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Impact of Cross-Sectional Changes in a Beam Tube on Beam Dynamics*

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Abstract

Cross-sectional transitions are seen in accelerators. The beam tube dimensions may vary from one section (of dipole, quadrupole, or undulator, etc.) to another. This paper studies the impact of these discontinuities on single bunch as well as on multiple bunches. The transitions contribute coupling impedances that may significantly affect the single bunch behavior when the beam tube dimensions are small (as in the case of an undulator). They also serve as parasitic rf cavities that have a set of rf modes with high quality factors. When a bunch of particles is passing by, these modes may get excited and lead to instability and/or emittance dilution in the bunches that follow. Several possible cures are discussed.

I. INTRODUCTION

In the design of storage rings, a beam tube of uniform size is usually prefered. But sometimes, cross-sectional transitions in a beam tube are inevitable. There are various reasons for this to happen. One is due to some specific applications of a storage ring, such as a synchrotron light source, in which the beam tube height becomes extremely small in the undulator sections in order to achieve a high magnetic field. Another possible reason is based on economic considerations, in particular for machines using superconducting magnets, such as the Superconducting Super Collider (SSC), of which large magnet apertures are expensive.

This non-uniform beam tube may have significant effects on the behavior of the particle beam circulating in it. For instance, the collective instabilities and the emittance dilution. In this paper, these effects are studied by analysing, in the time domain, the wake fields generated by a bunched beam, and in the frequency domain, the rf modes of the parasitic cavities formed from the cross-sectional variations.

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There are two distinctive cases. Firstly, for the synchrotron light sources, the transverse coupling impedance, $Z_\perp$, can be dominated by these cross-sectional transitions because of the fact that $Z_\perp$ is roughly inversely proportional to the third power of the beam tube size. This may become the bottleneck of single bunch current intensity in such machines (e.g., the Advanced Photon Source (APS) at the Argonne National Laboratory.). Secondly, for a machine equipped with superconducting magnets, such as the SSC, the concern is different type. In order to reduce the wall impedance, the beam tube is coated with a thin copper layer. The resistivity of the layer is very low at liquid helium temperature. This means that the quality factor, Q, of the rf modes of the parasitic cavities formed from the cross-sectional transitions will be very high ($\sim 10^5$). Therefore, once these modes get excited by a traversing bunch, they can stay there for a long while before being damped and will have impact on other bunches. The result may be a substantial emittance dilution.

II. IMPACT ON A SINGLE BUNCH

The effect of a bunch on itself mediated by the environment can be studied by analyzing the wakefields within the bunch envelope. This has been discussed in detail in several references [1, 2, 3]. A number of tools has been developed for computing the wakefields (or, equivalently, the impedance) associated with the structure of the cross-sectional transitions. These include numerical simulations, boundary perturbation, and scalings. The results obtained from the different approaches are in agreement. Therefore, it is believed that this phenomenon is relatively understood. Table 1 lists the results calculated for the two machines. One is the APS. In each of its 34 undulator sections, the vertical beam tube half-height is 0.4 cm. This explains why their transverse impedance is so dominant. Another is the SSC. In the present design, the quadrupole aperture (4 cm) is different from that of the dipole (5 cm). There would be about 1000 transitions if a beam tube of different cross section sizes were used. Contrary to the APS, the impedance due to the transitions in the SSC is insignificant because of the relatively large beam tube radius in the quads (1.65 cm).

Table 1. Impedance of Transitions

Machine	$Z_{\parallel}/n$ Ω	% †	$Z_{\perp}$ MΩ/m	% †
APS	0.03	3	0.06	40
SSC	0.008	2	0.2	1

† The percentage of the total machine impedance.

