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Abstract

The present status of the strong CP problem is briefly reviewed in a heuristic way. A crisis in EDMN calculation is explained. The equation of vacuum alignment obtained by the author and collaborators last year put a constraint on strong CP parameters. Thus the strong CP will be forced to vanish in one of the three scenarios characterized by axion, zero quark mass, and vanishing quark condensate.

I shall try to explain briefly the strong CP problem and its possible solutions.

The strong CP problem is a serious flaw of the standard model (SM), especially of its strong interaction section. This problem has been attacked for 15 years. Some solutions have been found, but none of them is conclusive. If it turns out that none of them works, it might mean that there is a deep defect in our basic understanding of SM.

The electroweak section of SM with three generations of quarks may explain the observed CP-violating process $K_L \to 2\pi$ very well. The essential quantity that appears in the calculation is the rephasing (vector-like) invariant[1] of the Kobayashi-Maskawa matrix:

$$t = S_1^2 S_2 S_3 C_1 C_2 C_3 \sin\delta. \tag{1}$$

If the decay width of the kaon is proportional to S_1^2, then the expected CP-violating rate $\epsilon = (K_L \to 2\pi)/(K_S \to 2\pi)$ is about $S_2 S_3 C_1 C_2 C_3 \sin\delta$. According to the present knowledge collected from other experiments, this value is about 10^{-3}, compared with the experimental value of $\epsilon = 2.7 \times 10^{-3}$. It is remarkable that the correct order of magnitude of ϵ can be obtained so easily.

In contrast to this success of the electroweak theory, the possible strong CP-violating effects—such as the electric dipole moment of the neutron (EDMN), described by the allowed parameter $\bar{\theta}$ (to be defined later)—have been ruled out to a very high precision. This requires $\bar{\theta}$ to be extremely small. The question of why $\bar{\theta}$ should be so small is studied under the title "Strong CP Problem."

Limited by space, let us concentrate on a QCD model with only one quark. The mass term of the fermion is usually written as $-\bar{\psi} m \psi$. Since QCD is part of SM and there is CP violation in SM anyway, the following mass term is in general allowed:

$$\mathcal{L}_m = -\bar{\psi}\tilde{m}\psi, \tag{2}$$

where $\tilde{m} = m e^{i\gamma_5\phi}$ is called the fermion mass with a chiral phase, or sometimes, the complex mass. Please be careful not to confuse $\tilde{m}$ with the effective complex mass of a decaying particle, $m + i\gamma/2$. The γ_5 part of Eq. (2) is P- and T-odd and is hermitian. The intrusion of the new parameter ϕ did not cause attention until the importance of another term in the pure gauge part of the QCD Lagrangian was noticed.[2,3] This term is called the θ-term:

$$\mathcal{L}_\theta = \theta G\tilde{G}, \tag{3}$$

where

$$G\tilde{G} = \frac{1}{32\pi^2}\varepsilon_{\mu\nu\varrho\sigma}G_a^{\mu\nu}G_a^{\varrho\sigma}. \tag{4}$$

This term is also P- and T-odd. Furthermore, the two terms are related by chiral rotation due to the triangle anomaly.[4] That is, when

$$\psi \longrightarrow \psi' = e^{i\alpha\gamma_5/2}\psi \ , \tag{5}$$

we obtain

$$\phi \longrightarrow \phi' = \phi + \alpha, \quad \theta \longrightarrow \theta' = \theta - \alpha. \tag{6}$$

Note that

$$\bar{\theta} = \theta + \phi \tag{7}$$

will not be changed under chiral rotations. Therefore, it is impossible to "turn away" CP violation terms by a chiral rotation once $\bar{\theta}$ is fixed. Besides, chiral rotation is not the symmetry of the system. The general equivalence of the different Lagrangian related by chiral rotation is under question, unless a corresponding adjustment of vacuum is made (see later).

The question of what physical effects could be due to the above strong CP violation is somehow subtle. Because the strong CP-violating terms do not change the flavors of the quarks as the weak interaction does, then the strong CP violation will certainly not be the leading effect in weak decays. Attention has been focused on the process $\eta \to 2\pi$ and on EDMN; both need P- and T-violation to happen. Since EDMN has been experimentally narrowed down to a very small number, it was claimed that the measured bound on EDMN placed a stringent bound on the value of $\bar{\theta}$. However, the celebrated calculation to relate EDMN to $\bar{\theta}$ by Crewther et al.[5] (CDVW) has recently been criticized by Banerjee, Chattergee, and Mitra[6] and by Gupta, McKellar, and Wu.[7] The method of CDVW can be sketched as follows. They first shift $\bar{\theta}$ to the mass term, so that superficially all strong CP comes from the mass term. They then use the chiral perturbation theory (the current algebra) to calculate EDM in terms of $\bar{\theta}$. In doing so, the possible complex mass of the neutron caused by the complex mass of the constituent quarks has not been consistently handled. Putting it succinctly, there is a risk that the final result would take the following form:

$$\bar{N}(\tilde{m}_N/m_N)(m_N + ig\pi\gamma_5 + a\sigma_{\mu\nu}F^{\mu\nu} + ...)N. \tag{8}$$

It seems that there is a $ia\gamma_5 \sin\phi\sigma_{\mu\nu}F^{\mu\nu}$ term in this formula, which can be identified as EDMN. However, this is fake because the common phase (the phase of the mass) is

protected by a perturbative symmetry of the original QCD-effective Lagrangian. This phase will disappear if a suitable wave function of the neutron is chosen which satisfies the Dirac equation with a complex neutron mass. Therefore, unless non-perturbative effects are explicitly included, it is impossible to produce a non-zero EDMN.

