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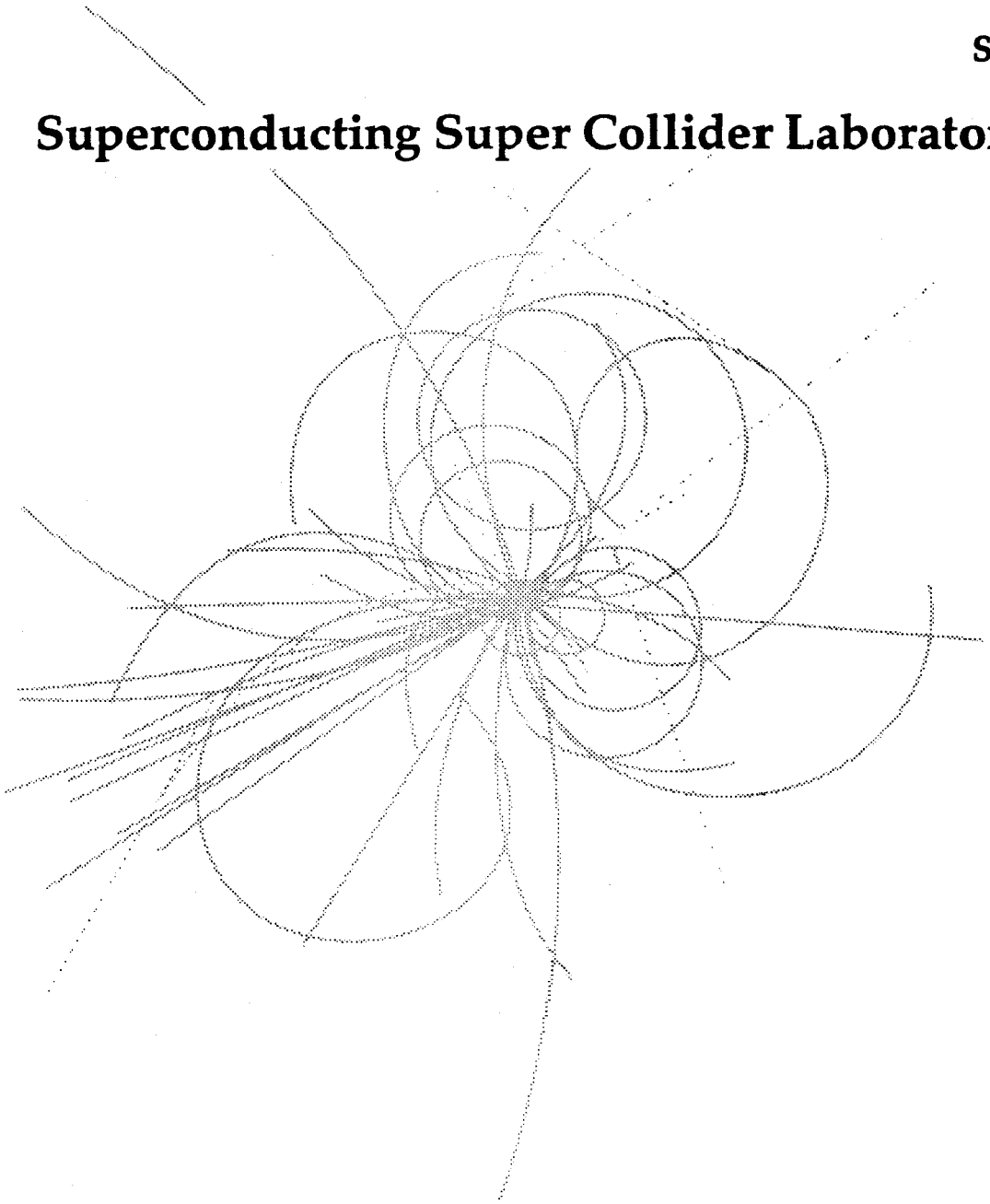

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Emittance Growth Caused by Magnet Vibrations in the SSC

G. V. STUPAKOV

Abstract

Magnet vibrations may be caused by ground motions in the collider. The vibrations can be characterized as high-frequency (in the range of 1 kHz) or low-frequency (in the range of Hz or tens of Hz) noise. High-frequency noise resonates with the betatron oscillations causing fast emittance growth of the beam. Analytical results indicate a tolerable vibration level that is not harmful for the beams. Estimations for the ambient low-frequency noise show that in typical conditions it is of no danger for the collider.

