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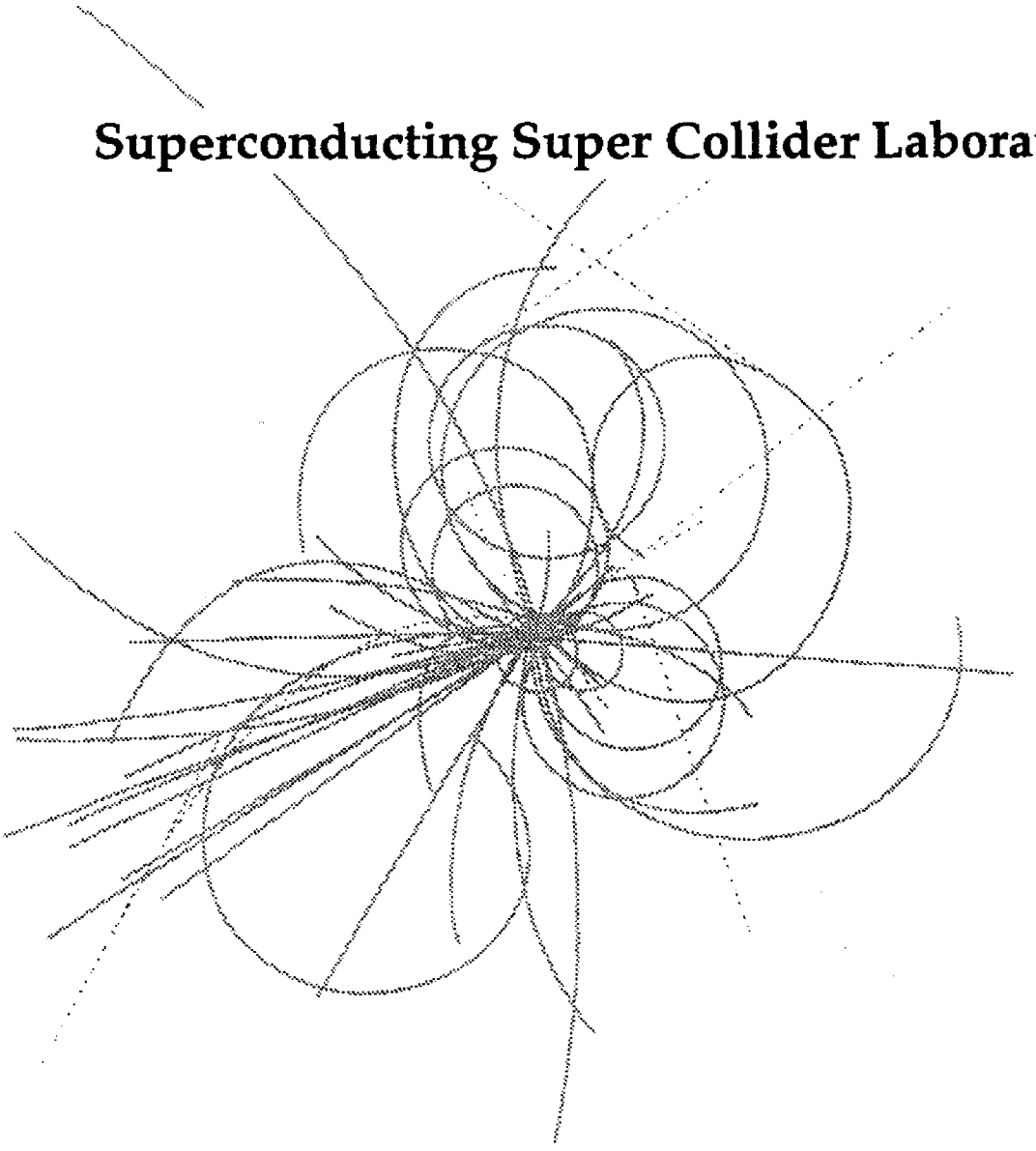

Bus Bar Conductor for the SSC Magnets

G. López and G. Snitchler

May, 1990

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Accelerator Division
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* Operated by Universities Research Association, Inc., for the U.S. Department of Energy under Contract No. DE-AC02-89ER40486.

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ABSTRACT

The effect of large thermal disturbances in superconducting cables has been studied. These effects depend on the copper-to-superconductor ratio of the cable, and the fraction of perimeter of the cable which is in contact with the Helium. The approach used brings about a direct closed expression for the amount of copper needed for a cryogenically stable and safe bus bar conductor.

1. INTRODUCTION

Cryogenic Stabilization (CS), provides effective cooling to a superconductor with a normal zone produced, after a large disturbance will collapse into a superconducting state. The criteria which provides CS has been formulated by several researchers from 1965 to 1969[1] and the best known and used criterion is "the equal-area theorem." The main objective is to provide enough copper for heat generated per unit time to be less than the maximum heat transferred to the helium flow producing, consequently, CS. Although this is a well understood

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problem, a new aspect will be presented for the solution of CS which in turn allows the estimate of the maximum disturbance that a conductor can sustain without burning out. In particular, this study is applied to the SSC bus bar conductor.

2. TEMPERATURE EVOLUTION AND THE FUNCTION V

Let us assume that the flow of the Helium (gas or/and liquid according with its thermodynamics state) produces a constant temperature, θ_o, outside the conductor cable when the temperature of the conductor is lower than 20K. This conductor has a total area given by

$$A = a_{cu} + a_{sc} \tag{1}$$

where a_{cu} and a_{sc} are the copper matrix and superconducting (NbTi) areas. The fraction of copper to superconducting areas, λ, is given by

$$\lambda = \frac{a_{cu}}{a_{sc}} \tag{2}$$

and the fraction of the perimeter in contact with the Helium, ξ, is defined through the following expression

$$P = 2\xi\sqrt{\pi A} \tag{3}$$

if the conductor is cylindrical.

