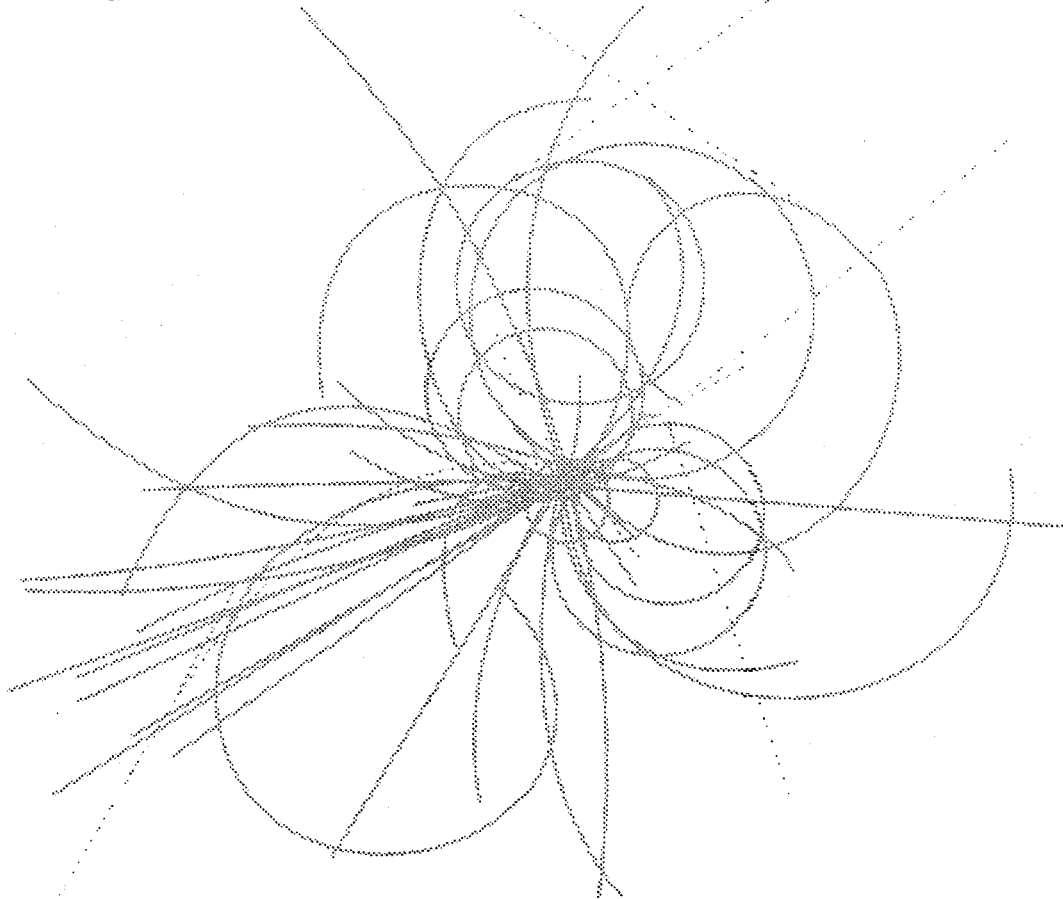


# Superconducting Super Collider Laboratory



## High Frequency Breakdown Voltage

Thanh Duy Chu

March 1992



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Thanh Duy Chu

Superconducting Super Collider Laboratory\*  
2550 Beckleymeade Ave.  
Dallas, TX 75237

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Thanh Duy Chu

### Abstract

This report contains information about the effect of frequency on the breakdown voltage of an air gap at standard pressure and temperature, 76 mm Hg and 0°C, respectively. The frequencies of interest are 47 MHz and 60 MHz. Additionally, the breakdown in vacuum is briefly considered.

The breakdown mechanism is explained on the basis of collision and ionization. The presence of the positive ions produced by ionization enhances the field in the gap, and thus determines the breakdown. When a low-frequency voltage is applied across the gap, the breakdown mechanism is the same as that caused by the DC or static voltage. However, when the frequency exceeds the first critical value  $f_c$ , the positive ions are trapped in the gap, increasing the field considerably. This makes the breakdown occur earlier; in other words, the breakdown voltage is lowered. As the frequency increases two decades or more, the second critical frequency,  $f_{ce}$ , is reached. This time the electrons start being trapped in the gap. Those electrons that travel multiple times across the gap before reaching the positive electrode result in an enormous number of electrons and positive ions being present in the gap. The result is a further decrease of the breakdown voltage. However, increasing the frequency does not decrease the breakdown voltage correspondingly. In fact, the associated breakdown field intensity is almost constant (about 29 kV/cm). The reason is that the recombination rate increases and counterbalances the production rate, thus reducing the effect of the positive ions' concentration in the gap.

The theory of collision and ionization does not apply to the breakdown in vacuum. It seems that the breakdown in vacuum is primarily determined by the irregularities on the surfaces of the electrodes. Therefore, the effect of frequency on the breakdown, if any, is of secondary importance.

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## 1.0 INTRODUCTION

The breakdown voltage between electrodes in air at atmospheric pressure is discussed when the frequency of the applied voltage varies. The frequency range of interest is 47–100 MHz, especially the two frequencies of 47 MHz and 60 MHz. Additionally, the breakdown in vacuum is briefly mentioned. However, before the effect of frequency on the breakdown voltage is examined, it is advisable to have an overview of the static or DC breakdown mechanism.

## 2.0 THE STATIC OR DC BREAKDOWN MECHANISM

Assume a uniform field  $E$  is applied to the electrodes, and somehow there is a free electron in the gap. Under the influence of the field  $E$ , the electron moves toward the anode, gaining more and more energy on the way. If the field  $E$  is strong, the electron has enough energy to ionize an air molecule by collision, thereby freeing another electron. The second electron travels to the anode together with the first one, leaving behind it a positive ion. Then the two electrons collide with other air molecules, yielding even more electrons and positive ions. The process goes on and on. When the electrons reach the anode, they have a long tail of positive ions whose density is highest at the anode (Figure 1).

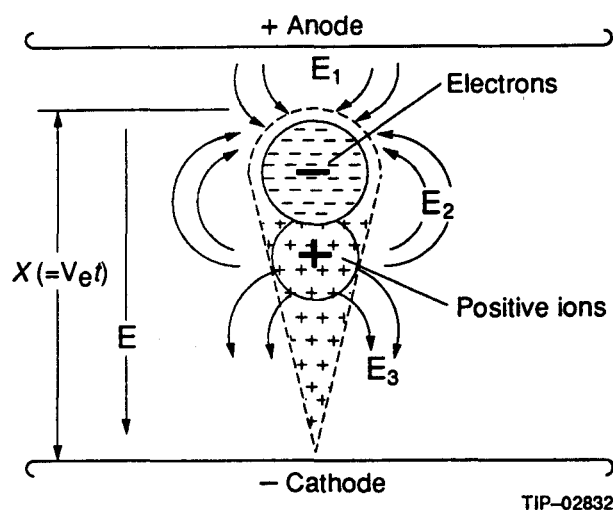


Figure 1. Distribution of Charge Carriers in an Avalanche and Their Contribution to the Applied Uniform Electric Field.  $E_1 > E_2$ ;  $E_2 < E$ ;  $E_3 > E$ .<sup>1</sup>

Note that an electron is much lighter than a positive ion, so the electron moves faster in the same field. Because most of the electrons produced are swept into the anode, the ionization of electrons alone cannot cause a breakdown.<sup>2</sup>

When the applied field is higher, positive ions are allowed to ionize the air molecules by collision—producing more electrons and positive ions—or to knock electrons out of the negative cathode. Those newborn electrons are enormous, and they in turn yield more electrons and ions through collision.

