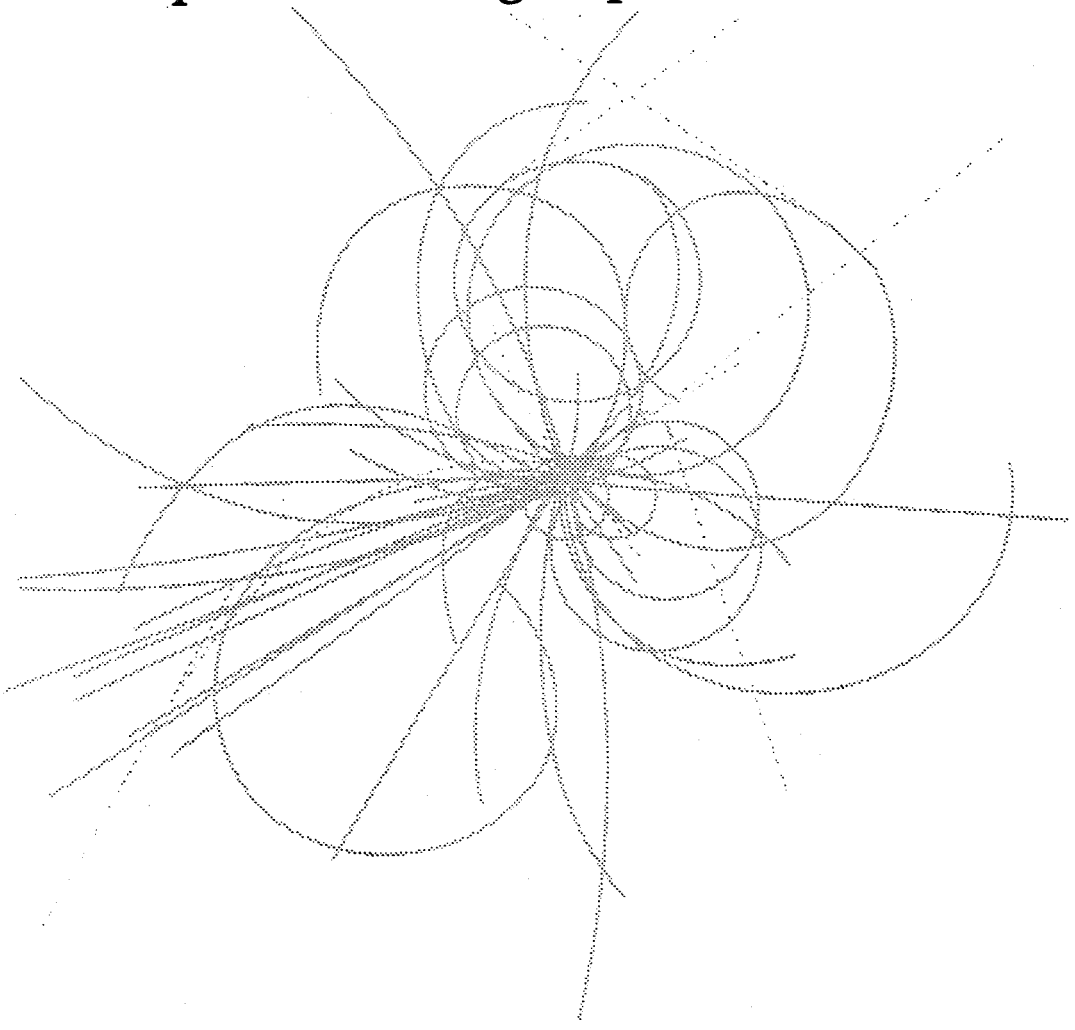
SSCL-442

SSCL-442

Superconducting Super Collider Laboratory



Head-On and Long Range Beam-Beam Tune Shifts Spread in the SSC

G. López

May 1991

SSCL-442

Head-On and Long Range Beam-Beam Tune Shifts Spread in the SSC

G. López

Accelerator Division
Superconducting Super Collider Laboratory*
2550 Beckleymeade Avenue
Dallas, Texas 75237

May 1991

*Operated by the Universities Research Association, Inc. for the U.S. Department of Energy under Contract No. DE-AC02-89ER40486.

SSCL-442

Head-On and Long Range Beam-Beam Tune Shifts Spread in the SSC

G. López

Abstract

The head-on and long range incoherent tune shifts for the Superconducting Super Collider (SSC) are estimated using the numerical integration of the analytical expression coming from the first order in the perturbation strength. The variation of the tune shift as a function of the displacements of the charged particle in the vertical and horizontal planes is studied with the nominal parameters for the SSC. A scaling expression is obtained for the parameters involved in the beam-beam tune shifts which allows prediction of the effect in the incoherent tune shift spread under changes in these parameters.

CONTENTS

FIGURES vii
1.0 INTRODUCTION 1
2.0 ANALYTICAL APPROXIMATION FOR HEAD–ON AND LONG-RANGE TSS .. 1
3.0 HEAD-ON AND LONG RANGE TUNE SHIFT FOR THE LOW-B IR 6
3.1 Head-On Tune Shift 6
3.2 Long Range Tune Shift 9
3.3 Maximum Long Range Tune Shift 12
3.4 Maximum Tune Shift Dependence 16
4.0 TOTAL TUNE SHIFT SCALING PARAMETRIZATION 17
5.0 CONCLUSIONS 18
ACKNOWLEDGMENTS 21
REFERENCES 23

FIGURES

1. Surface to be Integrated in (6)-(7) when $d_y = 0$, $a_x = a_y = 5$ 4
2. Surface to be Integrated in (6) when $d_y = 7.5$, $a_x = a_y = 5$ 5
3. Surface to be Integrated in (7) when $d_y = 7.5$, $a_x = a_y = 5$ 5
4. Relative Horizontal Head-On Tune Shift. 6
5. Relative Vertical Head-On Tune Shift 7
6. Relative Head-On Tune Shift Space. 8
7. Head-On Tune Shift Spread with the Nominal SSC Parameters for a Low-β IR 8
8. Relative Horizontal Long Range Tune Shift. 10
9. Relative Vertical Long Range Tune Shift 10
10. Relative Long Range Tune Shift Space. 11
11. Long Range Tune Shift Spread with the Nominal SSC Parameters for One Low-β IR 11
12. Total Tune Shift Spread with One Low-β IR using the SSC Nominal Parameters (α = 75 μrad) 12
13. Maximum Relative Long Range Tune Shift ($a_x = 0$, $a_y = 7$) 13
14. Maximum Long Range Tune Shift for the SSC in One Low-β IR ($a_x = 0$, and 1: $a_y = 7$; 2: $a_y = 6$; 3: $a_y = 5$) 14
15. Maximum Long Range Tune Shift for the SSC in One Low-β IR ($a_x = 0$, and 1: α = 50 μrad; 2: α = 75 μrad; 3: α = 120 μrad) 14
16. Beam-Beam Vertical Dynamics Aperture as a Function of Crossing Angle (Low-β IR) 15
17. Total Tune Shift Spread using the SSC Nominal Parameters. One Low-β IR (α = 120 μrad) 16

1.0 INTRODUCTION

The tune shift spread (TSS) in a collider machine is one of the most important parameters in its design and operation. The spread must be limited to those values that avoid crossing dangerous resonances in the operation tune space of the machine. This crossing may produce instabilities in the beam, limiting the performance of the machine (beam lifetime and luminosity)[1] and creating radiation problems is the detectors themselves (beam-halo).[2] Therefore, it is desirable to keep this TSS as small as possible for the Superconducting Super Collider (SSC). The most important contribution to the TSS comes from the beam-beam interaction (head-on and long range). This TSS is due to the kicks given to the particles in one bunch by another bunch because of the nonlinear (in general) force felt by the particles of the two interacting bunches. One kick corresponds to the head-on collision (when actually the two interacting bunches are in the same line), which produces a negative TSS in both transverse directions. The other kick comes from the long range interaction in the intersection region (IR), where the beams have not yet been separated into different beam tubes. Assuming that the crossing angle is in the vertical plane (y), this kick produces a negative tune shift in the horizontal plane (x) and a positive tune shift in the vertical plane, for y-amplitudes oscillate a little less than the separation of the close orbits of the two bunches, D_y.

For a beam with gaussian density distribution in both transversal planes, the exact expression for the tune shift experienced by test particles due to head-on collision is well known.[3] This expression will be used in the study of the TSS for the SSC.

2.0 ANALYTICAL APPROXIMATION FOR HEAD–ON AND LONG RANGE TSS

In general, the kick given to a particle (which has horizontal (x) and vertical (y) displacement from its own closed orbit) due to beam-beam interaction is given in the horizontal and vertical planes by

$$\Delta x' = -\left(\frac{4\pi\xi}{\beta}\right)\frac{2\sigma^2}{r^2}\left(1-\exp\left(-r^2/2\sigma^2\right)\right)x \tag{1a}$$

and

$$\Delta y' = -\left(\frac{4\pi\xi}{\beta}\right)\frac{2\sigma^2}{r^2}\left(1-\exp\left(-r^2/2\sigma^2\right)\right)(y+D_y)\ , \tag{1b}$$

where β is the Courant-Snyder beta function, σ is the standard deviation in the gaussian distribution of the particles, r is defined by

$$r^2 = x^2 + (y+D_y)^2, \tag{2}$$

and ξ (called the tune shift parameter) is defined as

$$\xi = -\frac{N r_p \beta}{4\pi\gamma\sigma^2} . \tag{3}$$

N is the number of particles in the bunch, r_p is the classical radius of proton (e^2/mc^2) which has the value 1.5348×10^{-18} m, and γ is the relativistic factor, $\gamma^{-1} = \sqrt{1 - v^2/c^2}$.

