ripple only affects the horizontal motion; therefore, only the horizontal motion will be considered here. The magnetic field B_y in the horizontal direction is the sum of the dipole field and the quadrupole field:

$$B_y = B_y(0,0) + \frac{\partial B_y}{\partial x}x. \tag{3}$$

$B_y(0,0)$ represents the dipole field, which is designed to be uniform in the x-direction. $\frac{\partial B_y}{\partial x}x$ represents the quadrupole field, which is designed to be linear in the x-direction.

Suppose that the dipole field has a ripple:

$$B_y(0,0) = \overline{B}_o + \Delta B(s,t), \tag{4}$$

where $\bar{B}_o$ represents the time invariant part and $\Delta B(s,t)$ represents the ripple. Since the transverse motion amplitude x is very small compared with the curvature ρ. Only the first order linear approximation is considered. Hill's equation becomes

$$\frac{d^2x}{ds^2} + \left(\frac{1}{\rho^2} + \frac{\frac{\partial B_y}{\partial x}}{B_\rho}\right)x = -\frac{\Delta B(s,t)}{(B_\rho)}. \tag{5}$$

If the time variant ripple is zero, the betatron oscillation is homogeneous. Hill's equation has a solution:

$$x(s) = \beta^{\frac{1}{2}}(s)\Big(A\cos\psi(s) + \delta\Big), \tag{6}$$

where

$$\psi(s) = \int_0^s \frac{ds}{\beta(s)}. \tag{7}$$

The tune of oscillation is defined

$$\nu = \frac{1}{2\pi} \oint \frac{ds}{\beta(s)}. \tag{8}$$

When the dipole field has a time variant ripple, the betatron oscillation becomes a forced oscillation:

$$x(s) = \beta^{\frac{1}{2}}(s)[A \cos \psi(s) + \delta] + f(s), \tag{9}$$

$f(s)$ depends on the force function; therefore, the particle motion is affected by the external force.

3.0 RIPPLE DISTRIBUTION ALONG THE RING

Let us examine the force function of the SSC collider. According to Reference 3, there will be ten power supplies distributed along the ring, as shown in Figure 2.

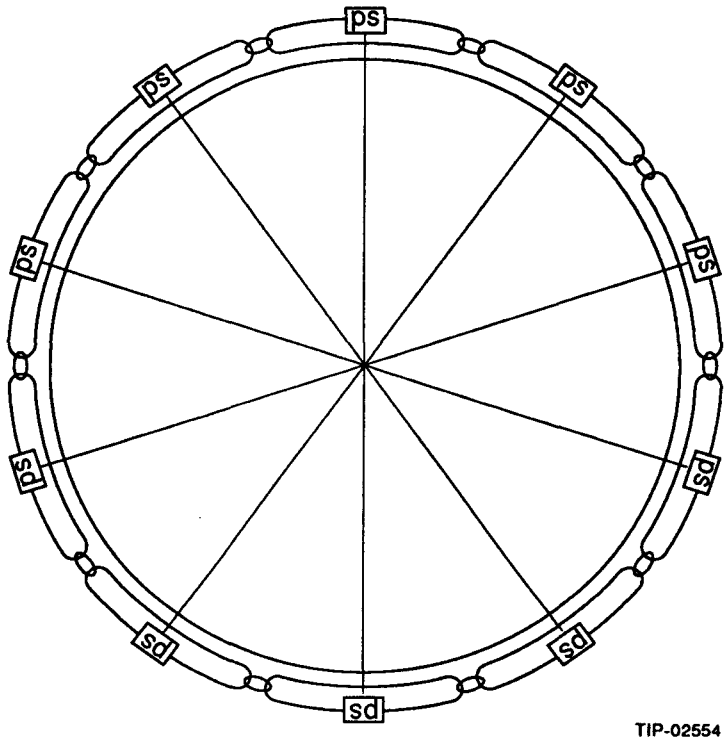

Figure 2. Collider Power Supply Distribution.