Being slower, the positive ions seem to be stationary while the electrons are crossing the gap. The ions then form a space charge near the anode. The field caused by this accumulation of positive ions increases the field in the gap. Consequently, the gap current grows rapidly and leads to breakdown. "... The criterion of breakdown may depend on the concentration of positive ions, rather than on the total number formed within the gap. As a result, the concentration of electrons in the electron avalanche will play a major part in the initiation of breakdown."<sup>3</sup>



## 3.0 THE EFFECT OF FREQUENCY ON THE BREAKDOWN VOLTAGE

### 3.1 The Critical Frequency

When the frequency of the applied voltage varies, we have to consider the distances the positive ions and electrons can travel before the cycle reversal—that is, in a half cycle of the voltage. Before we go further, let's define mobility. The mobility of a positive ion is its velocity in an electric field. Mobility varies directly with the field intensity  $E$ , so the mobility constant is defined as the velocity in cm/sec at atmospheric pressure and  $0^\circ\text{C}$  per V/cm of electric field. A positive ion has the mobility constant of 1.32, so in the field of 30,000 V/cm it has the velocity of  $30,000 \times 1.32$ , or 39,600 cm/sec. However, the mobility of an electron does not vary directly with the field, so it does not have the mobility constant. Its mobility is a function of  $E/p$ , where  $E$  is the field intensity, and  $p$  is the pressure. According to Compton in *Physical Review* of 1923, an electron has a velocity of 13,800,000 cm/sec at  $E$ , equal to 30,000 V/cm. In the same condition, the electron moves 348 times faster than the positive ion in this example.<sup>2</sup>

As the frequency increases, a point is reached at which the ions cannot reach the cathode before this electrode reverses its polarity. The ions remain in the gap, and because the avalanche continues, they form a space charge with growing density. The space charge then distorts the field in the gap. According to the above breakdown criterion suggested by Pim,<sup>3</sup> it is the space charge that lowers the breakdown voltage in comparison with the DC breakdown value. The frequency at which the breakdown voltage begins to decrease below the static value is called the first "critical frequency,"  $f_c$ .

If the frequency is raised even more, another phenomenon occurs. At this time, the electrons themselves cannot have enough time to travel to the positive electrode in the half cycle. Assume one electron ionizes once for every thousand collisions with air molecules. Because the mean free path of an electron (the distance the electron travels before colliding with a molecule) at atmospheric pressure is about 0.00005 cm, the electron will ionize 20 times per centimeter. The number of new electrons produced by that single electron is given by  $e^{20d}$ , where  $d$  is the gap width in centimeters. Because that electron cannot reach the positive electrode during the previous half cycle, it has to travel the distance of  $d$  cm once more to reach the new positive electrode. It will produce a total of  $e^{2 \cdot 20d}$  new electrons and the same number of positive ions. The density of the already existing positive ions space charge will then be highly increased and will in effect distort the field in the gap considerably. Therefore, the breakdown voltage is expected to be lowered even more. The corresponding frequency is called the second "critical frequency,"  $f_{ce}$ . With a further increase in frequency, however, the electrons become trapped in the gap, as do the positive ions, and the electrons cancel out the space charge of positive ions. In other words, the recombination rate goes up and finally counterbalances the production rate. This reduces the field intensity in the gap. Consequently, the breakdown voltage begins to rise toward the static value.

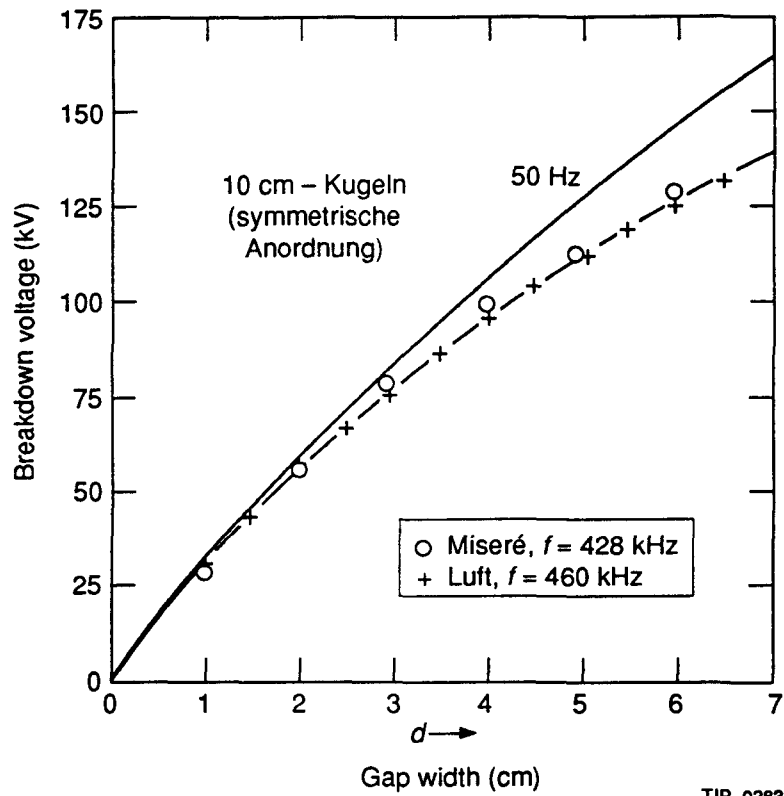
### 3.2 The Critical Gap Width

#### 3.2.1 Previous Investigations

On considering the effect of the frequency on the breakdown voltages, researchers are also interested in the "critical gap" in which, at a given frequency, the high-frequency breakdown voltage begins to decrease below the static value as the gap width is increased. Experiments have shown the existence of this critical gap.