Out of any resonance, the tune shifts are given to the first order of perturbation strength by

$$\Delta\nu_x = -\frac{\beta}{2\pi a_x \sigma}\langle \Delta x' \sin(\phi_x)\rangle \tag{4a}$$

and

$$\Delta\nu_y = -\frac{\beta}{2\pi a_x \sigma}\langle \Delta x' \sin(\phi_y)\rangle \tag{4b}$$

where a_x and a_y are the x and y amplitudes of the particle measured in units of the beam size, σ; the brackets $\langle\,\rangle$ represent the average over the betatron phases, ϕ_x and ϕ_y. Using (1), (2), and the following expressions for the displacement of the particle:

$$x = a_x \sigma(s) \sin(\phi_x(s) + 2\pi\nu_x t) \tag{5a}$$

and

$$y = a_y \sigma(s) \sin(\phi_y(s) + 2\pi\nu_y t) \tag{5b}$$

where t is the turn number, (ν_x, ν_y) is the operational tune of the machine, and s represents the azimuthal position of the collision, it follows that

$$\frac{\Delta\nu_x}{\xi} = \frac{1}{\pi^2}\int_0^{2\pi}\int_0^{2\pi} \frac{\left(1 - e^{-(a_x^2 s_x^2 + a_y^2 s_y^2 + 2a_y d_y s_y + d_y^2)/2}\right) s_x^2 d\psi_x d\psi_y}{(a_x^2 s_x^2 + a_y^2 s_y^2 + 2a_y d_y s_y + d_y^2)} \tag{6}$$

and

$$\frac{\Delta\nu_y}{\xi} = \frac{1}{\pi^2}\int_0^{2\pi}\int_0^{2\pi} \frac{\left(1-e^{-(a_x^2 s_x^2 + a_y^2 s_y^2 + 2a_y d_y s_y + d_y^2)/2}\right)\left(s_y^2 + \frac{d_y s_y}{a_y}\right) d\psi_x d\psi_y}{a_x^2 s_x^2 + a_y^2 s_y^2 + 2a_y d_y s_y + d_y^2}, \tag{7}$$

where $d_y = D_y/\sigma$ is the relative separation between the orbits of the colliding bunches, $s_i = \sin(\psi_i)$ and $\psi_i = \phi_i + 2\pi\nu_i t$ for $i = x, y$. Using the following identities:

$$\sin^2(\psi) = \frac{1}{2}(1 - \cos(2\psi)) , \tag{8a}$$

$$\int_0^1 e^{pu} du = \frac{1 - e^p}{p} , \tag{8b}$$

$$I_\nu(z) = \frac{(z/2)^\nu}{\Gamma(\nu + 1/2)\,\Gamma(1/2)}\int_0^{2\pi} e^{z\cos\theta}\sin^{2\nu}(\theta)\, d\theta , \tag{8c}$$

$$\int f^n(z)\, e^{pf(z)} dz = \frac{\partial^n}{\partial p^n}\int e^{pf(z)} dz , \tag{8d}$$

it is not difficult to see that expressions (5a) and (5b) can be written as

$$\frac{\Delta\nu_x}{\xi} = \int_0^1 du\; e^{-(\alpha_1 + \alpha_2 + d_y^2/2)u}[I_0(\alpha_1 u) - (I_1(\alpha_1 u)]$$
$$\left[I_0(\alpha_2 u) + \sum_{n=1}^{\infty}\frac{(a_y d_y u)^{2n} n!}{2^n (2n)!}\sum_{k=0}^{n}\frac{(-1)^k}{k!(n-k)!}\frac{d^k I_0 \alpha_2 u)}{d(\alpha_2 u)^k}\right] \tag{9}$$

and

$$\frac{\Delta\nu_y}{\xi} = \int_0^1 du\; e^{-(\alpha_1 + \alpha_2 + d_y^2/2)u} I_0(\alpha_1 u)\Bigg[I_0(\alpha_2 u) - I_1(\alpha_2 u)$$
$$+ \sum_{n=1}^{\infty}\frac{(a_y d_y u)^{2n}(n+1)!}{2^n (2n)!}\sum_{k=0}^{n+1}\frac{(-1)^k}{k!\,(n+1-k)!}\frac{d^k I_0(\alpha_2 u)}{d(\alpha_2 u)^k}$$
$$- \sum_{m=1}^{\infty}\frac{a_y^{2m-2} d_y^{2m} u^{2m-1} m!}{2^{m-1}(2m-1)!}\sum_{k=0}^{m}\frac{(-1)^k}{k!\,(m-k)!}\frac{d^k I_0(\alpha_2 u)}{d(\alpha_2 u)^k}\Bigg] , \tag{10}$$

where $\alpha_i = a_i^2/4$, $i = x, y$, and $I_\nu(z)$ are the Bessel functions of imaginary argument. Using $n = 5$, $m = 5$, it is enough to have a good approximation. In the calculation of the TSS for the SSC, the integration of (6) and (7) as well as (9) and (10) was done and compared.

The surface generated by the function to integrate in (6) and (7) has the following symmetry with respect to the phase advance variables, ψ_x and ψ_y:

$$f_i = (\psi_x) + \pi,\ \psi_y + \pi) = f_i\,(\psi_x,\ \psi_y);\ i = x,\ y$$

for head-on tune shift; but for long range tune shift, the symmetries are

$$f_i = (\psi_x) + \pi,\ \psi_y = f_i\,(\psi_x, \psi_y)$$

and

$$f_i\,(\psi_x, \psi_y + \pi) \neq f_i\,(\psi_x, \psi_y);\ i = x,\ y \quad .$$

These symmetries are shown in Figure 1 for the head-on case, and in Figure 2 and Figure 3 for the long range case.

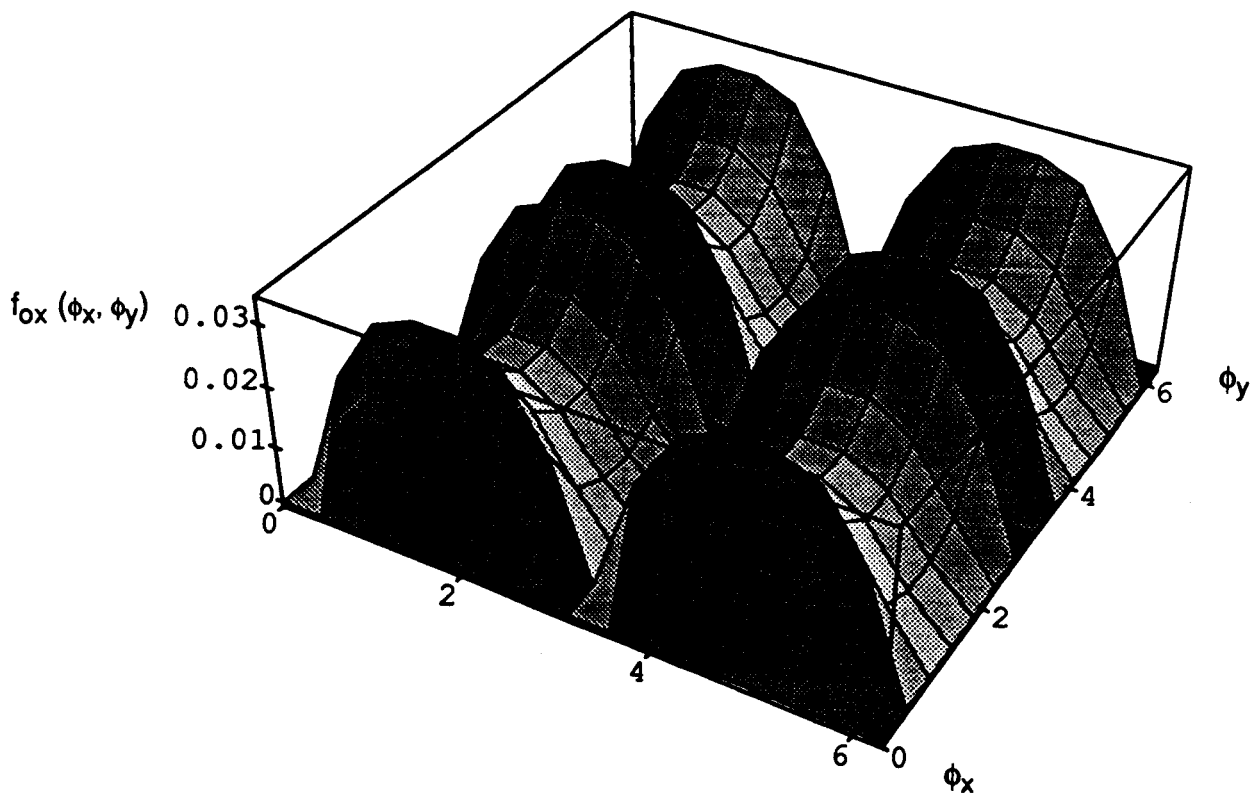


Figure 1. Surface to be Integrated in (6)-(7) when $d_y = 0$, $a_x = a_y = 5$.

