Impedance Scaling and Synchrotron Radiation Intercept

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Abstract

This paper presents several scalings in 2-D and 3-D impedance calculations. Most of the scalings are empirical and are found by using the boundary perturbation method and numerical simulations. As an application of these scalings, the impedance of one type of synchrotron radiation intercept is calculated. The results are then compared with that of several other types of intercept designs.
1.0 INTRODUCTION

Beam-environment coupling impedances are important parameters in the design of modern large accelerators. Calculations of the impedances are usually quite involved. In order to reduce the amount of computational work and to obtain quick estimations, it is desirable to have certain scalings, even approximate ones. In Section 2, several methods that are commonly used in impedance calculations are compared and are found to be in general agreement. In Section 3, consideration is given to several parametric scalings in 2-D cases. Two geometric scalings in 3-D are discussed in Section 4. The next section introduces the so-called synchrotron radiation intercept, which is really a 3-D structure that must be computed using 3-D codes such as MAFIA.1 The scalings have greatly simplified the computations of this structure. The results of several different designs of intercepts are compared.

2.0 COMPARISON OF DIFFERENT METHODS

In the past years, several impedance computation methods have been developed including field matching, boundary perturbation, transmission line and numerical simulations. A comparison of the results obtained from these methods reveals that they are in general agreement.

(a) Field matching vs. TBCI:2

Henke calculates the impedance of a small pillbox with beampipes (Figure 1) using the field matching method.3 Figure 2 depicts the spectra that he obtained. TBCI is run in order to compute the wake potentials of the same structure. The wakes are then Fourier-transformed to obtain the impedance as shown in Figure 3. They compare to each other reasonably well, except for some spurious peaks as shown in Figure 3 that remain to be understood.

Figure 1. A Small Pillbox with Beampipes ($b = 10 \text{ cm}, \ 2g = 0.05b, \ 2\varepsilon = 0.1b$).
Figure 2. Henke's Impedance Spectra of the Pillbox in Figure 1.

Figure 3. Impedance Spectra Obtained from TBCI and Fourier Transform. The dashed line in 3(b) is the result given by the transmission line method.

(b) TBCI vs. transmission line method:

The simple structure shown in Figure 1 can be approximated by a transmission line, in which the longitudinal impedance at low frequencies is purely inductive and can be roughly estimated by the formula

$$Z_\parallel = j\omega \cdot \frac{\mu_0}{2\pi} \cdot \frac{4\epsilon}{b} \ (\Omega) ,$$

in which $\mu_0 = 4\pi \times 10^{-7}$ $H/m$. This is a straight line with a slope equal to $(\mu_0/2\pi) \cdot (4\epsilon/b)$, as shown by the dashed line in Figure 3(b). It agrees with the TBCI results up to the cutoff of the beampipe.
(c) TBCI vs. boundary perturbation (BP):

The boundary perturbation technique is basically an analytic tool. Unlike the field matching method, this tool can be applied to various types of geometries provided that the perturbation is not too large. For details of this method, see References 4–6. As an example, Figure 4 shows a periodic structure which is typical in the beampipe of the storage ring of synchrotron radiation sources (e.g., ALS, APS, ESRF and SPring-8). The longitudinal and transverse wake potentials of a Gaussian bunch of rms length $\sigma$ traversing this structure are, respectively,

$$W_{\|}(s)^{m=0}(V/pC) = -1.8\pi \sum_{p=1}^{\infty} p^{|2c_p|^2}$$

$$W_{\perp}(s)^{m=1}(V/pC \cdot m) = -\frac{360\pi}{b_0^2} \sum_{p=1}^{\infty} p^{|2c_p|^2}$$

in which

$$c_p = -j \frac{2\epsilon}{\pi b_0} \cdot \sin \left( \frac{p \pi s}{L} \right) \cdot \frac{1}{p^2} \quad \text{for } p = \pm 1, \pm 3, ...$$

and

$$k_{mn} = \frac{\pi p}{L} + \frac{L x_{mn}^2}{4\pi p b_0^2},$$

$$k_{mn}' = \frac{\pi p}{L} + \frac{L x_{mn}'^2}{4\pi p b_0^2}.$$
3.0 PARAMETRIC SCALINGS IN 2-D

Assuming a rotationally symmetric obstacle in an otherwise smooth beampipe, the basic parameters that would determine the impedance of this discontinuity are the average radius, the depth and the width of the obstacle, and the slope of its edges. When these parameters vary, the changes of the impedance obey (approximately) certain power laws.\(^1\) Use the obstacle in Figure 4 as an illustration. A typical set of parameters are \(b_0 = 1.8 \text{ cm}, \epsilon = 0.2 \text{ cm}, \theta = 1/2 \) (in unit of \(\pi/2\)) and \(L = 8 \text{ cm}\).