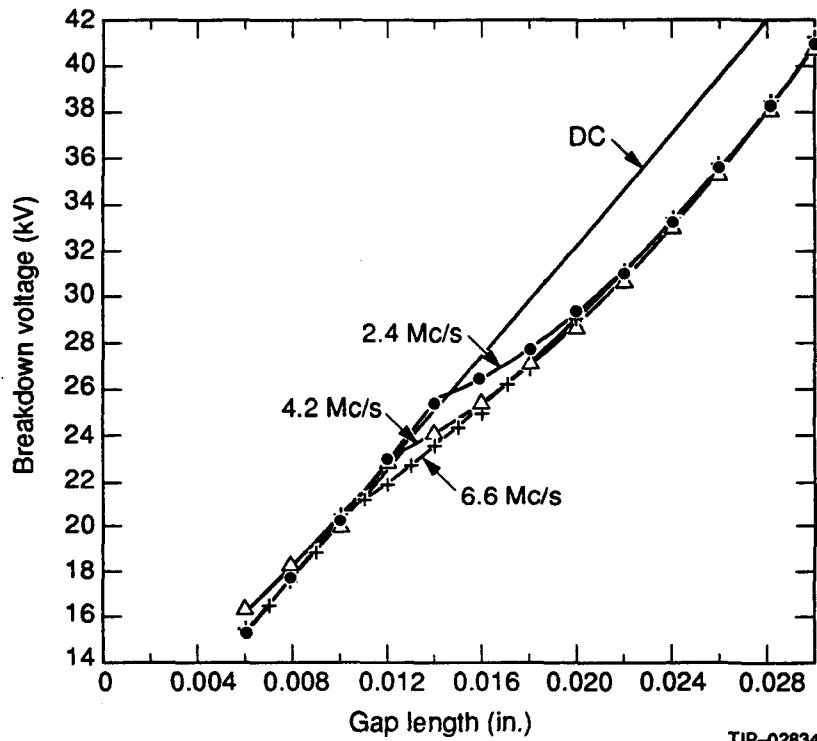
Misere and Luft, working on 10-cm-sphere electrodes at frequencies around 450 KHz, came up with the curves shown in Figure 2. For comparison, the breakdown values at 50 Hz are included on the graph. The critical gap is approximately 1.5 cm. Note that gaps smaller than the critical gap have breakdown values almost identical to DC values.<sup>4</sup>

Curves obtained by Bright showed the similar proofs of critical gaps when he studied short gaps in nitrogen.<sup>5</sup> (See Figure 3.)



TIP-02833

Figure 2. Gap Width versus Breakdown Voltage Obtained by Miseré and Luft. (10-cm-Kugeln Symmetrische Anordnung; 10-cm-sphere gap symmetrically arranged.)



TIP-02834

Figure 3. Breakdown Voltage Curves in Nitrogen as a Function of Gap Length between Spheres of 2.0-cm diameter.

Recalling the explanation for the critical frequency in Section 3.1, we can reason in the same way for the effect on the breakdown values as gap width increases. At a given frequency, a point is reached where the gap is so long that positive ions cannot travel to the positive electrode during the half cycle of the voltage. They become trapped in this critical gap and form the space charge, the field of which increases the field in the gap; then the breakdown voltage begins to decrease from the static values. This critical gap width,  $g_c$ , is associated with the critical frequency,  $f_c$ . This gap is clearly shown in the experimental curves issued by Reukema.<sup>2</sup> (Figure 4).

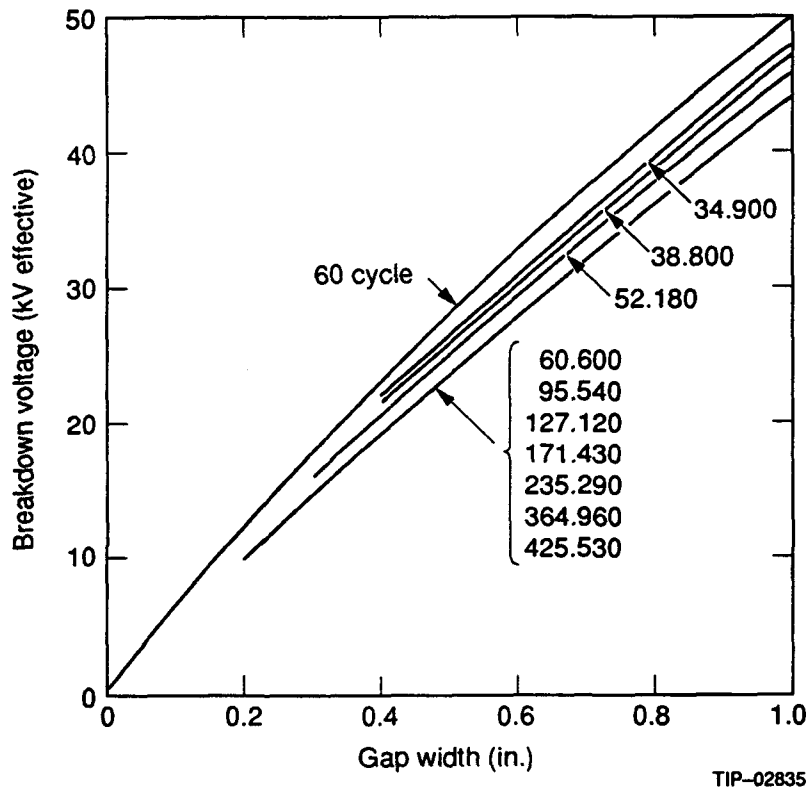
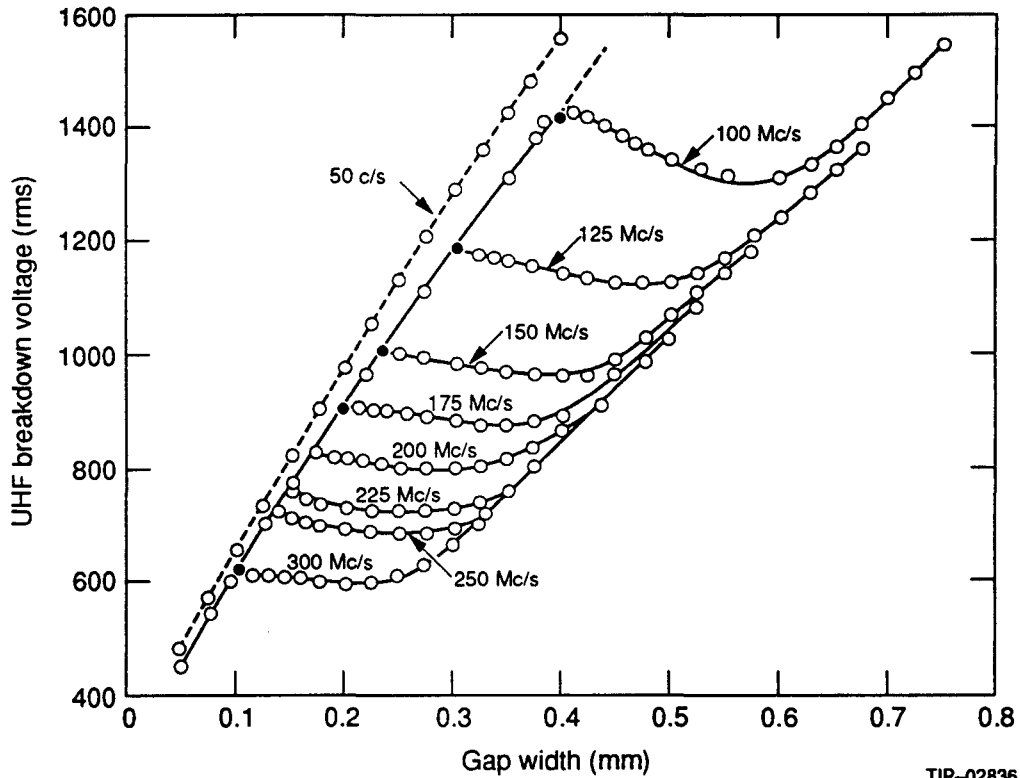


Figure 4. Gap Width versus Breakdown Voltage Obtained by Reukema.

