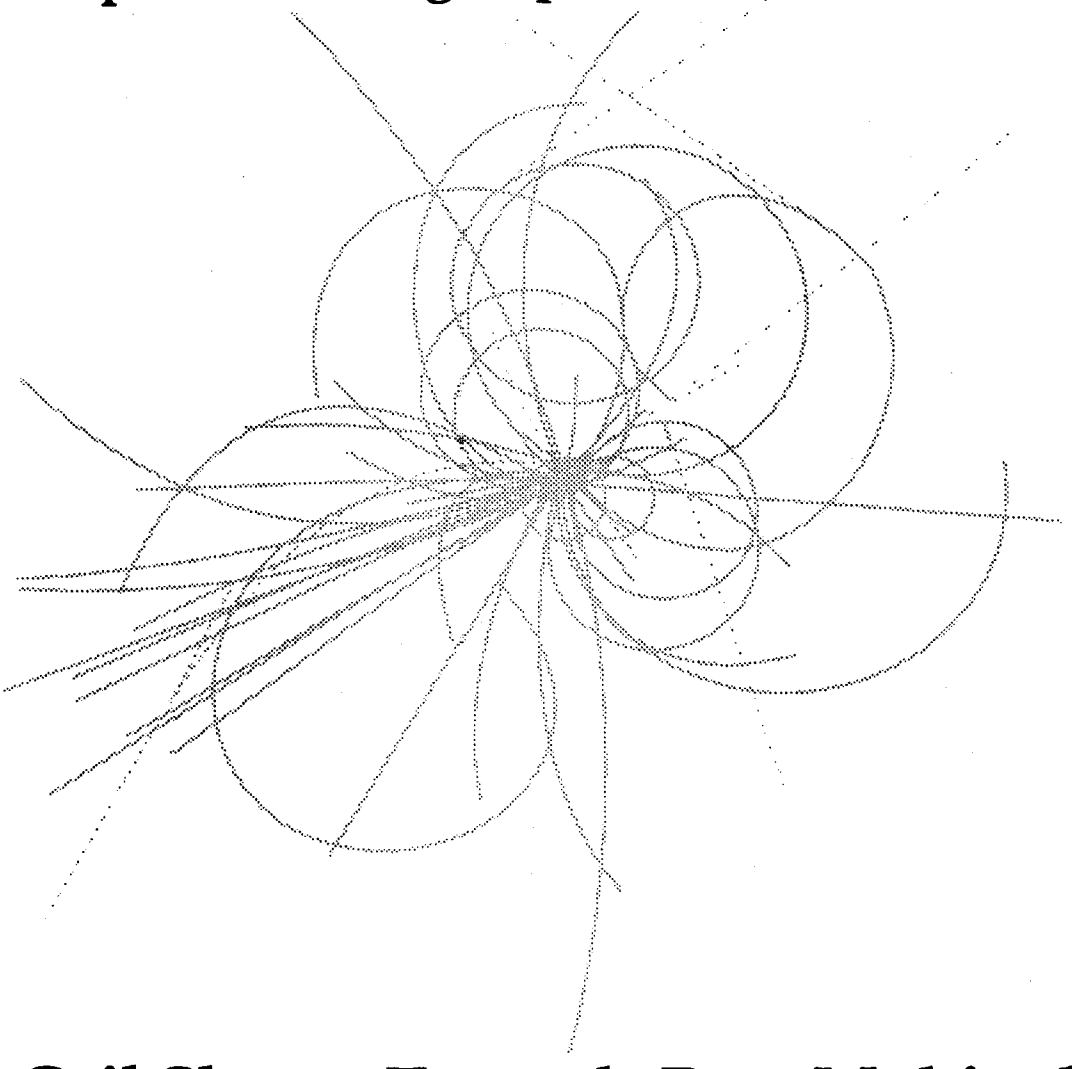
SSCL-399 Rev.1

SSCL-399 Rev.1

Superconducting Super Collider Laboratory



Coil Shapes Towards Pure Multipoles in Circular Regions (A Numerical Approach)

V. Thiagarajan

April 1991

SSCL-399 Rev.1

Coil Shapes Towards Pure Multipoles in Circular Regions (A Numerical Approach)

V. Thiagarajan

Accelerator Design and Operations Division
Superconducting Super Collider Laboratory*
2550 Beckleymeade Avenue
Dallas, TX 75237

April 1991

* Operated by the Universities Research Association, Inc., for the U.S. Department of Energy under Contract No. DE-AC02-89ER40486.

SSCL-399 Rev.1

Coil Shapes Towards Pure Multipoles in Circular Regions (A Numerical Approach)

V. Thiagarajan

Abstract

Coil shapes to produce pure multipole fields in circular regions have been studied and numerically evaluated. Coil shapes have been assumed as functions of unknown coefficients (and prescribed constraining parameters) and cosine functions. The coefficients have been numerically determined to produce the required multipole, simultaneously reducing the other multipoles to zero, or to negligible values.

CONTENTS

	FIGURES	vii
	TABLES	viii
1.0	INTRODUCTION	1
2.0	MULTIPOLE EXPANSION	1
3.0	COIL SHAPES	2
4.0	NUMERICAL DETERMINATION OF COIL SHAPES	2
5.0	CASE I—DIPOLE	3
6.0	CASE II—QUADRUPOLE	7
7.0	CASE III—SEXTUPOLE	8
8.0	CASE IV—HIGHER ORDER POLES	11
9.0	RESULTS AND DISCUSSION	11
10.0	CONCLUSION	18
	ACKNOWLEDGMENTS	23
	REFERENCES	25
	APPENDIX: LISTING OF DIPOLE3.F, QUAD1.F, AND SEXT1.F	27

FIGURES

1. Dipole Coil Profiles 14
2. Quadrupole Coil Profiles 15
3. Sextupole Coil Profiles 16
4. Potential Distribution—Dipole (case no. 2) 17
5. Potential Distribution—Quadrupole (case no. 2) 20
6. Potential Distribution—Sextupole (case no. 2) 21

TABLES

1. Coefficients for Dipole Coil Profile 12
2. Coefficients for Quadrupole Coil Profile 12
3. Coefficients for Sextupole Coil Profile 13
4. Harmonic Coefficients for Dipole Case 2 18
5. Harmonic Coefficients for Quadrupole Case 2 19
6. Harmonic Coefficients for Sextupole Case 2 19

1.0 INTRODUCTION

Coil shapes to produce approximately pure multipole fields in circular regions were studied earlier with cosine expressions for the shape of coils.[1] The required multipole was found to be of the order of Δ (with $\Delta < 1.0$); the multipole impurities were found to be of the order of higher powers of Δ. It is possible to reduce the impurities further by assuming coil shapes as a linear combination of the profiles (used in Reference 1) along with weight coefficients; these weight coefficients could be determined to reduce the multipole impurities to zero in a numerical sense. Such a procedure has been studied here, and the numerical method used to obtain these coefficients is described in this report.

2.0 MULTIPOLE EXPANSION

Let there be a current source $I = ja(da)(d\phi)$, located at the source co-ordinates a, ϕ with the current density being j. Let the coil be bounded by the constant radius a_1 of the circular field region on the inside and a radius $a_2(\phi)$ on the inside. The multipole expansion for the 2-D potential A_z at the field co-ordinates (r, θ) for $r < a$ is:[1,2]

$$\begin{aligned} A_z = \frac{\mu_o}{2\pi} \int_0^{2\pi} j \Bigg[& -ra_1 \left[1 - \left(\frac{a_1}{a_2}\right)^{-1}\right] \cos(\phi - \theta) - \frac{r^2}{2} \ln\left(\frac{a_1}{a_2}\right) \cos 2(\phi - \theta) \\ & + \frac{r^3}{3a_1} \left[1 - \left(\frac{a_1}{a_2}\right)\right] \cos 3(\phi - \theta) + \frac{r^4}{8{a_1}^2} \left[1 - \left(\frac{a_1}{a_2}\right)^2\right] \cos 4(\phi - \theta) \\ & + \frac{r^5}{15{a_1}^3} \left[1 - \left(\frac{a_1}{a_2}\right)^3\right] \cos 5(\phi - \theta) + \frac{r^6}{24{a_1}^4} \left[1 - \left(\frac{a_1}{a_2}\right)^4\right] \cos 6(\phi - \theta) \\ & + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \Bigg] \, d\phi. \end{aligned} \tag{1}$$

The 2-dimensional magnetic field is given by

$$\vec{B} = \nabla X A. \tag{2}$$

The components of $\vec{B}$ in the cylindrical co-ordinate system are

$$\begin{aligned} B_r &= \frac{1}{r} \frac{\partial A_z}{\partial \theta} \\ B_\theta &= -\frac{\partial A_z}{\partial r}. \end{aligned} \tag{3}$$

The cartesian components of the B field can be obtained from the following equation:

$$\begin{aligned} B_x &= B_r \cos\theta - B_\theta \sin\theta \\ B_y &= B_r \sin\theta + B_\theta \cos\theta. \end{aligned} \tag{4}$$

3.0 COIL SHAPES

It has been shown[1] that the following coil shapes and current distributions lead, approximately, to the required multipole:

Dipole:

$$a_2 = a_1(1 + \Delta|\cos\phi|) \tag{5}$$

$$j = j_o \frac{\cos\phi}{|\cos\phi|}. \tag{5.1}$$

Quadrupole:

$$a_2 = a_1 e^{\Delta|\cos(2\phi)|} \tag{6}$$

$$j = j_o \frac{\cos 2\phi}{|\cos 2\phi|}. \tag{6.1}$$

Sextupole and higher order poles:

$$a_2 = a_1 \left[1. - \Delta|\cos m\phi|\right]^{\frac{-1.}{m-2}} \tag{7}$$

$$j = j_o \frac{\cos m\phi}{|\cos m\phi|}. \tag{7.1}$$

In Equation (7), $m = 3$ leads to a sextupole, $m = 4$ leads to an octupole, etc.

The parameter Δ in Equations (5) to (7) must be $\ll 1.0$ to keep the design multipole predominant (of the order of Δ), and the multipole impurities small (of the order of higher powers of Δ).

4.0 NUMERICAL DETERMINATION OF COIL SHAPES

We will label each term in the integral (1) as I_1, I_2, I_3, etc.; further, each one of these terms can be decomposed into a term containing $\cos\theta$ and a term containing $\sin\theta$; we will label these terms (or, their integrals over ϕ) I_1C, I_1S, I_2C, I_2S, etc. If I_1C were non-zero and the rest were zero, pure dipole field would result. Similarly, if we keep I_2C non-zero

and the rest zero, pure quadrupole field results. Higher order multipoles can be achieved in a similar fashion by suitably specializing on the terms in integral (1). We can try to achieve the required multipole design, by using a linear combination of the profiles given in Equations (5) to (7) as follows:

$$\frac{a_2}{a_1} = A_1 + A_2|\cos\phi| + A_3 e^{A_4|\cos 2\phi|} + \frac{A_5}{(1 - A_6|\cos 3\phi|)} + \frac{A_7}{(1 - A_8|\cos 4\phi|)^{0.5}} + \frac{A_9}{(1 - A_{10}|\cos 5\phi|)^{\frac{1}{3}}} + \frac{A_{11}}{(1 - A_{12}|\cos 6\phi|)^{0.25}} + \frac{A_{13}}{(1 - A_{14}|\cos 7\phi|)^{0.2}} + \dots . \tag{8}$$

The task is to determine the coefficients A_1, A_2, etc., to keep one of the integrals non-zero and simultaneously reduce the others to zero.

5.0 CASE I—DIPOLE

Suppose we wish to design a numerically pure dipole. The integrand is a convergent series since the nth term is proportional to $\frac{1}{n(n-2)}$. We can truncate the series at some convenient point, hence the expression "numerically pure dipole." The current distribution is given by Equation (5.1). The coefficients A_2, A_4, A_6, A_8, A_{10}, A_{12}, and A_{14} in Equation (8) correspond to the parameter Δ in Equations (5) to (7), and they must be less than unity. In order to retain control over these during the numerical procedure, we will input these values. A_2 alone (out of this set) will be determined numerically for a reason that will be apparent later. The aim, therefore, is to determine A_1, A_2, A_3, A_5, A_7, A_9, A_{11}, and A_{13} to achieve a numerically pure dipole.

If we input typical numbers of the order of 1 for the coefficients to be determined and substitute Equations (5a) and (8) into Equation (1) and carry out the integration term by term, we can ascertain that all the terms are zero except I_1C, I_3C, I_5C, I_7C, etc. Further, if we keep I_1C non-zero and reduce I_3C, I_5C, I_7C, etc., to zero by appropriately choosing the coefficients A_2, A_3, A_5, A_7, etc., we will have a pure dipole. Practically, we must truncate the series at some point; therefore, we will have a numerically pure dipole.

We know from the approximate solution for a dipole coil profile (Equation (5)) that the coil has a maximum thickness at $\phi = 0$, and that it has a minimum thickness at $\phi = \frac{\pi}{2}$. We will force these conditions to obtain A_1 and A_2, and numerically determine A_3, A_5, A_7, etc., in order to reduce I_3C, I_5C, I_7C, etc. to zero. We will stop with coefficient A_{14} and reduce integrals to zero up to $I_{13}C$; beyond that the terms will be negligible anyway. The following equations can be written using the foregoing ideas.