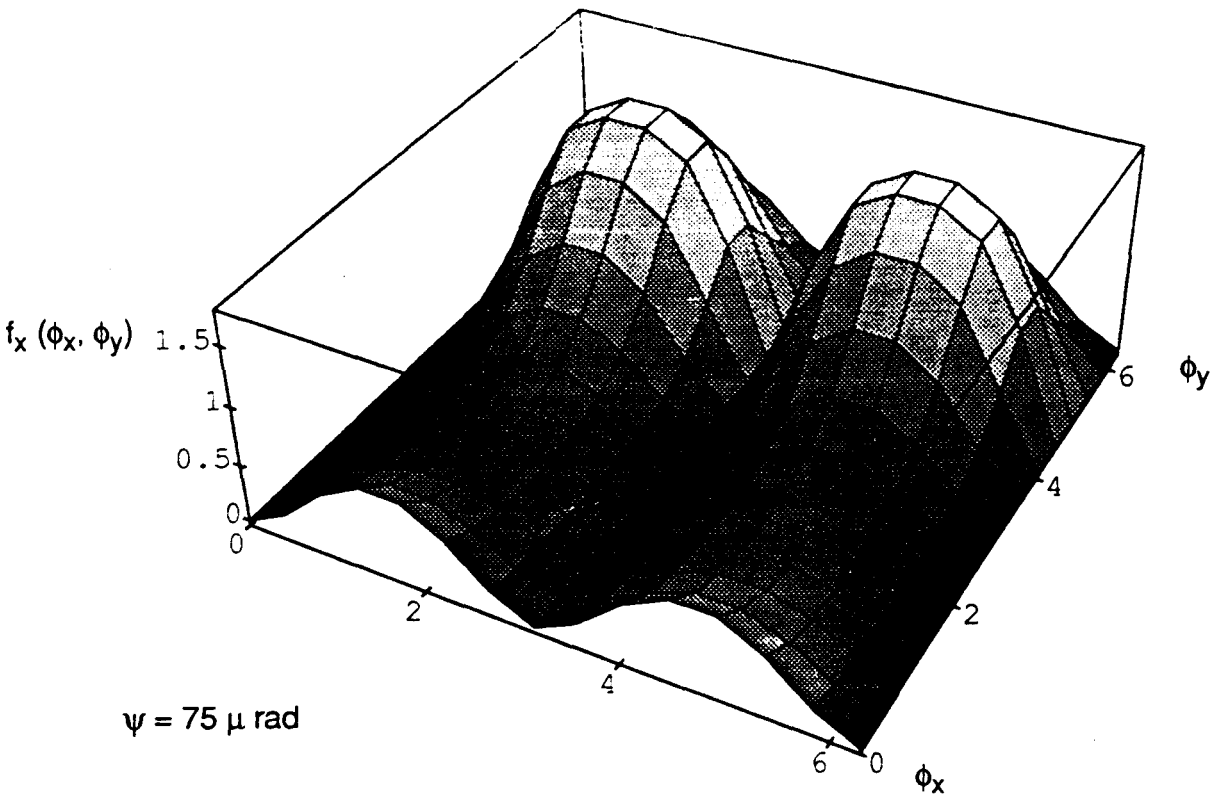


Figure 2. Surface to be Integrated in (6) when $d_y = 7.5$, $a_x = a_y = 5$..

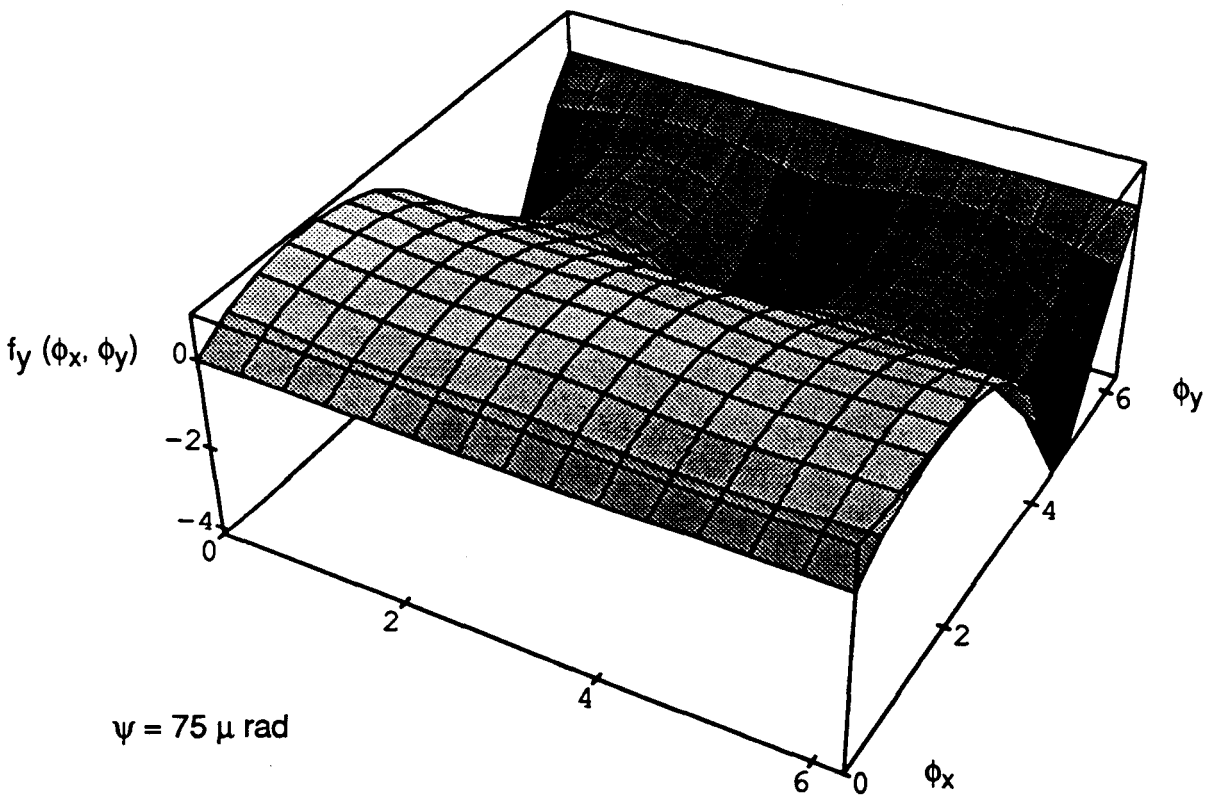


Figure 3. Surface to be Integrated in (7) when $d_y = 7.5$, $a_x = a_y = 5$.

3.0 HEAD-ON AND LONG RANGE TUNE SHIFT FOR THE LOW-β IR

Using the above analytical approximation and making a numerical integration of these expressions, the TSS can be calculated for the nominal parameters of the SSC collider. These calculations will be made mainly for the low-β IR, but the medium-β IR will be considered at the end of this section and in the next section.

3.1 Head-On Tune Shift

Making $d_y = 0$, relative tune shifts predicted by (6) and (7) or by (9) and (10) are shown in Figure 4 and Figure 5 as a function of the amplitudes. The relative tune shift space is shown in Figure 6.