Each power supply covers a sector of one-tenth of the circumference. Since a sector has an average length of 8.712 km, the transmission line effect has to be considered. Suppose

each magnet has an equivalent circuit as shown in Figure 3; the ripple from the power supply will have a distribution along the transmission line.[6] For a ripple component with a certain frequency of ω_r, the amplitude will exponentially decay as the ripple component travels though the line and its phase shift will increase, *i.e.*,

$$\Delta B(s,t) = B_m e^{-\alpha s} \cos(\omega_r t - \beta s) \tag{10}$$

where B_m is the peak magnetic field at the power supply output, α and β are functions of the circuit parameters.

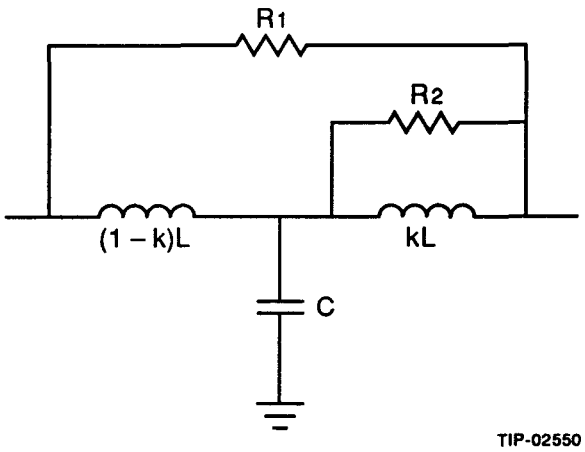

Figure 3. The Equivalent Circuit of the Magnet.

The power supply ripple has a very wide frequency spectrum. Each frequency component will have its own distribution along the ring. The distribution of the total ripple is the sum of distributions of all the frequency components.

Assume that each of the ten power supplies generates identical ripple waveform when they are connected to the same commercial power line with the same fire angles and the same load conditions. The picture of the ripple distribution (amplitude of the ripple) along the ring can be predicted as shown in Figure 4.

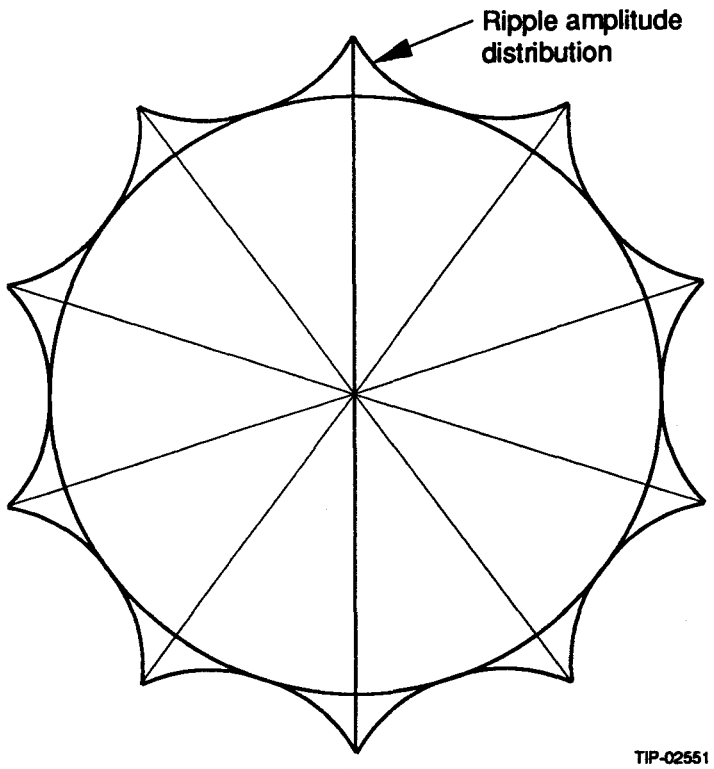

Figure 4. The Ripple Distribution Along the Ring.

4.0 RIPPLE EFFECT ON PARTICLE MOTION

To understand the beam behavior, it is necessary to solve Hill's equation with a force function described in the preceding section. Before beginning complex computations, let us imagine the physical pictures of the particle motion. Consider two extreme cases: Case one: ripple distribution is uniform along the ring and Case two: ripple concentrates on one point.