The first equation is obtained from the condition

$$\frac{a_2}{a_1} = \alpha \quad \text{at} \quad \phi = \pm\frac{\pi}{2}$$

where α will be input (usually $1.0 < \alpha < 2.0$).

From Equation (8),

$$\begin{aligned}\frac{a_2}{a_1} = \alpha = A_1 + A_2 + A_3 e^{A_4} + \frac{A_5}{(1-A_6)} + \frac{A_7}{(1-A_8)^{\frac{1}{2}}} \\ + \frac{A_9}{(1-A_{10})^{\frac{1}{3}}} + \frac{A_{11}}{(1-A_{12})^{\frac{1}{4}}} + \frac{A_{13}}{(1-A_{14})^{\frac{1}{5}}}.\end{aligned} \tag{9}$$

The second equation is obtained from the condition

$$\frac{a_2}{a_1} = \beta \quad \text{at} \quad \phi = 0,$$

where β is usually 1.0. Using this in Equation (8),

$$\frac{a_2}{a_1} = \beta = A_1 + A_3 e^{A_4} + A_5 + \frac{A_7}{(1-A_8)^{\frac{1}{2}}} + A_9 + \frac{A_{11}}{(1-A_{12})^{\frac{1}{4}}} + A_{13}. \tag{10}$$

We need six more equations to determine A_3, A_5, A_7, A_9, A_{11}, and A_{13}. They are:

$$\int_0^{2\pi} I_3 C \; d\phi = 0. \tag{11}$$

$$\int_0^{2\pi} I_5 C \; d\phi = 0. \tag{12}$$

$$\int_0^{2\pi} I_7 C \; d\phi = 0. \tag{13}$$

$$\int_0^{2\pi} I_9 C \; d\phi = 0. \tag{14}$$

$$\int_0^{2\pi} I_{11}C \; d\phi = 0. \tag{15}$$

$$\int_0^{2\pi} I_{13}C \; d\phi = 0. \tag{16}$$

Thus, if we input α, β, A_4, A_6, A_{10}, A_{12}, and A_{13}, and solve Equations (9) through (16) and obtain the coefficients A_1 to A_{13}, we will have a coil profile for a numerically pure dipole with the term I_1C in Equation (1) (which is proportional to $\cos\theta$) nonzero, and the rest of the integrals zero (or, negligible in the case of $I_{15}C$, $I_{17}C$, etc.). We could add more terms to the coil profile given by Equation (8) and reduce $I_{15}C$, etc., to zero; but this will be unnecessary, since these terms will be negligible anyway.

Equations (9) and (10) are linear in the unknown coefficients; Equations (11) to (16) are nonlinear. We need to linearize these equations about an assumed solution (initial guess), and successive iterations will converge to the real solution. As an example we will linearize Equation (11) which is of the form

$$\int_0^{2\pi} \frac{\cos\phi}{|\cos\phi|} \cos 3\phi \left(1 - \frac{a_1}{a_2}\right) d\phi = 0. \tag{11.1}$$

Constant terms like j_o have been divided out in writing Equation (11.1). This equation is non-linear in A_3, etc., since it contains the reciprocal of $\frac{a_2}{a_1}$.

Let

$$F(A_3, \ldots, A_{13}) = \frac{\cos\phi}{|\cos\phi|} \cos 3\phi \left(1 - \frac{a_1}{a_2}\right),$$

with $\frac{a_1}{a_2}$ given by Equation (8). Let the initial guesses for A_3, A_5, ..., A_{13} be termed A_{3i}, A_{5i},..., A_{13i}, etc.

Linearization yields the following:

$$\int_0^{2\pi} \left[F\Big|_i + \frac{\partial F}{\partial A_3}\Big|_i (A_3 - A_{3i}) + \frac{\partial F}{\partial A_5}\Big|_i (A_5 - A_{5i}) + \frac{\partial F}{\partial A_7}\Big|_i (A_7 - A_{7i}) \right.$$
$$\left. + \frac{\partial F}{\partial A_9}\Big|_i (A_9 - A_{9i}) + \frac{\partial F}{\partial A_{11}}\Big|_i (A_{11} - A_{11i}) + \frac{\partial F}{\partial A_{13}}\Big|_i (A_{13} - A_{13i}) \right] d\phi = 0.$$

The $\Big|_i$ indicates that the initial values A_{3i}, A_{5i}, etc., should be used in evaluating what immediately precedes $\Big|_i$. Or,

$$
\begin{aligned}
& A_3 \int_0^{2\pi} \frac{\partial F}{\partial A_3}\bigg|_i d\phi + A_5 \int_0^{2\pi} \frac{\partial F}{\partial A_5}\bigg|_i d\phi + A_7 \int_0^{2\pi} \frac{\partial F}{\partial A_7}\bigg|_i d\phi \\
& \quad + A_9 \int_0^{2\pi} \frac{\partial F}{\partial A_9}\bigg|_i d\phi + A_{11} \int_0^{2\pi} \frac{\partial F}{\partial A_{11}}\bigg|_i d\phi + A_{13} \int_0^{2\pi} \frac{\partial F}{\partial A_{13}}\bigg|_i d\phi \\
& = \int_0^{2\pi} \Bigg[-F\bigg|_i + A_{3i} \frac{\partial F}{\partial A_3}\bigg|_i + A_{5i} \frac{\partial F}{\partial A_5}\bigg|_i + A_{7i} \frac{\partial F}{\partial A_7}\bigg|_i \\
& \quad + A_{9i} \frac{\partial F}{\partial A_9}\bigg|_i + A_{11i} \frac{\partial F}{\partial A_{11}}\bigg|_i + A_{13i} \frac{\partial F}{\partial A_{13}}\bigg|_i \Bigg] d\phi.
\end{aligned}
\tag{11a}
$$

Similarly, Equations (12) to (16) can be linearized to obtain Equations (12a) to (16a). Equations (9), (10), and (11a) to (16a) constitute eight linear equations for the unknowns A_1, A_2, A_3, A_5, A_7, A_9, A_{11}, and A_{13}, and they can be solved by matrix inversion. The resulting solution for the coefficients will be the input for the next iteration. This procedure can be repeated until the solution converges and satisfies Equations (9) to (16).

A code DIPOLE3.F was written for the numerical solution. This employs trapezoidal integration and requires the following input:

1. $\alpha = \dfrac{a_2}{a_1}$ at $\phi = \dfrac{\pi}{2}$ $\quad (1.0 < \alpha < 2.0)$
2. $\beta = \dfrac{a_2}{a_1}$ at $\phi = 0$ $\quad (\beta =$ usually 1.0)
3. A_4N, A_6N, A_8N, $A_{10}N$, $A_{12}N$, $A_{14}N$ $\quad$ (input coefficients)
4. A_3N, A_5N, A_7N, A_9N, $A_{11}N$, $A_{13}N$

   (initial guesses for the coefficients to be solved for)
5. N—Number of integration steps in a quadrant
6. $IMAX$ $(2 < IMAX \leq 6)$

   (maximum number of coefficients to be solved for).

6.0 CASE II—QUADRUPOLE

Suppose we wish to design a quadrupole. The current distribution is given by Equation (6.1). We know from the approximate solution of coil profile for a quadrupole (Equation (6)), that it will consist of four crescent-type shapes. Therefore, we need to force the coil profile at three points and calculate the rest of the coefficients. Let us use A_1, A_2, and A_3 to force the profile to required value at $\phi = 0, \frac{\pi}{4}$, and $\frac{\pi}{2}$, and input A_4, A_6, A_8, A_{10}, and A_{12}, then calculate A_5, A_7, A_9, and A_{10} to reduce four undesired integrals to zero. If we substitute Equations (8) and (6.1) into (1) and carry out the integration, we find all the terms except I_2C, I_4C, I_6C, I_8C, etc., are zero. In order to preserve the quadrupole field, we need I_2C to be non-zero and the rest to be zero.

Hence, we will evaluate A_5, A_7, A_9, A_{11} so as to reduce I_4C, I_6C, I_8C, and $I_{10}C$ to zero. The first two equations are obtained from the conditions

$$\frac{a_2}{a_1} = \beta \text{ at } \phi = 0 \text{ and } \frac{\pi}{2} \quad (1.0 < \beta < 2.0).$$

Substituting in Equation (8),

$$\begin{aligned}\left.\frac{a_2}{a_1}\right|_{\phi=0} &= A_1 + A_2 + A_3 e^{A_4} + \frac{A_5}{(1. - A_6)} \\ &+ \frac{A_7}{(1. - A_8)^{0.5}} + \frac{A_9}{(1 - A_{10})^{\frac{1}{3}}} + \frac{A_{11}}{(1 - A_{12})^{0.25}} = \beta.\end{aligned} \tag{17}$$

$$\left.\frac{a_2}{a_1}\right|_{\phi=\frac{\pi}{2}} = A_1 + A_3 e^{A_4} + A_5 + \frac{A_7}{(1 - A_8)^{0.5}} + A_9 + \frac{A_{11}}{(1 - A_{12})^{0.25}} = \beta. \tag{18}$$

The third equation is obtained from the following condition:

$$\frac{a_2}{a_1} = \alpha \text{ at } \phi = \frac{\pi}{4} \quad (\alpha = 1.0, \text{usually}).$$

Substituting in Equation (8),

$$\left.\frac{a_2}{a_1}\right|_{\phi=\frac{\pi}{4}} = A_1 + \frac{A_2}{\sqrt{2}} + A_3 + \frac{A_5}{(1 - \frac{A_6}{\sqrt{2}})} + \frac{A_7}{(1 - A_8)^{0.5}} + \frac{A_9}{(1 - \frac{A_{10}}{\sqrt{2}})^{\frac{1}{3}}} + A_{11} = \alpha. \tag{19}$$

We need four more equations. They are:

$$\int_0^{2\pi} I_4C \; d\phi = 0 \tag{20}$$

$$\int_0^{2\pi} I_6 C \; d\phi = 0 \tag{21}$$

$$\int_0^{2\pi} I_8 C \; d\phi = 0 \tag{22}$$

$$\int_0^{2\pi} I_{10} C \; d\phi = 0. \tag{23}$$

Equations (20) to (23) are nonlinear in the unknown coefficients A_1, A_2, A_3, A_5, A_7, A_9, A_{11}, and A_{13}. They can be linearized about an assumed solution with a procedure as described in obtaining Equation (11a) from Equation (11). Let us call the resulting equations (20a) through (23a). These equations, along with Equations (17) to (19), can be solved iteratively to obtain the coefficients and the coil profile.

A code QUAD1.F was written for the numerical solution. This employs trapezoidal integration and requires the following input:

1. $\alpha = \left.\dfrac{a_2}{a_1}\right|_{\phi=\frac{\pi}{4}} \qquad (\alpha = 1.0)$
2. $\beta = \left.\dfrac{a_2}{a_1}\right|_{\phi=0 \text{ or } \frac{\pi}{2}} \qquad (1.0 < \beta < 2.0)$
3. A_4N, A_6N, A_8N, $A_{10}N$, $A_{12}N$

   (input coefficients)
4. A_1N, A_2N, A_3N, A_5N, A_7N, A_9N, $A_{11}N$

   (initial guesses for the coefficients to be solved for)
5. N—Number of integration steps in one octant
6. $IMAX =$ Number of coefficients to be solved for

   $(2 < IMAX \leq 6)$.

7.0 CASE III—SEXTUPOLE

The current distribution for a sextupole is given by Equation (7.1). The coil profile will consist of six crescent-like shapes; we need to force the coil profile at three points to given values and calculate the rest of the coefficients. Let us use A_1, A_2, and A_5 to force

the profile to required values at $\phi = 0, \frac{\pi}{2}$, and $\frac{\pi}{6}$. We will also input A_4, A_6, A_8, etc., and calculate A_3, A_7, A_9, etc., so as to reduce the undesired integrals to zero. If we substitute Equations (7.1) and (8) into (1) and carry out the integration, we find that all the terms except I_1C, I_3C, I_5C, etc., are zero. In order that the field be that of a pure sextupole, I_3C must be non-zero and the rest must be zero. Therefore, we will evaluate A_3, A_7, A_9, A_{11}, etc., so as to reduce I_1C, I_5C, I_7C, etc., to zero. The integrals I_3C, I_9C, $I_{15}C$, etc., become predominant in the sextupole design. Hence, we will add two more terms to Equation (8) and retain coefficients up to A_{18}.