1.0 INTRODUCTION

The effects of ground motion in the SSC have been intensively studied in recent years.[1-4] The main concern of these studies has been a low-frequency (tens of Hz and lower) motion of the quadrupole magnets. This motion causes a distortion of closed orbits of the beams so that the two beam centers can miss each other at the interaction point. In addition, slow motion can gradually increase the emittance of the beams.[4,5]

This paper addresses two kinds of problems. The first one relates to high-frequency vibrations of the quadrupoles (in the range of one kHz) that can resonantly excite coherent betatron oscillations.[6] Due to nonlinearity of these oscillations, the betatron frequency depends on their amplitude. This dependence triggers a phase mixing mechanism[7] which quickly transforms coherent betatron oscillations into increased emittance of the beams.* The primary results of this study agree with a recent independent study of the Novosibirsk group.[8]

The other problem considered is the emittance growth caused by low-frequency motions of the quadrupoles. Using a small parameter $T\omega$, where T is the particle revolution period in the collider and ω is the frequency of the ground motion, one finds an amplitude of the betatron oscillations around a displaced closed orbit driven by the slowly-moved quadrupoles. Using the equation for this amplitude one can estimate the emittance growth induced by nonlinear phase mixing. Though results differ from the previous considerations,[4,5] the general conclusion is that the increase of emittance caused by the slow motion is typically small and does not deteriorate the beam performance.

2.0 GENERAL EQUATIONS

Consider a quadrupole magnet of the length l. Let it be displaced from its original position by the distance $\Delta(t)$ which is a function of time t The displacement of the quadrupole perturbs the magnetic field on a particle orbit by a quantity $\delta B(t) = B'\Delta(t)$ where B' stands for the magnetic field gradient. The governing equation for particle motion takes the form

$$y'' + K(s)y = F(s,t)\,, \tag{1}$$

where y is the position offset of a particle,

$$F(s,t) = \frac{e\delta B}{cP}h(s) = \frac{\Delta(t)}{f_q l}h(s) \tag{2}$$

$f_q \equiv cP/elB'$ is the focal length of the quadrupole, P is the particle longitudinal momentum, and $h(s)$ denotes the function that is equal to unity inside the quadrupole and takes zero values outside of it. The function $h(s)$ indicates that the magnetic field on the orbit is perturbed only inside the quadrupole.

Since the length of the quadrupole is much smaller than the betatron period, the function $h(s)$ can be replaced by a delta–function (thin-lens approximation) according to

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*The main source of nonlinearity in the SSC stems from beam-beam interaction and gives the characteristic time for the phase mixing of the order of a second.

$$\frac{1}{l}h(s) \rightarrow \delta(s - s_0)\,, \tag{3}$$

where s_0 stands for the quadrupole position. Putting Eq. (3) into Eq. (2) yields

$$y'' + K(s)y = \frac{\Delta(t)}{f_q}\delta(s - s_0)\,. \tag{4}$$

It is convenient to transform from y and s to new variables η and ϕ:

$$\eta = \beta^{-1/2} y\,, \qquad \phi = \nu^{-1}\psi(s)\,, \tag{5}$$

where $\psi(s)$ is the betatron phase. From Eq. (4), using the relation $d\phi/ds = 1/\nu\beta$ne finds

$$\frac{d^2\eta}{d\phi^2} + \nu^2\eta = \nu\beta_0^{1/2}\frac{\Delta(t)}{f_q}\sum_{m=-\infty}^{\infty}\delta\left(\phi - 2\pi m - \phi_0\right), \tag{6}$$

where β_0 and ϕ_0 refer to the position of the quadrupole and the sum on the right-hand side indicates that due to periodicity, the values of ϕ which differ by 2π correspond to the same position on the orbit. Each delta-function in Eq. (6) represents a kick that the particle experiences during successive passings through the displaced quadrupole.