4.1 Uniform Distribution of the Ripple

If the ripple distribution along the ring is uniform, *i.e.*,

$$\Delta B(s,t) = B_m \cos(\omega_r t), \text{ for all } s \tag{11}$$

then a single particle moving in the ring with speed c will see a kicking force in Hill's equation:

$$\frac{\Delta B(s,t)}{B_\rho} = \frac{B_m}{B_\rho} \cos\left(\omega_r \frac{s}{c}\right), \tag{12}$$

where $t = \frac{s}{c}$.

Suppose that the betatron oscillation has a frequency $f_\beta = \frac{\omega_\beta}{2\pi}$, when the ripple is zero,

$$x = A_\beta \cos\left(\omega_\beta \frac{s}{c}\right). \tag{13}$$

With the ripple effect, the motion becomes

$$x = A_\beta \cos(\omega_\beta t + \theta) + A_r \cos(\omega_r t) \tag{14}$$

$$A_r = \frac{c^2}{\rho(\omega_\beta^2 - \omega_r^2)} \frac{B_m}{B_o} \tag{15}$$

$$\approx \frac{c^2}{\rho\omega_\beta^2} \frac{B_m}{B_o}. \tag{16}$$

The motion is shown in Figure 5. In this case, the transverse motion is perturbed by the ripple. The oscillation amplitude increase is A_r. Since the betatron frequency is in the level of 340 kHz, far away from the ripple frequency, the motion, therefore, will not grow infinitely.

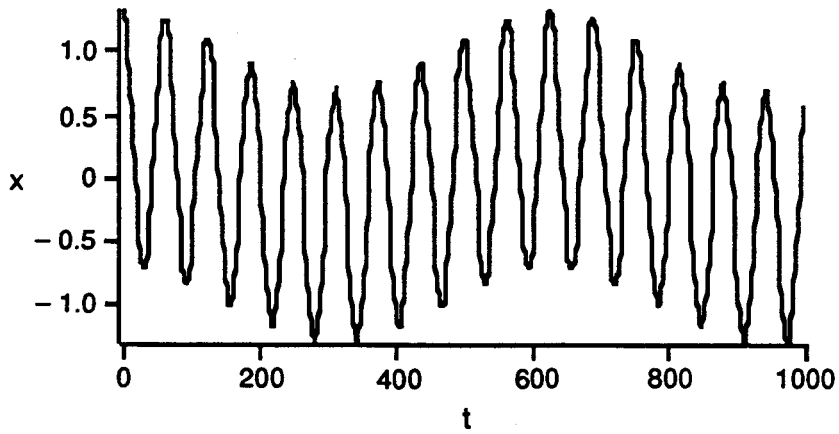

Figure 5. The Particle Motion with Uniform Ripple Distribution.

4.2 Ripple at One Point

The ripple appears only at one point, say $s = 0$,

$$\Delta B(s,t) = \begin{cases} B_m cos(\omega_r t) & s = 0 \\ 0 & s \neq 0. \end{cases} \tag{17}$$

At $s = 0$, the particle appears at $t = n\frac{C}{c}$, $n = 0, 1 \ldots$ where C is the collider circumference. The particle sequence forms a sinusoidal function with a frequency equal to the fractional part of the betatron oscillation, *i.e.*,

$$x(t) = A\cos(\omega_{\beta_\text{frac}} t), \tag{18}$$

$t = n\frac{C}{c}$, $n = 0, 1, \ldots$. The motion will be

$$x = A_{\beta_\text{frac}} \cos(\omega_{\beta_\text{frac}} t + \theta) + A_r \cos(\omega_r t) \tag{19}$$

$$A_r = \frac{c^2}{\rho(\omega^2_{\beta_\mathrm{frac}} - \omega_r^2)} \frac{B_m}{B_o}. \tag{20}$$

The frequency of the fractional part of the betatron oscillation is in the same scale as the ripple frequency. When the betatron frequency is the same as one frequency component of the ripple, the transverse motion will grow to infinity. When the betatron frequency is close to one component of the ripple frequency, the transverse motion will have a frequency beat, as shown in Figure 6. The amplitude A_r is proportional to $\frac{1}{\omega^2_{\beta_\mathrm{frac}} - \omega_r^2}$; it will grow out of the beam tube when the two frequencies are close enough.