Reukema has worked on gaps up to 2.5 cm (1 in.) in the range of 20–425 KHz. It is observed that there is not much change in the breakdown voltage versus frequency until it approaches 20 KHz. Then there is a gradual decrease in the breakdown values in the range of 20–60 KHz. However, there is little change as the frequency rises to 425 KHz, which is the highest test frequency. In fact, on the graph we find a single curve for the whole bunch of frequencies in the range of 60.6–425.530 KHz. In other words, the breakdown voltage increases with constant gradient as the gap width increases. (In this case the voltage gradient approximates 17 kV/cm.)

As the gap width increases, the electrons in turn are caught in the gap. We expect further decrease in the breakdown value at the second critical gap,  $g_{ce}$ , which is associated with the critical frequency,  $f_{ce}$ . Indeed, in the above experiment Reukema<sup>2</sup> also predicted that there would be further decrease in breakdown values at frequency equal to 6 MHz for a 1-cm gap, which accounts for  $f_{ce}$  and  $g_{ce}$ .

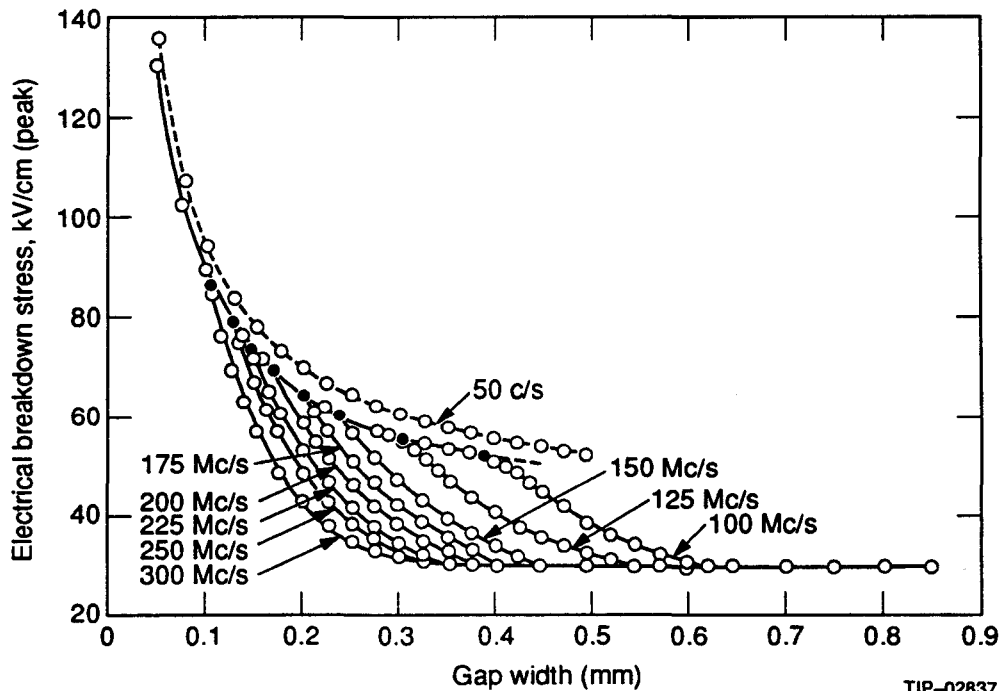
Pim<sup>3</sup> has worked on parallel plane gaps up to 1 mm with a field of frequencies between 100 MHz and 300 MHz (Figure 5). Breakdown voltages versus gap width at various frequencies at atmospheric pressure are shown. Breakdown voltage follows the normal rising curve, which agrees quantitatively with the above values obtained by Reukema (10–15% lower than the low-frequency values), until the critical gap is achieved. Then the breakdown voltage drops suddenly, passes a minimum, and finally rises with a constant gradient.



TIP-02836

Figure 5. Variation of Ultra-High-Frequency Breakdown Voltage with Gap Width at Atmospheric Pressure.

The corresponding curves of breakdown field intensity versus the gap width are shown in Figure 6. Gaps smaller than the critical gap have a breakdown field 10–15% lower than the 50 Hz values. At the critical gap, the field decreases almost double, finally arriving at a constant value of 29 kV/cm as the gap increases at all frequencies.<sup>3</sup>



TIP-02837

Figure 6. The Electrical Breakdown Strength of Air at Ultra-High Frequencies.<sup>3</sup>

The above theory is further confirmed by the experiment of Rohde and Wedemeyer. They worked on 1-cm-sphere gap spacing 2 mm apart. The breakdown voltage is made relative to its value at 50 Hz for comparison (Figure 7).

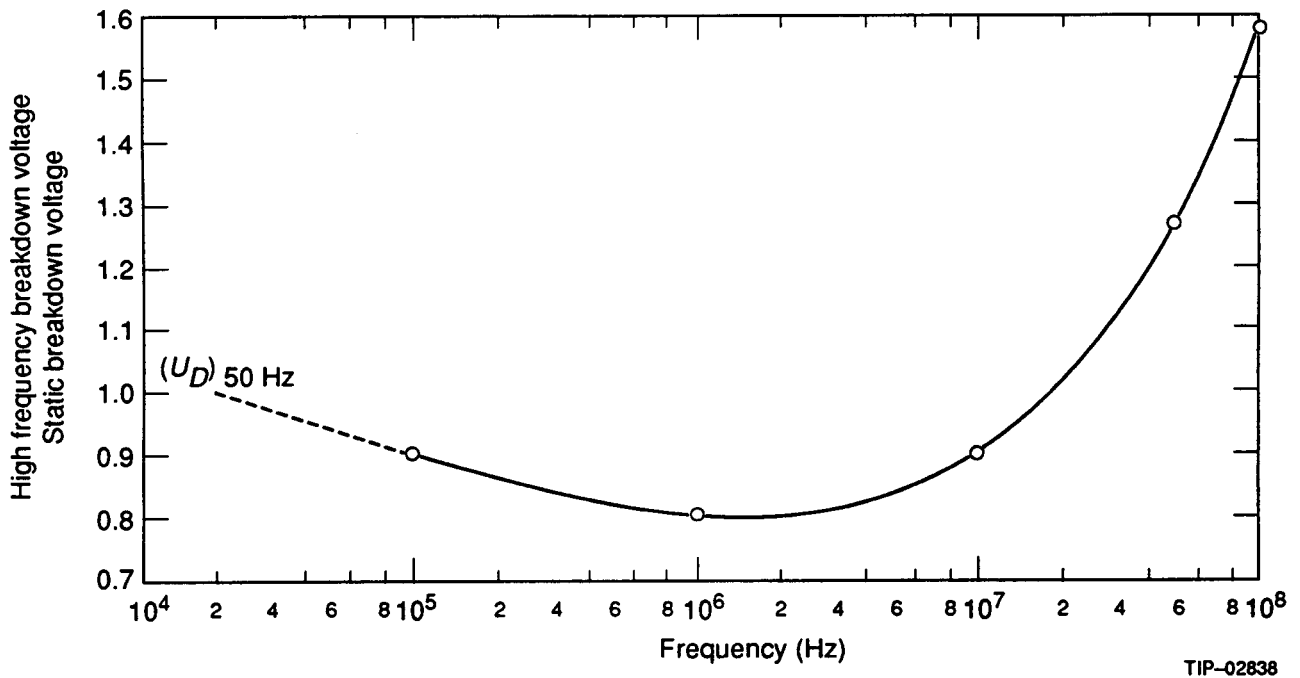


Figure 7. Ratio of High-Frequency Breakdown Voltage to Static Breakdown Voltage as a Function of Frequency for an Air Gap.<sup>4</sup>

Two observations are worth noting:

1. Around 20–40 KHz the breakdown potential starts reducing. This must be associated with the critical frequency,  $f_c$ .
2. When the frequency increases, as expected, the breakdown potential reduces further, passes a minimum at around 1 MHz, and rises toward the static value. Note that the breakdown voltage is finally becoming greater than the static value.