- Scaling I:

\[
k_\perp = B \theta^\alpha \quad (\alpha = 0.7 \pm 0.1)
\]

The transverse loss \(k_\perp\) varies with the taper angle \(\theta\) by the power law described by Equation (7), in which \(B\) and \(\alpha\) are constants.\(^7\) The value of \(\alpha\) is close to 0.7. There is

\(^1\)The width of obstacle plays a somewhat different role, which will not be discussed here.
a small variation (±0.1) according to different bunch lengths \( \sigma \) and parameters \( b_0 \) and \( \epsilon \). Figure 6 demonstrates the usage of this scaling. For given \( \sigma, b_0 \) and \( \epsilon \), first calculate \( k_{\perp} \) for \( \theta = 1 \) (i.e. \( \pi/2 \)), which gives \( B \). Then from the loss for \( \theta = 1/2 \) (i.e. \( \pi/4 \)), obtain \( \alpha \). These two constants determine the solid curve in Figure 6. When TBCI is used to compute the losses for other values of \( \theta \), they just sit on this curve as shown in Figure 6. This greatly simplifies the work, in particular, for small \( \theta \). It is known that TBCI has difficulty in treating the long tapered structures.\(^8\) It should be pointed out, though, that this scaling may not be very accurate when the bunch length becomes too small.

- Scaling II:
  \[ k \sim \epsilon^\beta \quad (1.5 < \beta \leq 2) \quad (8) \]
  When the obstacle depth \( \epsilon \) varies, both the longitudinal and the transverse losses \( k \) vary as described by Equation (8). Here one may distinguish two cases. If the angle \( \theta \) varies with \( \epsilon \) such that the taper length \( g \) keeps a constant, \( \beta \) exactly equals 2. This can be seen immediately in Equations (2)-(4). If, on the other hand, \( \theta \) is fixed while \( \epsilon \) and \( g \) vary, the value of \( \beta \) is between 1.5 and 2. See Figure 7.

- Scaling III:
  \[ k_{\perp} \sim b_0^\gamma \quad (-3 < \gamma < -2.3) \quad (9) \]
  When the average radius \( b_0 \) varies, the transverse loss is inversely proportional to the second-to-third power of \( b_0 \). This is seen in Figure 8.

The results are summarized in Figure 9. Various derivations can be obtained by combining these scalings. For example, in the design of the undulators, one needs to know the dependence of \( k_{\perp} \) on the undulator aperture \( b \) (\( \equiv b_0 - \epsilon \)) when the beam chamber radius \( a \) (\( \equiv b_0 + \epsilon \)) and the taper angle \( \theta \) are fixed. It can be readily shown that the scaling of this variation is
\[ k_{\perp} \sim \left( \frac{a + b_1}{a + b_2} \right)^\gamma \cdot \left( \frac{a - b_1}{a - b_2} \right)^\beta \quad (10) \]
in which \( b_1 \) and \( b_2 \) are two values of \( b \). This scaling has been used in the design of the 7-GeV Advanced Photon Source (APS) at ANL, in which the transverse impedance is dominated by the undulator aperture.

### 4.0 GEOMETRIC SCALINGs IN 3-D

When the obstacles are real 3-D structures, the impedance calculations usually have to invoke 3-D codes. However, for certain 3-D geometry, there are two simple scalings which
Figure 6. An Exhibit of Scaling I (cf. Reference 6).

Figure 7. An Exhibit of Scaling II. When both $\epsilon$ and $\theta$ vary while $g$ keeps a constant, $\beta$ is exactly $2$ (solid curve). For fixed $\theta$, $\beta$ is between $1.5$ and $2$ (dashed line).

can provide us with quick estimates from 2-D results. Assume a flag type 3-D structure as in Figure 10(a), and its 2-D counterpart, which has a rotational symmetry, in Figure 10(b).

- Scaling IV:

<table>
<thead>
<tr>
<th>2-D vs. 3-D</th>
</tr>
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<tbody>
<tr>
<td>$k_z \sim \text{area ratio}$</td>
</tr>
<tr>
<td>$k_y \sim 1$</td>
</tr>
</tbody>
</table>
Figure 8. An Exhibit of Scaling III.