Suppose that the superconducting cable carries a current density, J_o, and that due to a large disturbance it reaches a homogeneous temperature, θ, bringing it out into a normal conductor state. The temperature evolution of the conductor is described by the equation

$$(\delta c)\frac{dq}{dt} = \rho(\theta)J_o^2 - Ph(\theta - \theta_o)\frac{(\theta - \theta_o)}{A} \tag{4}$$

where $\rho(\theta)$ is the resistivity of the copper, δ is the density, c is the specific heat, and $h(\theta - \theta_o)$ is the heat transfer coefficient. The quantity (δc) represents the

average taken over the conductor components and it is given by

$$(\delta c) = \frac{\lambda(\delta c)_{cu}}{(1+\lambda)} + \frac{(\delta c)_{sc}}{(1+\lambda)} \,. \tag{5}$$

Using the above expressions and defining the variable σ as

$$\sigma = \theta - \theta_o \tag{6}$$

equation (4) can be written as

$$(\delta c)\frac{d\theta}{dt} = V(\xi, \lambda, \sigma) \tag{7}$$

where the function V has the following form:

$$V(\xi, \lambda, \sigma) = \frac{\rho(\theta_o + \sigma)I_o^2}{a_{sc}^2\lambda^2} - \frac{2\sqrt{\pi}\xi\sigma h(\sigma)}{\sqrt{a_{sc}(1+\lambda))}} \,. \tag{8}$$

Given the fraction of perimeter in contact with the Helium, ξ, the copper to superconductor ratio, λ, the magnitude of the disturbance in the conductor, σ, we can determine whether the conductor will recover its superconducting state by using relation (7). As time evolves beyond the initiation of a normal zone, the temperature will rise for positive V, fall for negative V, and remain constant for $V = 0$, the equilibrium temperature. The equilibrium stationary point will be stable if the temperature of the conductor does not stray far from this value based on other perturbations. Figure 1 illustrates the variation of the function, $\sigma h(\sigma)$, with respect to the variable σ, and Figure 2 shows the variation of the function $\rho(\theta)I_o^2$ with respect to the temperature θ of the conductor assuming a current of $I_o = 6500$ A and copper with a residual resistivity ratio (RRR) approximately 80. The heat generation starts at the temperature θ_g which is given by the following expression

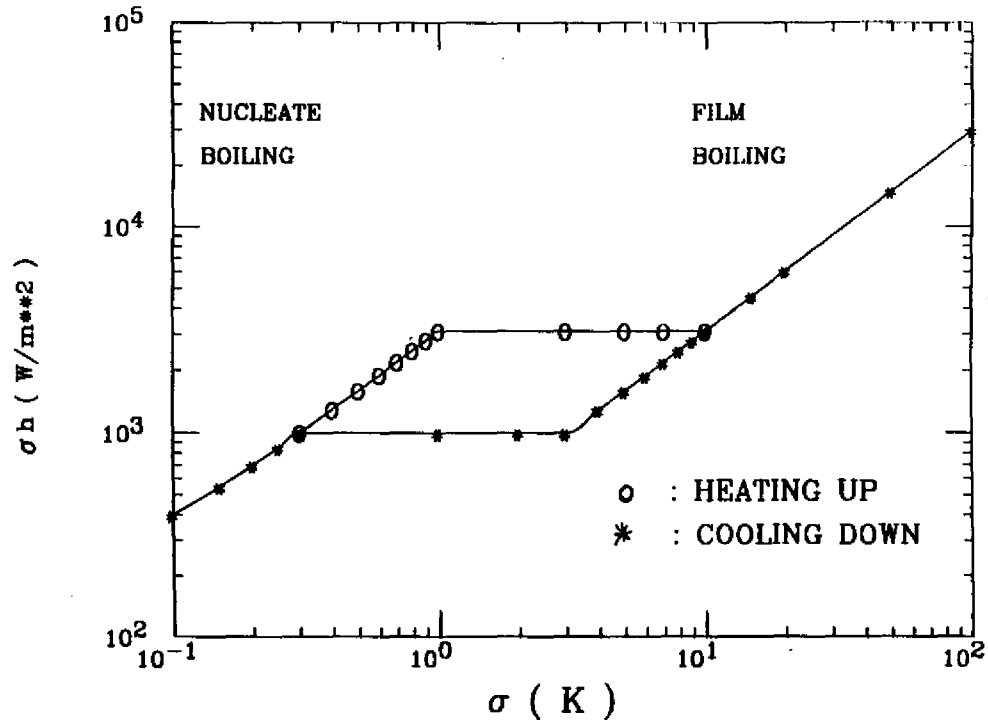

Figure 1.

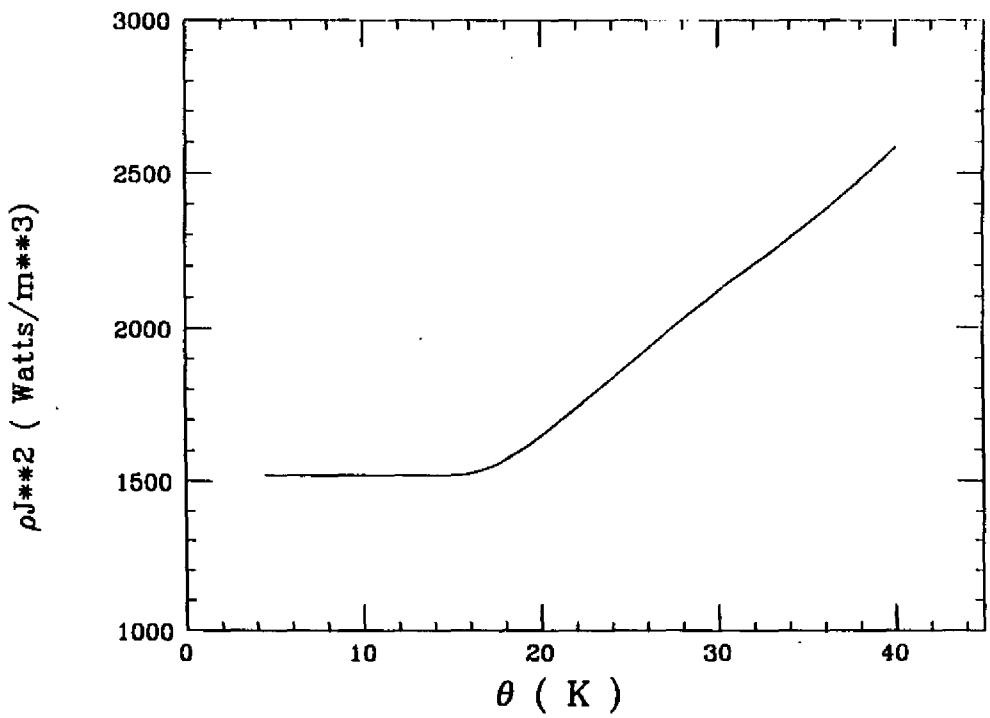

Figure 2.

$$\theta_g(B) = \theta_c(b) - (\theta_c(b) - \theta_o)\frac{J_o}{J_{co}(B)} \tag{9}$$

where θ_c is the critical temperature at zero current and J_{co} is the critical current density of the superconducting cable at the bath temperature, θ_o. These quantities depend on the magnetic field, B, as seen in Figure 3. The critical temperature and the critical current density can be calculated using proper expressions appearing in Ref. 2.