The first equation is obtained from the following condition:

$$\frac{a_2}{a_1} = \alpha \text{ at } \phi = 0 \text{ with } (1.0 < \alpha < 2.0).$$

Substituting in Equation (8),

$$\begin{aligned}\left.\frac{a_2}{a_1}\right|_{\phi=0} &= \alpha = A_1 + A_2 + A_3 e^{A_4} \\ &+ \frac{A_5}{(1-A_6)} + \frac{A_7}{(1-A_8)^{0.5}} + \frac{A_9}{(1-A_{10})^{\frac{1}{3}}} \\ &+ \frac{A_{11}}{(1-A_{12})^{0.25}} + \frac{A_{13}}{(1-A_{14})^{0.2}} + \frac{A_{15}}{(1-A_{16})^{\frac{1}{6}}} + \frac{A_{17}}{(1-A_{18})^{\frac{1}{7}}}.\end{aligned} \tag{24}$$

The second equation results from the following:

$$\frac{a_2}{a_1} = \beta \text{ at } \phi = \frac{\pi}{2} \quad (\beta = 1.0)$$

$$\begin{aligned}\left.\frac{a_2}{a_1}\right|_{\phi=\frac{\pi}{2}} &= \beta = A_1 + A_3 e^{A_4} + A_5 + \frac{A_7}{(1-A_8)^{0.5}} + A_9 \\ &+ \frac{A_{11}}{(1-A_{12})^{0.25}} + A_{13} + \frac{A_{15}}{(1-A_{16})^{\frac{1}{6}}} + A_{17}.\end{aligned} \tag{25}$$

The third equation is obtained from

$$\frac{a_2}{a_1} = \eta \text{ at } \phi = \frac{\pi}{6} \quad (\eta = 1.0)$$

$$\begin{aligned}\left.\frac{a_2}{a_1}\right|_{\phi=\frac{\pi}{6}} &= \eta = A_1 + A_2\frac{\sqrt{3}}{2} + A_3 e^{\frac{A_4}{2}} + A_5 + \frac{A_7}{\left(1-\frac{A_8}{2}\right)^{0.5}} + \frac{A_9}{\left(1-A_{10}\frac{\sqrt{3}}{2}\right)^{\frac{1}{3}}} \\ &+ \frac{A_{11}}{(1-A_{12})^{0.25}} + \frac{A_{13}}{\left(1-A_{14}\frac{\sqrt{3}}{2}\right)^{0.2}} + \frac{A_{15}}{\left(1-A_{16}\frac{\sqrt{3}}{2}\right)^{\frac{1}{6}}} + A_{17}.\end{aligned} \tag{26}$$

In order to determine A_3, A_7, A_9, through A_{17}, we need seven more equations. They are:

$$\int_0^{2\pi} I_1 C \; d\phi = 0 \tag{27}$$

$$\int_0^{2\pi} I_5 C \; d\phi = 0 \tag{28}$$

$$\int_0^{2\pi} I_7 C \; d\phi = 0 \tag{29}$$

$$\int_0^{2\pi} I_9 C \; d\phi = 0 \tag{30}$$

$$\int_0^{2\pi} I_{11} C \; d\phi = 0 \tag{31}$$

$$\int_0^{2\pi} I_{13} C \; d\phi = 0 \tag{32}$$

$$\int_0^{2\pi} I_{15} C \; d\phi = 0. \tag{33}$$

Equations (27) to (33) can be linearized about an initial solution to obtain linearized Equations (27a) to (33a). With α, β, η and A_4, A_6, through A_{18} input along with initial guesses for A_3, A_7, through A_{17}, Equations (24) to (26) and (27a) to (33a) may be solved to obtain the unknown coefficients; the iteration could be repeated until the solutions converge.

A code SEXT1.F was written for the numerical solution; it requires the following input:

1. $\alpha = \left.\frac{a_2}{a_1}\right|_{\phi=0}$ $(1.0 < \alpha < 2.0)$

2. $\beta = \left.\frac{a_2}{a_1}\right|_{\phi=\frac{\pi}{2}}$ $(\beta = 1)$

3. $\eta = \left.\frac{a_2}{a_1}\right|_{\phi=\frac{\pi}{6}}$ $(\eta = 1)$

4. A_4N, A_6N, $A_8N \ldots A_{18}N$

   (input coefficients)

5. A_1N, A_2N, A_3N, $A_5N \ldots A_{17}N$

   (initial guesses for input coefficients)

6. N—Number of integration steps in a 60° sector

7. $IMAX$—number of coefficients to be solved for

   $(2 < IMAX \leq 7)$.

8.0 CASE IV—HIGHER ORDER POLES

Numerical solutions for higher order poles may be treated in a similar fashion. A few trials may be required to determine the number of terms to be included in Equation (8).

9.0 RESULTS AND DISCUSSION

Numerical computations were done for the profiles of coils using DIPOLE3.F, QUAD1.F, and SEXT1.F, and the field computations were done using the code PE2D.[3] Three different cases were run with DIPOLE3.F, with $\alpha = 1.2, 1.3$, and 1.4, and with $\beta = 1.0$ and IMAX= 6. The coefficients A_4, A_6, A_8, A_{10}, A_{12}, A_{14} were input as 0.2, and A_1, A_2, A_3, A_5, A_7, A_9, A_{11}, A_{13} were calculated for the three cases. The resulting coefficients for dipole coil profiles are shown in Table 1.

Three cases were run using QUAD1.F with $\alpha = 1.2, 1.3, 1.4$, and with $\beta = 1.0$ and IMAX= 4. The coefficients A_4, A_6, A_8, A_{10}, A_{12} were input, and A_1, A_2, A_3, A_5, A_7, A_9, A_{11}, A_{13} were calculated. The coefficients for the quadrupole coil profile are shown in Table 2. Three cases were run using SEXT1.F, with $\alpha = 1.2, 1.3, 1.4$, with $\beta = \eta = 1.0$, and with IMAX= 7. The coefficients A_4, A_6, A_8, A_{12}, A_{14}, A_{16}, A_{18} were input as 0.1 or 0.2, and A_1, A_2, A_3, A_5, A_7, A_9, A_{11}, A_{13}, A_{15}, A_{17} were calculated. The coefficients for the sextupole coil profile are shown in Table 3. The coil profiles for dipole, quadrupole, and sextupole are shown in Figures 1, 2, and 3, respectively.

Table 1. Coefficients for Dipole Coil Profile.

	Case 1 $\alpha = 1.2$	Case 2 $\alpha = 1.3$	Case 3 $\alpha = 1.4$
A_1	1.06477	1.18392	1.38247
A_2	0.209775	0.321917	0.438888
A_3	0.0313991	0.0695509	0.122094
A_4	0.2	0.2	0.2
A_5	−0.0150988	−0.032749	−0.056678
A_6	0.2	0.2	0.2
A_7	0.029944	0.0641391	0.108447
A_8	0.2	0.2	0.2
A_9	−0.0247618	−0.054636	−0.101836
A_{10}	0.2	0.2	0.2
A_{11}	−0.00677304	−0.0423667	−0.118235
A_{12}	0.2	0.2	0.2
A_{13}	−0.0895804	−0.208396	−0.36931
A_{14}	0.2	0.2	0.2

Table 2. Coefficients for Quadrupole Coil Profile.

	Case 1 $\beta = 1.2$	Case 2 $\beta = 1.3$	Case 3 $\beta = 1.4$
A_1	0.174461	−0.180633	−0.518068
A_2	0.225794E−3	0.372139E−3	0.52793E−3
A_3	0.981432	1.53405	2.11447
A_4	0.2	0.2	0.2
A_5	−0.905029E−3	−0.136362E−2	−0.178955E−2
A_6	0.2	0.2	0.2
A_7	0.13077	0.303228	0.531166
A_8	0.2	0.2	0.2
A_9	0.599738E−5	−0.404481E−3	−0.104305E−2
A_{10}	0.2	0.2	0.2
A_{11}	−0.30121	−0.690685	−1.18746
A_{12}	0.2	0.2	0.2

Table 3. Coefficients for Sextupole Coil Profile.

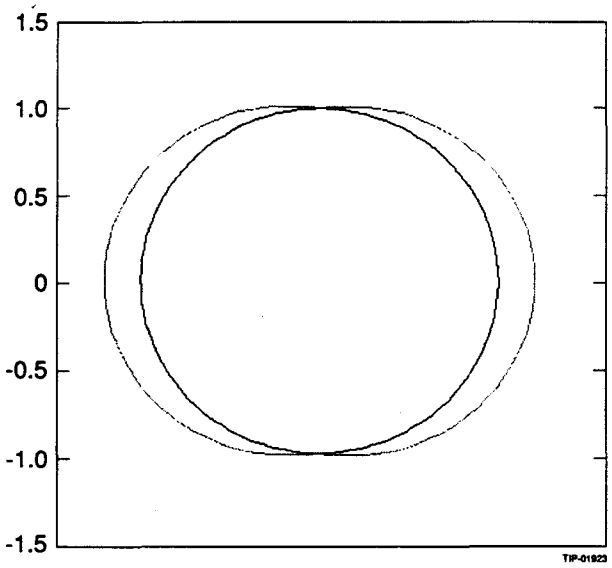
	Case 1 $\alpha = 1.2$	Case 2 $\alpha = 1.3$	Case 3 $\alpha = 1.4$
A_1	0.678023	0.582428	0.254828
A_2	−0.938618E−4	−0.143104E−3	−0.188468E−3
A_3	−0.137324E−2	−0.228336E−2	−0.328689E−2
A_4	0.1	0.1	0.1
A_5	0.885485	1.38448	1.8938
A_6	0.2	0.2	0.2
A_7	−0.358939E−2	−0.570224E−2	−0.751794E−2
A_8	0.1	0.1	0.1
A_9	−0.542441E−2	−0.103715E−1	−0.0159266
A_{10}	0.1	0.1	0.1
A_{11}	0.826461	2.0251	3.60455
A_{12}	0.1	0.1	0.1
A_{13}	−0.181084E−2	−0.131475E−2	0.412246E−3
A_{14}	0.1	0.1	0.1
A_{15}	−0.011583	−0.19992E−1	−0.279737E−1
A_{16}	0.1	0.1	0.1
A_{17}	−1.3877	−3.0055	−4.79383
A_{18}	0.1	0.1	0.1

Field computations were done for the dipole for case no. 2 (with $\alpha = 1.3$), using the code PE2D.[3] A current density of 3.5E8 amps/m^2 was used. Dirichlet boundary condition was applied at a radius of 6.0 cm. The computed potential distribution is shown in Figure 4. The field in the center was 1.288 T.

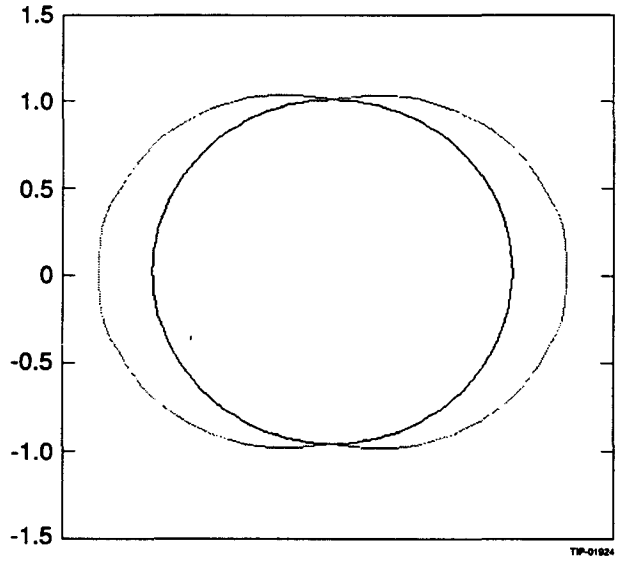
The field at a radius of 1.0 cm was used to conduct a harmonic analysis using the following equation:

$$B_y + iB_x = B_0 \sum_{n=0}^{\infty} (b_n + ia_n)(x + iy)^n. \tag{34}$$

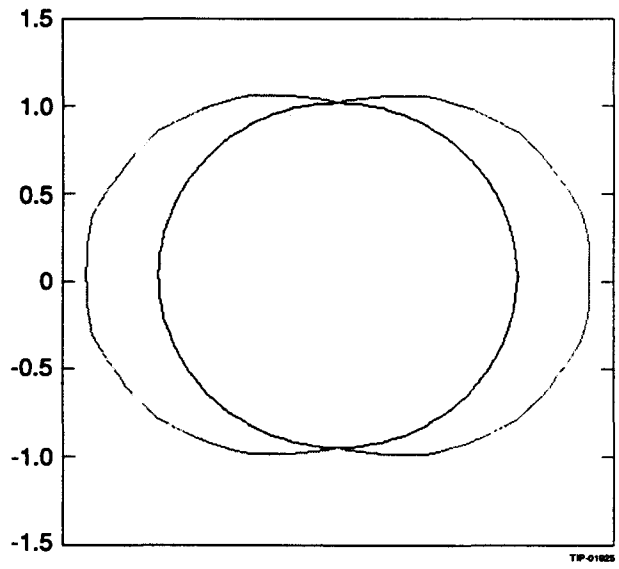
B_0 in the above equation was assumed to be 1.0, and the values of x and y were input in centimeters. The resulting coefficients are shown in Table 4. If the dipole field were pure, the coefficient b_0 will be non-zero, and the rest of the coefficients will be zero.



