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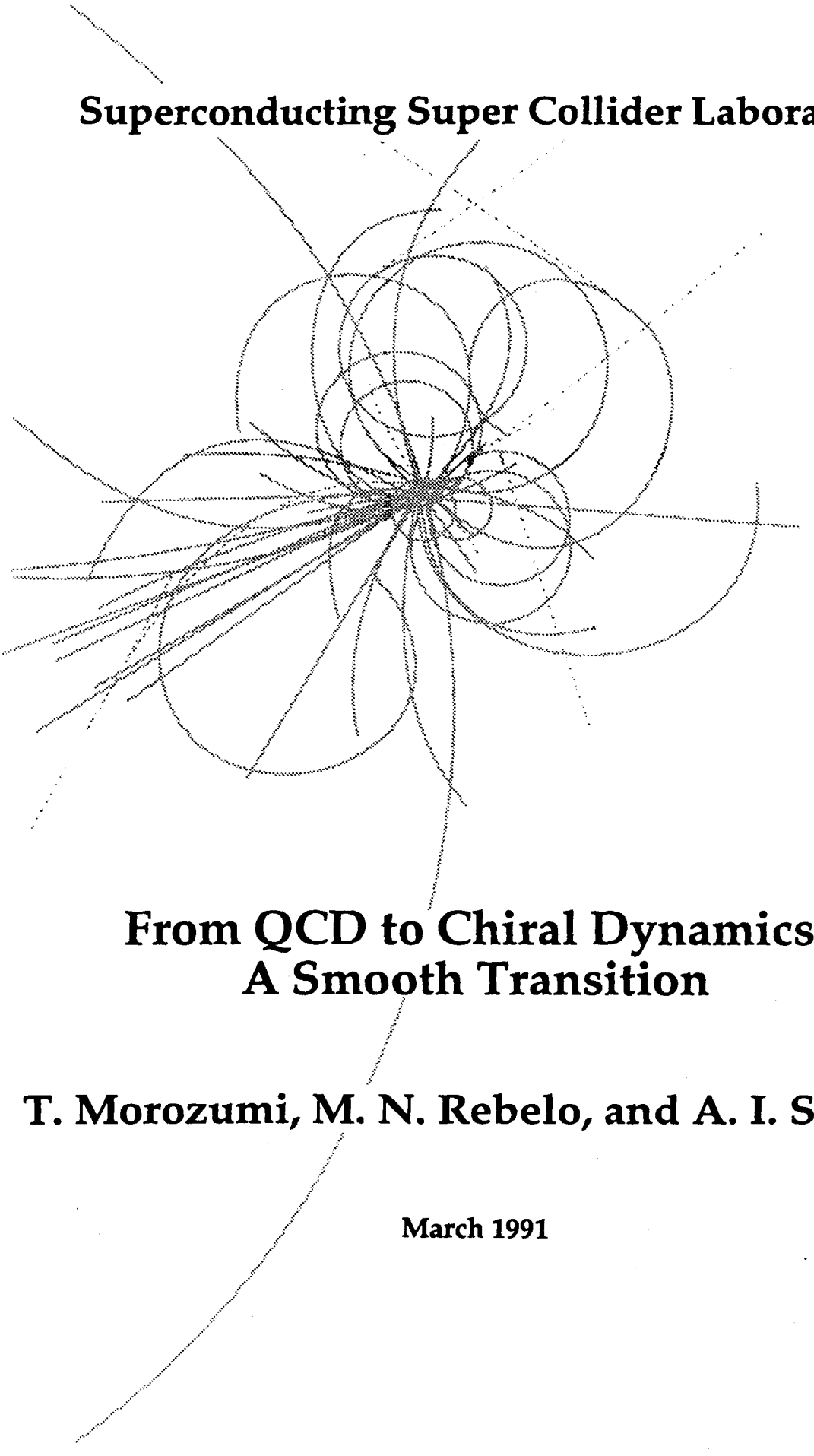

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# From QCD to Chiral Dynamics: A Smooth Transition

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**Abstract**

We study the transition from perturbative QCD to Chiral Dynamics (linear $\sigma$ model), which describe physics between $M_W$ and $\mu$, and between $\mu$ and $M_\pi$, respectively. This is done within the context of K decays. The smoothness of the transition is shown by comparing the explicit form of the anomalous dimension matrices in both pictures and by checking the approximate cancellation of the $\mu$ dependence.