Due to the circular motion of the particle, the variable ϕ changes with time t so that ϕ increases by 2π after each period $T = 2\pi/\Omega$ (Ω is the rotational frequency). The function $\phi(t)$ has the form

$$\phi = \Omega t + \delta\phi(t)\,, \tag{7}$$

where $\delta\phi(t)$ is a small quantity, $\delta\phi(t) \ll 1$, that describes deviation of $\phi(t)$ from the linear dependence. In what follows, neglect $\delta\phi$ and keep only the linear term in Eq. (7). As a detailed analysis shows, the account for $\delta\phi$ would add only small corrections to the final result. Using t as an independent variable instead of ϕ, $\phi = \Omega$, from Eq. (6) we obtain

$$\frac{d^2\eta}{dt^2} + \nu^2\Omega^2\eta = \nu\Omega\beta_0^{1/2}\frac{\Delta(t)}{f_q}\sum_{m=-\infty}^{\infty}\delta(t - mT - t_0)\,. \tag{8}$$

3.0 HIGH-FREQUENCY VIBRATIONS

Assume now that $\Delta(t)$ is a random function with a given autocorrelation $K(\tau)$,

$$K(\tau) = \langle\Delta(t)\Delta(t - \tau)\rangle\,, \tag{9}$$

where the angular brackets stand for the ensemble averaging. An important characteristic of a random process is its spectral density $S(\omega)$ given by Fourier transform of the autocorrelation function,

$$S(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} K(\tau)e^{i\omega t}dt\ , \quad K(\tau) = \int_{0}^{\infty} S(\omega)\cos\omega t d\omega \tag{10}$$

Using the identity

$$\sum_{m=-\infty}^{\infty} \delta(t - mT - t_0) = \frac{1}{T}\sum_{m=-\infty}^{\infty} e^{im\Omega(t-t_0)} \tag{11}$$

one can write down a formal solution to Eq. (8)

$$\eta(t) = \frac{\beta_0^{1/2}}{Tf_q}\int_0^t \Delta(\xi - t_0)\sin\nu\Omega(t - \xi + t_0)\sum_{m=-\infty}^{\infty} e^{im\Omega\xi}d\xi + \eta_0\cos(\nu\Omega t + \theta_0)\,, \tag{12}$$

where η_0 and θ_0 are the initial amplitude and phase of η. Because $\eta(t)$ is a random function, an appropriate measure of the amplitude of the oscillations is the averaged value $\langle\eta^2(t)\rangle$. As will be shown below, this quantity grows linearly with time so in the following calculations one can neglect the initial value η_0 in Eq. (12). Taking the square of Eq. (12) and performing the averaging one finds

$$\langle\eta^2(t)\rangle = \frac{\beta_0}{2T^2f_q^2}\int_0^t d\xi'\int_0^t d\xi'' K(\xi' - \xi'')\left[\cos\nu\Omega(\xi' - \xi'') - \cos\nu\Omega(2(t + t_0) - (\xi' + \xi''))\right]$$

$$\times \sum_{m,n=-\infty}^{\infty} e^{i\Omega(m\xi' + n\xi'')}. \tag{13}$$

Consider a limit of large t, $t \to\infty$. First, transform to new variables of integration ζ and μ, $\zeta = \xi' - \xi''$, $\mu = (\xi' + \xi'')/2$. For $m + n = 0$ resonant terms appear in Eq. (13) which linearly grow with time.* Keeping only these terms after straightforward calculations one finds

$$\langle\eta^2(t)\rangle = \frac{\pi\beta_0 t}{2T^2f_q^2}\sum_{n=-\infty}^{\infty} S(\Omega(\nu - n))\,. \tag{14}$$

*Formally, resonant terms also appear for $m + n \pm 2\nu = 0$, however this condition never holds for an accelerator because it requires ν to be equal to a half integer.

4.0 LOW-FREQUENCY VIBRATIONS

To study the low-frequency limit, return to Eq. (8) and define a complex variable z and dimensionless time ζ,

$$z = \frac{1}{\sqrt{\beta_0}}(\eta - i\frac{d\eta}{dt}), \quad \zeta = t\nu\Omega. \tag{15}$$

With ζ chosen as a time variable, the betatron oscillations proceed with a unit frequency. Transforming from η and t to z and ζ, Eq. (8) takes the form

$$\frac{dz}{d\zeta} - iz = -i\varepsilon(\zeta)\sum_{m=-\infty}^{\infty} \delta(\zeta - \zeta_0 - \mu m), \tag{16}$$

where $\mu = 2\pi\nu$ and $\varepsilon = \Delta f_q$ is the displacement normalized to the focal length of the quadrupole.