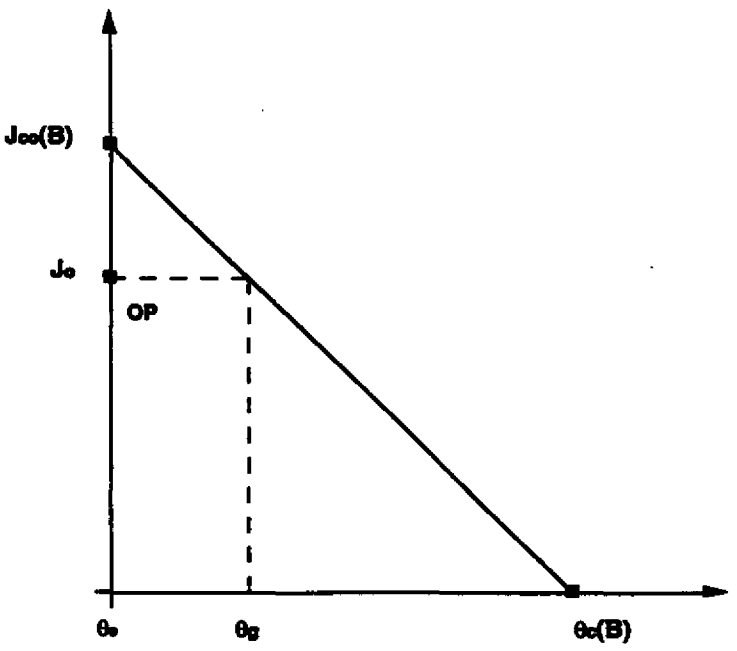

Figure 3.

3. ANALYSIS OF LARGE DISTURBANCES

Using the dependency of $\rho(\theta)I_o^2$ and $h(\sigma)$ with respect to the temperature and the superconducting area of $a_{sc} = 11.785 \times 10^{-6}$, a study can be made of the variation of the function V with respect to any of the variables ξ, λ, or σ to determine the region where V is negative. In the Figures 4, 5, and 6 observe the variation of V with respect to the variable λ for several perturbations, σ, and for each fraction of the perimeter in contact with Helium, ξ. The crossing point shown in the figures, as the function V changes from a positive value to a negative value, represents the minimum λ_1 and will yield the stable stationary temperature given by $\theta_o + \sigma_1$ provided that If σ_1 is less than 38.5 K. The temperature change σ_1 is the value associated with the curve (ξ) in Figure 7 and was calculated numerically using the relation (13). The temperature, $\theta_o + \sigma_1$, is a stable stationary value due to the choice of λ_1 for a small perturbation $\delta\sigma$. The value $\theta_o + \sigma_1 + \delta\sigma$ will be a stationary temperature for a copper to superconductor ratio of λ_1^*. According to Figures 4, 5, and 6, $\lambda_1^* < \lambda_1$, this implies that for values of σ between temperatures $\theta_o + \sigma_1$ and $\theta_o + \sigma_1 + \delta\sigma$, the function $V(\xi, \lambda, \sigma)$ will be negative and the temperature of the conductor will return to $\theta_o + \sigma_1$. If the value of σ_1 is higher than 38.5 K, then the above condition reverses and the function $V(\xi, \lambda, \sigma)$ would be positive, yielding an increasing temperature and making an unstable stationary point. As a result, a perturbation of this magnitude in the conductor will destabilize the conductor temperature and the cable will burn out. As shown in Figures 4, 5, and 6, if a stable stationary temperature close to θ_g (in order to have a fast recovery of the superconducting state) is required, the copper-to-superconductor ratio, λ, must be very high.

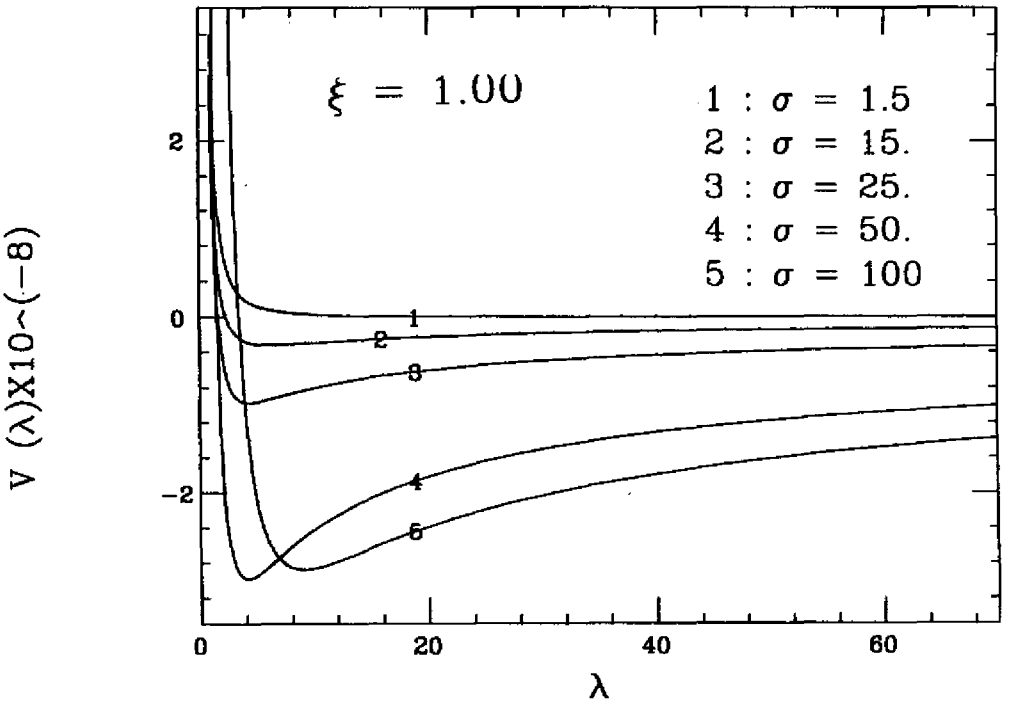

Figure 4.