III. Impact on Multiple Bunches

The effects of multiple bunches versus a single bunch are different. The beam tube consists of a sequence of large and small pipes, connected by tapered or untapered transitions. The large pipes can be regarded as parasitic cavities with certain eigenfrequencies and eigenmodes. These modes may get excited by a traversing bunch. When the mode frequencies are below the cutoff frequency of the small pipe, they will be trapped. Otherwise, they will propagate along the beam tube. If the quality factor of the trapped modes are high enough, they will stay there for a long time and will affect the behavior of the beam.

The SSC serves as a good example to demonstrate these multiple bunch effects. Assume the radius of the large pipe is 2 cm, the small one 1.6 cm. The conductivity, σ, of copper at liquid helium temperature ($\sim$ 4 K) in a high magnetic field (6.6 T) is taken to be $1.8 \times 10^9\ \Omega^{-1}m^{-1}$ (corresponding to RRR = 30). The first cutoff of the TM wave in the large pipe is 5.7 GHz, and in the small pipe 7.2 GHz. The skin depth and quality factor at these frequencies are, respectively,

$$\begin{aligned} \delta &= \sqrt{\frac{2}{\mu\sigma\omega}} \sim 0.15 \quad \mu\text{m}, \\ Q &= \frac{V}{S\delta} \cdot G \sim 10^5, \end{aligned}$$

in which V is the volume of the cavity, S the total surface area, and G the geometrical factor of the order of unity. The bunch separation in the SSC is 60 MHz. Therefore, an rf mode of a frequency of 6 GHz and a quality factor of 10^5 would be seen by 1000 bunches.

The modes are computed by URMEL [4]. Some of them are shown in Figure 1 and listed in Tables 2 and 3. A question exists whether or not these modes would be excited when a bunch of particles pass by. The answer is not straightforward and depends on whether or not the bunch contains high frequency components. For electron machines, the bunch distribution is Gaussian. One may readily compute the high frequency part from its spectrum. But, unfortunately, for proton machines, the bunches are not really rigid Gaussian. It has been observed in the Tevatron at the Fermilab that 8 GHz lines are seen in the spectrum of a proton bunch which has an rms length of about 60 cm. This may come from the fine structures within the bunch [5]. If, on the other side, one assumes a parabolic distribution for a proton bunch, then the sharp discontinuities at both ends of the bunch will certainly produce high frequency components. In a real machine, the discontinuities in a bunch distribution may well be generated by the filamentation. No matter what the explanation is, that there are high frequency components in a proton bunch is an acknowledged fact.

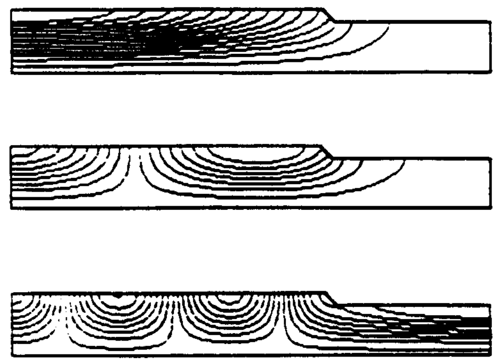

Figure 1: The $\vec{E}$ lines of several TM_{01} modes.

Table 2. Sample Monopole Modes

Mode Type TM_{mnp}	f MHz	R/Q Ω
TM_{011}	5761	0.002
TM_{012}	5852	0.008
TM_{013}	5999	0.037
TM_{014}	6197	0.006
TM_{015}	6436	0.016
TM_{016}	6698	0.045
TM_{017}	7155	8.213
TM_{018}	7230	4.938
TM_{019}	7319	10.17

Table 3. Sample Dipole Modes

Mode Type TE/TM_{mnp}	f MHz	R/Q * Ω
TE_{111}	4551	0.000
TE_{113}	5457	0.001
TE_{115}	6258	0.003
TM_{111}	9156	0.000
TM_{113}	9661	0.001
TM_{115}	10629	0.000
TM_{117}	11437	2.815
TM_{119}	11977	0.859

*Integrated at 3 mm displacement from the axis.