$\alpha = 1.2$

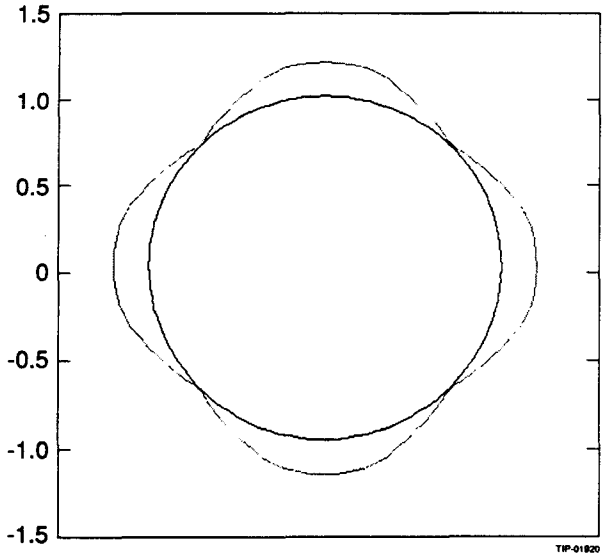


$\alpha = 1.3$

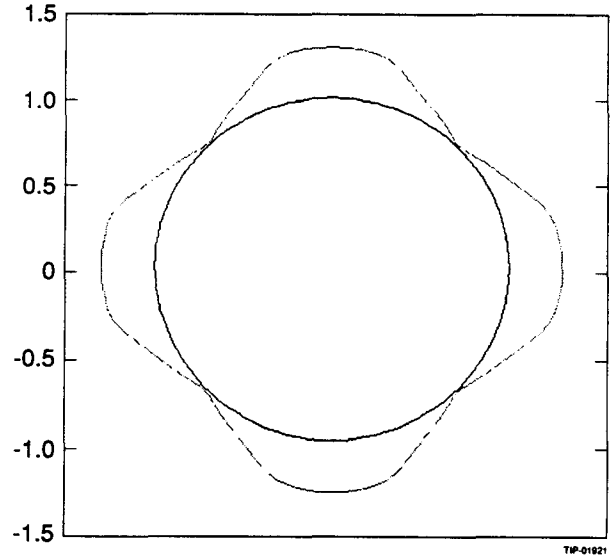


$\alpha = 1.4$

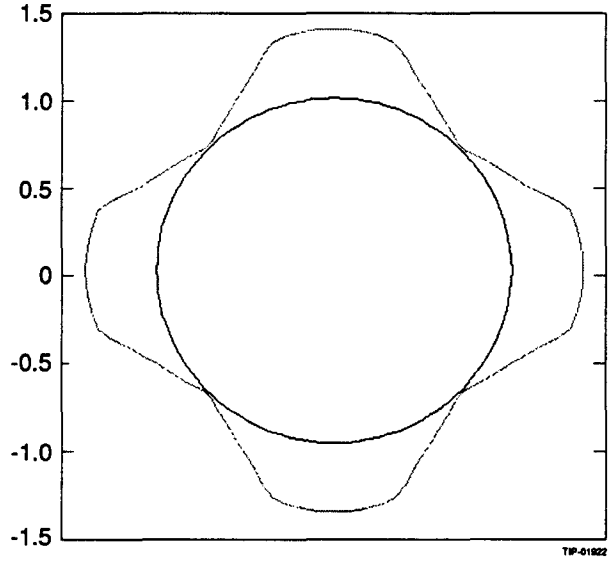
Figure 1. Dipole Coil Profiles.



$\beta = 1.2$

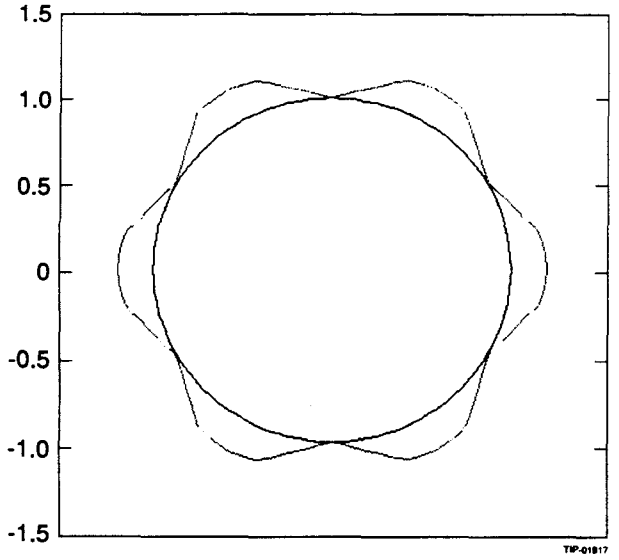


$\beta = 1.3$

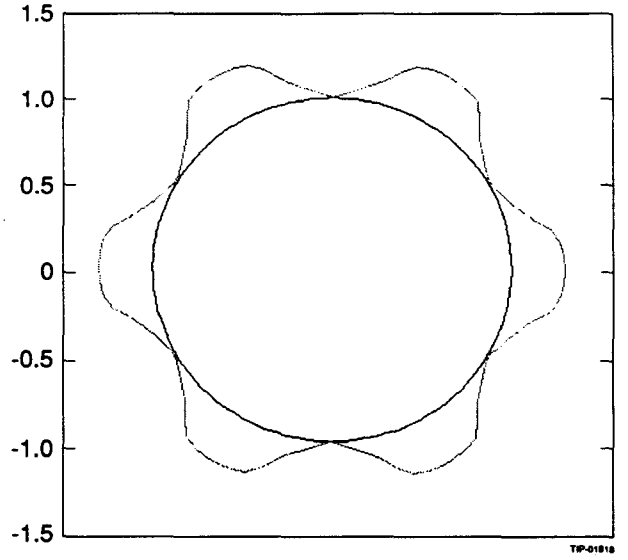


$\beta = 1.4$

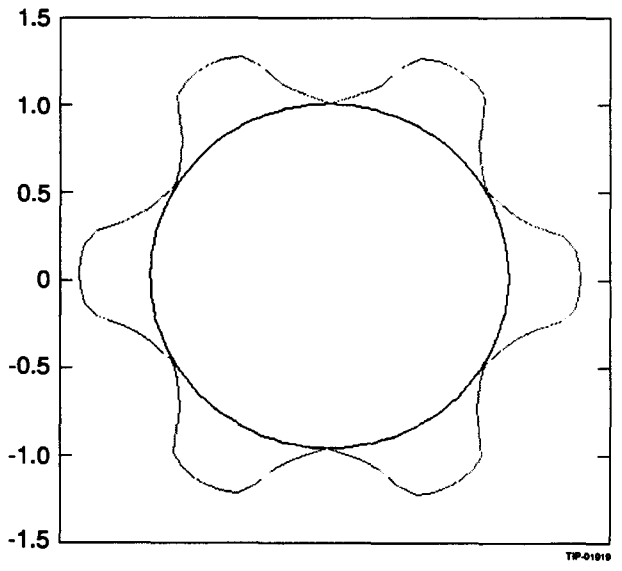
Figure 2. Quadrupole Coil Profiles.



$\alpha = 1.2$



$\alpha = 1.3$



$\alpha = 1.4$

Figure 3. Sextupole Coil Profiles.

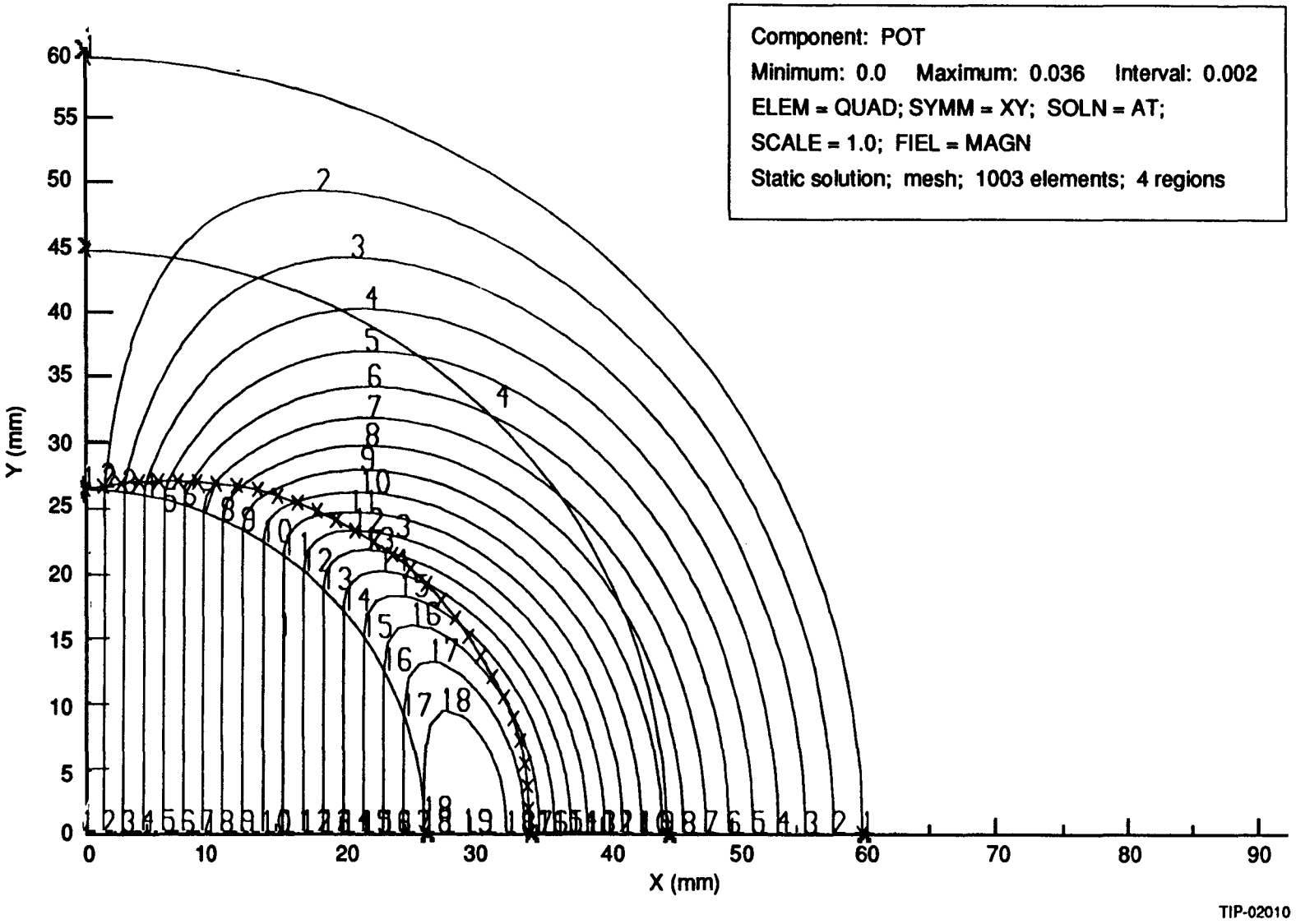


Figure 4. Potential Distribution—Dipole (case no. 2).

It is seen that $b_0 = -1.288$, $b_2 = 0.5544\text{E}-3$, $b_4 = -0.1319\text{E}-4$, etc. This shows that the multipole impurities are about 0.04%.

Table 4. Harmonic Coefficients for Dipole Case 2.

n	b(n)	a(n)
0	−1.288	0.1126E−6
1	−0.1905E−6	0.3351E−13
2	0.5544E−3	−0.1454E−9
3	−0.3008E−6	0.105E−12
4	−0.1319E−4	0.5766E−11
5	−0.4258E−6	0.2230E−12
6	0.1342E−5	−0.8214E−12
7	−0.5433E−6	0.3814E−12
8	0.1458E−4	−0.1146E−10
9	−0.6469E−6	0.5579E−12
10	0.2307E−4	−0.2219E−10

Field computations for the quadrupole case no. 2 (with $\beta = 1.3$) were done using PE2D (current density = 3.5E8, Dirichlet boundary condition at 6.0 cm). The potential distribution is shown in Figure 5. The gradient was found to be 52.18 T/m. Harmonic analysis was performed using the potentials at 1.0 cm; the resulting coefficients are shown in Table 5. It is seen that $b(1) = -0.5218$, $b(5) = 0.8048\text{E}-5$. The rest of the coefficients are still less. If the quadrupole were pure, $b(1)$ would be non-zero and the rest would be zero. The impurities are found to be less than 0.02%.

Field computations were done for the sextupole case no. 2 (with $\alpha = 1.3$). Dirichlet boundary conditions were applied at a radius of 5.0 cm, and a current density of 3.5E8 amps/m^2 was used. The potential distribution is shown in Figure 6. Harmonic analysis with the potentials at 1.0 cm (Table 6) shows that $b(2) = 0.1708$, $b(8) = 0.3947\text{E}-4$; the rest of the coefficients are much less. If the sextupole were pure, $b(2)$ would be non-zero and the rest would be zero. It is seen that the impurities are about 0.02%.

10.0 CONCLUSION

It is possible to achieve pure multipole fields in circular regions by adjusting the shape of the coils using Equation (8). The error can be minimized by increasing the number of terms in this equation and by reducing to zero as many of the undesired integrals (in Equation 1) as possible.