### 3.2.2 Extrapolation

Knowledge of the critical gap permits the calculation of the critical gaps in the frequency range of interest (47–100 MHz) from Pim's curves (Figure 5).

Analyzing the curve, we find that the critical gap, at a given frequency, must lie on the curve KA. In other words, the gap's coordinates will satisfy the equation of the curve KA. We are going to approximate a straight line for the curve KA using "the least square curve fitting" method based on empirical data from Pim's curve. We choose eight points on the curve KA: A, B, C, D, E, F, G, and K, corresponding to breakdown points at 100 MHz, 125 MHz, 150 MHz, 175 MHz, 200 MHz, 225 MHz, 250 MHz, and 300 MHz, respectively (Table 1).

**TABLE 1. GAP WIDTHS AND BREAKDOWN VOLTAGES FOR EIGHT POINTS ON CURVE KA.**

POINT	GAP WIDTH (cm)	BREAKDOWN VOLTAGE (rms V)
	0.040	1427
A	0.031	1200
B	0.240	1013
C	0.020	907
D	0.017	833
E	0.015	773
F	0.014	733
G	0.010	640
K		

A program in Fortran gives the equation of the straight line that most fits the curve KA:

$$\text{Breakdown Voltage (in rms V)} = 26423.46 \text{ gap}_c \text{ (in cm)} + 375.948 \quad (1)$$

The critical gap at a given frequency also matches the following equation given by Pim:

$$g_{ce} = \frac{2kE}{7wp} \left[ 6 - \sqrt{15 - \frac{21p^2 B_0}{AE^2}} \right] \quad (2)$$

where

$k$  = proportional factor between velocity and  $(E/p)$

$v = k (E/p)$

$k = 3.9 \times 10^5 \text{ (cm/sec) (mm Hg) (V/cm)}$

$E$  = peak electric field (V/cm)

$w = 2\pi f$

$p$  = gas pressure (mm of mercury) ( $p = 760$  mm Hg at atmospheric pressure)

$B$  = loss coefficient representing electron loss due to recombination, spread of electron radial outward.  $B = 0.0116$  in DC condition.  $B = 0.0133$  at atmospheric pressure and  $E = 34$  kV/cm.  $B$  is proportional to gas pressure. Assume  $B = 0.0133$  in our calculation.

$A$  = empirical data =  $1.58 \times 10^{-5} \text{ (mm Hg)/(V/cm)}^2$ .

Of course, we have:

$$\text{Breakdown Voltage} = \frac{Eg_{ce}}{\sqrt{2}} \quad (3)$$

because the breakdown voltage value is an rms value, and  $E$  is the peak value.

A solution to  $g_{ce}$  is obtained by solving the three simultaneous equations (1), (2), and (3). (See Table 2.) This is done in a program in Fortran using iteration methods as follows:

1. Guess a value for  $E$  in V/cm.
2. Use this  $E$  value to calculate  $g_{ce}$  in Eq. (2).
3. Use this value of  $g_{ce}$  to calculate the breakdown voltage in Eq. (1).
4. Use this value of breakdown voltage to calculate  $E$  in Eq. (3).
5. Compare the value of  $E$  in Step 4 and the guessed value in Step 1. If the difference is greater than 0.01, let  $E$  equal  $E_1$ , and repeat Steps 2-5.

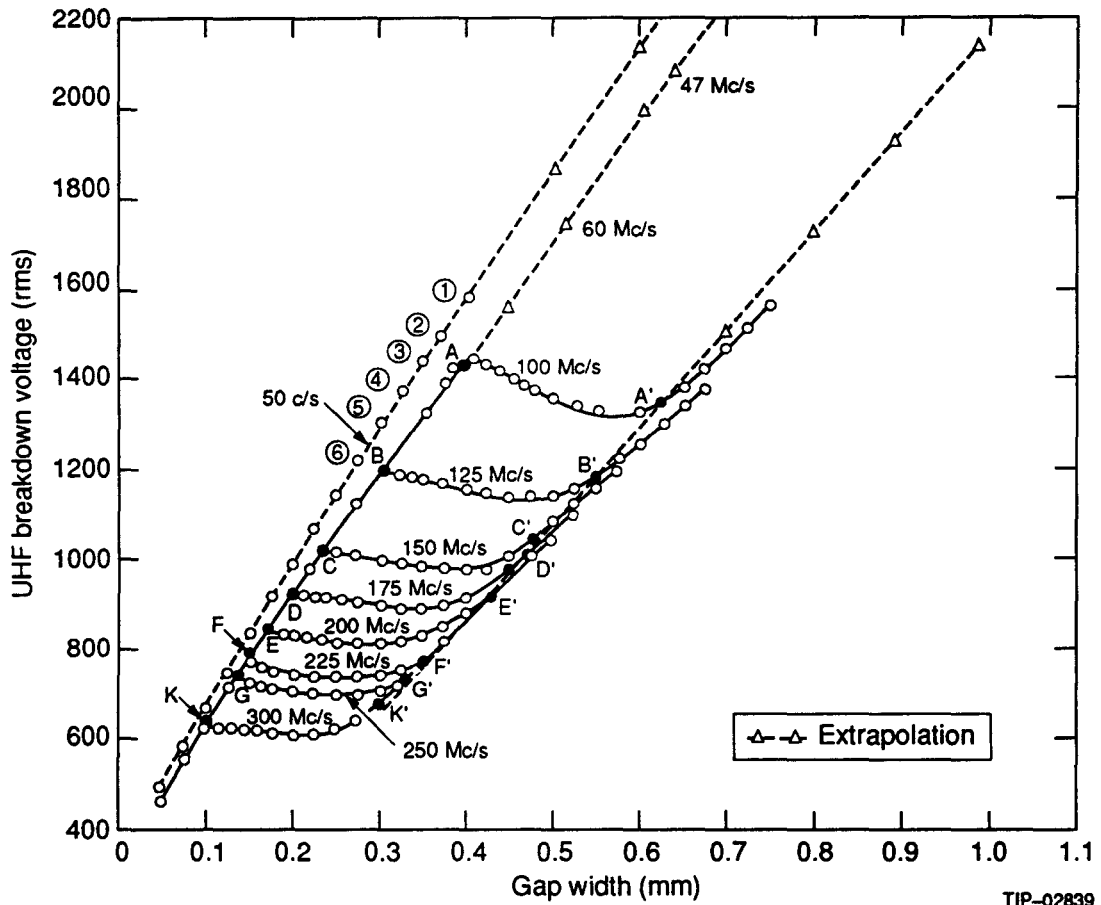
**TABLE 2. CRITICAL GAP AT GIVEN FREQUENCIES IN THE RANGE OF 47-100 MHZ.**

<b>FREQUENCY (MHz)</b>	<b>CRITICAL GAP (cm)</b>	<b>BREAKDOWN VOLTAGE (rms V)</b>	<b>BREAKDOWN FIELD INTENSITY (peak kV/cm)</b>
47	0.064	2066	45.67
50	0.060	1972	46.16
60	0.051	1728	47.74
70	0.045	1554	49.28
75	0.042	1485	50.03
80	0.040	1424	50.77
90	0.036	1322	52.20
100	0.033	1240	53.60