Though the result of CDVW is criticized, it does not mean that the strong CP effects do not exist. Common wisdom tells us that if we can establish a meaningful relation among some theoretical parameters, such as mass and the strong CP parameters, these parameters must not be redundant ones. They must have some effects. I shall introduce you to such a relation called the equation of vacuum alignment (EVA) established by Huang, Viswanathan, and Wu[8] (HVW). If strong CP does contribute to EDMN, it is expected, from a dimensional argument, that

$$EDMN \sim e\bar{\theta}/m_N \sim 10^{-14}\bar{\theta}\ e \cdot cm. \tag{9}$$

To meet the experimental bound, $\bar{\theta}$ has to be extremely small:

$$\bar{\theta} \leq 10^{-11}. \tag{10}$$

Now let us discuss the promised equation of vacuum alignment. The EVA can be obtained by the use of invariance of the functional under chiral transformation, as all fermion fields are integrated out in the functional. It reads

$$< G\tilde{G} >=< \bar{\psi}_L i\gamma_5 f\varphi\psi_R + h.c. > \tag{11}$$

where $me^{i\phi}$ is replaced by $f\varphi$, with φ the Higgs field and f the Yukawa coupling constant. Let us specify the vacuum by the following equations:

$$< \psi >=< G_{\mu\nu} >= 0, \quad < \bar{\psi}_L\psi_R >= \frac{1}{2}C_d \neq 0, \quad < \varphi >= ve^{ia} \tag{12}$$

with C_d the dynamical condensate of the quark. Ngee Pong discussed this quantity in this session. By choosing C_d to be real and negative, as people usually do, we actually choose a specific vacuum orientation in the chiral frame. Generally speaking, C_d can have an arbitrary phase and be non-zero even when $m \to 0$. When $m \neq 0$, we renormalize C_d by

subtracting the contribution due to the current mass. With Eq. (12), Eq. (11) is expressed at the tree level of the Higgs interactions as

$$< G\tilde{G} >= mC_d \sin\phi, \tag{13}$$

where

$$m = |f|v, \quad f = |f|e^{i\phi_f}, \quad \phi = \phi_f + a. \tag{14}$$

Eq. (13) is the EVA of the question. Slightly different equation for light quarks only has been found by CDVW and by 't Hooft[9] using low-energy effective theories of QCD. Unfortunately, their equation was not seriously considered in the calculation of strong CP effects mentioned previously.

We find from EVA that the values of the phase of the mass ϕ are constrained (so is θ) if $\bar{\theta}$ is fixed. The strong-interaction dynamics come into play in EVA as represented by C_d, the dynamical condensate. The vacuum specification of Eq. (12) accompanies EVA and makes it impossible to shift $\bar{\theta}$ arbitrarily without changing the phase of C_d at the same time. As we pointed out before,[10] it is impossible to shift the strong CP completely to the θ-term without changing the condition of C_d being real at the same time. Different Lagrangians related by chiral rotations are generally not equivalent unless corresponding rotations of the vacuum are taken into account by changing the phase of C_d.

EVA also provides three possible scenarios in which $< G\tilde{G} >$ is forced to vanish. The first is the famous Peccei-Quinn (PQ) scenario.[3] The so-called PQ symmetry makes the phase a of the Higgs field an arbitrary parameter. In this one-quark model the PQ symmetry can be reached by one Higgs field. But when there are quarks with two different electric charges, two Higgs fields are needed to meet the PQ symmetry. One can then always choose a to make $\phi = 0$. A consequence of PQ symmetry is the necessity of the ghost particle called axion,[12] which is a pseudo-scalar particle predicted but not found after an exhaustive ten-year search. "Invisible" axion models have been invented, but they are complicated and unappealing.

The second scenario is $m = 0$ (e.g., for the u quark). Since there is no reason why the u quark should not obtain a small mass, this scenario is regarded as unnatural.

The third scenario, newly proposed by HVW, is $C_d = 0$ (e.g., for a heavy quark—the b or t quark). Of course, the t quark should not be too heavy, if we assume it is the one to take the responsibility. Because if m_t is too large, it will meet the condition for the $t\bar{t}$ condensate to form due to the Yukawa-like interaction, as described by Professor Nambu

at this conference. The third scenario needs phase transition in dynamical chiral symmetry breakdown, when the current mass of the quark increases to exceed a certain value. While C_d for the light quarks must be non-zero as indicated by the success of the current algebra, C_d might vanish when the current mass of the quark becomes too heavy. Proof of this phase transition[12] requires a deep commitment to the strong interaction dynamics. The solution of the strong CP problem (if there is a problem) probably lies in the dynamics of QCD itself if the phase transition does exist.

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