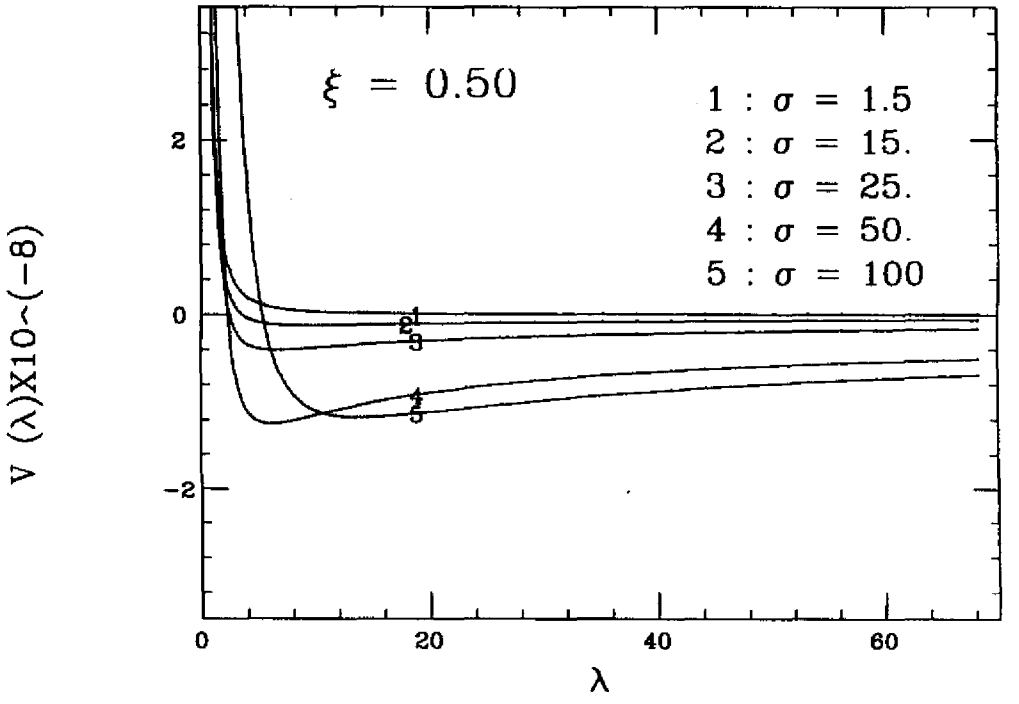

Figure 5.

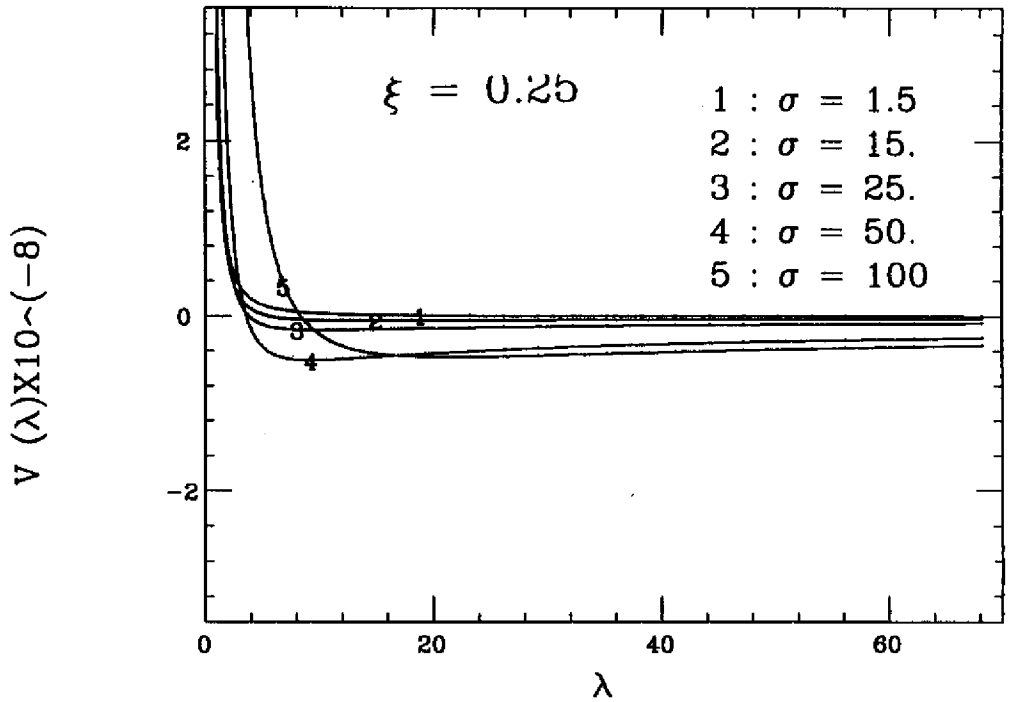

Figure 6.