A general solution to Eq. (16) is the sum of the general solution $z_{gen.}$ of the homogeneous equation and a particular solution $z_{part.}$ of the inhomogeneous one:

$$z(\zeta) = z_{part.} + z_{gen.} \tag{17}$$

where

$$z_{gen.}(\zeta) = z_0 e^{i\zeta} \tag{18}$$

and

$$z_{part.}(\zeta) = -i\int_{-\infty}^{\zeta} d\xi\varepsilon(\xi)e^{i(\zeta-\xi)}\sum_{m=-\infty}^{\infty} \delta(\xi - \zeta_0 - \mu m) =$$

$$-i\sum_{m=-\infty}^{N} \varepsilon(\mu m + \zeta_0)e^{i(\zeta - \zeta_0 - \mu m)} \tag{19}$$

where N stands for the integer part of the ratio $(\zeta - \zeta_0)\mu$. N refers to the kick number at the last passing through the quadrupole and changes by unity each turn. For the sake of simplicity, assume that there are no initial coherent betatron oscillations in the beam and put $z_0 = 0$ in what follows.

To proceed further, suppose that the beam is injected into the lattice at $\zeta = 0$. Since the quadrupole motion before the injection does not influence the beam dynamics at $\zeta > 0$, one can put $\varepsilon(\zeta) = 0$ for $\zeta < 0$. Having this in mind one can utilize Laplace transform from $\varepsilon(\zeta)$ to $\hat{\varepsilon}(p)$,

$$\hat{\varepsilon}(p) = \int_0^{\infty} d\zeta \varepsilon(\zeta) e^{-p\zeta}, \quad \varepsilon(\zeta) = \frac{1}{2\pi i}\int_{s-i\infty}^{s+i\infty} dp\hat{\varepsilon}(p) e^{p\zeta}, \tag{20}$$

to obtain

$$z_{\text{part.}}(\zeta) = -\frac{1}{2\pi} e^{i(\zeta-\zeta_0)} \int_{s-i\infty}^{s+i\infty} dp\hat{\varepsilon}(p) \frac{e^{-iN\mu(1+ip)}}{1-e^{i\mu(1+ip)}}. \tag{21}$$

The integration in Eq. (21) is performed along a straight line Re $(p) = s$, so that the poles of the integrand be located in the left part of the complex plane p.

Now, take into account that $\Delta(t)$ is a slow function with a characteristic time of change, $t_\varepsilon \sim \pi/\omega$, much larger then the rotational period,

$$t_\varepsilon \gg T. \tag{22}$$

The analysis performed in Appendix A shows that the integral of Eq. (21) in the limit given by Eq. (22) splits into three parts, $z_{\text{part}} = z_1 + z_2 + z_3$. The first one is

$$z_1 = \frac{1}{2}\varepsilon(\mu N) \frac{e^{i(\zeta-\zeta_0-\mu N-\pi\nu)}}{\sin \pi\nu}. \tag{23}$$

To gain an insight to the origin of this term, take its real part and return back to dimensional variables:

$$y_1 = \frac{\Delta(NT)\sqrt{\beta\beta_0}}{2f_q \sin \pi\nu} \cos(\psi - \pi\nu - [\psi_0 + \mu N]), \tag{24}$$

where ζ identifies with the betatron phase ψ (see Eqs. (5) and (7)). Equation (24) now represents a well known expression for a perturbed closed orbit resulting from a shift of a quadrupole. Note, that according to Eq. (24) the closed orbit slowly evolves in time corresponding to the position of the quadrupole at the moments of the last kick, $t = NT$.

The second term in z_{part} is given by the following formula

$$z_2 = -\frac{i\pi\nu}{2\sin^2 \pi\nu} e^{i(\zeta-\zeta_0-\mu N)} \frac{d\varepsilon}{d\zeta}(\mu N). \tag{25}$$

This term, being proportional to the time derivative of the displacement Δ, describes excitation of the betatron oscillations of the beam due to time variation of Δ. A rough estimate of the ratio $|z_1/z_2|$ gives a value of the order of $T/t_\varepsilon \ll 1$; that is the amplitude of these oscillations is much smaller than the setoff of the closed orbit.