Understanding non-leptonic K meson decays has been a challenge for more than three decades. There has been considerable progress recently. Within the context of the standard model of $SU(3) \otimes SU(2) \otimes U(1)$ strong and electroweak interactions we now know the Hamiltonian:[1]

$$H_W = -\frac{G_F}{\sqrt{2}} sin\theta_c cos\theta_c \sum_{i=1}^{6} R_i(\mu) Q_i(\mu) + h.c., \tag{1}$$

where $\theta_c$ denotes the Cabibbo angle; $Q_i(\mu)$ are a complete set of 4-fermi (dimension 6) operators and $R_i(\mu)$ are Wilson coefficients. Starting from a simple $(V-A) \otimes (V-A)$ 4-fermi interaction of quarks at an energy scale of $M_W$, the Hamiltonian (1) contains QCD corrections due to physics in the energy scale between $M_W$ and $\mu$. The challenge of understanding non-leptonic decays has now been reduced to that of computing the matrix elements of the quark operators $Q_i(\mu)$ between physical meson states. This is not a simple task. The matrix element for the $K \to$ f decay, $< f|Q_i(\mu)|K >$, which is evaluated at the energy scale of $M_\pi$ and $M_K$, contains the hadronic correction from physics below $\mu$.

To evaluate the decay matrix element $< f|Q_i(\mu)|K >$, we must have the theory of the strong interaction below the energy scale $\mu$. For the QCD computation of $R_i(\mu)$ to be reliable, we cannot take $\mu$ to be much smaller than 1 GeV. In order to describe the strong interaction physics below 1 GeV, we use a chiral Lagrangian based on the linear $\sigma$ model.

Since $\mu$ is an artifically introduced energy scale at which we change from the quark picture to the meson picture, the physical amplitude for the $K \to f$ decay cannot depend on $\mu$. If the linear $\sigma$ model is indeed a correct extension of QCD to low energy, the $\mu$ dependence in $R_i(\mu)$ governed by QCD must cancel the $\mu$ dependence of $< f|Q_i(\mu)|K >$ governed by the $\sigma$ model. In this note we present a first indication for such cancellation.[2] It has been shown that the linear $\sigma$ model gives a satisfactory description of meson physics below 1 GeV.[3]

In a previous paper[4] we pointed out that the matrix element of the $Q_6$ operator has an additional enhancement factor of $2 \sim 3$ to that computed using

the vacuum saturation approximation. Based on this point, we suggested an explanation for the $\Delta I = 1/2$ rule. The above result was based on the tree level computation. In this paper we study the $\mu$ dependence of the physical $K \to \pi\pi$ amplitude by computing the counterterms associated with the loop correction to the matrix element.

At low energy, we assume that the strong interaction is described by the $SU(3)_L \otimes SU(3)_R$ linear $\sigma$ model:[3]

$$\begin{aligned} L_{st} = & Tr(\partial_\mu M \partial^\mu M^\dagger) + Z_m m^2 Tr(MM^\dagger) - Z_\lambda \frac{\lambda}{4}\mu^{2\epsilon} Tr(MM^\dagger MM^\dagger) \\ & - (Z_\lambda - 1)\frac{\lambda}{8}\mu^{2\epsilon}\Big(Tr(MM^\dagger)\Big)^2 + \frac{1}{4{G_1}^o} Tr[m^o(M + M^\dagger)], \end{aligned} \tag{2}$$

where M transforms like $(3, \overline{3})$ under $SU(3)_L \otimes SU(3)_R$, and $m^o$ is a bare diagonal current quark mass matrix, $m^o_{ij} = m^o_i \delta_{io}$ where $m^o_1 = m^o_2 = m_{ou}$, $m^o_3 = m_{os}$, assuming isospin invariance. ${G_1}^o$ is renormalized as $G_1(\mu) = (Z_{mf})^{-1}{G_1}^o$ where $Z_{mf}$ is a fermion mass renormalization constant in QCD, because the ratio $\frac{m^o}{{G_1}^o}$ is not renormalized in the linear $\sigma$ model. For simplicity we adopt the large $N_c$ approximation and drop interactions like $(Tr(MM^\dagger))^2$ and detM in the tree level. The counterterms are given by $Z_\lambda = Z_m = 1 + \frac{3\lambda}{16\pi^2\epsilon}$ in the one loop level ($\epsilon = 2 - \frac{n}{2}; n$ is the dimension of integration). As for $H_W$, in the same approximation, only $Q_1$, $Q_2$, $Q_4$ and $Q_6$ contribute to the K decays. We write these operators in terms of bosons. At the tree level,