Finally, we use the same method, "the least square curve fitting," to:

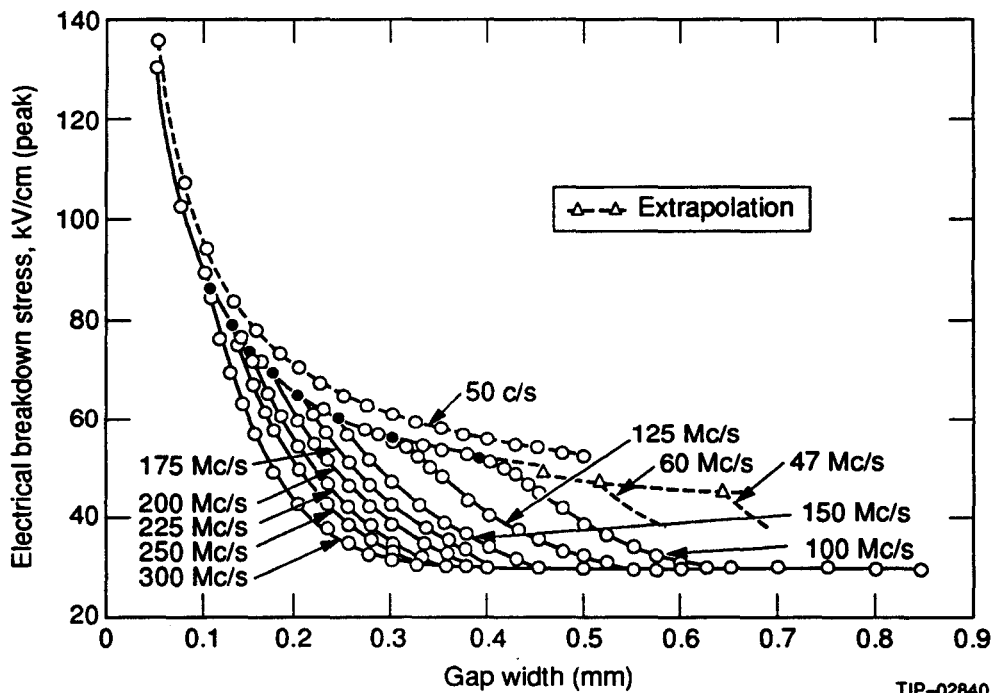
1. Establish the approximate line for the 50-Hz-breakdown voltage versus gap width in frequencies 47-100 MHz.
2. Establish the approximate line for the "limit" line K'A' so that we can predict the gap range, at a given frequency, in which the breakdown voltage is reduced substantially.

All the parts of curves obtained by extrapolation are included on the original curves from Pim for comparison (Figures 8-10).



TIP-02839

Figure 8. Variation of Ultra-High-Frequency Breakdown Voltage with Gap Width at Atmospheric Pressure (with Extrapolated Values).



TIP-02840

Figure 9. Electrical Breakdown Strength of Air at Ultra-High Frequencies (with Extrapolated Values).<sup>3</sup>



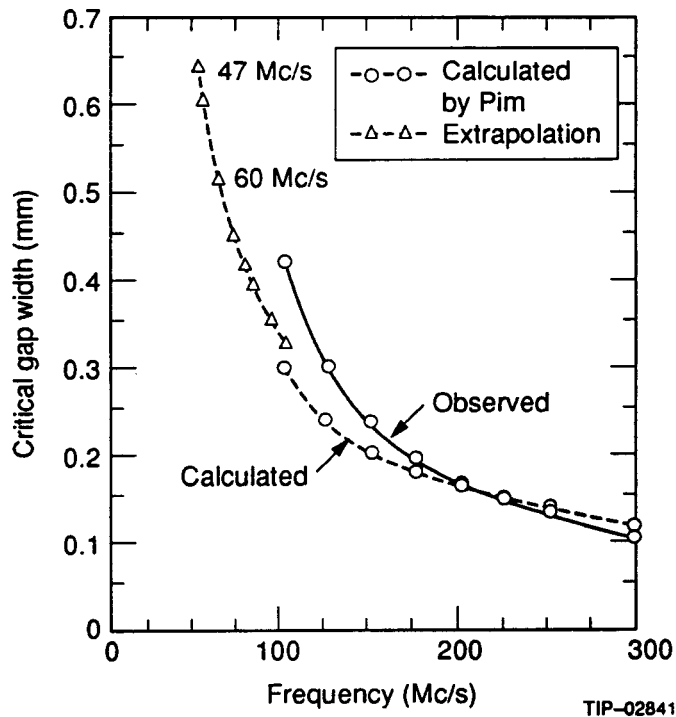


Figure 10. Variation of Critical Gap Width with Frequency (Calculated by Pim and Extrapolated).

#### 4.0 BREAKDOWN IN VACUUM

Breakdown in vacuum cannot be explained by the above theory, which states that breakdown is caused by the ionization electrons, positive ions, and molecules through collisions; that is, unless there are gas molecules, no breakdown can occur.<sup>6</sup> "When the pressure and gas density in a system are so low that the electron mean free path is much larger than the spacing of conductors, electron multiplication by impact ionization of the gas molecules cannot take place."<sup>7</sup> Meek also agrees on this point.<sup>5</sup>

In reality, breakdown is observed at all times in vacuum if the field intensity is very high. Sparks appear in good vacuum ( $10^{-6}$ – $10^{-7}$  mm Hg) when the field intensity is 100–1000 kV/cm.

In the absence of gas ionization, breakdown can occur from the electrodes' effects. "Milikan, Eyring, and Mackeowen studied these sparks [in vacuum  $10^{-6}$ – $10^{-7}$  mm Hg] and came to the conclusion that they resulted from intense localized electron emission at the surfaces of electrodes due to the 'pulling of electrons out of metal.'"<sup>6</sup> In fact, there is a protrusion or an irregularity on the surface of the electrode where the field intensity is much higher than that at any other point. At this point, the electron can get out of the surface forming the current; the local temperature rises, so the gas forms a film at the electrode, and the metal point vaporizes. Then there are enough gas and metal vapors to be ionized, forming the path for the breakdown. After the breakdown, the irregularity is destroyed. Another irregularity with the higher breakdown potential will be the next target. Consequently, electrodes that have been flashed in vacuum many times will have higher breakdown potential. In other words, the smoother the surfaces of the electrodes, the higher the breakdown potential.