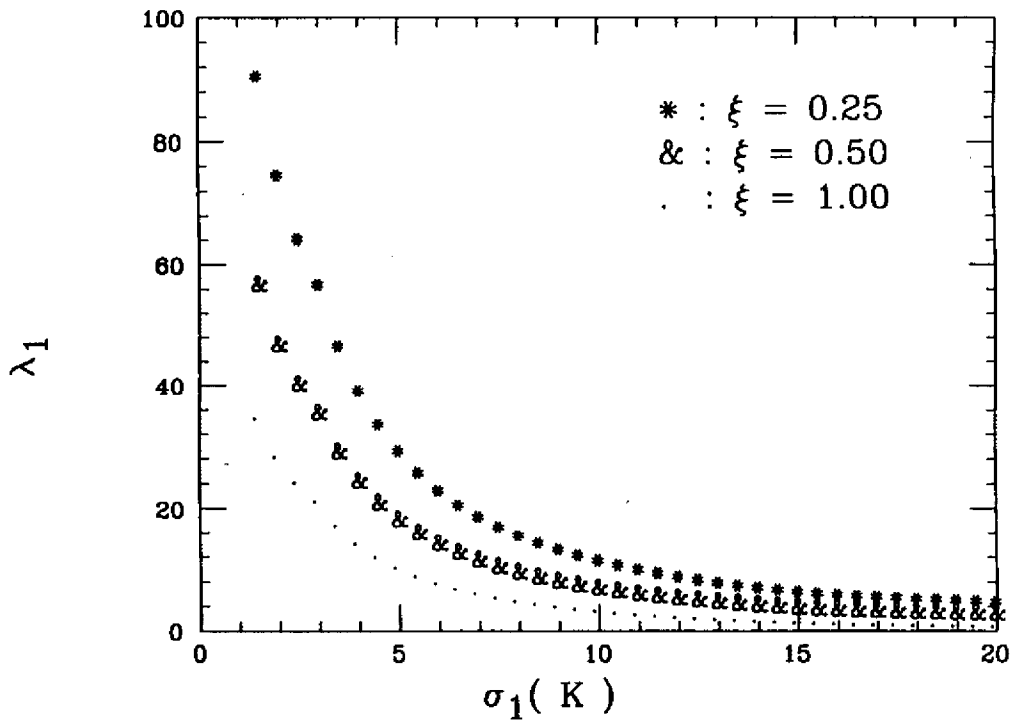

Figure 7.

Given the stable stationary temperature θ_1 of the conductor and the fraction of the perimeter in contact with Helium, the values of λ_1 such that $V = 0$ can be determined by solving the equation

$$\frac{\rho(\theta_o + \sigma_1)I_o^2}{a_{sc}^2\lambda_1^2} - \frac{2\sqrt{\pi}\xi\sigma_1 h(\sigma_1)}{\sqrt{a_{sc}(1+\lambda_1)}} = 0 \,. \tag{10}$$

The solution of this equation can be determined by finding, equivalently, the roots of the polynomial

$$\lambda_1^4 - q^2\lambda_1 - q^2 = 0 \,, \tag{11}$$

where q is defined by

$$q = \frac{\rho(\theta_o + \sigma_1)I_o^2}{[a_{sc}^{3/2}2\sqrt{\pi}\xi\sigma_1 h(\sigma_1)]} \,. \tag{12}$$

The positive root of (11) represents the physical solution of (10) and is given by

$$\lambda_1(\sigma_1, \xi) = \frac{\sqrt{R^{2/3} - 4q^2/3}}{2R^{1/6}}\left[1 + \sqrt{1 + \frac{2q^2R^{1/2}}{(R^{2/3} - 4q^2/3)^{3/2}}}\right] \tag{13}$$

where R has the following expression

$$R = \frac{q^4}{2}\left[1 + \sqrt{1 + \frac{256}{27q^2}}\right] \,. \tag{14}$$

The values of λ_1 are plotted in Figure 7 as a function of the stable stationary values of σ_1 (they are stable because $\sigma_1 < 38.5$ K) and for several values of the fraction of the perimeter in contact with Helium. As seen from this plot, if obtaining a stable stationary temperature close to θ_g is desired, which is close to θ_o, the copper to superconductor ratio λ_1 would be too high. A safe and appropriate stable stationary value can be established in an interval of temperatures where the resistivity is constant, between 5 K and 12 K.

Figures 8, 9, and 10 are plots of V as a function of the perturbation allowed σ for several values of λ and for a given fraction ξ. These plots show the possible conductor stability which is characterized by λ and ξ. Some of the curves have two points of intersection with the line $V = 0$, the first intersection is a stable stationary value as can be easily seen from the plots.

Figures 11, 12, and 13 are the numerical solution of the equation (7) where the evolution of the temperature of the conductor has been plotted as a function of time. The initial temperature represents the perturbation suffered by the conductor. For a perturbation reaching values higher than 20 K the cooling process could be less efficient because the flow of the Helium may not be fast enough. If this happens, then the approximation of the constant bath temperature, θ_o, is no longer valid and we should consider the increasing temperature of the Helium should be which is flowing on the surface of the conductor. However, as an approximation to this effect, the heat transfer coefficient could be reduced by a factor of γ which depends on the temperature σ is shown in the Figure 14. This factor was included in the above calculation. More experimental and theoretical analysis are needed to determine whether or not this effect may occur for each particular case. Assuming that the above pessimistic situation occurs in the bus bar conductor for the SSC magnets and having a fraction of perimeter in contact with the Helium of $\xi = 0.25$, as shown in Figures 7, 10, and 13 that a value of copper-to-superconducting ratio of $\lambda = 15$ or higher, the bus bar cable is capable of recovering by itself into a superconducting state after a large disturbance of $\sigma = 100$ K, and will obtain a stable stationary temperature of 10.5 K.