Finally, the third term in z_{part} is

$$z_3 = -\frac{1}{2i\pi\nu} e^{i(\zeta - \zeta_0)} \sum_{k=-\infty}^{k=\infty} \hat{\varepsilon}(p_k) , \tag{26}$$

where $p_k = i(1 - k/\nu)$ and k is an integer. As is shown in Appendix A, this term is related with the behavior of $\varepsilon(\zeta)$ at $\zeta = 0$. It corresponds to the oscillations that the beam acquires at the moment of injection to a slightly perturbed closed orbit. If ε and its first derivative with respect to time are equal to zero at $\zeta = 0$, $|z_3|$ appears to be much less that $|z_2|$. Since long-time effects of the quadrupole motion are of most interest, one can discard this term in what follows. Note only that this term may be of importance in computer simulations of low frequency ground motions; the result of the simulations may be influenced by the smoothness of the start at $t = 0$.

Using Eqs. (17), (18), (23), (24) one is now able to find how the beam oscillates around a new closed orbit:

$$z_{\text{oscill.}}(\zeta) = z(\zeta) - z_{\text{closed orbit}} = z(\zeta) - z_1 =$$

$$\frac{i\pi\nu}{2\sin^2 \pi\nu} e^{i(\zeta - \zeta_0 - \mu N)} \frac{d\varepsilon}{dt}(\mu N) = \Delta z(N) e^{i\zeta} , \tag{27}$$

where

$$\Delta z(N) = -\frac{i\pi\nu}{2\sin^2 \pi\nu} e^{-i(\zeta_0 + \mu N)} \frac{d\varepsilon}{dt}(\mu N) . \tag{28}$$

Equation (27) shows that the beam experiences coherent betatron oscillations whose amplitude and phase change in time (remember that N increases by one each turn).

5.0 DISCUSSION

As seen from Eq. (14) the largest perturbations of the beam are produced by the lenses with the smallest focal length located at high-β positions. At the SSC, these are the quadrupoles focussing the beam to the low-β interaction point. Typical values for these lenses are:[9] $f_q \approx 20$ m, $\beta_0 \approx 8000$ m. The lowest resonant frequency that contributes to Eq. (14) is $\Omega\{\nu\}$, where the braces denote the distance from ν to the nearest integer. For nominal SSC parameters, $\Omega = 3.44$ kHz and $\{\nu\} = 0.22$ this frequency is $\Omega\{\nu\} = 760$ Hz. Putting the numbers into Eq. (14) for the low-β interaction region ($\beta = 0.5$ m) and require $\sqrt{\langle y^2(t)\rangle_{av}}$ to be less than 2 μm (that is roughly a half of the beam rms radius at IP) after 10 hours of operation one obtains a constraint on a tolerable level of vibrations of the magnets:

$$S(760 \text{ Hz}) < 10^{-12} \mu\text{m}^2/\text{Hz} . \tag{29}$$

Another estimate can be made if one chooses a typical quadrupole with $f_q \approx 60$ m, located at $\beta_0 \approx 200$ m and takes into account that there are about 800 quadrupoles in each ring. At this high frequency the vibrations of different magnets are uncorrelated, so one has to sum up all the contributions to $\langle y^2(t) \rangle_{av}$ from separate quadrupoles. With the same requirements for $\sqrt{\langle y^2(t) \rangle_{av}}$, one finds

$$S(760\ \text{Hz}) < 0.8 \cdot 10^{-12}\,\mu\text{m}^2/\text{Hz}\,. \tag{30}$$

Up to now, there were no detailed measurements of ground motion performed in this range of frequencies at the SSC site. To compare the level of vibrations given by Eq. (30) with experimental data, use the results of Novosibirsk group who measured ground motion in a wide frequency range in the UNK tunnel near Protvino.[8] They found that, depending on environmental conditions, S can reach as high as $10^{-9}\mu\text{m}^2/\text{Hz}$ for the frequency of a few hundred Hz. This level of vibrations is three orders of magnitude larger than is tolerable for the SSC.

The preceding estimates assume that the magnets vibrate with the same amplitude as the ground. Strictly speaking, this is not true and the magnet motion will significantly depend on the properties of supports and mechanical structures connected to the body of the magnet. An important issue of magnet response to the high-frequency ground motion requires further investigations.

ACKNOWLEDGMENTS

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APPENDIX A

In this Appendix we calculate the following integral:

$$z_{\text{part.}}(z) = -\frac{1}{2\pi} e^{i(\zeta - \zeta_0)} \int_{s-i\infty}^{s+i\infty} dp\,\hat{\varepsilon}(p) \frac{e^{-iN\mu(1+ip)}}{1 - e^{i\mu(1+ip)}} . \tag{A.1}$$

First, close the contour of integration by a half circle through the left-hand side of the complex plane as is shown in Figure A-1.