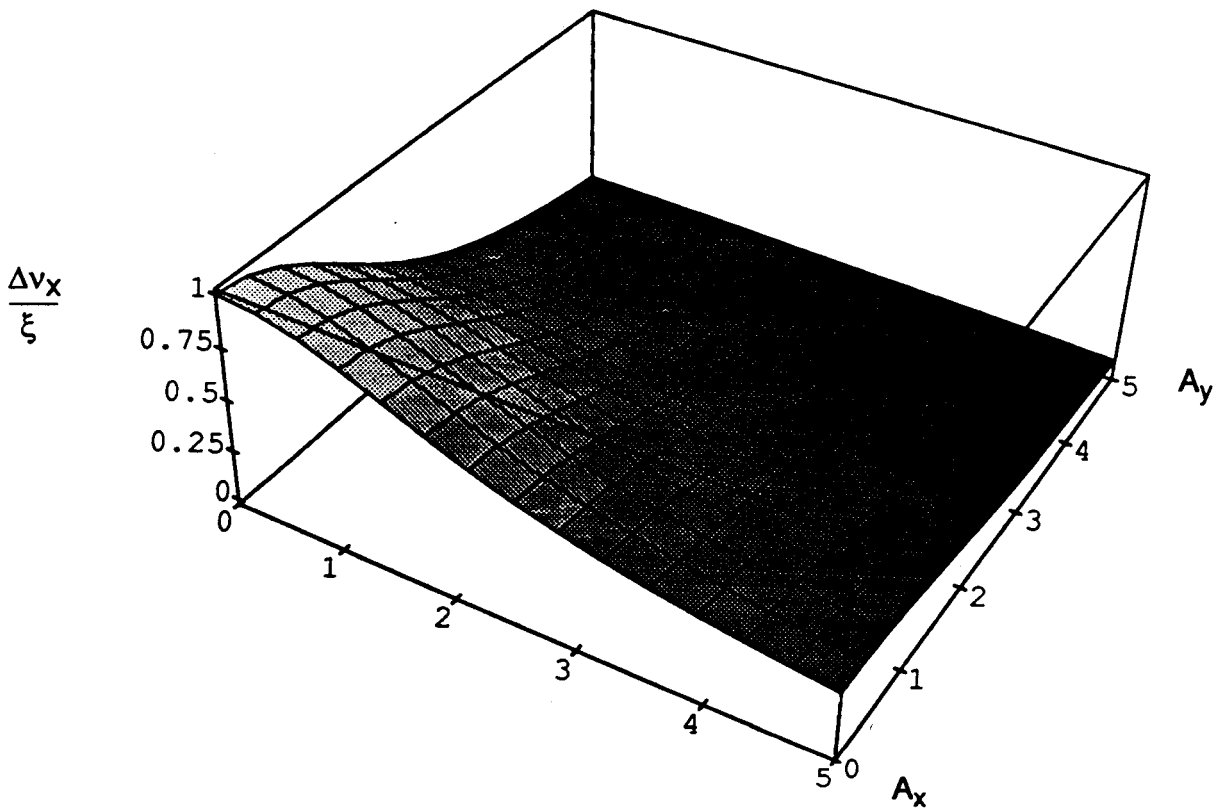


Figure 4. Relative Horizontal Head-On Tune Shift.

Using the relation between the beam size and the Curant-Snyder parameters, the beta function (β) and the normalized emittance (ε),

$$\sigma = \sqrt{\varepsilon\beta/\gamma}\,, \tag{11}$$

the tune shift parameter (3) is expressed as

$$\xi = \frac{N r_p}{4\pi\varepsilon} \ . \tag{12}$$

With the nominal values for the SSC[4]

$$N = 7.5 \times 109$$

and

$$\varepsilon = 10^{-6}\,\mathrm{m}\,,$$

the value of the tune shift parameter obtained is

$$\xi = -0.000916\,. \tag{13}$$

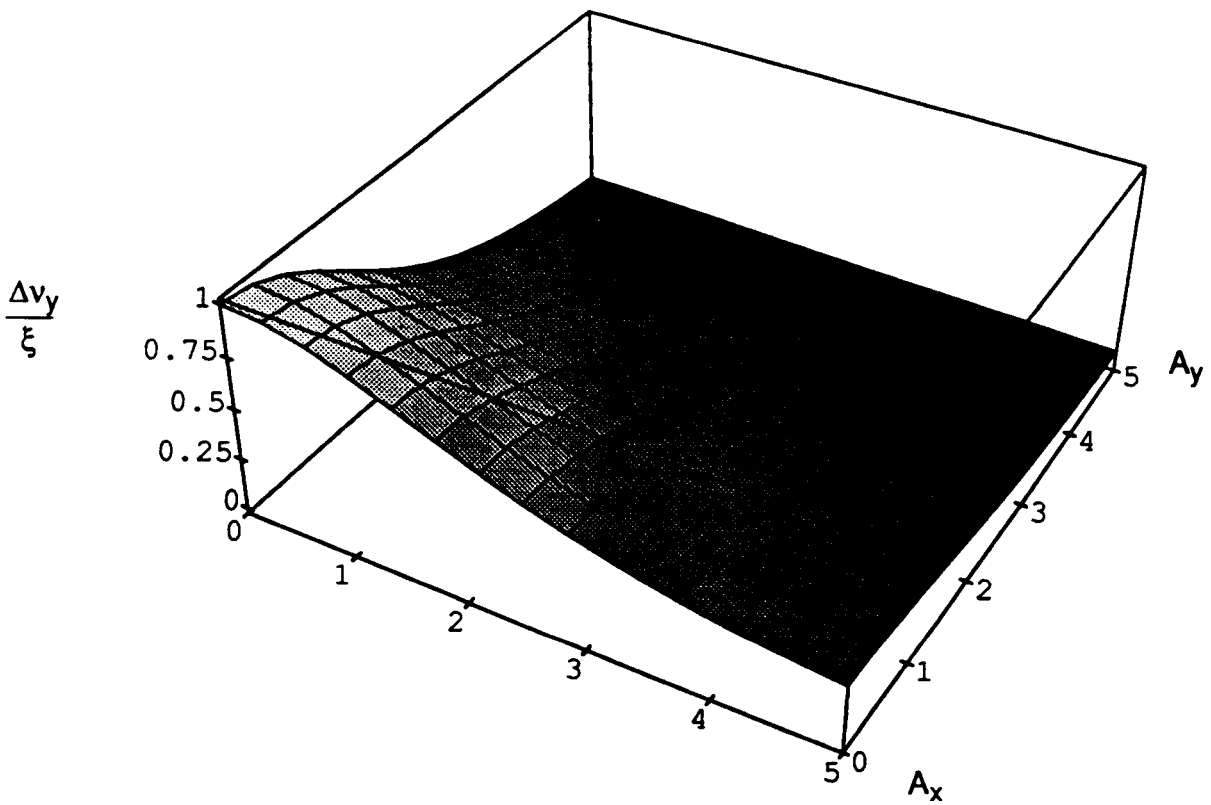


Figure 5. Relative Vertical Head-On Tune Shift.

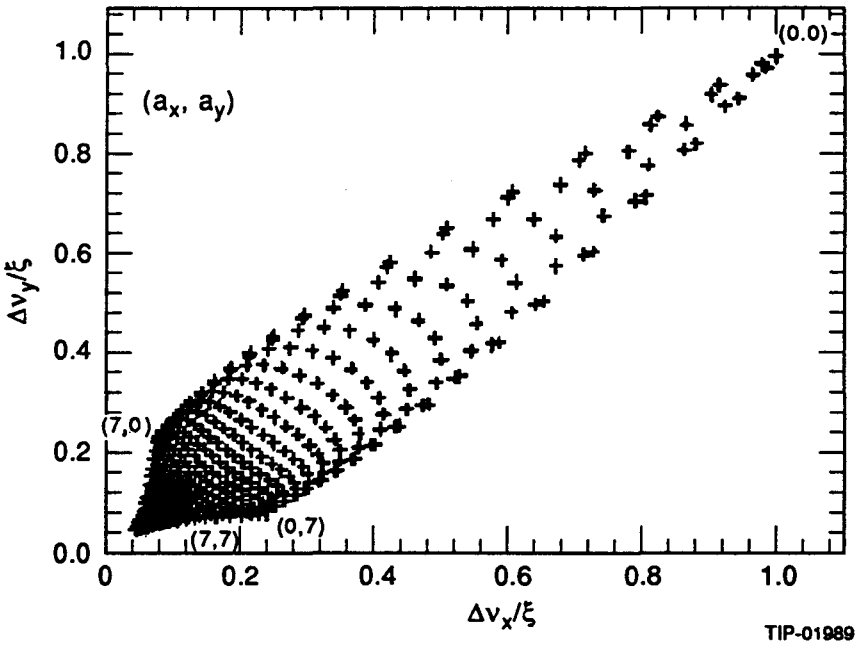


Figure 6. Relative Head-On Tune Shift Space.

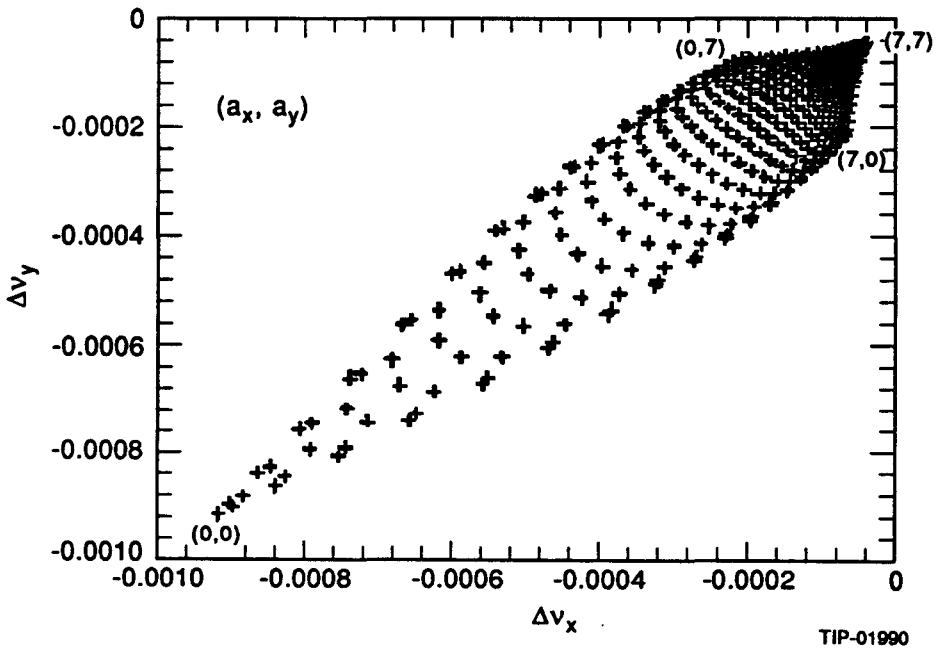


Figure 7. Head-On Tune Shift Spread with the Nominal SSC Parameters for a Low-β IR.