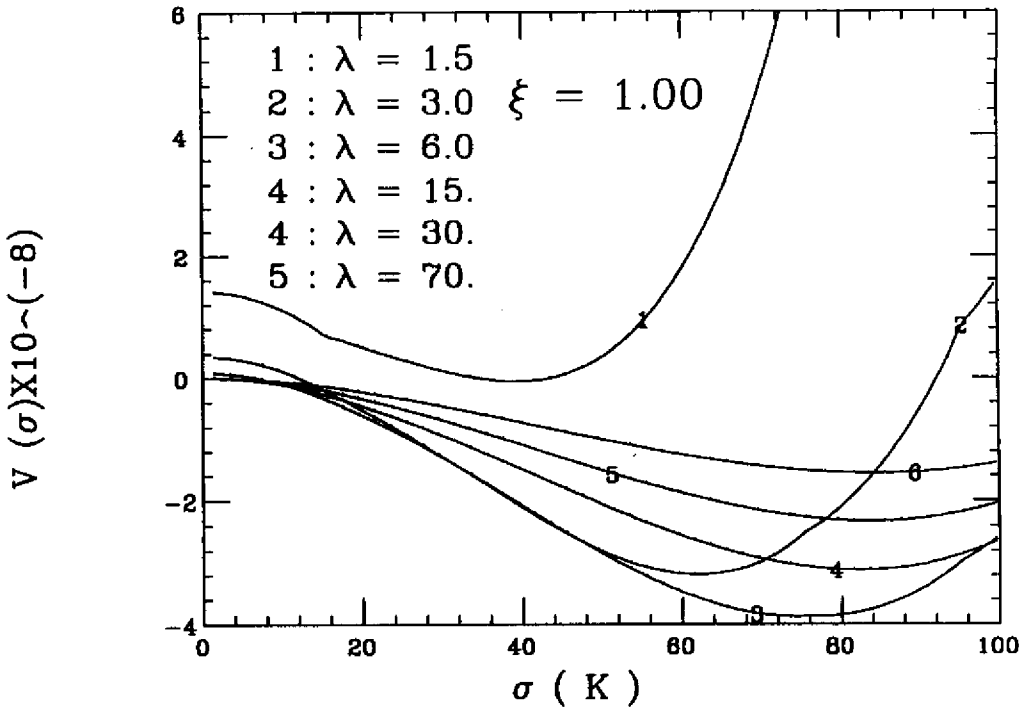

Figure 8.

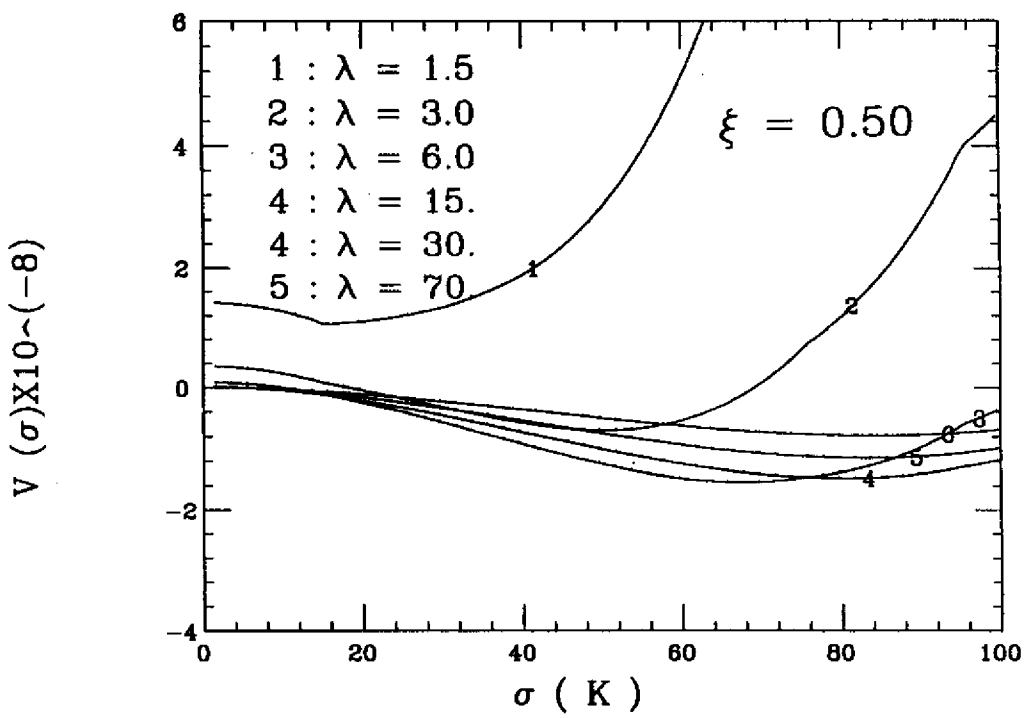

Figure 9.

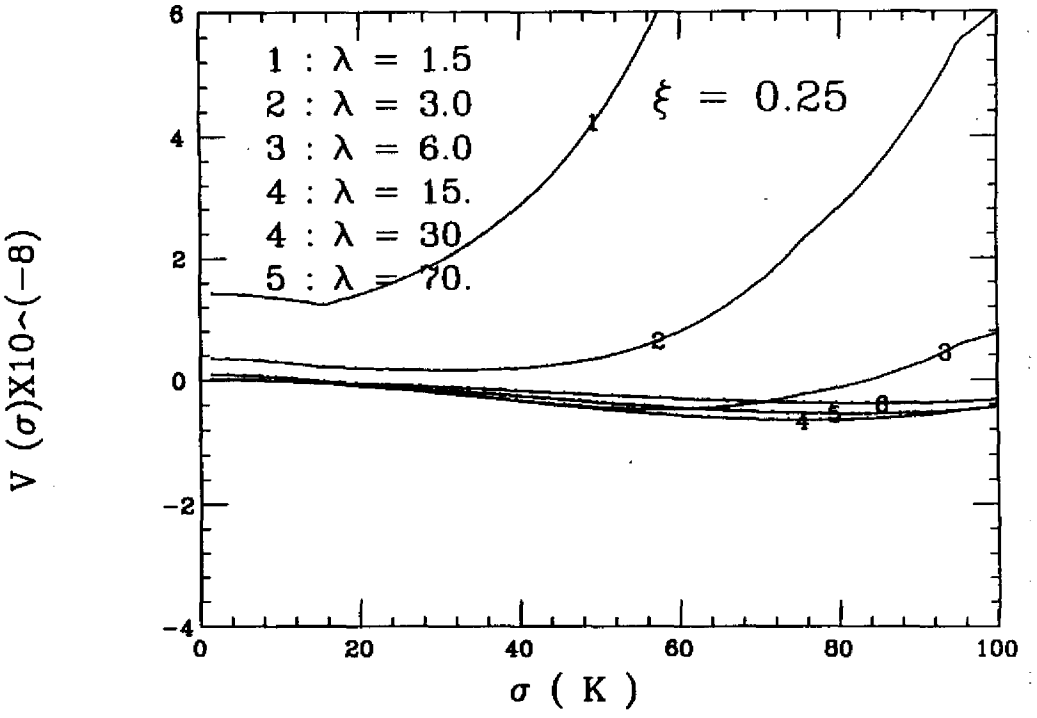

Figure 10.