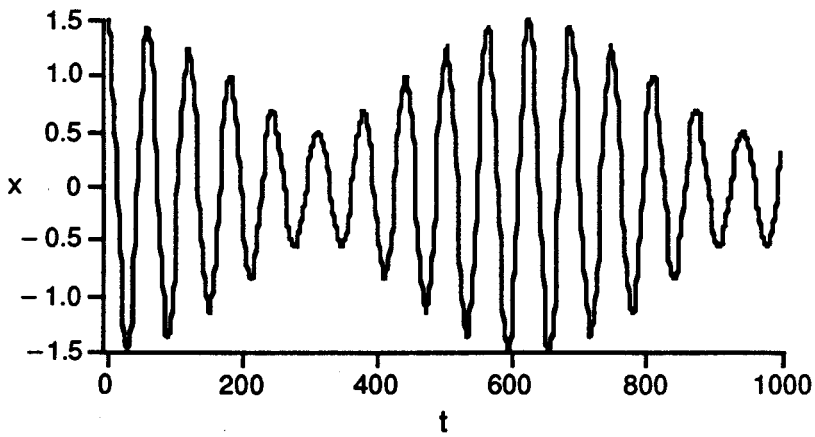

Figure 6. The Particle Motion with Single Point Ripple Distribution.

4.3 Real Collider Ripple Distribution

In reality, the ripple distribution is neither a uniform distribution nor a single point concentration. From the preceding discussion, we may surmise that a symmetrical ripple distribution is better than an asymmetrical ripple distribution. The ripple effect of the collider ring will be worse than the uniform case and better than the single point case. However, to determine the actual effect, we need to input the ripple information into the collider lattice and do the particle tracking.

5.0 WORK TO BE DONE

1. Measure the circuit parameters of the superconducting magnets. Because of the eddy currents in the beam tube and the iron yoke, the circuit parameters vary with the frequency. Since the power supply ripple has a very wide spectrum, it is necessary to find the circuit parameters over a wide frequency range.
2. Develop a transmission line circuit SPICE model for the ring magnet string. Though some work has been done for the Tevatron,[6] the SSC Collider magnets and lattice are very different from those of the Tevatron; therefore, a different model is needed.

3. Simulate the ripple distribution along the ring over the whole ripple spectrum. First, we must find out the power supply ripple spectrum; then, we use the SPICE program to simulate the distribution of each frequency component; finally, we integrate the distribution functions over the whole frequency spectrum to get the real ripple distribution.
4. Integrate the ripple distribution into the lattice simulation program. Since the SSC collider will have ten symmetrical sectors, we integrate the ripple force along one sector for sector mapping (mapping is a type of simulation). The whole system mapping is the equivalent of ten sector maps.
5. Track the particle with the influence of the ripple using the Zmap program.[4] Find the ripple tolerance of the beam. Some frequencies are more sensitive than others. For the most sensitive frequency, determine what amplitude can be tolerated.
6. Develop a power supply topology with minimum ripple effect on the beam dynamics. Further, it might be necessary to conceive a power supply physical location picture so that the ripple effect is minimized.

REFERENCES

1. D. Edwards and M. Syphers, "An Introduction to the Physics of Particle Accelerators," SSC Laboratory 1990.
2. M. Furman and F. Schmidt, "Reduction of the Dynamical Aperture due to Tune Modulation," Report SSCL-A-6.
3. J. Sanford and D. Matthews, "Site-Specific Conceptual Design of the Superconducting Super Collider," 1990, SSC Lab.
4. T. Sen, Private communication.
5. K. Kauffmann, Private communication.
6. B. Shafer, Private communication.
7. D. Talman, Private communication.
8. T. Jackson, Private communication.
9. J. Gannon, Private communication.