Table 5. Harmonic Coefficients for Quadrupole Case 2.

n	b(n)	a(n)
0	0.1807E−7	−0.5889E−14
1	−0.5218	0.3400E−6
2	−0.2913E−6	0.2848E−12
3	−0.2864E−6	0.3733E−12
4	−0.3178E−6	0.5178E−12
5	−0.8048E−5	−0.1573E−10
6	−0.4006E−6	0.9138E−12
7	−0.4440E−6	0.1157E−11
8	−0.487E−6	0.1428E−11
9	0.6660E−6	−0.2170E−11
10	−0.5691E−6	0.2040E−11

Table 6. Harmonic Coefficients for Sextupole Case 2.

n	b(n)	a(n)
0	0.1480E−8	−0.4826E−15
1	0.1892E−7	−0.1233E−13
2	−0.1708	0.1670E−6
3	−0.1078E−6	0.1405E−12
4	−0.9185E−7	0.1496E−12
5	−0.9376E−7	0.1833E−12
6	−0.1001E−6	0.2284E−12
7	−0.1082E−6	−0.2822E−12
8	0.3947E−4	−0.1157E−9
9	−0.1264E−6	0.4118E−12
10	−0.1358E−6	0.4869E−12

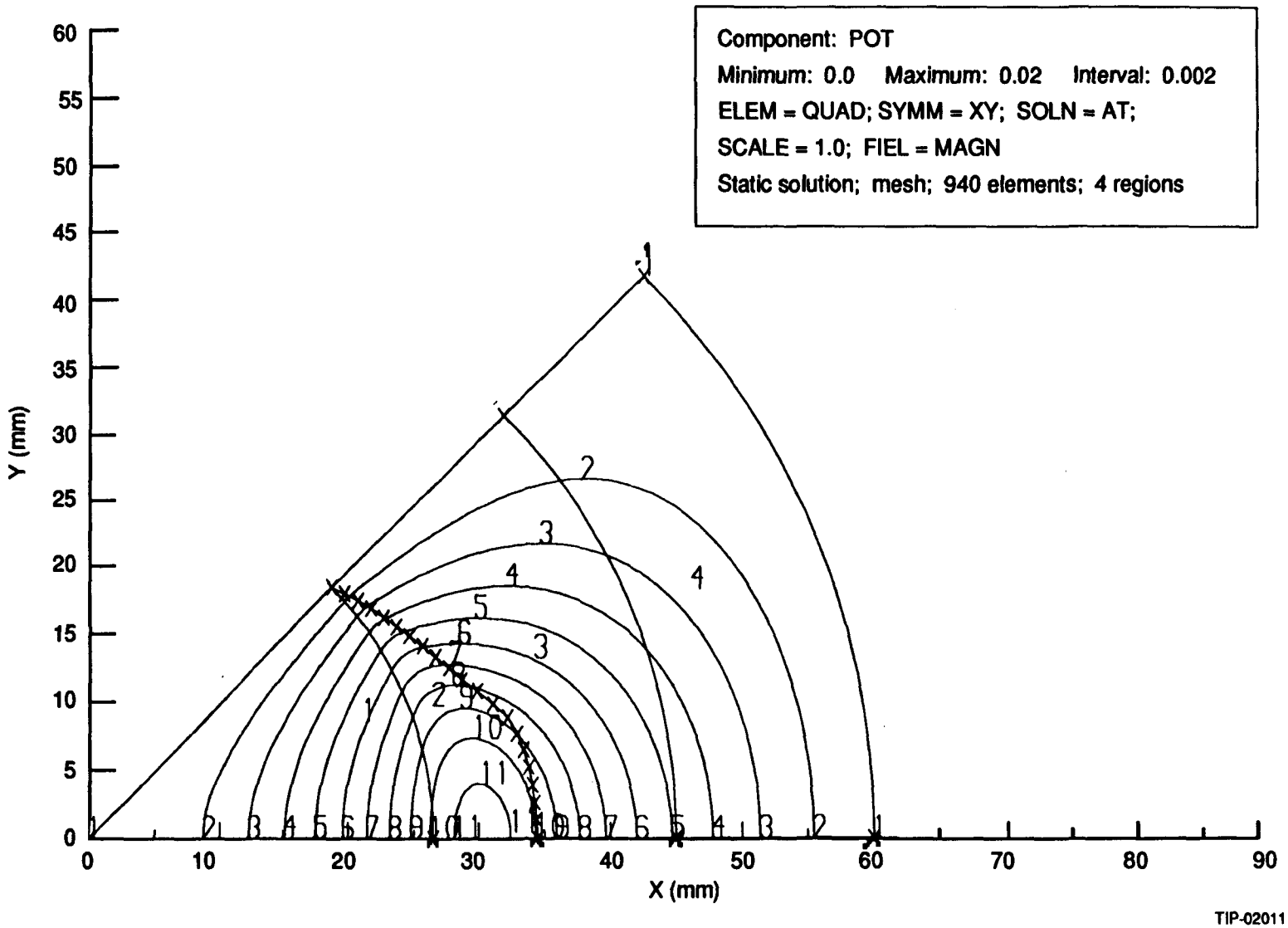


Figure 5. Potential Distribution—Quadrupole (case no. 2).

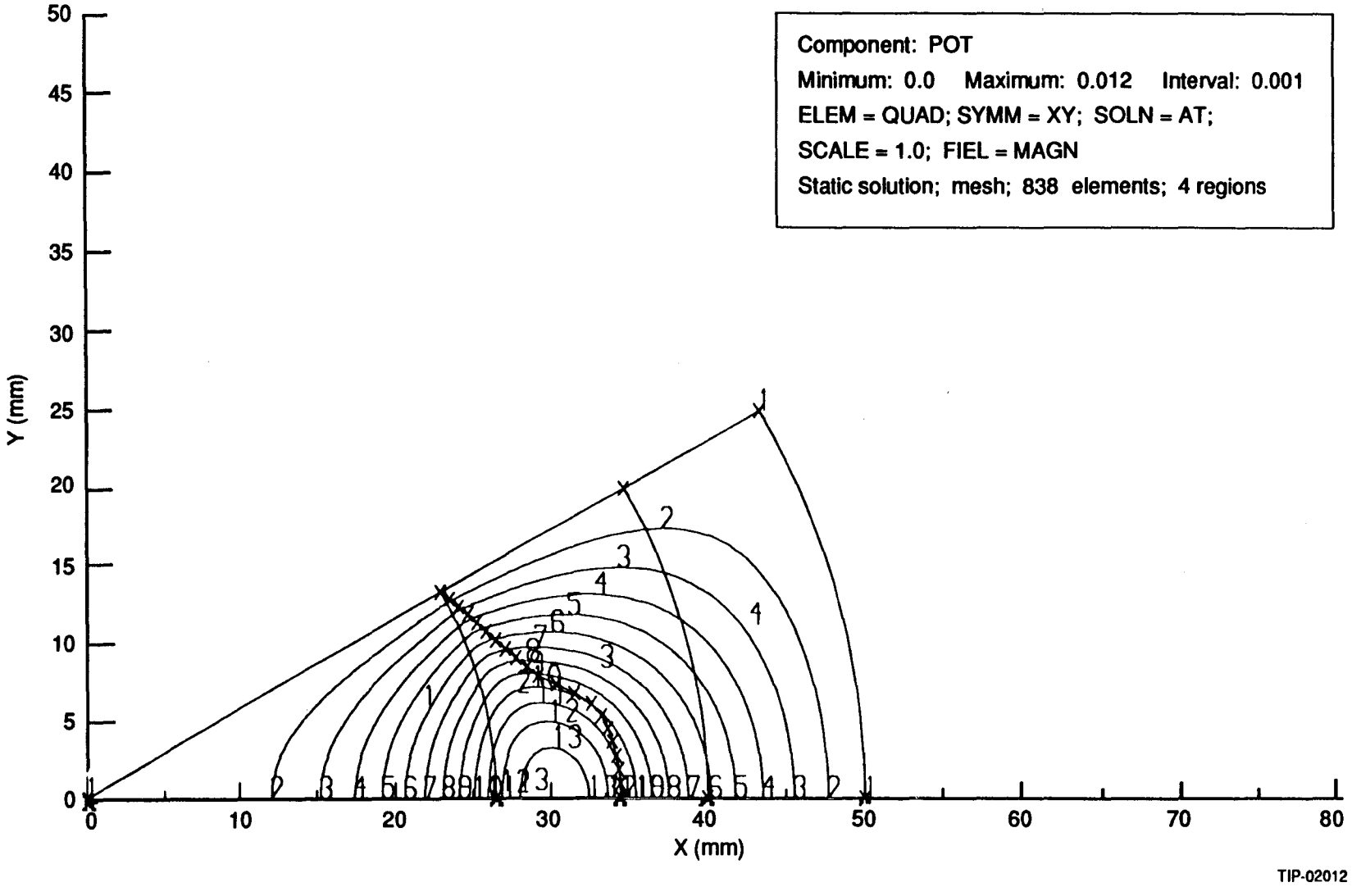


Figure 6. Potential Distribution—Sextupole (case no. 2).

ACKNOWLEDGMENTS

The author acknowledges encouragement from Richard York, John Skaritka, Michael Syphers, and Don Edwards.

REFERENCES

1. V. Thiagarajan, "Coil Shapes Towards Pure Multipoles in Circular Regions," SSCL-N-733, SSC Laboratory, Dallas, Texas.
2. P. Schmuser, "Superconducting Accelerator Magnets," DESY HERA 89-01, January 1989.
3. *PE 2D Reference Manual*, Version 8.2, Vector Fields Limited, 24, Bankside, Kidlington, Oxford OX5 1JE, England.