$$\begin{aligned} Q^o_1 &= -4(L_\mu)_{21}(L^\mu)_{13}, \quad & Q^o_2 &= -4(L_\mu)_{23}(L^\mu)_{11}, \\ Q^o_4 &= -4(L_\mu L^\mu)_{23}, & Q^o_6 &= -\frac{1}{2({G_1}^o)^2}(MM^\dagger)_{23}\,, \end{aligned} \tag{3}$$

where $L_\mu = M\partial_\mu M^\dagger - \partial_\mu M M^\dagger$.

To compute the loop correction, we introduce counterterms:

$$Q_i(\mu) = Q^o_i + (Z_{iJ} - \delta_{iJ})Q^o_J, \tag{4}$$

where $Q_J^o$ is a set of operators needed to remove all divergences. The weak Hamiltonian now becomes:

$$H_W = -\frac{G_F}{\sqrt{2}} sin\theta_c cos\theta_c \sum_{i=1}^{6} R_i(\mu) Z_{iJ}(\mu) Q_J^o + h.c. \tag{5}$$

Since $R_i(\mu)$ is computed in the mass independent renormalization scheme, we must adopt a compatible procedure in computing $Z_{iJ}(\mu)$.

It is well known[5] that $Z_{iJ}(\mu)$ can be obtained from counterterms computed in the symmetric phase of the linear $\sigma$ model. Here we use the dimensional regularization scheme with the minimal subtraction prescription, i.e., keep only the terms which are proportional to $\frac{1}{\epsilon}$. This determines the entire $\mu$ dependence of $Q_i(\mu)$, since neither the spontaneous chiral symmetry breaking, quadratic divergence and quartic divergence, nor the finite part introduce additional $\mu$ dependence.

In Figure 1 we show the relevant divergent diagrams which contribute to the $\mu$ dependence of the matrix elements for the $K \to \pi\pi$ decay.

The result is

$$\begin{aligned}
Q_1(\mu^2) &= Q_1^o + \frac{3\lambda}{64\pi^2\epsilon}(Q_2^o + Q_2^{o\prime} - 2Q_2^{o\prime\prime}), \\
Q_2(\mu^2) &= Q_2^o + \frac{3\lambda}{64\pi^2\epsilon}(Q_1^o + Q_1^{o\prime} - 2Q_1^{o\prime\prime}) + \frac{\lambda}{64\pi^2\epsilon}Q^o, \\
Q_4(\mu^2) &= Q_4^o, \\
Q_6(\mu^2) &= Q_6^o + ((Z_{mf})^2 - 1)Q_6^o,
\end{aligned} \tag{6}$$

with

$$
\begin{aligned}
&Q_1^{o\prime} = -4(\bar{L_\mu})_{23}(\bar{L^\mu})_{11}, \quad Q_2^{o\prime} = -4(\bar{L_\mu})_{21}(\bar{L^\mu})_{13}, \\
&Q_4^{o\prime} = -4(\bar{L_\mu}\bar{L^\mu})_{23}, \quad \bar{L_\mu} = \partial_\mu(MM^\dagger), \\
&Q_1^{o\prime\prime} = -4[(\partial^\mu M\partial_\mu M^\dagger)_{23}(MM^\dagger)_{11} + (\partial^\mu M\partial_\mu M^\dagger)_{11}(MM^\dagger)_{23}], \\
&Q_2^{o\prime\prime} = -4[(\partial^\mu M\partial_\mu M^\dagger)_{21}(MM^\dagger)_{13} + (\partial^\mu M\partial_\mu M^\dagger)_{13}(MM^\dagger)_{21}], \\
&Q_4^{o\prime\prime} = -4(\partial^\mu M\partial_\mu M^\dagger MM^\dagger + MM^\dagger\partial^\mu M\partial_\mu M^\dagger)_{23}, \\
&Q_6^{o\prime} = -4[2m^2(MM^\dagger MM^\dagger)_{23} - (\partial^\mu MM^\dagger M\partial_\mu M^\dagger)_{23}], \\
&Q_6^{o\prime\prime} = -4[m^2(\partial^\mu M\partial_\mu M^\dagger)_{23} - m^4(M^\dagger M)_{23}]\mu^{-2\epsilon}, \\
&Q_6^{o\prime\prime\prime} = 4(MM^\dagger MM^\dagger MM^\dagger\mu^{2\epsilon})_{23}, \\
&Q^o = Q_4^o + Q_4^{o\prime} + 2Q_4^{o\prime\prime} + 2Q_6^{o\prime} + \frac{4}{\lambda}Q_6^{o\prime\prime} + \lambda Q_6^{o\prime\prime\prime}, \\
&Z_{mf} = 1 - \frac{9\alpha}{8\pi\epsilon},
\end{aligned}
\tag{7}
$$