At first, it was believed that high field intensity reduced the potential barriers of metal atoms so that the electrons were liberated. If this were true, the effect would have to be temperature-dependent. But experiments conducted above 1500°C showed that the results were independent of temperature. Later a theory was developed in which all electrons, including the degenerate ones, can leak out under the influence of the external high field. These electrons consist of not only the ones in the outermost band, but the degenerate ones in the inner bands as well. Therefore, it was concluded that the results are not temperature-dependent.<sup>6</sup>

## 5.0 CONCLUSION

In summary, the critical gap may be determined at any desired frequency. The two frequencies of interest are 47 MHz and 60 MHz, at which the estimated critical gaps are 0.64 mm and 0.512 mm, respectively, and the estimated breakdown potentials are 2066 rms V and 1728 rms V, respectively. It is suggested that at gaps smaller than the critical gap, the value of breakdown stress should be 10–15% lower than the static value. For gaps much longer than the critical gap, the value of the breakdown stress should be 29 kV/cm.<sup>3</sup>

Since the breakdown in vacuum originates from irregularities on the surfaces, these irregularities are dominant factors. Moreover, the vacuum breakdown cannot be explained by the theory of ionization due to collisions. The effect of frequency on the breakdown in this case, if any, is of secondary importance.

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## APPENDIX

```

C THIS PROGRAM DETERMINES THE EQUATION OF A STRAIGHT LINE
C REPRESENTING A NUMBER OF EMPIRICAL DATA.
C
C VARIABLES USED.
    INTEGER N, I, INDEX
    REAL SUMX, SUMY, SUMXY, SLOPE, B, RESSUM,
    +     X(20), Y(20), ESTY(20), RES(20), RESMAX
C
C
C NUMBER OF DATA USED
    READ(1,*) N
C PRINT THE HEADING AND THE COLUMNS OF THE OUTPUT
    WRITE(5,100) 'THE LEAST SQUARES LINE FITTING'
100  FORMAT('-',20X,A26)
    WRITE(5,200) 'NUMBER OF EXPERIMENTAL POINTS: ', N
200  FORMAT('0',20X,A33,I2)
    WRITE(5,300) 'EXPERIMENTAL', 'EXPERIMENTAL',
    +           'ESTIMATED Y', 'RESIDUAL'
300  FORMAT('0',A12,13X,A12,13X,A9,15X,A9)
    WRITE(5,400) 'X-VALUE', 'Y-VALUE', 'Y-VALUE'
400  FORMAT(1X,A7,18X,A7,18X,A7)
C
C READ THE EMPIRICAL DATA INTO ARRAYS X(I), Y(I)
    READ(1,*)(X(I), Y(I), I = 1,N)
C FIND THE EQUATION OF THE STRAIGHT LINE  $Y = A * X + B$ 
C A IS THE SLOPE; B IS THE Y-INTERCEPT
C  $SUMX = X_1 + X_2 + \dots + X_N$ 
C  $SUMY = Y_1 + Y_2 + \dots + Y_N$ 
C  $SUMXY = X_1*Y_1 + X_2*Y_2 + \dots + X_N*Y_N$ 
C  $SUMSQX = X_1*X_1 + X_2*X_2 + \dots + X_N*X_N$ 
C  $SLOPE A = (SUMX*SUMY - N*SUMXY) / (SUMX*SUMY - N*SUMSQX)$ 
C  $Y-INTERCEPT B = (SUMY - SLOPE*SUMX) / N$ 
    SUMX = 0.000
    SUMY = 0.000
    SUMXY = 0.000
    SUMSQX = 0.000
    DO 500 I = 1,N
        SUMX = SUMX + X(I)
        SUMY = SUMY + Y(I)
        SUMXY = SUMXY + X(I)*Y(I)
        SUMSQX = SUMSQX + X(I)**2
500  CONTINUE
    SLOPE = (SUMX*SUMY - FLOAT(N)*SUMXY) /
    +       (SUMX**2 - FLOAT(N)*SUMSQX)
    B = (SUMY - SLOPE*SUMX) / FLOAT(N)
C COMPUTE THE RESIDUAL OF EACH POINT, WHICH IS THE
C DIFFERENCE BETWEEN THE ACTUAL AND ESTIMATED VALUES OF Y
    RESSUM = 0.000
    DO 600 I = 1,N
        ESTY(I) = SLOPE*X(I) + B
        RES(I) = Y(I) - ESTY(I)
        RESSUM = RESSUM + RES(I)**2
600  CONTINUE
C FIND THE LEAST ACCURATE POINTS WHOSE RESIDUALS ARE MAXIMUM.
C INDEX INDICATES THE SUBSCRIPTS OF THESE POINTS IN RES(I)

```

```

C RESMAX IS THE CORRESPONDING VALUE OF RES( INDEX )
  RESMAX = ABS( RES( I ) )
  INDEX = 1
  DO 700 I = 1, N
    IF ( ABS( RES( I ) ).GT. RESMAX ) THEN
      RESMAX = ABS( RES( I ) )
      INDEX = I
    ENDIF
700  CONTINUE
C PRINT THE RESULTS.
  DO 800 I = 1, N
    WRITE( 5, 900 ) X( I ), Y( I ), ESTY( I ), RES( I )
900  FORMAT( '0', F6.4, 17X, F8.3, 15X, F8.3, 15X, F8.3 )
800  CONTINUE
    WRITE( 5, 1000 ) 'THE LINEAR EQUATION IS : '
1000 FORMAT( '0', 20X, A23 )
    WRITE( 5, 2000 ) 'Y = ', SLOPE, 'X + ', B
2000 FORMAT( 1X, 20X, A5, F12.3, A5, F8.3 )
    WRITE( 5, 3000 ) ' THE LEAST ACCURATE POINT IS: '
3000 FORMAT( '0', 20X, A29 )
    WRITE( 5, 4000 ) 'X = ', X( INDEX ), ' Y = ', Y( INDEX )
4000 FORMAT( 20X, A5, F8.3, A10, F8.3 )
    WRITE( 5, 5000 ) 'THE RESIDUAL SUM IS: ', RESSUM
5000 FORMAT( '0', 20X, A22, F8.3 )
  STOP
  END

```

THE LEAST SQUARES LINE FIT

NUMBER OF EXPERIMENTAL POINTS: 8

EXPERIMENTAL X-VALUE	EXPERIMENTAL Y-VALUE	ESTIMATED Y-VALUE	RESIDUAL
.0400	1427.000	1432.887	-5.887
.0310	1200.000	1195.076	4.924
.0240	1013.000	1010.112	2.888
.0200	907.000	904.418	2.582
.0170	833.000	825.147	7.853
.0150	773.000	772.300	.700
.0140	733.000	745.877	-12.877
.0100	640.000	640.183	-.183

THE LINEAR EQUATION IS

$$Y = 26423.460 X + 375.948$$

THE LEAST ACCURATE POINT IS:

$$X = .014 \quad Y = 733.000$$

THE RESIDUAL SUM IS: 301.918

THIS IS THE DATA TABLE TO APPROXIMATE THE EQUATION  
OF THE STRAIGHT LINE REPRESENTING THE BREAKDOWN vs GAPS.