APPENDIX

LISTING OF DIPOLE3.F, QUAD1.F, AND SEXT1.F

```
c23456789012345678901234567890123456789012345678901234567890123456789012
      PROGRAM DIPOLE3
c DESCRIPTION OF INPUT:  1.0< ALPHA <2.0; BETA= USUALLY 1.0;
C VALUES OF A4N, A6N, A8N, A10N, A12N, A14N SHUOULD BE << 1.0 SAY 0.01 TO 0.5
C A1N,A2N ARE CALCULATED TO SATISFY ALPHA AND BETA AND ARE NOT INCLUDED IN IMAX;
C INITIAL GUESS FOR A3N, A5N, A7N, A9N, A11N, A13N SHOULD BE < 1.0 SAY 0.2;
C N= NUMBER OF STEPS IN TRAPEZOIDAL INTEGRATION SAY 1001, 10001 ETC.;
C IMAX= NUMBER OF CALCULATED COEFFICIENTS A3N THROUGH A13N AND 2<IMAX<6
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION RHS(11),C(11,11)
      COMMON A(11,11),R(11),IMAX
      READ(*,*) ALPHA,BETA,A4N,A6N,A8N,A10N,A12N,A14N
      READ(*,*) A3N,A5N,A7N,A9N,A11N,A13N,N,IMAX
      WRITE (*,*) ALPHA,BETA,A4N,A6N,A8N,A10N,A12N,A14N
      WRITE(*,*)  A3N,A5N,A7N,A9N,A11N,A13N,N,IMAX
      PAI=2.*DASIN(1.0D0)
      DELPHI=PAI/(2.*(N-1))
C II= NUMBER OF NEWTON-RAPHSON ITERATIONS
      DO 16 II=1,10
      PHI=0.
      DO 15 I=1,IMAX
      R(I)=0.
      DO 17 J=1,IMAX
      A(I,J)=0.
  17  CONTINUE
  15  CONTINUE
      T3=0.
      T5=0.
      T7=0.
      T9=0.
      T11=0.
      T13=0.
      T15=0.
      T17=0.
      DO 18 J=1,4
      IF (J.EQ.1) FB=1.
      IF(J.NE.1) FB=FB*(-1)**(J-1)
      DO 20 K=1,N
      FA=2.*DELPHI/2.
      IF(K.EQ.1.OR.K.EQ.N) FA=DELPHI/2.
      COSP=DCOS(PHI)
      DCOSP=DABS(COSP)
      COS2P=DCOS(2.*PHI)
      DCOS2P=DABS(COS2P)
      COS3P=DCOS(3.*PHI)
      DCOS3P=DABS(COS3P)
      COS4P=DCOS(4.*PHI)
      DCOS4P=DABS(COS4P)
      COS5P=DCOS(5.*PHI)
      DCOS5P=DABS(COS5P)
      COS6P=DCOS(6.*PHI)
      DCOS6P=DABS(COS6P)
      COS7P=DCOS(7.*PHI)
      DCOS7P=DABS(COS7P)
      COS9P=DCOS(9.*PHI)
      COS11P=DCOS(11.*PHI)
      COS13P=DCOS(13.*PHI)
      COS15P=DCOS(15.*PHI)
      COS17P=DCOS(17.*PHI)
      GAM=BETA+(ALPHA-BETA)*DCOSP
      DELTA=DEXP(A4N*DCOS2P)-DEXP(A4N)
      SLAM=1./(1.-A6N*DCOS3P)-DCOSP*A6N/(1.-A6N)-1.
C23456789012345678901234567890123456789012345678901234567890123456789012
      PSI=1./(1.-A8N*DCOS4P)**0.5-1./(1.-A8N)**0.5
      THETA=1./(1.-A10N*DCOS5P)**(1./3.)-DCOSP/(1.-A10N)**(1./3.)-1.
     1+DCOSP
      ETA=1./(1.-A12N*DCOS6P)**0.25-1./(1.-A12N)**0.25
      SMU=1./(1.-A14N*DCOS7P)**0.2-DCOSP/(1.-A14N)**0.2+DCOSP-1.
      A21=GAM+A3N*DELTA+A5N*SLAM+A7N*PSI+A9N*THETA+A11N*ETA+A13N*SMU
      A12=1./A21
      A122=A12**2
      A123=A12**3
      A124=A12**4
      A125=A12**5
      A126=A12**6
      TA=FA*FB*COS3P*A122
      A(1,1)=A(1,1)+TA*DELTA
      A(1,2)=A(1,2)+TA*SLAM
      A(1,3)=A(1,3)+TA*PSI
```

```
      A(1,4)=A(1,4)+TA*THETA
      A(1,5)=A(1,5)+TA*ETA
      A(1,6)=A(1,6)+TA*SMU
      R(1)=R(1)+FA*FB*(A12-1.+A122*(A3N*DELTA+A5N*SLAM+A7N*PSI+A9N*
     1THETA+A11N*ETA+A13N*SMU))*COS3P
      TC=3.*A124
      TA=FA*FB*COS5P*TC
      TB=1-A123
      A(2,1)=A(2,1)+TA*DELTA
      A(2,2)=A(2,2)+TA*SLAM
      A(2,3)=A(2,3)+TA*PSI
      A(2,4)=A(2,4)+TA*THETA
      A(2,5)=A(2,5)+TA*ETA
      A(2,6)=A(2,6)+TA*SMU
      R(2)=R(2)+TA*(-TB+TC*(DELTA*A3N+SLAM*A5N+PSI*A7N+THETA*A9N+
     1ETA*A11N+SMU*A13N))/TC
      TC=5.*A126
      TA=FA*FB*COS7P*TC
      TB=1.-A125
      A(3,1)=A(3,1)+TA*DELTA
      A(3,2)=A(3,2)+TA*SLAM
      A(3,3)=A(3,3)+TA*PSI
      A(3,4)=A(3,4)+TA*THETA
      A(3,5)=A(3,5)+TA*ETA
      A(3,6)=A(3,6)+TA*SMU
      R(3)=R(3)+TA*(-TB+TC*(DELTA*A3N+SLAM*A5N+PSI*A7N+THETA*A9N
     1+ETA*A11N+SMU*A13N))/TC
      TC=7.*A126*A122
      TA=FA*FB*COS9P*TC
      TB=1.-A125*A122
      A(4,1)=A(4,1)+TA*DELTA
      A(4,2)=A(4,2)+TA*SLAM
      A(4,3)=A(4,3)+TA*PSI
      A(4,4)=A(4,4)+TA*THETA
      A(4,5)=A(4,5)+TA*ETA
      A(4,6)=A(4,6)+TA*SMU
      R(4)=R(4)+TA*(-TB+TC*(DELTA*A3N+SLAM*A5N+PSI*A7N+THETA*A9N
     1+ETA*A11N+SMU*A13N))/TC
      TC=9.*A125*A125
      TA=FA*FB*COS11P*TC
      TB=1.-A125*A124
      A(5,1)=A(5,1)+TA*DELTA
      A(5,2)=A(5,2)+TA*SLAM
      A(5,3)=A(5,3)+TA*PSI
      A(5,4)=A(5,4)+TA*THETA
      A(5,5)=A(5,5)+TA*ETA
      A(5,6)=A(5,6)+TA*SMU
      R(5)=R(5)+TA*(-TB+TC*(DELTA*A3N+SLAM*A5N+PSI*A7N+THETA*A9N
     1+ETA*A11N+SMU*A13N))/TC
      TC=11.*A126*A126
      TA=FA*FB*COS13P*TC
      TB=1.-A126*A125
      A(6,1)=A(6,1)+TA*DELTA
      A(6,2)=A(6,2)+TA*SLAM
      A(6,3)=A(6,3)+TA*PSI
      A(6,4)=A(6,4)+TA*THETA
      A(6,5)=A(6,5)+TA*ETA
      A(6,6)=A(6,6)+TA*SMU
      R(6)=R(6)+TA*(-TB+TC*(DELTA*A3N+SLAM*A5N+PSI*A7N+THETA*A9N
     1+ETA*A11N+SMU*A13N))/TC
c23456789012345678901234567890123456789012345678901234567890123456789012
      PHI=PHI+DELPHI
      T3=T3+FA*FB*COS3P*(1.-A12)
      T5=T5+FA*FB*COS5P*(1-A123)
      T7=T7+FA*FB*COS7P*(1.-A125)
      T9=T9+FA*FB*COS9P*(1.-A125*A122)
      T11=T11+FA*FB*COS11P*(1.-A125*A124)
      T13=T13+FA*FB*COS13P*(1.-A125*A126)
      T15=T15+FA*FB*COS15P*(1.-A125*A126*A122)
      T17=T17+FA*FB*COS17P*(1.-A126*A126*A123)
 20   CONTINUE
 18   CONTINUE
      WRITE (*,106) (A(I,1),A(I,2),A(I,3),A(I,4),A(I,5),A(I,6),I=1,IMAX)
106   FORMAT(6(1X,D12.6))
      DO 120 IK=1,IMAX
      DO 122 JK=1,IMAX
      C(IK,JK)=A(IK,JK)
 122  CONTINUE
```

```
  120 CONTINUE
      WRITE (*,*) (R(I),I=1,IMAX)
      CALL MATINV(RHS)
      DO 100 I=1,IMAX
      TA=0.
      DO 102 J=1,IMAX
      TA=TA+C(I,J)*RHS(J)
102   CONTINUE
       WRITE (*,104)TA
104    FORMAT(8X,3HTA=,D12.6)
100   CONTINUE
      TA=0.
      A3N=RHS(1)
      A5N=RHS(2)
      A7N=RHS(3)
      A9N=RHS(4)
      A11N=RHS(5)
      A13N=RHS(6)
      A1N=BETA-A3N*DEXP(A4N)-A5N-A7N/(1.-A8N)**0.5-A9N-A11N/
     1(1.-A12N)**0.25-A13N
      A2N=ALPHA-BETA-A5N*A6N/(1.-A6N)-A9N/(1.-A10N)**(1./3.)+A9N
     1-A13N/(1.-A14N)**0.2+A13N
      WRITE(*,32)A1N,A2N,A3N,A4N,A5N
      WRITE(*,34)A6N,A7N,A8N,A9N,A10N
      WRITE (*,40) A11N,A12N,A13N,A14N
 32   FORMAT(/,4X,3HA1=,D12.6,4X,3HA2=,D12.6,4X,3HA3=,D12.6,4x,3HA4=,
     1D12.6,4X,3HA5=D12.6)
 34   FORMAT(/,4X,3HA6=,D12.6,4X,3HA7=,D12.6,4X,3HA8=,D12.6,4X,
     13HA9=,D12.6,4X,4HA10=,D12.6)
      WRITE (*,36) T3,T5,T7,T9
 36   FORMAT(2X,3HT3=,D12.6,4X,3HT5=,D12.6,4X,3HT7=,D12.6,4X,
     13HT9=,D12.6)
      WRITE(*,38) T11,T13,T15,T17
38    FORMAT(2X,4HT11=,D12.6,4X,4HT13=,D12.6,4X,4HT15=,D12.6,4X,
     14HT17=,D12.6)
40    FORMAT(/,4X,4HA11=,D12.6,4X,4HA12=,D12.6,4X,4HA13=,D12.6,
     14X,4HA14=,D12.6)
 16   CONTINUE
      END
      SUBROUTINE MATINV(RHS)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION B(11,11),RHS(11)
      COMMON A(11,11),R(11),IMAX
      RHS(1)=R(1)
      DO 5 J=1,IMAX
      B(1,J)=A(1,J)
 5    CONTINUE
      II=1
 10   CONTINUE
      IIP=II+1
      DO 20 I=IIP,IMAX
      Y=DABS(A(I,II))
      IF (Y.LT.1.D-50) GO TO 75
      RHS(I)=R(II)-R(I)*A(II,II)/A(I,II)
      GO TO 76
 75   RHS(I)=R(I)
 76   CONTINUE
      DO 30 J=IIP,IMAX
      IF (Y.LT.1.D-50) GO TO 77
      B(I,J)=A(II,J)-A(I,J)*A(II,II)/A(I,II)
      GO TO 79
 77   CONTINUE
      B(I,J)=A(I,J)
 79   CONTINUE
 30   CONTINUE
 20   CONTINUE
      DO 28 I=1,IMAX
      R(I)=RHS(I)
      DO 29 J=1,IMAX
      A(I,J)=B(I,J)
 29   CONTINUE
```

```
28    CONTINUE
      II=II+1
      IF (II.EQ.IMAX) GO TO 40
      GO TO 10
40    CONTINUE
      RHS(IMAX)=RHS(IMAX)/B(IMAX,IMAX)
      DO 50 IN=2,IMAX
      I=IMAX+1-IN
      X=0.
      IP=I+1
      DO 70 J=IP,IMAX
      X=X+B(I,J)*RHS(J)
70    CONTINUE
      RHS(I)=(RHS(I)-X)/B(I,I)
50    CONTINUE
      RETURN
      END
```

```
c23456789012345678901234567890123456789012345678901234567890123456789012
      PROGRAM QUAD1
C DESCRIPTION OF INPUT DATA: ALPHA =USUALLY 1.0; 1.0<BETA<2.0;
C VALUES OF A4N,A6N,A8N,A10N,A12N,A14N SHOULD BE << 1.0 SAY 0.01 TO 0.5;
C A1N,A2N,A3N ARE CALCULATED TO SATISFY ALPHA AND BETA AND ARE NOT INCLUDED
C IN IMAX; INITIAL GUESS FOR A5N,A7N,A9N,A11N,A13N SHOULD BE <1.0 SAY 0.2;
C N= NUMBER OF STEPS IN THE TRAPEZOIDAL INTEGRATION  SAY 1001, 10001 ETC.;
C IMAX=NUMBER OF CALCULATED COEFFICIENTS A5N THROUGH A13N AND 2<IMAX≤5
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION RHS(11),C(11,11)
      COMMON A(11,11),R(11),IMAX
      READ(*,*) ALPHA,BETA,A2N,A4N,A6N,A8N,A10N,A12N,A14N
      READ(*,*) A1N,A3N,A5N,A7N,A9N,A11N,A13N,N,IMAX
      WRITE (*,*) ALPHA,BETA,A2N,A4N,A6N,A8N,A10N,A12N,A14N
      WRITE(*,*)  A1N,A3N,A5N,A7N,A9N,A11N,A13N,N,IMAX
      PAI=2.*DASIN(1.0D0)
      DELPHI=PAI/(4.*(N-1))