3.2 Long Range Tune Shift

In this case, $d_y \neq 0$ and the tune shift parameter (12) has to be multiplied by the number of long range interactions for the two low-β IR,

$$2(2L/S_B) ,$$

where S_B is the separation between the colliding bunches. This defines a new parameter,

$$\xi_L = \frac{4L}{S_B} \xi , \tag{14}$$

for the long range tune shifts. Using (11) and the relation between the separation, D_y, the long range interaction length, L, and the crossing angle, α ($D_y = \alpha L$), the relative separation can be written as

$$d_y = D_y \sqrt{\frac{\gamma}{\varepsilon\beta}} = \alpha L \sqrt{\frac{\gamma}{\varepsilon\beta}} . \tag{15}$$

Assume a pure drift space during the long range interaction, where the beta function is given by

$$\beta(L) = \beta^* + \frac{L^2}{\beta^*} \cong \frac{L^2}{\beta^*} ; \tag{16}$$

then expression (15) can be written in the form

$$d_y = \alpha \sqrt{\beta^* \frac{\gamma}{\varepsilon}} . \tag{17}$$

For the low-β IR, $\beta^* = 0.5$ m, and at the highest energy, $\gamma = 2 \times 10^4$, this parameter has the simple expression

$$d_y = 10^5 \alpha . \tag{18}$$

Using the nominal crossing angle, $\alpha = 75$ μrad, Figure 8 and Figure 9 show the relative tune shifts (with respect to ξ_L) as a function of the amplitude a_x for several fixed amplitudes a_y. The relative tune shift space is shown in Figure 10. For the nominal values, $L = 82.5$ m and $S_B = 5$ m ($4L/S_B = 66$ long range collisions), the long range TSS is shown in Figure 11.

Finally, the total TSS (head-on plus long range) is shown in Figure 12 for the above nominal values.

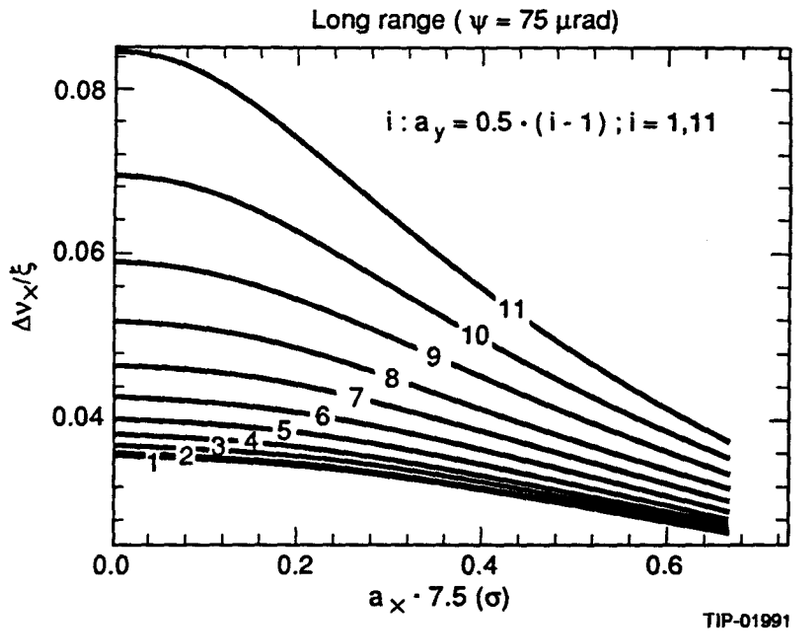


Figure 8. Relative Horizontal Long Range Tune Shift.

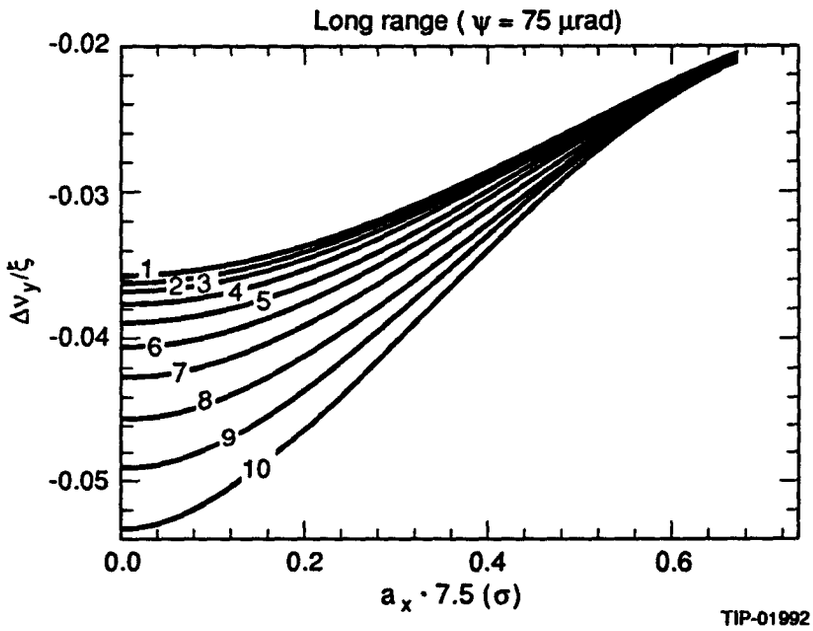


Figure 9. Relative Vertical Long Range Tune Shift.

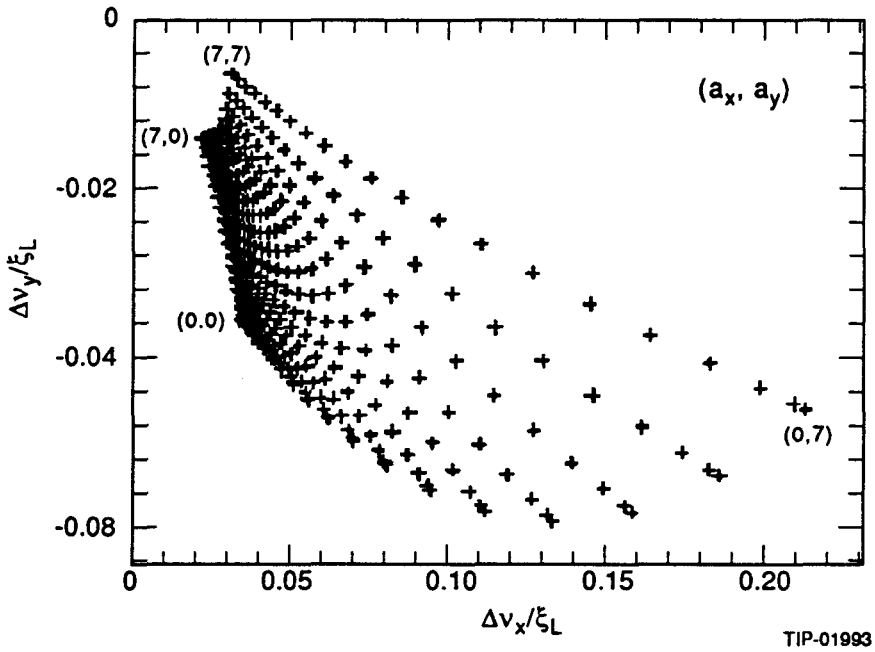


Figure 10. Relative Long Range Tune Shift Space.

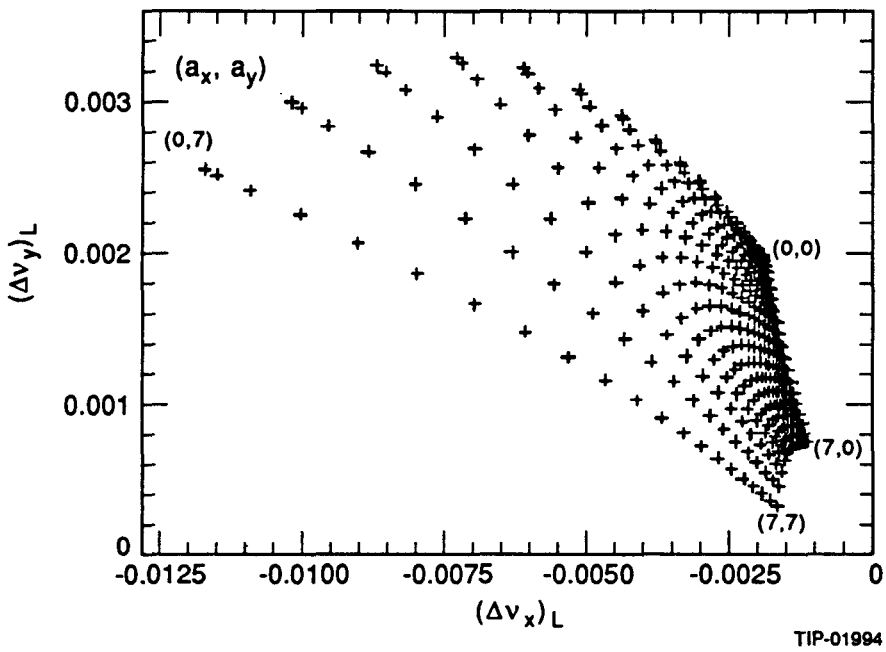


Figure 11. Long Range Tune Shift Spread with the Nominal SSC Parameters for One Low-β IR.