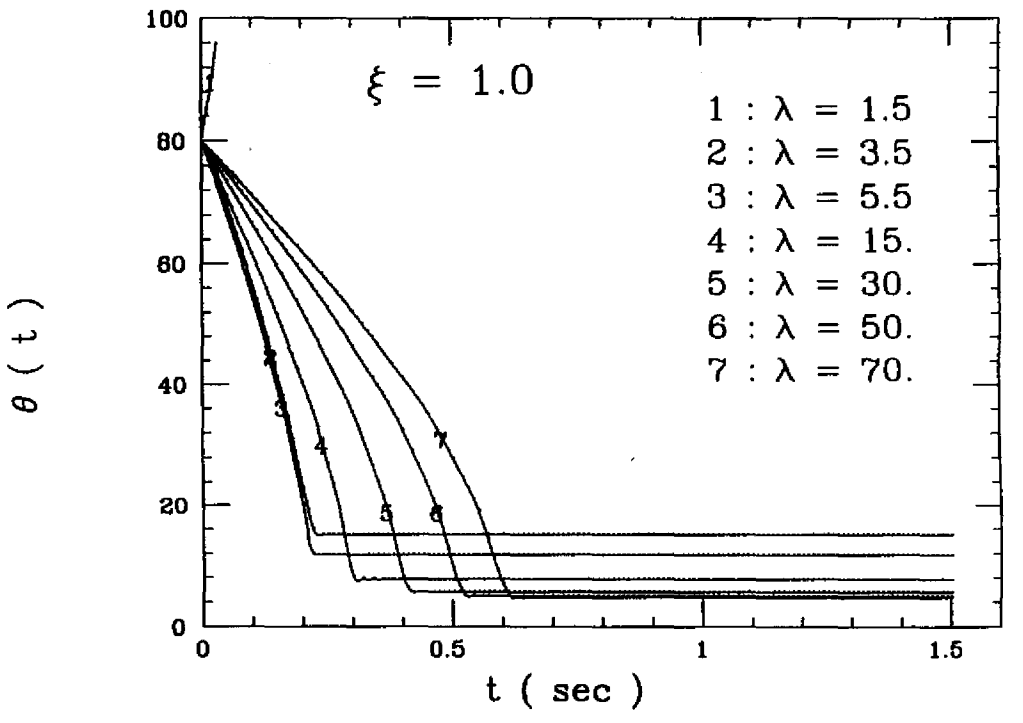

Figure 11.

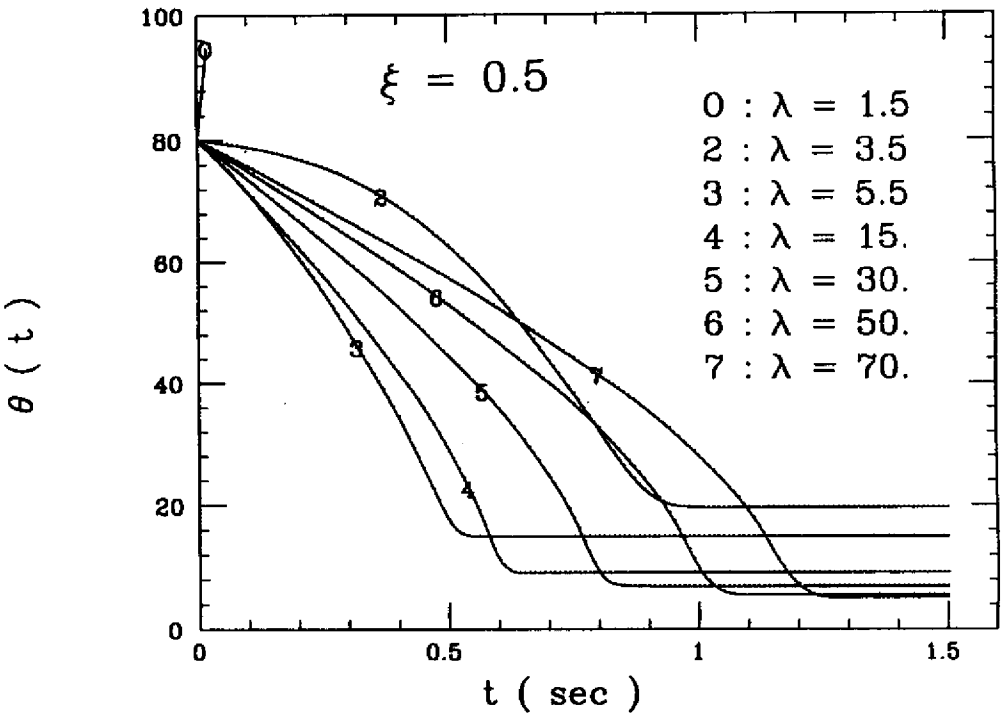

Figure 12.

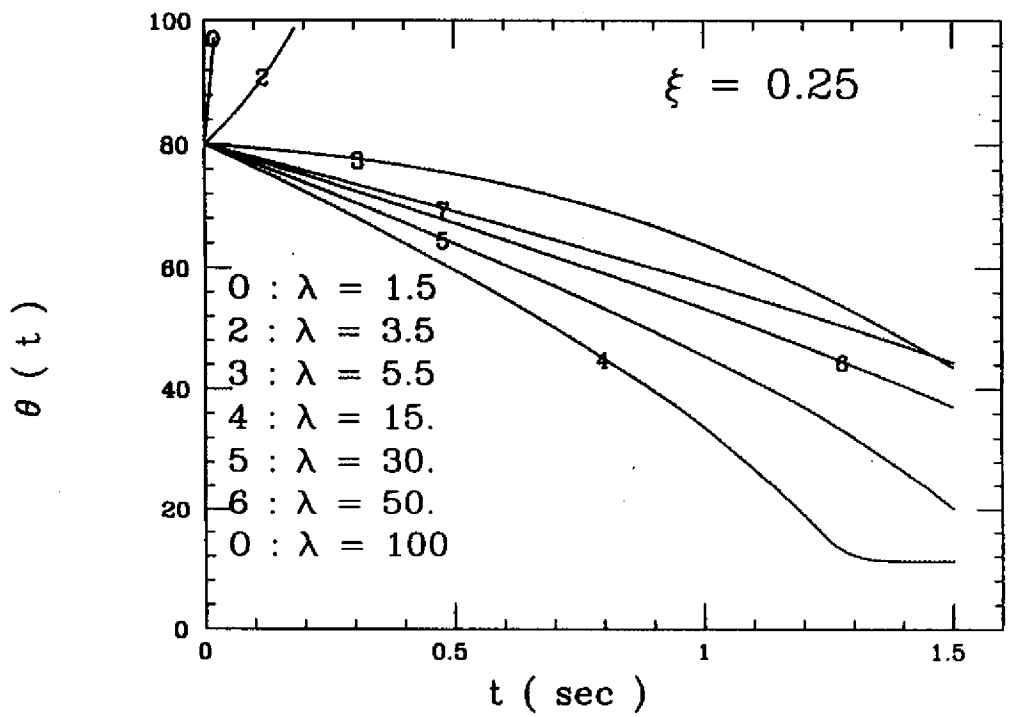

Figure 13.