where $Q^o$ is the sum of the pure $\Delta I = 1/2$ operators in the symmetric phase. In the leading order of $\frac{1}{N_c}$ expansion, the counterterm for $Q_6^o$ does not come from the loop diagrams but the renormalization of ${G_1}^o$. $Q_4^o$ is not renormalized in the same order, because it was born after the loop corrections of $Q_2^o$. Note that $Q_1^o$ and $Q_2^o$ induce an explosive number of new operators, "$Q^{New}$", from the strong interaction correction. All of them have to be kept since $< f|Q^{New}|K >$ can be written in terms of $< f|Q_i^o|K >$, $(i = 1,2,4,6)$, when the chiral symmetry is spontaneously broken. Indeed, the operator relations given in Equation (6) lead an equation for any matrix element of $K \to \pi\pi$ amplitudes:

$$
\begin{aligned}
&< Q_1(\mu^2) > =< Q_1^o > + \frac{3\lambda}{64\pi^2\epsilon} < Q_2^o >, \\
&< Q_2(\mu^2) > =< Q_2^o > + \frac{3\lambda}{64\pi^2\epsilon} < Q_1^o > \\
&\qquad + \frac{3\lambda}{128\pi^2\epsilon}(1 + \frac{2}{3}\frac{M_K^2}{M_\sigma^2 - M_K^2}) < Q_4^o > \\
&\qquad + \frac{3\lambda}{128\pi^2\epsilon}\left(\frac{F_\pi}{F_K}\right)^2(\frac{m_{os}^2}{2M_K^2} - \frac{m_{os}^2}{M_\sigma^2}) < Q_6^o >, \\
&< Q_4(\mu^2) > =< Q_4^o >, \quad < Q_6(\mu^2) > =< Q_6^o > - \frac{9\alpha}{4\pi\epsilon} < Q_6^o >
\end{aligned}
\tag{8}
$$

In deriving Equations (8), we use $< Q_4^o >=< Q_2^o - Q_1^o >$ that holds for all the tree level matrix elements in $K \to \pi\pi$ decays. We are now in a position to discuss the $\mu$ dependence of the S matrix:

$$\begin{aligned} \mu\frac{d}{d\mu}S_{matrix} &\sim \sum_i (\mu\frac{d}{d\mu}R_i < Q_i > + R_i\mu\frac{d}{d\mu} < Q_i >) \\ &= \sum_{ij} \left(R_i\gamma_{ij} < Q_j > - R_i\gamma_{ij}^M < Q_j >\right). \end{aligned} \tag{9}$$

The anomalous dimension matrix $\gamma_{ij}^M$ in the $Q_1, Q_2, Q_4$, and $Q_6$ can be obtained from Equation (9). We list our result together with that of QCD:

$$\gamma = \frac{3\alpha}{2\pi}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & \frac{1}{9} & \frac{1}{9} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}, \quad \gamma^M = \frac{3\lambda}{32\pi^2}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & a & b \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c \end{pmatrix}, \tag{10}$$

where

$$\begin{aligned} a &= \frac{1}{2}\left(1 + \frac{2}{3}\frac{M_K^2}{M_\sigma^2 - M_K^2}\right), \\ b &= \frac{1}{2}\left(\frac{m_{os}^2}{2M_K^2} - \frac{m_{os}^2}{M_\sigma^2}\right)\left(\frac{F_\pi}{F_K}\right)^2, \\ c &= -\frac{48\pi\alpha}{\lambda}. \end{aligned} \tag{11}$$