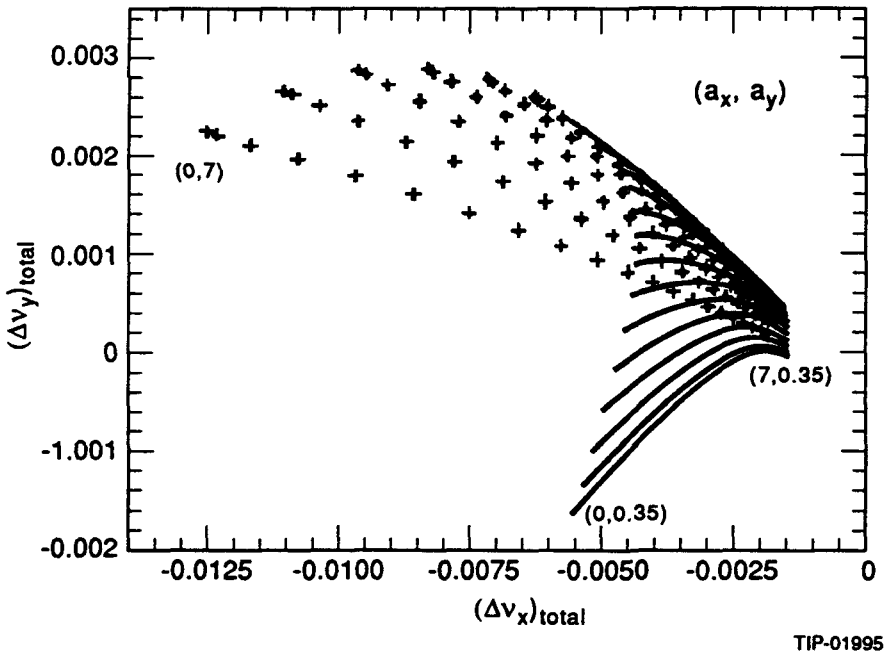


Figure 12. Total Tune Shift Spread with One Low-β IR using the SSC Nominal Parameters (α = 75 μrad).

3.3 Maximum Long Range Tune Shift

The relative horizontal long range tune shift reaches its maximum value when the horizontal amplitude is zero and the vertical amplitude has its maximum allowed value ($a_x = 0$ and $a_y = a_y^{max}$). This is almost true for the vertical plane (see Figure 12) so the condition can be used as a good approximation for the maximum vertical tune shift. This fact simplifies the analysis of the maximum relative tune shift as a function of the crossing angle. Using the above conditions in (6) and (7), it follows that

$$\left(\frac{\Delta\nu_x}{\xi}\right)_L^{max} = \frac{1}{\pi}\int_0^{2\pi} \frac{\left(1 - e^{-(a_y^{max} s_y + d_y)^2/2}\right) d\psi_y}{(a_y^{max} s_y + d_y)^2} \tag{19a}$$

and

$$\left(\frac{\Delta\nu_y}{\xi}\right)_L^{max} \cong \frac{2}{\pi}\int_0^{2\pi} \frac{\left(1 - e^{-(a_y^{max} s_y + d_y)^2/2}\right)\left(s_y^2 + \frac{d_y}{a_y^{max}} s_y\right) d\psi_y}{(a_y^{max} s_y + d_y)^2} \,. \tag{19b}$$

The relative long range tune shift as a function of d_y, for the nominal value $a_y^{max} = 7$ can be seen in Figure 13. The maximum long range tune shift expected for the SSC as a function of d_y and for several maximum amplitudes a_y^{max} is shown in Figure 14.

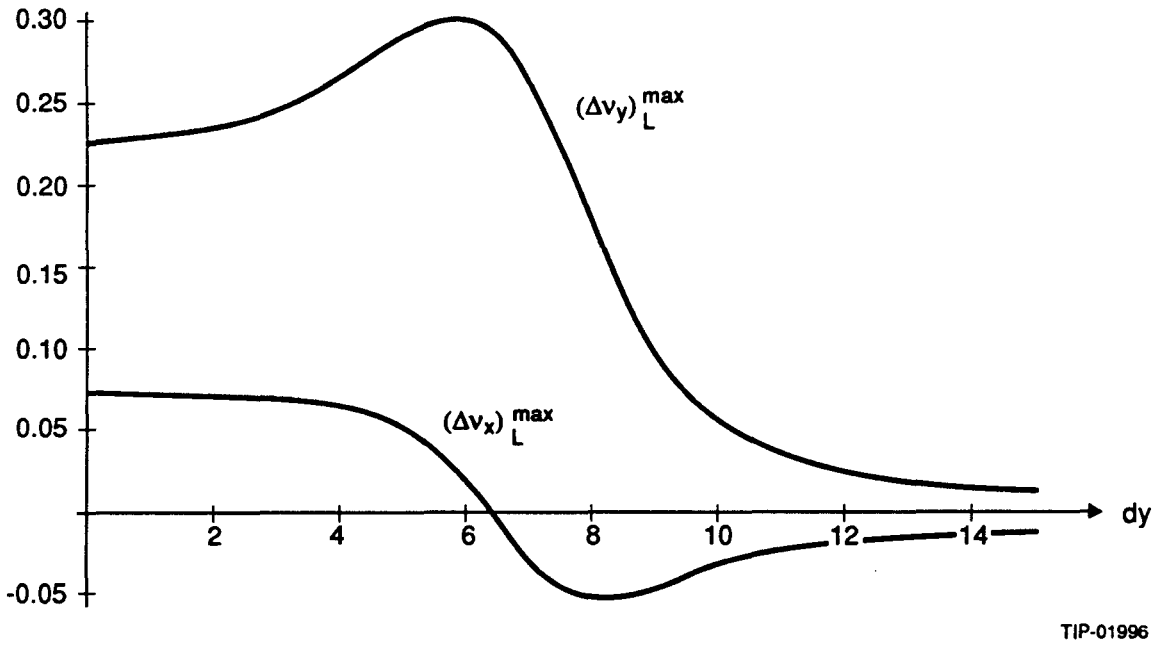


Figure 13. Maximum Relative Long Range Tune Shift ($a_x = 0$, $a_y = 7$).

Figure 15 shows the maximum tune shifts as a function of the maximum amplitude for several crossing angles. As the figure shows, there is no limitation in amplitude of the horizontal oscillations of the particle which could come from the restriction in the maximum long range tune shift. However, this restriction may bring about limitations in the vertical oscillations of the particle which come from the maximum horizontal long range tune shift (19a). This maximum beam-beam tune shift limit appeared initially from the observations of the beam-beam instabilities in the ISR and SPS at CERN.[1] There is not yet a quantitative explanation for these instabilities,[7] and due to the lack of an adequate theory, gross safe estimations based on the experience with low energy and low intensity machines (ISR, SPS, and Tevatron) must be made.[8]

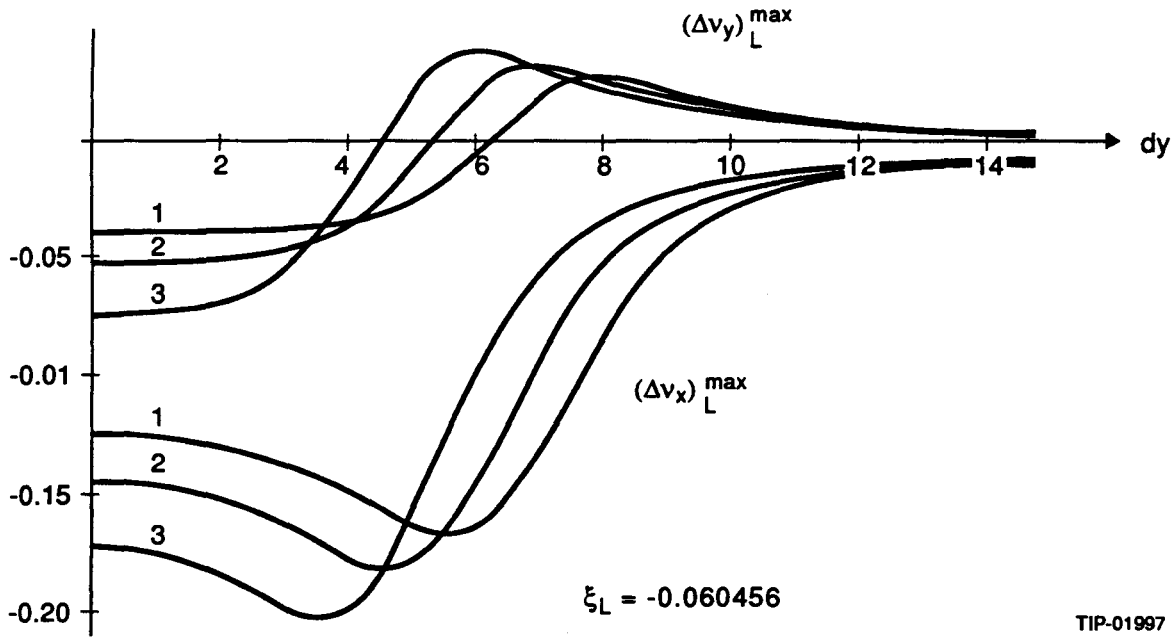


Figure 14. Maximum Long Range Tune Shift for the SSC in One Low-β IR ($a_x = 0$, and 1: $a_y = 7$; 2: $a_y = 6$; 3: $a_y = 5$).