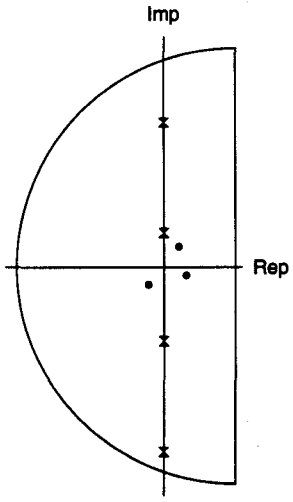

Figure A-1. Complex Plane *p* with the Contour of Integration for the Calculation of the Integral (A.1). Shown by dots are the poles of the function $\hat{\varepsilon}(p)$ and the crosses show locations of the poles of the denominator in (A.1).

The contribution coming from the half-circle tends to zero when the radius of the circle goes to infinity because the integrand exponentially vanishes at infinity at the left-hand side part of the complex plane. Now, the integral can be calculated as a sum of the residues of the integrand. There are two different types of poles in our problem at which the residues should be computed. First, in general, the function $\hat{\varepsilon}(p)$ has poles, shown by dots in Figure 1. The location of these poles in the complex plane, $p = p_i^{(\varepsilon)}$, $i = 1,\ldots,i_{\max}$, can be roughly related with the characteristic time t_ε of the function $\Delta(t)$

$$\left|p_i^{(\varepsilon)}\right| \sim \frac{1}{\Omega \nu \tau_\varepsilon} . \tag{A.2}$$

Second, there is a set of poles connected with the zeroth of the denominator in Eq. (A.1). These are located at

$$p_k = i\,(l - k/\nu) , \tag{A.3}$$

where k is an arbitrary integer.

When calculating the residues at the points $p = p_i^{(\varepsilon)}$ we will take into account that according to Eqs. (A.2) and (22) $\mu p \ll 1$. Using this small parameter we expand the factor $\exp(-\mu p)$ in the denominator of the integrand:

$$\frac{1}{1-e^{i\mu\nu(1+ip)}} \approx \frac{1}{1-e^{i\mu\nu}} - \frac{\mu p e^{i\mu\nu}}{\left(1-e^{i\mu\nu}\right)^2} . \tag{A.4}$$

Putting the first term from Eq. (A.4) back into Eq. (A.1) and performing the integration, one finds a contribution denoted by z_1,

$$z_1 = \frac{1}{2}\varepsilon(\mu N)\frac{e^{i\left(\zeta - \zeta_0 - \mu N - \pi\nu\right)}}{\sin \pi\nu} , \tag{A.5}$$

while the second term from the right-hand side of Eq. (A.4) gives the contribution which we denote by z_2,

$$z_2 = -\frac{i\pi\nu}{2\sin^2\pi\nu} e^{i\left(\zeta - \zeta_0 - \mu N\right)} \frac{d\varepsilon}{d\zeta}(\mu N) . \tag{A.6}$$

Finally, calculating the contribution of the poles p_k, one finds the contribution z_3,

$$z_3 = -\frac{1}{2i\pi\nu} e^{i\left(\zeta - \zeta_0\right)} \sum_{k=-\infty}^{k=\infty} \hat{\varepsilon}(p_k) . \tag{A.7}$$

In Eq. (A.7) the modulus of p_k, $|p_k| \gg k/\nu$, is much larger than the characteristic scale of the function $\hat{\varepsilon}(p)$ in the complex plane (which is of the order of Eq. (A.2)). That means that a good approximation for $\hat{\varepsilon}(p_k)$ is given by asymptotic expansion of the function $\hat{\varepsilon}(p)$ in the limit $|p| \to \infty$. As is known from the theory of Laplace transform, this limit is related with the behavior of the function $\varepsilon(\zeta)$ and its derivative at $\zeta = 0$,

$$\hat{\varepsilon}(p) \propto \frac{\text{const}}{p^{n+1}} , \tag{A.8}$$

where n is the number of the first nonzero derivative* of the function $\varepsilon(\zeta)$ at $\zeta = 0$. Using Eq. (A.8) one finds that

$$|z_3| \sim \left(\frac{T}{t_\varepsilon}\right)^{n-1} |z_2| . \tag{A.9}$$

*If $\varepsilon(0)$ does not vanish, $n = 0$.

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