      DO 16 II=1,40
      PHI=0.
      DO 15 I=1,IMAX
      R(I)=0.
      DO 17 J=1,IMAX
      A(I,J)=0.
   17 CONTINUE
   15 CONTINUE
      T4=0.
      T6=0.
      T8=0.
      T10=0.
      T12=0.
      T14=0.
      T16=0.
      T18=0.
      ST=1./2**0.5
      DO 18 J=1,8
      IF (J.EQ.1) FB=1.
      IF(J.NE.1) FB=FB*(-1)**(J-1)
      DO 20 K=1,N
      FA=2.*DELPHI/2.
      IF(K.EQ.1.OR.K.EQ.N) FA=DELPHI/2.
      COSP=DCOS(PHI)
      DCOSP=DABS(COSP)
      COS2P=DCOS(2.*PHI)
      DCOS2P=DABS(COS2P)
      COS3P=DCOS(3.*PHI)
      DCOS3P=DABS(COS3P)
      COS4P=DCOS(4.*PHI)
      DCOS4P=DABS(COS4P)
      COS5P=DCOS(5.*PHI)
      DCOS5P=DABS(COS5P)
      COS6P=DCOS(6.*PHI)
      DCOS6P=DABS(COS6P)
      COS7P=DCOS(7.*PHI)
      DCOS7P=DABS(COS7P)
      COS8P=DCOS(8.*PHI)
      COS10P=DCOS(10.*PHI)
      COS12P=DCOS(12.*PHI)
      COS14P=DCOS(14.*PHI)
      COS16P=DCOS(16.*PHI)
      COS18P=DCOS(18.*PHI)
C23456789012345678901234567890123456789012345678901234567890123456789012
      SA=DEXP(A4N)
      SB=DEXP(A4N*DCOS2P)
      A2N=-A5N*A6N/(1.-A6N)+A9N*(1.-1./(1.-A10N)**(1./3.))+A13N*(1.-
     11./(1.-A14N)**0.2)
      A3N=(ALPHA-BETA-A2N*ST+A5N*(1.-1./(1.-A6N*ST))+A9N*(1.-1./
     1(1.-A10N*ST)**(1./3.))+A11N*(1./(1.-A12N)**0.25-1.)+A13N*(1.-1./
     2(1.-A14N*ST)**0.2))/(1.-SA)
      A1N=BETA-A3N*SA-A5N-A7N/(1.-A8N)**0.5-A9N-A11N/(1.-A12N)**0.25
     1-A13N
      A21=A1N+A2N*DCOSP+A3N*SB+A5N/(1.-A6N*DCOS3P)+A7N/(1.-A8N*
     1DCOS4P)**0.5+A9N/(1.-A10N*DCOS5P)**(1./3.)+A11N/(1.-A12N*DCOS6P)
     2**0.25+A13N/(1.-A14N*DCOS7P)**0.2
      GAM=1./(1.-A6N*DCOS3P)-1.0+(1.-1./(1.-A6N*ST)-A6N*ST/(1.-A6N))*
     1(SB-SA)/(1.-SA)-DCOSP*A6N/(1.-A6N)
      DELTA=1./(1.-A8N*DCOS4P)**0.5-1./(1.-A8N)**0.5
      TX=(1.-A10N*ST)**(1./3.)
      TY=(1.-A10N)**(1./3.)
      SLAM=1./(1.-A10N*DCOS5P)**(1./3.)+(1.-1./TX-ST*(1./TY-1.))*
```

```
     1(SB-1.)/(1.-SA)-1./TX+(DCOSP-ST)*(1.-1./TY)
      PSI=1./(1.-A12N*DCOS6P)**0.25-1.+(1.-1./(1.-A12N)**0.25)*(SB-1.)
     1/(1.-SA)
      TX=(1.-A14N*ST)**0.2
      TY=(1.-A14N)**0.2
      ETA=1./(1.-A14N*DCOS7P)**0.2-1./TX+(DCOSP-ST)*(1.-1./TY)+(1./TX
     1-1.-ST*(1.-1./TY))*(SB-1.)/(1.-SA)
      A12=1./A21
      A122=A12**2
      A123=A12**3
      A124=A12**4
      A125=A12**5
      A126=A12**6
      TA=FA*FB*COS4P
      TC=2.*A123
      TB=1.-A122
      A(1,1)=A(1,1)+TA*TC*GAM
      A(1,2)=A(1,2)+TA*TC*DELTA
      A(1,3)=A(1,3)+TA*TC*SLAM
      A(1,4)=A(1,4)+TA*TC*PSI
      A(1,5)=A(1,5)+TA*TC*ETA
      R(1)=R(1)+TA*(-TB+TC*(GAM*A5N+DELTA*A7N+SLAM*A9N+PSI*A11N+
     1ETA*A13N))
      TC=4.*A125
      TA=FA*FB*COS6P
      TB=1-A124
      A(2,1)=A(2,1)+TA*TC*GAM
      A(2,2)=A(2,2)+TA*TC*DELTA
      A(2,3)=A(2,3)+TA*TC*SLAM
      A(2,4)=A(2,4)+TA*TC*PSI
      A(2,5)=A(2,5)+TA*TC*ETA
      R(2)=R(2)+TA*(-TB+TC*(GAM*A5N+DELTA*A7N+SLAM*A9N+PSI*A11N+
     1ETA*A13N))
      TC=6.*A126*A12
      TA=FA*FB*COS8P
      TB=1.-A126
      A(3,1)=A(3,1)+TA*TC*GAM
      A(3,2)=A(3,2)+TA*TC*DELTA
      A(3,3)=A(3,3)+TA*TC*SLAM
      A(3,4)=A(3,4)+TA*TC*PSI
      A(3,5)=A(3,5)+TA*TC*ETA
      R(3)=R(3)+TA*(-TB+TC*(GAM*A5N+DELTA*A7N+SLAM*A9N+PSI*A11N+
     1ETA*A13N))
      TA=FA*FB*COS10P
      TC=8.*A123*A126
      TB=1.-A122*A126
      A(4,1)=A(4,1)+TA*TC*GAM
      A(4,2)=A(4,2)+TA*TC*DELTA
      A(4,3)=A(4,3)+TA*TC*SLAM
      A(4,4)=A(4,4)+TA*TC*PSI
      A(4,5)=A(4,5)+TA*TC*ETA
      R(4)=R(4)+TA*(-TB+TC*(GAM*A5N+DELTA*A7N+SLAM*A9N+PSI*A11N+
     1ETA*A13N))
      TA=FA*FB*COS14P
      TC=12.*A126*A125*a122
      TB=1.-A126*A126
      A(5,1)=A(5,1)+TA*TC*GAM
      A(5,2)=A(5,2)+TA*TC*DELTA
      A(5,3)=A(5,3)+TA*TC*SLAM
      A(5,4)=A(5,4)+TA*TC*PSI
      A(5,5)=A(5,5)+TA*TC*ETA
      R(5)=R(5)+TA*(-TB+TC*(GAM*A5N+DELTA*A7N+SLAM*A9N+PSI*A11N+
     1ETA*A13N))
c23456789012345678901234567890123456789012345678901234567890123456789012
      PHI=PHI+DELPHI
      T4=T4+FA*FB*COS4P*(1.-A122)
      T6=T6+FA*FB*COS6P*(1-A124)
      T8=T8+FA*FB*COS8P*(1.-A126)
      T10=T10+FA*FB*COS10P*(1.-A125*A123)
      T12=T12+FA*FB*COS12P*(1.-A125*A125)
      T14=T14+FA*FB*COS14P*(1.-A126*A126)
      T16=T16+FA*FB*COS16P*(1.-A126*A126*A122)
      T18=T18+FA*FB*COS18P*(1.-A126*A126*A124)
 20   CONTINUE
 18   CONTINUE
      WRITE (*,106) (A(I,1),A(I,2),A(I,3),A(I,4),A(I,5),A(I,6),I=1,IMAX)
```

```
106   FORMAT(6(1X,D12.6))
      DO 120 IK=1,IMAX
      DO 122 JK=1,IMAX
      C(IK,JK)=A(IK,JK)
 122  CONTINUE
 120  CONTINUE
      WRITE (*,*) (R(I),I=1,IMAX)
      CALL MATINV(RHS)
      DO 100 I=1,IMAX
      TA=0.
      DO 102 J=1,IMAX
      TA=TA+C(I,J)*RHS(J)
102   CONTINUE
       WRITE (*,104)TA
104    FORMAT(8X,3HTA=,D12.6)
100   CONTINUE
      TA=0.
      A5N=RHS(1)
      A7N=RHS(2)
      A9N=RHS(3)
      A11N=RHS(4)
      A13N=RHS(5)
c23456789012345678901234567890123456789012345678901234567890123456789012
      WRITE(*,32)A1N,A2N,A3N,A4N,A5N,A6N,A7N
      WRITE(*,34)A8N,A9N,A10N,A11N,A12N,A13N,A14N
 32   FORMAT(/,4X,3HA1=,D12.6,4X,3HA2=,D12.6,4X,3HA3=,D12.6,4X,3HA4=,
     1D12.6,4X,3HA5=D12.6,4X,3HA6=,D12.6,4X,3HA7=,D12.6)
 34   FORMAT(/,4X,3HA8=,D12.6,4X,3HA9=,D12.6,4X,4HA10=,D12.6,4X,4HA11=,
     1D12.6,4X,4HA12=,D12.6,4X,4HA13=,D12.6,4X,4HA14=,D12.6)
      WRITE (*,36) T4,T6,T8,T10
 36   FORMAT(2X,3HT4=,D12.6,4X,3HT6=,D12.6,4X,3HT8=,D12.6,4X,
     14HT10=,D12.6)
      WRITE(*,38) T12,T14,T16,T18
38    FORMAT(2X,4HT12=,D12.6,4X,4HT14=,D12.6,4X,4HT16=,D12.6,4X,
     14HT18=,D12.6)
40    FORMAT(/,4X,4HA11=,D12.6,4X,4HA12=,D12.6,4X,4HA13=,D12.6,
     14X,4HA14=,D12.6)
 16   CONTINUE
      END
      SUBROUTINE MATINV(RHS)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION B(11,11),RHS(11)
      COMMON A(11,11),R(11),IMAX
      RHS(1)=R(1)
      DO 5 J=1,IMAX
      B(1,J)=A(1,J)
 5    CONTINUE
      II=1
 10   CONTINUE
      IIP=II+1
      DO 20 I=IIP,IMAX
      Y=DABS(A(I,II))
      IF (Y.LT.1.D-50) GO TO 75
      RHS(I)=R(II)-R(I)*A(II,II)/A(I,II)
      GO TO 76
 75   RHS(I)=R(I)
 76   CONTINUE
      DO 30 J=IIP,IMAX
      IF (Y.LT.1.D-50) GO TO 77
      B(I,J)=A(II,J)-A(I,J)*A(II,II)/A(I,II)
      GO TO 79
 77   CONTINUE
      B(I,J)=A(I,J)
 79   CONTINUE
 30   CONTINUE
 20   CONTINUE
      DO 28 I=1,IMAX
      R(I)=RHS(I)
      DO 29 J=1,IMAX
      A(I,J)=B(I,J)
 29   CONTINUE
 28   CONTINUE
      II=II+1
      IF (II.EQ.IMAX) GO TO 40
      GO TO 10
```

```
40    CONTINUE
      RHS(IMAX)=RHS(IMAX)/B(IMAX,IMAX)
      DO 50 IN=2,IMAX
      I=IMAX+1-IN
      X=0.
      IP=I+1
      DO 70 J=IP,IMAX
      X=X+B(I,J)*RHS(J)
70    CONTINUE
      RHS(I)=(RHS(I)-X)/B(I,I)
50    CONTINUE
      RETURN
      END
```

```
c23456789012345678901234567890123456789012345678901234567890123456789012
      PROGRAM SEXT1
c DESCRIPTION OF INPUT:  1.0< ALPHA <2.0; BETA=ETA= USUALLY 1.0;
C VALUES OF A4N, A6N, A8N, A10N, A12N, A14N, A16N, A18N SHOULD BE << 1.0 SAY
C A1N,A2N,A5N ARE CALCULATED TO SATISFY ALPHA, BETA AND ETA AND ARE NOT INCLUDED
C INITIAL GUESS FOR A3N, A7N, A9N, A11N, A13N, A15N AND A17N SHOULD BE < 1.0 SAY
C N= NUMBER OF STEPS IN TRAPEZOIDAL INTEGRATION SAY 1001, 10001 ETC.;
C IMAX= NUMBER OF CALCULATED COEFFICIENTS A3N THROUGH A17N AND 2<IMAX≤7
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION RHS(11),C(11,11)
      COMMON A(11,11),R(11),IMAX
      READ(*,*) ALPHA,BETA,ETA,A4N,A6N,A8N,A10N,A12N,A14N,A16N,A18N
      READ(*,*) A3N,A5N,A7N,A9N,A11N,A13N,A15N,A17N,N,IMAX
      WRITE (*,*) ALPHA,BETA,ETA,A4N,A6N,A8N,A10N,A12N,A14N,A16N,A18N
      WRITE(*,*)  A3N,A5N,A7N,A9N,A11N,A13N,A15N,A17N,N,IMAX
      PAI=2.*DASIN(1.0D0)
      DELPHI=PAI/(6.*(N-1))
C II= NUMBER OF NEWTON-RAPHSON ITERATIONS
      DO 16 II=1,10
      PHI=0.
      DO 15 I=1,IMAX
      R(I)=0.
      DO 17 J=1,IMAX
      A(I,J)=0.
  17  CONTINUE
  15  CONTINUE
      T1=0.
      T3=0.
      T5=0.
      T7=0.
      T9=0.
      T11=0.
      T13=0.
      T15=0.
      T17=0.
      T23=0.
      DO 18 J=1,12
      IF (J.EQ.1) FB=1.
      IF(J.NE.1) FB=FB*(-1)**(J-1)
      DO 20 K=1,N
      FA=2.*DELPHI/2.
      IF(K.EQ.1.OR.K.EQ.N) FA=DELPHI/2.
      COSP=DCOS(PHI)
      DCOSP=DABS(COSP)
      COS2P=DCOS(2.*PHI)
      DCOS2P=DABS(COS2P)
      COS3P=DCOS(3.*PHI)
      DCOS3P=DABS(COS3P)
      COS4P=DCOS(4.*PHI)
      DCOS4P=DABS(COS4P)
      COS5P=DCOS(5.*PHI)
      DCOS5P=DABS(COS5P)
      COS6P=DCOS(6.*PHI)