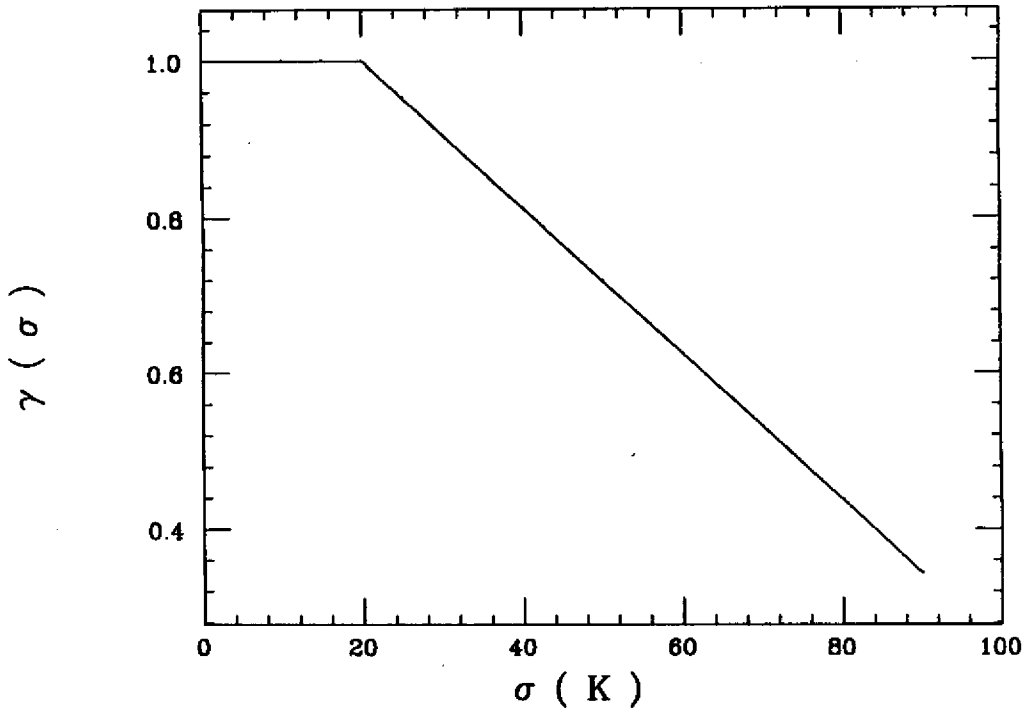

Figure 14.

4. IMPLICATION OF "THE EQUAL-AREA THEOREM"

By integrating Eq. (7) with respect to the temperature, equating the result to zero, and making some arrangements the following equation is obtained

$$\lambda_*^4 - q_*^2 \lambda_* - q_*^2 = 0 \tag{15}$$

which represents "the equal-area theorem" statement, where q_* is defined as

$$q_* = I_o^2 \frac{\int_{\sigma_g}^{\sigma_{max}} \rho(\theta_o + \sigma) d\sigma}{[a_{sc}^{3/2} 2\sqrt{\pi}\xi \int_{\sigma_g}^{\sigma_{max}} \sigma h(\sigma) d\sigma]} , \tag{16}$$

and σ_g, σ_{max} are given by

$$\sigma_g = \theta_g - \theta_o , \tag{17a}$$

$$\sigma_{max} = \theta_{max} - \theta_o \ , \tag{17b}$$

where θ_{max} is the maximum temperature allowed in the conductor. The solution of equation (15) has the same form as expression (13), by changing q by q_*. Of course, this integration is an average of the process, as can be seen by letting σ_{max} be less than 20^oK (the region where the resistivity is constant). Using the λ_* expression obtained from (15), when we change q by q_*, using the following values $\xi = 0.25$, $\sigma_g = 1.5$ K and $\sigma_{max} = 13$ K, and the heat transfer function shown in Figure 1, we obtain the value of

$$\lambda_* = 13.4 \tag{18}$$

for the bus bar conductor cable.

5. CONCLUSIONS

If a large disturbance raises the temperature of the superconducting cable up to values less than 100^oK, there might exist one, two, or no stationary temperature values for the conductor, depending on its copper to superconducting ratio, λ, and the fraction of its perimeter in contact with the Helium, ξ. If the unstable stationary value is reached by a conductor, this conductor will burn out eventually. A superconductor with the characteristics that produces a stable stationary temperature, in the range of temperatures considered for the disturbances, will be a safe conductor that recovers its superconducting state. A recovery will happen considering the conduction of the heat along the conductor, which causes the stable temperature to fall off to the bath temperature.

Knowing the fraction of conductor perimeter in contact with the Helium, ξ, the proper copper to superconducting ratio can be found that the conductor requires to have a stable stationary temperature, by using the expression (13) , as seen in Figure 7. Although this ratio depends also on the magnetic field in the conductor, this dependence has been neglected in the above calculation. Its main

effect is to increase the value of the resistivity which is equivalent to an increase in the value of the copper-to-superconducting ratio. If the magnetic field is too low, this effect is not important.

As shown above, a copper-to-superconducting ratio of 15 or higher is a safe value for this conductor, allowing it to recover its superconducting state for very large disturbances (as high as 100 K). Assuming that the fraction of the perimeter in contact with the Helium of the bus bar superconductor for the SSC is about 0.25.

The result can be generalized to the case where a conductor with more components, like solder or aluminum for example, by adding in (1) their contribution to the total area of the conductor.

Summarizing, the above approach gives a lower limit for the copper to super conducting ratio for cryogenic stable conductor, the upper limit depends on the mechanical or cost considerations.

ACKNOWLEDGEMENTS

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