We do not expect to obtain an exact cancellation because there are several aproximations done in the course of our calculation. We go up only to one loop level, we use the $\frac{1}{N_c}$ approximation, and the choice of our matching point $\mu \approx$ 1GeV may be too small for the QCD perturbation expansion to be valid and too large for the $\sigma$ model to be valid. In spite of all these points, the fact that the matrices $\gamma$ and $\gamma^M$ have the same general structure is very encouraging. The zeroes of the matrix elements agree with one another; for nonvanishing matrix elements, their signs also agree; features like $\gamma_{12} = \gamma_{21}$, $\gamma_{21} > \gamma_{24}, \gamma_{26}$ are common properties.

In Figure 2 we give a rough comparison of $\gamma_{12}^M$ and $\gamma_{12}$ using

$$\alpha(\mu) = \frac{2\pi}{9}\frac{1}{\ln\frac{\mu}{\Lambda_{QCD}}}, \quad \lambda(\mu) = \frac{8\pi^2}{3}\frac{1}{\ln\frac{\bar{\Lambda}}{\mu}}, \tag{12}$$

with $\Lambda_{QCD} = 0.4$ GeV and $\bar{\Lambda}$ fixed by choosing as matching point for $\gamma_{12}$ and $\gamma_{12}^M$ the value $\mu_0$ of 1GeV.

Considering the roughness of this computation, we find the result rather satisfying. The value for $\lambda(\mu)$ is somewhat small in order to obtain an $M_\sigma$ of 900MeV, but we remind the reader that $M_\sigma$ is rather sensitive to the presence of the $(Tr(MM^\dagger))^2$ interaction. Also, both $\alpha(\mu)$ and $\lambda(\mu)$ are computed with the lowest order $\beta$ functions.

We conclude with some speculative comments. It can be shown that both $\alpha(\mu)$ and $\lambda(\mu)$ may have a region of $\mu$ in which they are both approximately constants when the Borel summation[6] of the power series expansion for the $\beta$ functions are performed. It is therefore conceivable that there is a finite overlap region in which $\alpha(\mu) \propto \lambda(\mu)$. Our results suggest that a nonperturbative computation of $\gamma$ and $\gamma^M$ would lead to a smooth transition from QCD to the chiral Lagrangian we adopted. The corresponding figure may resemble that shown in Figure 2.

Note that the low energy running of the Wilson coefficients may lead to an additional enhancement (suppression) of $R_-(R_+)$, where $R_\pm = \frac{R_2 \pm R_1}{2}$. This may give a further enhancement of the $\Delta I = 1/2$ amplitude over the $\Delta I = 3/2$ amplitude.