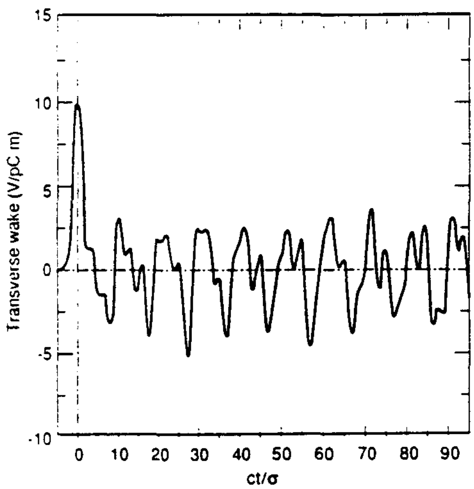

Figure 2: The transverse wakefields generated by a rigid Gaussian bunch.

For the purpose of carrying out the numerical estimation for the SSC, let us assume the bunch distribution is parabolic (approximated by half-cosine) with a full bunch length of a (in seconds).

$$\lambda(t) = \begin{cases} \frac{\pi}{2a}\cos\frac{\pi t}{a} & |t| \le \frac{a}{2} \ , \\ 0 & |t| > \frac{a}{2} \ . \end{cases}$$

Its spectrum is

$$\tilde{\lambda}(f) = f_0^2 \cdot \frac{\cos\frac{\pi f}{2f_0}}{f_0^2 - f^2} \ ,$$

in which

$$f_0 = \frac{1}{2a} \ .$$

For $a = 0.5$ ns, we have $f_0 = 1$ GHz. Thus, those components in the spectrum which are above $5f_0$ may be called the ***high frequency components*** and will be able to excite the rf modes in the cavities. These components account for about 10% out of a total of 100%.

Figure 2 shows the transverse wakefields generated by a rigid Gaussian bunch traversing a cavity. This bunch is short enough so that it contains abundant high frequency components (r.m.s. width 8 GHz). The peak of the wakes behind the bunch is about 5 V/pC·m. A very rough estimation is as follows.

For the SSC, each bunch has 0.75×10^{10} protons. This gives 1 nC. The average displacement from the axis is about 1 mm (closed orbit, survey error, etc.). Assume 10% high frequency component in a bunch. Then one bunch will generate $5 \times 10^3 \times 10^{-3} \times 0.1 = 0.5$ V in one cavity. There are 1000 bunches that will see each other. There are 1000 cavities. Therefore, the transverse kick per turn experienced by the protons will be

$$\Delta x' = \frac{0.5 \cdot \sqrt{1000} \cdot 1000}{E} \sim 10^{-9} \quad \text{rad},$$

in which $E = 20$ TeV is the proton energy. These continuous random kicks will result in emittance growth. The growth rate is

$$\begin{aligned} \dot{\epsilon} &= \bar{\beta} \cdot \overline{\Delta x'^2} \cdot f \\ &= 200 \times 10^{-18} \times 3 \times 10^4 \\ &= 6 \times 10^{-12} \quad \text{m-rad/s} \end{aligned}$$

in which $\bar{\beta}$ is the average betatron function (in meters), $\overline{\Delta x'^2}$ the average square of the kick, and f the revolution frequency (in Hz). When compared to the nominal value of the emittance, 5×10^{-11} m-rad, we see that the emittance would be doubled in approximately about 10 seconds.

IV. Conclusions

1. There are two possible consequences when adopting a non-uniform beam tube. The first is the single bunch instability due to large transverse impedances of these transitions. This is the case of the APS. The second is the probable transverse emittance dilution due to the multibunch effects, which is demonstrated using the SSC parameters.

2. The best cure is to use a uniform beam tube. When this is impossible, one has to adjust the beam parameters carefully to reach the desired value of the bunch intensity or to apply a feedback system to stabalize the beam.

3. If a concern exists for the trapped rf modes, dampers may be installed. Another conceivable solution is to randomely vary the beam tube lengths by a few centimeters in each section in order for the statistically averaged kicks to become smaller.

References

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