08		; NUMBER OF POINTS USED.
0.0400	1427.0	; POINT A.
0.0310	1200.0	; POINT B.
0.0240	1013.0	; POINT C.
0.0200	907.0	; POINT D.
0.0170	833.0	; POINT E.
0.0150	773.0	; POINT F.
0.0140	733.0	; POINT G.
0.0100	640.0	; POINT K.

THE LEAST SQUARES LINE FIT

NUMBER OF EXPERIMENTAL POINTS: 8

EXPERIMENTAL X-VALUE	EXPERIMENTAL Y-VALUE	ESTIMATED Y-VALUE	RESIDUAL
.0629	1350.000	1343.735	6.265
.0550	1173.000	1180.506	-7.506
.0479	1033.000	1033.806	-.806
.0450	980.000	973.886	6.114
.0425	913.000	922.231	-9.231
.0350	766.000	767.267	-1.267
.0325	726.000	715.612	10.388
.0300	660.000	663.957	-3.957

THE LINEAR EQUATION IS

$$Y = 20661.940 X + 44.099$$

THE LEAST ACCURATE POINT IS:

$$X = .032 \quad Y = 726.000$$

THE RESIDUAL SUM IS: 344.007

THIS IS THE DATA TABLE TO APPROXIMATE THE EQUATION  
OF THE STRAIGHT LINE REPRESENTING THE "LIMIT 29Kv/cm"  
CURVE

X	Y	POINT
.0629	1350.	POINT A'
.0550	1173.	POINT B'
.0479	1033.	POINT C'
.0450	980.	POINT D'
.0425	913.	POINT E'
.0350	766.	POINT F'
.0325	726.	POINT G'
.0300	660.	POINT K'

THE LEAST SQUARES LINE FIT

NUMBER OF EXPERIMENTAL POINTS: 6

EXPERIMENTAL X-VALUE	EXPERIMENTAL Y-VALUE	ESTIMATED Y-VALUE	RESIDUAL
.0400	1573.000	1571.476	1.524
.0375	1493.000	1502.419	-9.419
.0350	1440.000	1433.362	6.638
.0325	1367.000	1364.305	2.695
.0300	1300.000	1295.248	4.752
.0275	1220.000	1226.191	-6.191

THE LINEAR EQUATION IS  
 $Y = 27622.770 X + 466.565$

THE LEAST ACCURATE POINT IS:  
 $X = .038 \quad Y = 1493.000$

THE RESIDUAL SUM IS: 203.277

THIS IS THE DATA TABLE TO APPROXIMATE THE EQUATION  
 OF THE STRAIGHT LINE REPRESENTING THE 50Hz- BREAKDOWN  
 CURVE.

6		; NUMBER OF POINTS USED.
0.04	1573.	; POINT 1
0.0375	1493.	; POINT 2
0.0350	1440.	; POINT 3
0.0325	1367.	; POINT 4
0.03	1300.	; POINT 5
0.0275	1220.	; POINT 6



```

C This program computes the CRITICAL GAP verssus FREQUENCY
C using the iteration method.
      REAL      E, El, V, F, G, W, K, A, B, P, PI
C Guess the initial value of breakdown stress E
      WRITE(*,*) 'ENTER THE VALUE OF E'
      READ (*,*) E
      WRITE(*,*) 'ENTER THE VALUE OF F'
      READ (*,*) F
      K = 3.9E+5
      A = 1.58E-5
      B = 1.33E-2
      P = 760.
      PI = 3.1416
C the first iteration
      W = 2 * PI * F
100  G = ((2*K*E)/(7*W*P))*(6-SQRT(15-(21*P**2*B)/(A*E**2)))
      V = 26423.460 * G + 375.948
      El = V * 1.414 /G
C Create the WHILE DO loop to compare values of guessed E and
c calculated one after an iteration cycle.
      IF (ABS(El - E).GT.0.01 ) THEN
          E = El
          GO TO 100
      ENDIF
      WRITE (5,*) 'THE APPLIED FREQUENCY IN HZ: ', F
      WRITE (5,*) 'THE CRITICAL GAP IN CM: ', G
      WRITE (5,*) ' THE BREAKDOWN VOLTAGE IN VOLT RMS: ', V
      WRITE (5,*) 'THE PEAK BREAKDOWN STRESS IN VOLT/CM ', E
      STOP
      END

```

THE APPLIED FREQUENCY IN HZ: 47000000.0000000  
THE CRITICAL GAP IN CM: 6.397086E-002  
THE BREAKDOWN VOLTAGE IN VOLT RMS: 2066.2800000  
THE PEAK BREAKDOWN STRESS IN VOLT/CM 45672.6600000

THE APPLIED FREQUENCY IN HZ: 50000000.0000000  
THE CRITICAL GAP IN CM: 6.042679E-002  
THE BREAKDOWN VOLTAGE IN VOLT RMS: 1972.6330000  
THE PEAK BREAKDOWN STRESS IN VOLT/CM 46160.0400000

THE APPLIED FREQUENCY IN HZ: 60000000.0000000  
THE CRITICAL GAP IN CM: 5.118431E-002  
THE BREAKDOWN VOLTAGE IN VOLT RMS: 1728.4150000  
THE PEAK BREAKDOWN STRESS IN VOLT/CM 47748.5800000

THE APPLIED FREQUENCY IN HZ: 70000000.0000000  
THE CRITICAL GAP IN CM: 4.459188E-002  
THE BREAKDOWN VOLTAGE IN VOLT RMS: 1554.2200000  
THE PEAK BREAKDOWN STRESS IN VOLT/CM 49284.0200000

THE APPLIED FREQUENCY IN HZ: 75000000.0000000  
THE CRITICAL GAP IN CM: 4.195645E-002  
THE BREAKDOWN VOLTAGE IN VOLT RMS: 1484.5830000  
THE PEAK BREAKDOWN STRESS IN VOLT/CM 50032.8300000

THE APPLIED FREQUENCY IN HZ: 80000000.0000000  
THE CRITICAL GAP IN CM: 3.965079E-002  
THE BREAKDOWN VOLTAGE IN VOLT RMS: 1423.6590000  
THE PEAK BREAKDOWN STRESS IN VOLT/CM 50769.5900000

THE APPLIED FREQUENCY IN HZ: 90000000.0000000  
THE CRITICAL GAP IN CM: 3.580752E-002  
THE BREAKDOWN VOLTAGE IN VOLT RMS: 1322.1070000  
THE PEAK BREAKDOWN STRESS IN VOLT/CM 52208.5400000

THE APPLIED FREQUENCY IN HZ: 1.000000E+008  
THE CRITICAL GAP IN CM: 3.273082E-002  
THE BREAKDOWN VOLTAGE IN VOLT RMS: 1240.8100000  
THE PEAK BREAKDOWN STRESS IN VOLT/CM 53604.0600000