      DCOS6P=DABS(COS6P)
      COS7P=DCOS(7.*PHI)
      DCOS7P=DABS(COS7P)
      COS8P=DCOS(8.*PHI)
      DCOS8P=DABS(COS8P)
      COS9P=DCOS(9.*PHI)
      DCOS9P=DABS(COS9P)
      COS11P=DCOS(11.*PHI)
      COS13P=DCOS(13.*PHI)
      COS15P=DCOS(15.*PHI)
      COS17P=DCOS(17.*PHI)
      COS23P=DCOS(23.*PHI)
      SA=(1.-A6N)/A6N
      SB=2./(3.**0.5)
      SC=DEXP(A4N)
      SD=DEXP(A4N/2.)
      SE=DEXP(A4N*DCOS2P)
      GAM=SE-SC+SB*(SC-SD)*(DCOSP-SA/(1.-A6N*DCOS3P)+SA)
      SC=1./(1.-A8N)**0.5
      SD=1./(1.-A8N/2.)**0.5
      SE=SC-SD
```

```
c23456789012345678901234567890123456789012345678901234567890123456789012
      DELTA=1./(1.-A8N*DCOS4P)**0.5+DCOSP*SB*SE-(1./(1.-A6N*DCOS3P))*
     1SA*SB*SE+(-SC+SA*SE*SB)
      SC=1./(1.-A10N)**(1./3.)
      SD=1./(1.-A10N/SB)**(1./3.)
      SLAM=1./(1.-A10N*DCOS5P)**(1./3.)+DCOSP*SB*(1.-SD)+(1./
     1(1.-A6N*DCOS3P))*(-SA*SB*(1.-SD)-SA*(SC-1.))-1.+SA*(SC-1.)+SA*SB*
     2(1.-SD)
C23456789012345678901234567890123456789012345678901234567890123456789012
      PSI=1./(1.-A12N*DCOS6P)**0.25-1./(1.-A12N)**0.25
      SC=1./(1.-A14N/SB)**0.2
      SD=1./(1.-A14N)**0.2
      THETA=1./(1.-A14N*DCOS7P)**.2+DCOSP*SB*(1.-SC)+(1./(1.-A6N*DCOS3P
     1))*(-SA*SB*(1.-SC)-SA*(SD-1.))+(-1.+SA*(SD-1.)+SA*SB*(1.-SC))
      SC=1./(1.-A16N)**(1./6.)
      SD=1./(1.-A16N*0.5)**(1./6.)
      SE=SC-SD
      TAU=1./(1.-A16N*DCOS8P)**(1./6.)+DCOSP*SB*SE-SA*SB*SE/(1-A6N*
     1DCOS3P)-SC+SA*SB*SE
      SC=(1./(1.-A18N)**(1./7.))-1.
      SE=DEXP(A4N*DCOS2P)
      OMEGA=1./(1.-A18N*DCOS9P)**(1./7.)-SA*SC/(1.-A6N*DCOS3P)-1.+SA*SC
      A21=A1N+A2N*DCOSP+A3N*SE+A5N/(1.-A6N*DCOS3P)+A7N/(1.-A8N*DCOS4P
     1)**0.5+A9N/(1.-A10N*DCOS5P)**(1./3.)+A11N/(1.-A12N*DCOS6P)**0.25+
     2A13N/(1.-A14N*DCOS7P)**0.2+A15N/(1.-A16N*DCOS8P)**(1./6.)+A17N/
     3(1.-A18N*DCOS9P)**(1./7.)
      A12=1./A21
      A122=A12**2
      A123=A12**3
      A124=A12**4
      A125=A12**5
      A126=A12**6
      TA=FA*FB*COSP
      TB=(1.-A21)
      TC=-1.
      A(1,1)=A(1,1)+TA*TC*GAM
      A(1,2)=A(1,2)+TA*TC*DELTA
      A(1,3)=A(1,3)+TA*TC*SLAM
      A(1,4)=A(1,4)+TA*TC*PSI
      A(1,5)=A(1,5)+TA*TC*THETA
      A(1,6)=A(1,6)+TA*TC*TAU
      A(1,7)=A(1,7)+TA*TC*OMEGA
      R(1)=R(1)+TA*(-TB+TC*(A3N*GAM+A7N*DELTA+A9N*SLAM+A11N*
     1PSI+A13N*THETA+A15N*TAU+A17N*OMEGA))
      TA=FA*FB*COS5P
      TB=1-A123
      TC=3.*A124
      A(2,1)=A(2,1)+TA*TC*GAM
      A(2,2)=A(2,2)+TA*TC*DELTA
      A(2,3)=A(2,3)+TA*TC*SLAM
      A(2,4)=A(2,4)+TA*TC*PSI
      A(2,5)=A(2,5)+TA*TC*THETA
      A(2,6)=A(2,6)+TA*TC*TAU
      A(2,7)=A(2,7)+TA*TC*OMEGA
      R(2)=R(2)+TA*(-TB+TC*(A3N*GAM+A7N*DELTA+A9N*SLAM+A11N*
     1PSI+A13N*THETA+A15N*TAU+A17N*OMEGA))
      TA=FA*FB*COS7P
      TB=1.-A125
      TC=5.*A126
      A(3,1)=A(3,1)+TA*TC*GAM
      A(3,2)=A(3,2)+TA*TC*DELTA
      A(3,3)=A(3,3)+TA*TC*SLAM
      A(3,4)=A(3,4)+TA*TC*PSI
      A(3,5)=A(3,5)+TA*TC*THETA
      A(3,6)=A(3,6)+TA*TC*TAU
      A(3,7)=A(3,7)+TA*TC*OMEGA
      R(3)=R(3)+TA*(-TB+TC*(A3N*GAM+A7N*DELTA+A9N*SLAM+A11N*
     1PSI+A13N*THETA+A15N*TAU+A17N*OMEGA))
      TA=FA*FB*COS9P
      TB=1.-A126*A12
      TC=7.*A126*A122
      A(4,1)=A(4,1)+TA*TC*GAM
      A(4,2)=A(4,2)+TA*TC*DELTA
      A(4,3)=A(4,3)+TA*TC*SLAM
      A(4,4)=A(4,4)+TA*TC*PSI
      A(4,5)=A(4,5)+TA*TC*THETA
      A(4,6)=A(4,6)+TA*TC*TAU
      A(4,7)=A(4,7)+TA*TC*OMEGA
      R(4)=R(4)+TA*(-TB+TC*(A3N*GAM+A7N*DELTA+A9N*SLAM+A11N*
```

```
     1PSI+A13N*THETA+A15N*TAU+A17N*OMEGA))
      TA=FA*FB*COS11P
      TB=1.-A125*A124
      TC=9.*A125*A125
      A(5,1)=A(5,1)+TA*TC*GAM
      A(5,2)=A(5,2)+TA*TC*DELTA
      A(5,3)=A(5,3)+TA*TC*SLAM
      A(5,4)=A(5,4)+TA*TC*PSI
      A(5,5)=A(5,5)+TA*TC*THETA
      A(5,6)=A(5,6)+TA*TC*TAU
      A(5,7)=A(5,7)+TA*TC*OMEGA
      R(5)=R(5)+TA*(-TB+TC*(A3N*GAM+A7N*DELTA+A9N*SLAM+A11N*
     1PSI+A13N*THETA+A15N*TAU+A17N*OMEGA))
      TA=FA*FB*COS13P
      TB=1.-A126*A125
      TC=11.*A126*A126
      A(6,1)=A(6,1)+TA*TC*GAM
      A(6,2)=A(6,2)+TA*TC*DELTA
      A(6,3)=A(6,3)+TA*TC*SLAM
      A(6,4)=A(6,4)+TA*TC*PSI
      A(6,5)=A(6,5)+TA*TC*THETA
      A(6,6)=A(6,6)+TA*TC*TAU
      A(6,7)=A(6,7)+TA*TC*OMEGA
      R(6)=R(6)+TA*(-TB+TC*(A3N*GAM+A7N*DELTA+A9N*SLAM+A11N*
     1PSI+A13N*THETA+A15N*TAU+A17N*OMEGA))
      TA=FA*FB*COS15P
      TB=1.-A126*A126*A12
      TC=13.*A126*A126*A122
      A(7,1)=A(7,1)+TA*TC*GAM
      A(7,2)=A(7,2)+TA*TC*DELTA
      A(7,3)=A(7,3)+TA*TC*SLAM
      A(7,4)=A(7,4)+TA*TC*PSI
      A(7,5)=A(7,5)+TA*TC*THETA
      A(7,6)=A(7,6)+TA*TC*TAU
      A(7,7)=A(7,7)+TA*TC*OMEGA
      R(7)=R(7)+TA*(-TB+TC*(A3N*GAM+A7N*DELTA+A9N*SLAM+A11N*
     1PSI+A13N*THETA+A15N*TAU+A17N*OMEGA))
c23456789012345678901234567890123456789012345678901234567890123456789012
      PHI=PHI+DELPHI
      T1=T1+FA*FB*COSP*(1.-A21)
      T3=T3+FA*FB*COS3P*(1.-A12)
      T5=T5+FA*FB*COS5P*(1-A123)
      T7=T7+FA*FB*COS7P*(1.-A125)
      T9=T9+FA*FB*COS9P*(1.-A125*A122)
      T11=T11+FA*FB*COS11P*(1.-A125*A124)
      T13=T13+FA*FB*COS13P*(1.-A125*A126)
      T15=T15+FA*FB*COS15P*(1.-A125*A126*A122)
      T17=T17+FA*FB*COS17P*(1.-A126*A126*A123)
      T23=T23+FA*FB*COS23P*(1.-A126*A126*A126*A123)
 20   CONTINUE
 18   CONTINUE
      WRITE (*,106) (A(I,1),A(I,2),A(I,3),A(I,4),A(I,5),A(I,6),A(I,7),
     1I=1,IMAX)
106   FORMAT(7(1X,D12.6))
      DO 120 IK=1,IMAX
      DO 122 JK=1,IMAX
      C(IK,JK)=A(IK,JK)
 122  CONTINUE
 120  CONTINUE
      WRITE (*,*) (R(I),I=1,IMAX)
      CALL MATINV(RHS)
      DO 100 I=1,IMAX
      TA=0.
      DO 102 J=1,IMAX
      TA=TA+C(I,J)*RHS(J)
102   CONTINUE
       WRITE (*,104)TA
104    FORMAT(8X,3HTA=,D12.6)
```

```
100   CONTINUE
      TA=0.
      A3N=RHS(1)
      A7N=RHS(2)
      A9N=RHS(3)
      A11N=RHS(4)
      A13N=RHS(5)
      A15N=RHS(6)
      A17N=RHS(7)
      SC=DEXP(A4N)
      SD=1./(1.-A8N)**0.5
      SE=ETA-BETA
c23456789012345678901234567890123456789012345678901234567890123456789012
      A2N=SB*(SE+A3N*(SC-DEXP(A4N*0.5))+A7N*(SD-1./(1.-A8N*0.5)**0.5)+
     1A9N*(1.-1./(1.-A10N/SB)**(1./3.))+A13N*(1.-1./(1-A14N/SB)**0.2))
     2+A15N*SB*(1./(1.-A16N)**(1./6.)-1./(1.-A16N*0.5)**(1./6.))
      A5N=SA*(ALPHA-BETA-A2N-A9N*(1./(1.-A10N)**(1./3.)-1.)-A13N*
     1(1./(1.-A14N)**0.2-1.)-A17N*((1./(1.-A18N)**(1./7.))-1.))
      A1N=BETA-A3N*SC-A5N-A7N*SD-A9N-A11N/(1.-A12N)**0.25-A13N-A15N/
     1(1.-A16N)**(1./6.)-A17N
      WRITE(*,32)A1N,A2N,A3N,A4N,A5N
      WRITE(*,34)A6N,A7N,A8N,A9N,A10N
      WRITE (*,40) A11N,A12N,A13N,A14N
      WRITE (*,42)A15N,A16N,A17N,A18N
 32   FORMAT(/,4X,3HA1=,D12.6,4X,3HA2=,D12.6,4X,3HA3=,D12.6,4x,3HA4=,
     1D12.6,4X,3HA5=D12.6)
 34   FORMAT(/,4X,3HA6=,D12.6,4X,3HA7=,D12.6,4X,3HA8=,D12.6,4X,
     13HA9=,D12.6,4X,4HA10=,D12.6)
      WRITE (*,35) T1
 35   FORMAT(2X,3HT1=,D12.6)
      WRITE (*,36) T3,T5,T7,T9
 36   FORMAT(2X,3HT3=,D12.6,4X,3HT5=,D12.6,4X,3HT7=,D12.6,4X,
     13HT9=,D12.6)
      WRITE(*,38) T11,T13,T15,T17
      WRITE(*,44) T23
 38   FORMAT(2X,4HT11=,D12.6,4X,4HT13=,D12.6,4X,4HT15=,D12.6,4X,
     14HT17=,D12.6)
 40   FORMAT(/,4X,4HA11=,D12.6,4X,4HA12=,D12.6,4X,4HA13=,D12.6,
     14X,4HA14=,D12.6)
 42   FORMAT(/,4X,4HA15=,D12.6,4X,4HA16=,D12.6,4X,4HA17=,D12.6,
     14X,4HA18=,D12.6)
 44   FORMAT(4X,4HT23=,D12.6,/)
 16   CONTINUE
      END
      SUBROUTINE MATINV(RHS)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION B(11,11),RHS(11)
      COMMON A(11,11),R(11),IMAX
      RHS(1)=R(1)
      DO 5 J=1,IMAX
      B(1,J)=A(1,J)
 5    CONTINUE
      II=1
 10   CONTINUE
      IIP=II+1
      DO 20 I=IIP,IMAX
      Y=DABS(A(I,II))
      IF (Y.LT.1.D-50) GO TO 75
      RHS(I)=R(II)-R(I)*A(II,II)/A(I,II)
      GO TO 76
 75   RHS(I)=R(I)
 76   CONTINUE
      DO 30 J=IIP,IMAX
      IF (Y.LT.1.D-50) GO TO 77
      B(I,J)=A(II,J)-A(I,J)*A(II,II)/A(I,II)
      GO TO 79
 77   CONTINUE
      B(I,J)=A(I,J)
 79   CONTINUE
 30   CONTINUE
 20   CONTINUE
      DO 28 I=1,IMAX
      R(I)=RHS(I)
      DO 29 J=1,IMAX
      A(I,J)=B(I,J)
 29   CONTINUE
 28   CONTINUE
      II=II+1
      IF (II.EQ.IMAX) GO TO 40
```

```
      GO TO 10
40    CONTINUE
      RHS(IMAX)=RHS(IMAX)/B(IMAX,IMAX)
      DO 50 IN=2,IMAX
      I=IMAX+1-IN
      X=0.
      IP=I+1
      DO 70 J=IP,IMAX
      X=X+B(I,J)*RHS(J)
70    CONTINUE
      RHS(I)=(RHS(I)-X)/B(I,I)
50    CONTINUE
      RETURN
      END
```