Figure 9. Summary of Scalings I, II, and III.

Figure 10. (a) A Flag Type 3-D Structure. (b) The 2-D Counterpart which has a Rotational Symmetry.
The longitudinal loss is roughly proportional to the cross-section area of the obstacle, which is conceivable. The transverse one along the direction toward the obstacle, however, is not sensitive to the area.\(^2\) It is basically determined by the depth of the obstacle (the distance from the beampipe axis).

- Scaling \(V:\)

<table>
<thead>
<tr>
<th></th>
<th>One-sided</th>
<th>vs.</th>
<th>Symmetric (about (x-z) plane)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_x)</td>
<td>1</td>
<td>:</td>
<td>2.0</td>
</tr>
<tr>
<td>(k_y)</td>
<td>1</td>
<td>:</td>
<td>0.5</td>
</tr>
<tr>
<td>(k_z)</td>
<td>1</td>
<td>:</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Figure 11(a) is a 3-D view of one flag. Figure 11(b) shows two flags symmetric about the \(x-z\) plane. Due to the symmetry, it is understandable that when two flags exist, both the \(x\) and \(z\) components of the wakefields would be doubled, while the \(y\) component would be reduced. It is not clear, though, why it should be reduced by a factor of 2, which is a numerical output.

5.0 SYNCHROTRON RADIATION INTERCEPT AND ITS IMPEDANCE

Now employ the scalings discussed in the previous sections to study a specific structure—the synchrotron radiation intercept. It is a crucial component in upgrading the luminosity of huge hadron colliders, such as the SSC and LHC. One main limitation of the luminosity

\(^2\)The transverse loss along another direction is insignificant.
of both machines comes from the synchrotron radiation of the protons, which needs to be absorbed at very low temperatures. To achieve higher Carnot efficiency in the cryogenic system, it is suggested that an intercept be designed to absorb the radiation and to keep it at a temperature considerably higher than that of the superconducting magnets. There are several different types of intercepts, including the flag type (Figure 11), the dog-ear type (Figure 12), and the liner (Figure 13).

One concern about the intercept is how much additional impedance it would bring with it. We have used the scalings to analyze various designs of the flag type intercept. Figure 14 exhibits five variations. Their relative impedances are listed in Table 1. Furthermore, when the depth of the flag shrinks, its impedance is reduced as the scaling II, and is shown in Figure 15. Meanwhile, the number of the flags is increased in a linear way. It is thus concluded that the total impedance would be lower if the flags are smaller, symmetric and tapered on both sides.

<table>
<thead>
<tr>
<th>Table 1. Comparison of Various Designs of the Flag Type Intercept.</th>
</tr>
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<tbody>
<tr>
<td>a</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>1-flag</td>
</tr>
<tr>
<td>Taper angle</td>
</tr>
</tbody>
</table>

$k_x$ | 1 unit | 2 | 0.6 | 0.26 | 0.2 |
$k_y$ | 1 unit | 0.5 | 0.7 | 0.42 | 0.36 |

As another comparison, Table 2 lists the impedance of three different types of intercept relative to that of the bellows. Because of the simplicity and the small impedance of the liner, and for some other reasons, it has been given serious consideration.

<table>
<thead>
<tr>
<th>Table 2. Comparison of Various Types of Intercept.</th>
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<tbody>
<tr>
<td>Bellows</td>
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<tr>
<td>---------</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td>$k_x$</td>
</tr>
<tr>
<td>$k_y$</td>
</tr>
</tbody>
</table>

Note that comparisons of different materials for the posts (dielectric and metallic) and of different sizes for the slot (1, 2 and 3 mm) have been made.
6.0 CONCLUSIONS

Five impedance scalings have been demonstrated. They have been used in the design of two large accelerator facilities — the APS at ANL and the SSC. From our experience, they usually give quite good, quick estimates without the need of sophisticated computations.
Figure 15. Losses vs. Flag Depth. For small $\epsilon$, the curves tend to be quadratic ($\beta \approx 2$).

The study of the flag type intercept gives one such example. However, except in a few cases, most of them are empirical. A further analytical study by means of, say, the boundary perturbation method will be useful in order to gain more insight into these scalings.
REFERENCES

1. MAFIA is a group of time and spectral domain simulation codes developed by T. Weiland and his group in collaboration with several laboratories.

2. TBCI is a time domain simulation code written by T. Weiland.