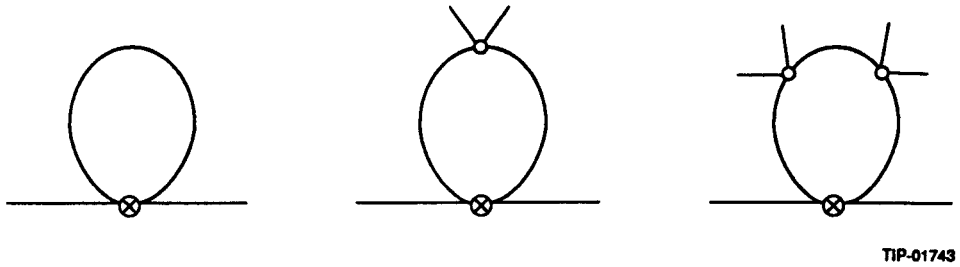


Figure 1

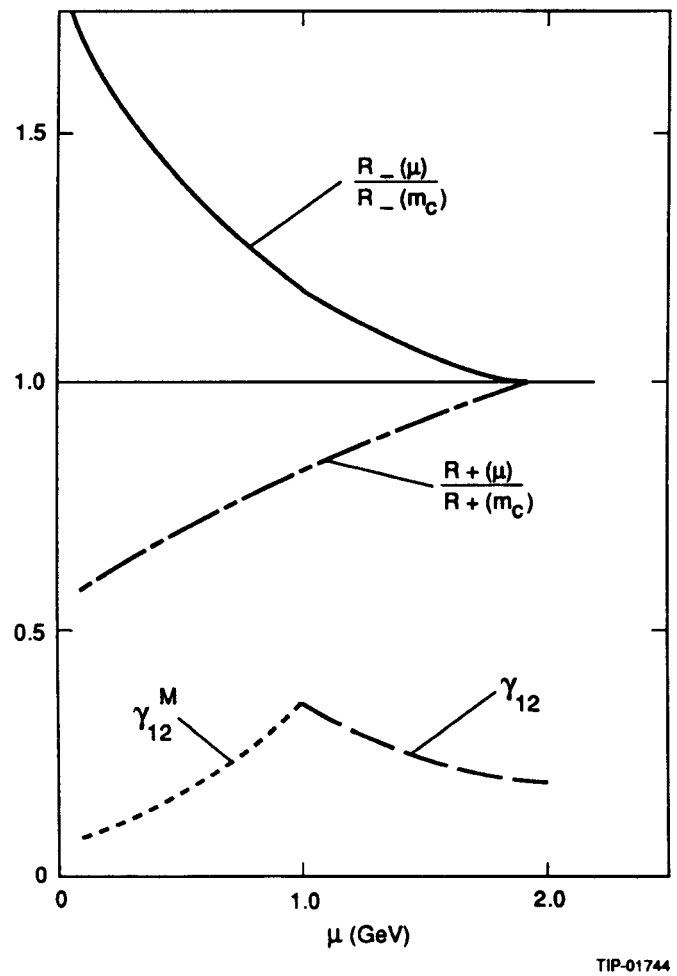


Figure 2

## Figure Captions

Figure 1:

The diagrams which give rise to counter terms. $\otimes$ and $\circ$ denote an insertion of weak interaction $(Q_1^0, Q_2^0)$, and strong interaction, respectively.

Figure 2:

The $\mu$ dependence of the anomolous dimensions $(\gamma_{12}, \gamma_{12}^M)$ and the Wilson coefficients $R_\pm$. The following values of the parameters are used: $\Lambda_{QCD} = 0.4GeV$, $\overline{\Lambda} = 1.99GeV$, $m_c = 2.0GeV$, and $\mu_0 = 1.0GeV$ (the maching point). We conjecture that the higher order contributions to $\gamma_{12}$ and $\gamma_{12}^M$ will make the transition between QCD and the linear $\sigma$ model smooth. Note also that we expect enhancement of $R_-$ over $R_+$ from physics below 1 GeV. This will further enhance the $\Delta I = 1/2$ amplitude over that of $\Delta I = 3/2$.

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## REFERENCES


1. F.J. Gilman and M.B. Wise, *Phys. Rev.* **D20** (1979) 2392; M.K. Gaillard and B.W. Lee, *Phys. Rev. Lett.* **33** (1974) 108; G. Altarelli and L. Maiani, *Phys. Lett.* **B52** (1974) 351; A.I. Vainshtein, V.I. Zakharov and M.A. Shifman, Sov. Phys. *JETP***45** (1977) 670.

2. W.A. Bardeen, A.J. Buras, and J.-M. Gérard, *Nucl. Phys.* **B293** (1987) 787; W.A. Bardeen, A.J. Buras, and J.-M. Gérard, *Phys. Lett.* **B192** (1987) 138.

3. J.L. Basdevant and B.W. Lee, *Phys. Rev.* **D2** (1970) 1680; L.-H. Chan and R. W. Haymaker, *Phys. Rev.* **D10** (1974) 4143.

4. T. Morozumi, C.S. Lim, and A.I. Sanda, *Phys. Rev. Lett.* **65** (1990) 404; after the publication of this paper E.P. Shabalin brought to our attention E.P. Shabalin, *Sov. J. Nucl. Phys.* **48** (1988) 172, in which a similar result is obtained.

5. For example, see D.J. Gross, "Applications of the Renormalization Group to High-Energy Physics" in *Methods in Field Theory*, Les Houches 1975 (North-Holland) 1976; T. Kugo, *Prog. Theor. Phys.* **57** (1977) 593.

6. A.I. Sanda, *Phys. Rev. Lett.* **42** (1979) 1658; N.N. Khuri, *Phys. Lett.* **B82** (1979) 83.