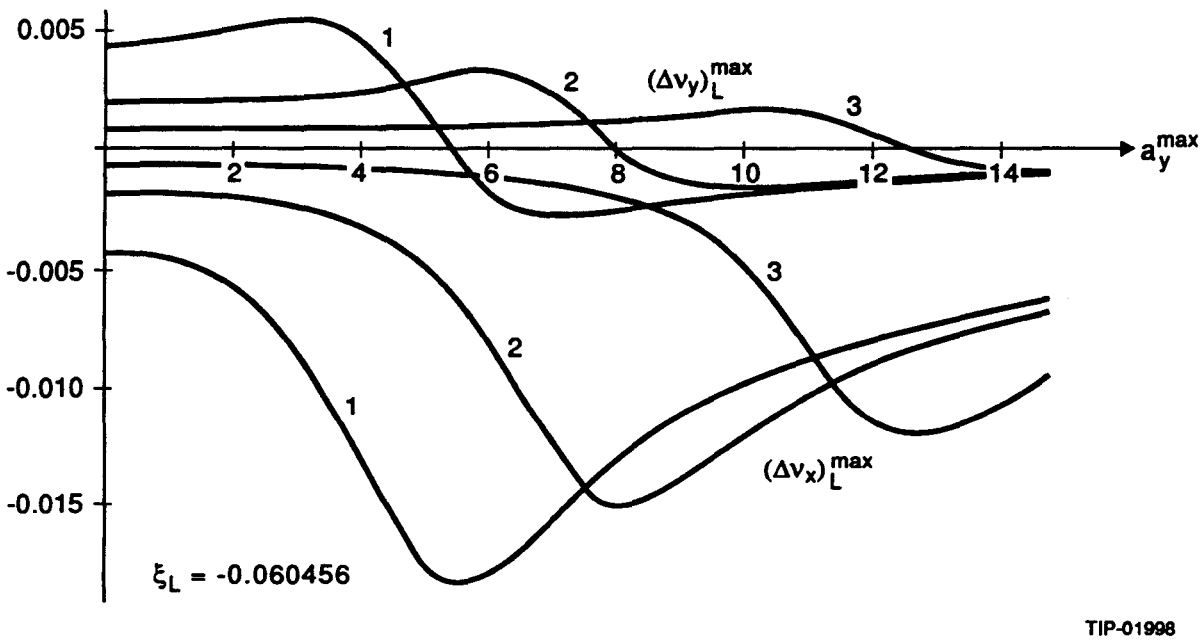


Figure 15. Maximum Long Range Tune Shift for the SSC in One Low-β IR ($a_x = 0$, and 1: $\alpha = 50$ µrad; 2: $\alpha = 75$ µrad; 3: $\alpha = 120$ µrad).

For example, if the maximum allowed long range tune shift for the SSC is restricted by $|(\Delta\nu)_L^{max}| \leq 0.012$, then the maximum relative vertical amplitude, a_y^{max}, such that $(\Delta\nu_x)_L^{max} = 0.012$, defines the beam-beam dynamic aperture. This dynamic aperture (relative to σ) as a function of the crossing angle can be seen in Figure 16.

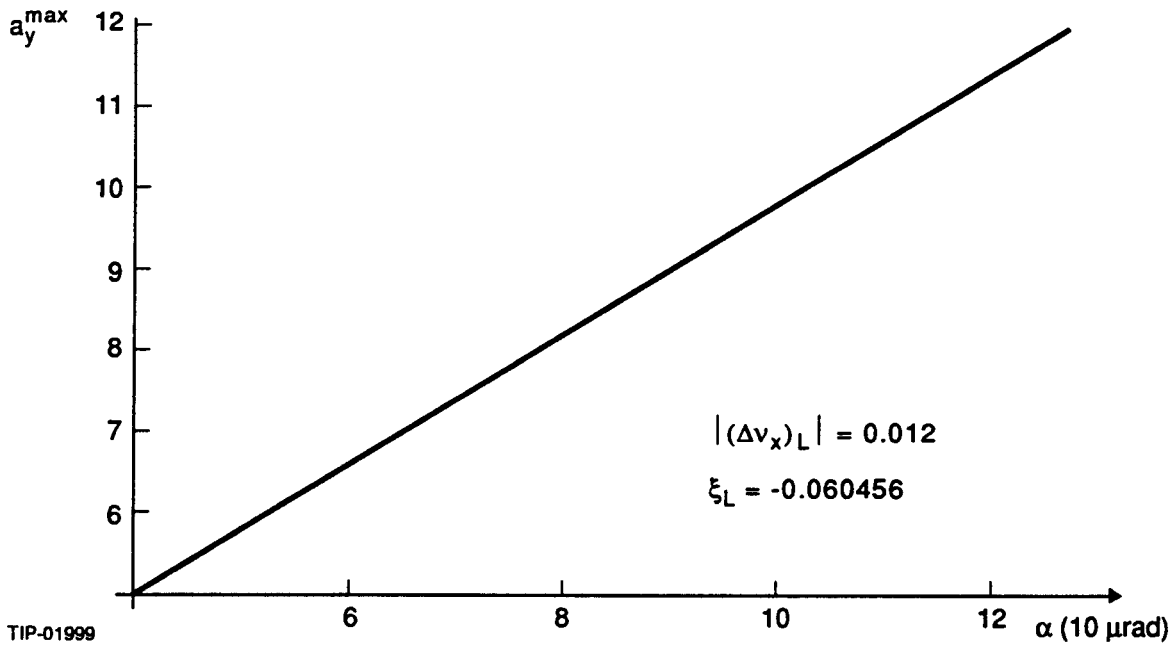


Figure 16. Beam-Beam Vertical Dynamics Aperture as a Function of Crossing Angle (Low-β IR).

The head-on tune shift limit (for $a_x = 0$) is obtained when $d_y = 0$. For the nominal crossing angle, $\alpha = 75$ μrad ($d_y = 7.5$), the maximum tune shift in the horizontal and vertical directions have these values:

$$(\Delta\nu_x)_L^{max} \cong -0.0125$$

and

$$(\Delta\nu_y)_L^{max} \cong +\ 0.003$$

This horizontal tune shift value may be large for the SSC. These calculations suggests that the maximum long range tune shift, in the low-β IR, can be reduced a lot and that the beam-beam vertical dynamical aperture can be much bigger, if the crossing angle is about 120 μrad or higher. Figure 17 shows the total tune shift spread using the nominal parameters of the SSC, but for a crossing angle of 120 μrad. As can be seen from this plot, the maximum tune shifts are reduced to small values (compare with Figure 12).

For the medium-β IR, the values $\beta^* = 10$ m and $d_y \cong 34$ bring about a very small long range tune shift, so the main contributor to the TSS in this region comes from the head-on tune shift.

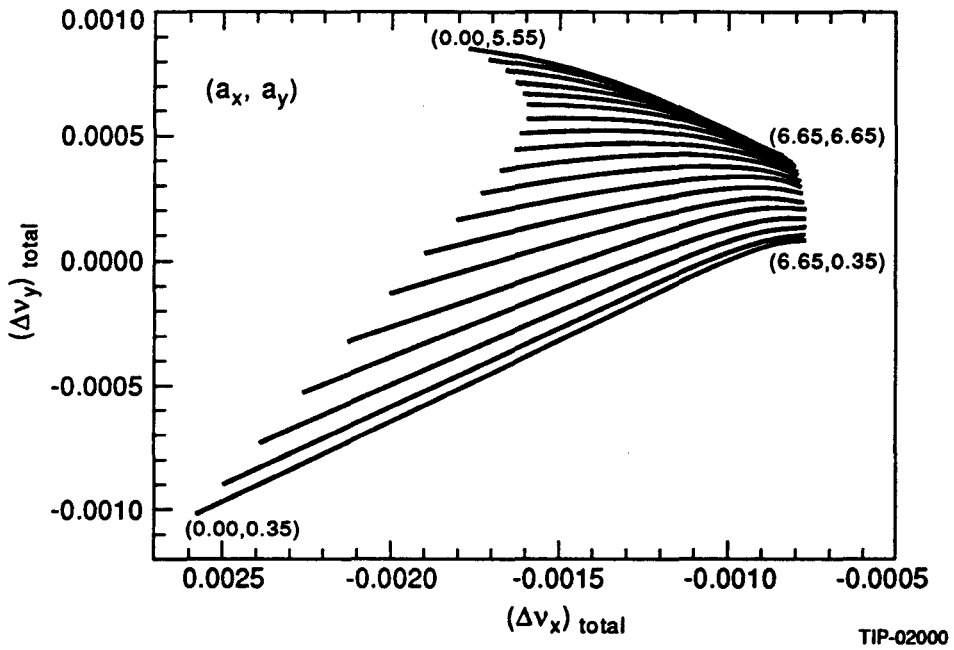


Figure 17. Total Tune Shift Spread using the SSC Nominal Parameters. One Low-β IR ($\alpha = 120$ μrad).

3.4 Maximum Tune Shift Dependence

There are three main parameters in the beam-beam tune shift relations: the head-on tune shift parameter, ξ, the long range tune shift parameter, ξ_L, and the relative separation parameter, d_y; they are related with the parameters of the machine and the beam itself as shown in expressions (12), (14), and (17). According to the above results, it is possible to estimate the maximum tune shift expected as a function of the physical parameters. For not very large vertical amplitudes ($a_y \leq 5d_y/6$ as a gross estimate from Figure 14), the maximum tune shifts have the following dependence:

$$(\Delta\nu_{x,y})_{ON}^{max} \sim -\frac{N}{\varepsilon} , \tag{20}$$

$$(\Delta\nu_x)_L^{max} \sim -\frac{N\beta(L)}{LS_B\gamma\alpha^2} , \tag{21a}$$

and

$$(\Delta\nu_y)_L^{max} \sim +\frac{N\beta(L)}{LS_B\gamma\alpha^2} \ . \tag{21b}$$

Therefore, the total maximum tune shift in the horizontal and vertical planes are

$$(\Delta\nu_x)_T^{max} = -N\left(\frac{f_1}{\varepsilon} + \frac{f_2\beta(L)}{LS_B\gamma\alpha^2}\right), \tag{22a}$$

and

$$(\Delta\nu_y)_T^{max} = -N\left(\frac{f_1}{\varepsilon} + \frac{f_2\beta(L)}{LS_B\gamma\alpha^2}\right), \tag{22b}$$

where f_1 and f_2 are integration factors.

In Sections 2.2 and 2.3, the β function was assumed to be of the form (16) for all the interaction length, L. Actually, there are four strong quadrupoles filling a total length of about 50 m (low-β IR). The beta function in the quadrupole magnets is of the form[5]

$$\beta_+(z) = \frac{E+}{2K} + \frac{1}{2K}\sqrt{E_+^2 - 4K}\,sin\,(2\sqrt{K}\,z) \tag{23a}$$

or

$$\beta_-(z) = \frac{4K + (E_- + \Omega e^{2\sqrt{K}\,z})^2}{4K\Omega e^{2\sqrt{K}\,z}} \tag{23b}$$

for a focusing or a defocusing effect, where K is related with the magnitude of the quadrupole gradient, and Ω and $E\pm$ are constant.

However, the separation between the two closed orbits of the interacting train of particles must be proportional to $\sqrt{\beta}$ since the orbits represent the motion of their relative synchronous particle. Then, the approximation (18) is expected to be good even in the magnets region.

4.0 TOTAL TUNE SHIFT SCALING PARAMETRIZATION

Using the approximation (16) in (22) and for $a_y < 5d_y/6$, the following relations result:

$$(\Delta\nu_x)_T^{max} = -N\left(\frac{f_1}{\varepsilon} + \frac{f_2L}{\beta^* S_B\gamma\alpha^2}\right) \tag{24a}$$

and

$$(\Delta\nu_y)_T^{max} = -N\left(\frac{f_1}{\varepsilon} + \frac{f_2 L}{\beta^* S_B \gamma \alpha^2}\right) . \tag{24b}$$

Written in terms of the nominal values for the SSC, considering two Low-β and two medium-β IR ($n_{low\,\beta} = n_{med\,\beta} = 2$), and using the information obtained above, these expressions have the form:

$$(\Delta\nu_x)_T^{max} = -N_o \left\{\frac{0.000916 n_o^*}{\varepsilon_o} + \frac{0.0125}{S_{B_o}\gamma_o}\left[\left(\frac{n_o L_o}{\beta_o^* \alpha_o^2}\right)_{low\,\beta} + \frac{1}{10}\left(\frac{n_o L_o}{\beta_o^* \alpha_o^2}\right)_{med\,\beta}\right]\right\} \tag{25a}$$

and

$$(\Delta\nu_y)_T^{max} = -N_o \left\{\frac{0.000916 n_o^*}{\varepsilon_o} - \frac{0.003}{S_{B_o}\gamma_o}\left[\left(\frac{n_o L_o}{\beta_o^* \alpha_o^2}\right)_{low\,\beta} + \frac{1}{10}\left(\frac{n_o L_o}{\beta_o^* \alpha_o^2}\right)_{med\,\beta}\right]\right\} , \tag{25b}$$

where the subindex (o) in N_o, ε_o, S_{Bo}, γ_o, L_o, β_o^*, α_o and $n_o^* = n_{olow\,\beta} + n_{omed\,\beta}$ means its relative values with respect to the nominal ($N = 7.5 \times 10^9$, $\varepsilon = 10^{-6}$ m, $S_B = 5$ m, $\gamma = 2 \times 10^4$ $L_{low\,\beta} = 82.5$ m ($L_{med\,\beta} = 150$ m), $\beta^*_{low\,\beta} = 0.5$ m ($\beta^*_{med\,\beta} = 10$ m), $\alpha = 75$ µrad, *and* $n_o^* = 4$) .

Making $\alpha_o \geq 1.6$ ($\alpha \geq 120$ µrad) and keeping all other parameters equal to one, the maximum TSS can be reduced to tolerable values for the SSC. However, by increasing the crossing angle, care must be taken to keep the ratio $\alpha\ \sigma_z/2\sigma$ small, where σ_z and σ are the longitudinal and transversal sigmas of the bunch ($\sigma_z = 0.075$ m, $\sigma = 5$ μm). This restriction is required to avoid syncro-betatron resonance oscillations of the beam and a decrease in its luminosity. Experiments[10, 11] seem to confirm these expectations. In this way, if the angle increases by a factor of 1.6 ($\alpha = 120$ µrad), the length of the bunch might decrease by the same factor ($\sigma_z = 0.0468$ m), keeping all the other parameters the same.

5.0 CONCLUSIONS

Using the first approximation in the incoherent[9] perturbative beam-beam strength, the tune shift spread was estimated for the SSC collider. The results suggest that this TSS may be large for the SSC collider with the nominal SSC parameters, and the beam-beam tune shift moves toward or from the line $\nu_x = \nu_y$ in the tune shift space (depending on the location of the selected operational point). This represents a potential danger for the particles to cross low order diagonal resonances (m $(\nu_y - \nu_x) = p$; p, m integers). Since the horizontal tune shift is much larger than the vertical one, when the crossing angle is on the vertical plane, the alternation of the crossing angle (horizontal plane) in the other Low-β IR of the SSC would increase the vertical tune shift, leaving the large horizontal tune shift the same. The whole effect would be large tune shift in both planes with the tune spread parallel to the line $\nu_x = \nu_y$ in the tune space. It also suggests that by increasing the crossing angle up to at least 120 µrad, it is possible to bring down the TSS to small values for

the SSC. These values represent the first approximation of the TSS. A serious computer simulation is required for a confident result .

ACKNOWLEDGMENTS

I wish to thank Dr. A.W. Chao and Dr. N. Metha for their valuable discussions, and Dr. R. Schwitters and Dr. Don Edwards for their support at the SSC Laboratory.

REFERENCES

1. L. R. Evans and J. Gareyte, "Beam-Beam Effects," CERN-SPS/86-8 (1986).
2. S. Peggs, *Particle Acc.*, Vol. **17**, 11-50 (1985).
3. J. Le Duff, VII Int. Acc. Conf., CERN (1971).
4. *Site-Specific Conceptual Design,* SSCL-SR-1056, p. 36 (1990).
5. G. Lopez, "Notes about Beta Function," SSC memo to M. Syphers, Feb. 15, 1991.
6. *Site-Specific Conceptual Design,* SSCL-SR-1056, p. 56 (1990).
7. A. W. Chao, *Proc* , "Nonlinear Dynamics Aspects of Particle Accelerators," **79** (1985).
8. A. W. Chao, Lecture Notes, "SSC Luminosity Limits and Upgrades," December 1989.
9. A. W. Chao, *AIP Conf. Proc.* , No. **127**, 202 (1983).
10. K. Cornelis, W. Herr and M. Meddahi, "Proton antiproton collisions at a finite crossing angle in the SPS," CERN-SL CH-1211 (1991).
11. A. A. Zholents, "Beam-Beam effects in electron-positron storage rings," Prepint 91-18, Novosibirsk (1991).